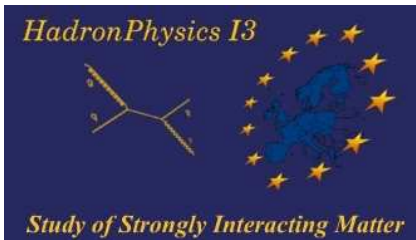




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# RADIATIVE AND SEMILEPTONIC KAON DECAYS IN CHPT

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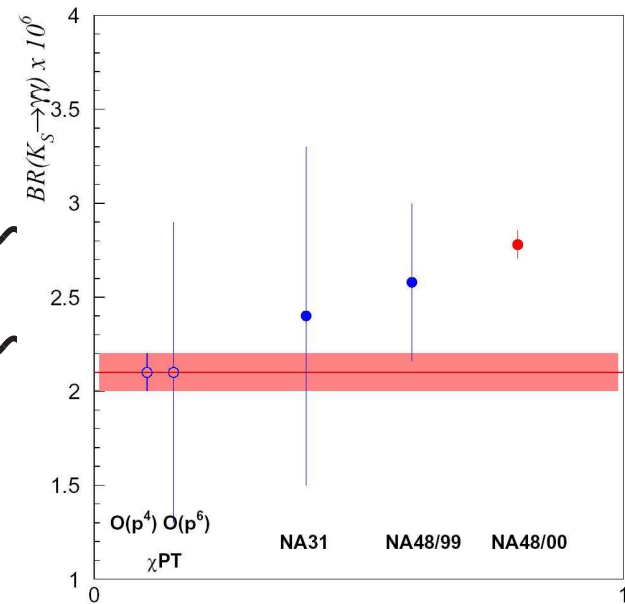
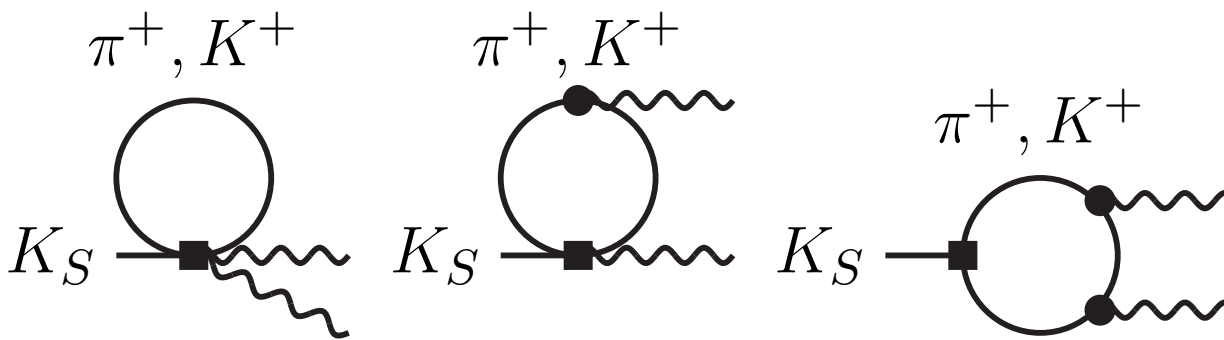
**Various ChPT:** <http://www.thep.lu.se/~bijnens/chpt.html>

# Overview

- A few comments about  $K_{S,L} \rightarrow \gamma\gamma$  and  $K_{S,L} \rightarrow \pi^0\gamma\gamma$
- Semileptonic Decays
- Radiative Semileptonic Decays
- Note: 2nd Eurodaphne report: hep-ph/9411311
- Note: tests of the anomaly including sign

# $K_S \rightarrow \gamma\gamma$

Well predicted by CHPT at order  $p^4$  from **Goity, D'Ambrosio, Espriu**



Prediction was:  $BR = 2.1 \cdot 10^{-6}$

NA48:  $2.78(6)(4) \cdot 10^{-6}$  (PLB 551 2003)

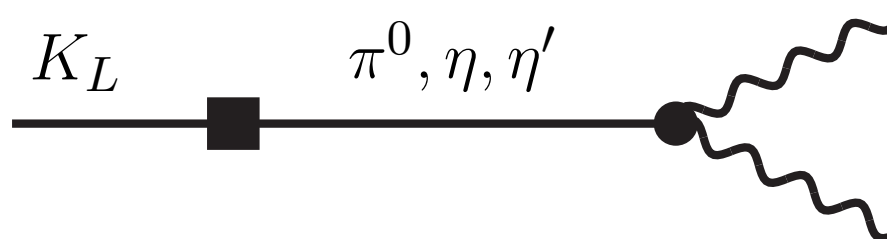
No full  $p^6$  calculation exists, FSI effects estimated

# Some other rare decays

- $K_L \rightarrow \gamma\gamma$

Needs work: main contribution is full of cancellations:

difficult



- $K_L \rightarrow \pi^0 \gamma\gamma$

$K_L \rightarrow \pi^0 \gamma\gamma$  OK predicted by CHPT

Main succes: events must be at high  $m_{\gamma\gamma}$

Rate still a problem

- $K_S \rightarrow \pi^0 \gamma\gamma$

Similar problems as in  $K_L \rightarrow \gamma\gamma$

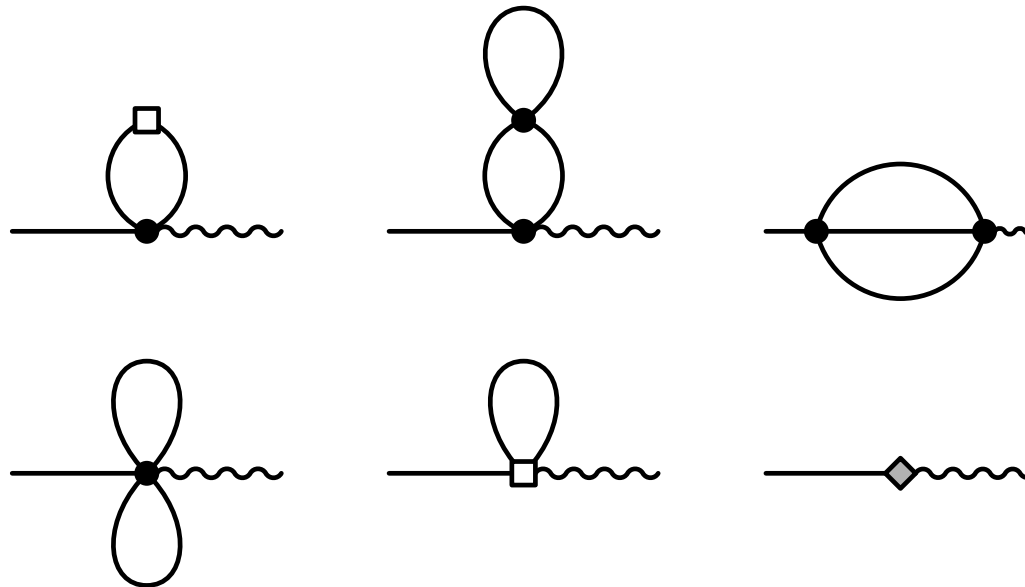
- Ecker, Pich, de Rafael, D'Ambrosio, . . .

# Semileptonic Decays

- $K \rightarrow \ell\nu$ : known to order  $p^6$
- $K \rightarrow \pi\ell\nu$ : known to order  $p^6$ , isospin breaking at  $p^6$  preliminary
- $K \rightarrow \pi\pi\ell\nu$ :  $F$ ,  $G$  and  $H$  known to  $p^6$ ,  $R$  only to  $p^4$ .
- $K \rightarrow \pi\pi\pi\ell\nu$ : known to  $p^2$

# $K_{\ell 2}$

Input for determining  $F_K$ .



Diagrams:

Amoros, JB, Talavera NPB 2000

Main use: determining  $L_5^r$

Typical convergence:

$$\frac{F_K}{F_\pi} = 1.22 = 11. + 0.162 + 0.058$$

$$\frac{F_\pi}{F_0} = 1 + 0.135 - 0.075$$

# $K_{\ell 3}$

- H. Leutwyler and M. Roos, Z.Phys.C25:91,1984.
- J. Gasser and H. Leutwyler, Nucl.Phys.B250:517-538,1985.
- J. Bijnens and P. Talavera, hep-ph/0303103, Nucl. Phys. B669 (2003) 341-362
- V. Cirigliano et al., hep-ph/0110153, Eur.Phys.J.C23:121-133,2002.
- J. Bijnens and K. Ghorbani, to be published.

# $K_{\ell 3}$ Definitions

$$K_{\ell 3}^+ : \quad K^+(p) \rightarrow \pi^0(p') \ell^+(p_\ell) \nu_\ell(p_\nu)$$

$$K_{\ell 3}^0 : \quad K^0(p) \rightarrow \pi^-(p') \ell^+(p_\ell) \nu_\ell(p_\nu)$$

$$K_{\ell 3}^+ : \quad T = \frac{G_F}{\sqrt{2}} V_{us}^* \ell^\mu F_\mu^+(p', p)$$

$$\ell^\mu = \bar{u}(p_\nu) \gamma^\mu (1 - \gamma_5) v(p_\ell)$$

$$F_\mu^+(p', p) = \langle \pi^0(p') | V_\mu^{4-i5}(0) | K^+(p) \rangle$$

$$= \frac{1}{\sqrt{2}} [(p' + p)_\mu f_+^{K^+ \pi^0}(t) + (p - p')_\mu f_-^{K^+ \pi^0}(t)]$$

**Isospin:**  $f_+^{K^0 \pi^-}(t) = f_+^{K^+ \pi^0}(t) = f_+(t)$

$$f_-^{K^0 \pi^-}(t) = f_-^{K^+ \pi^0}(t) = f_-(t)$$



# $K_{\ell 3}$ Definitions and $V_{us}$

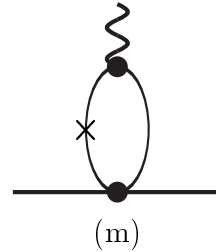
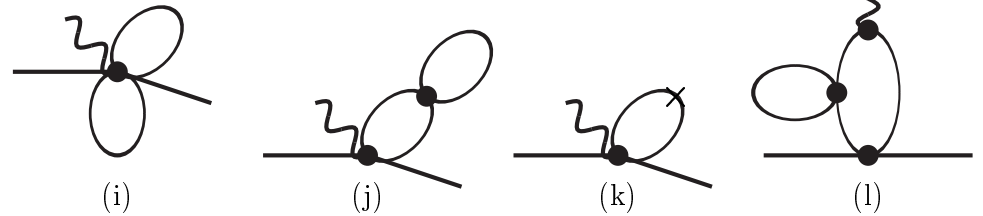
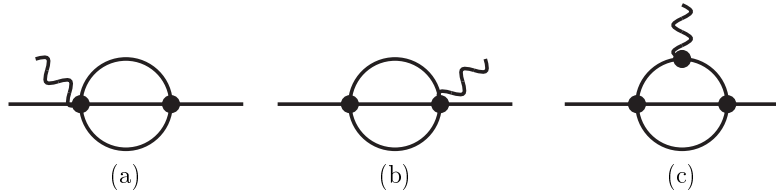
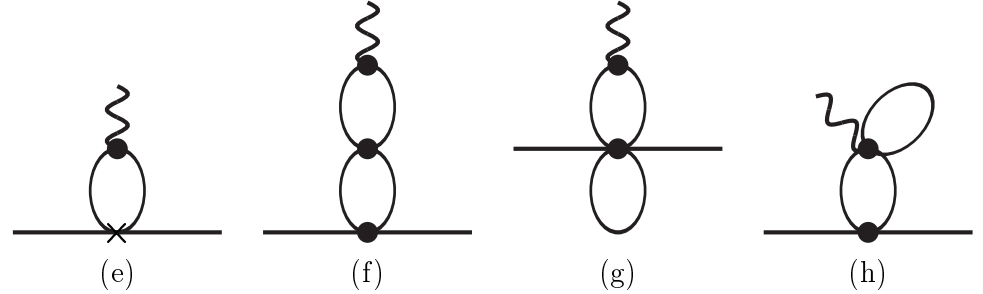
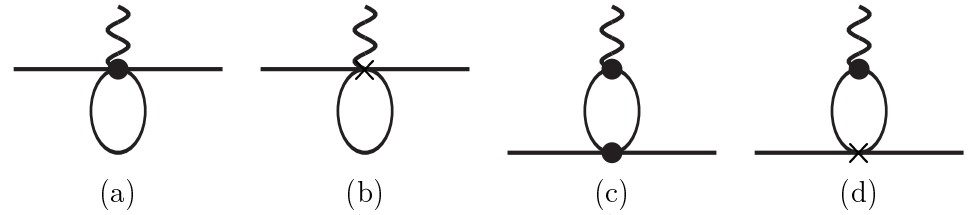
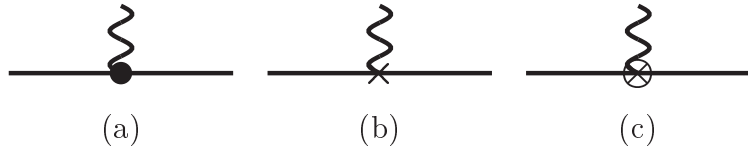
Scalar formfactor: 
$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Old Usual parametrization:

$$f_{+,0}(t) = f_+(0) \left( 1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right)$$

- $|V_{us}|$ :
- Know theoretically  $f_+(0) = 1 + \dots$ 
    - Short distance correction to  $G_F$  from  $G_\mu$   
Marciano-Sirlin
    - Ademollo-Gatto-Behrends-Sirlin theorem:  
 $(m_s - \hat{m})^2$
    - Isospin Breaking Leutwyler-Roos: see later
    - Radiative corrections: Cirigliano et al
  - Know experimentally  $f_+(0)$

# $K_{\ell 3}$ Diagrams



● :  $p^2$  vertex  
 × :  $p^4$  vertex  
 ⊗ :  $p^6$  vertex

# $f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} \\ + \frac{t^2}{F_\pi^4} (-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

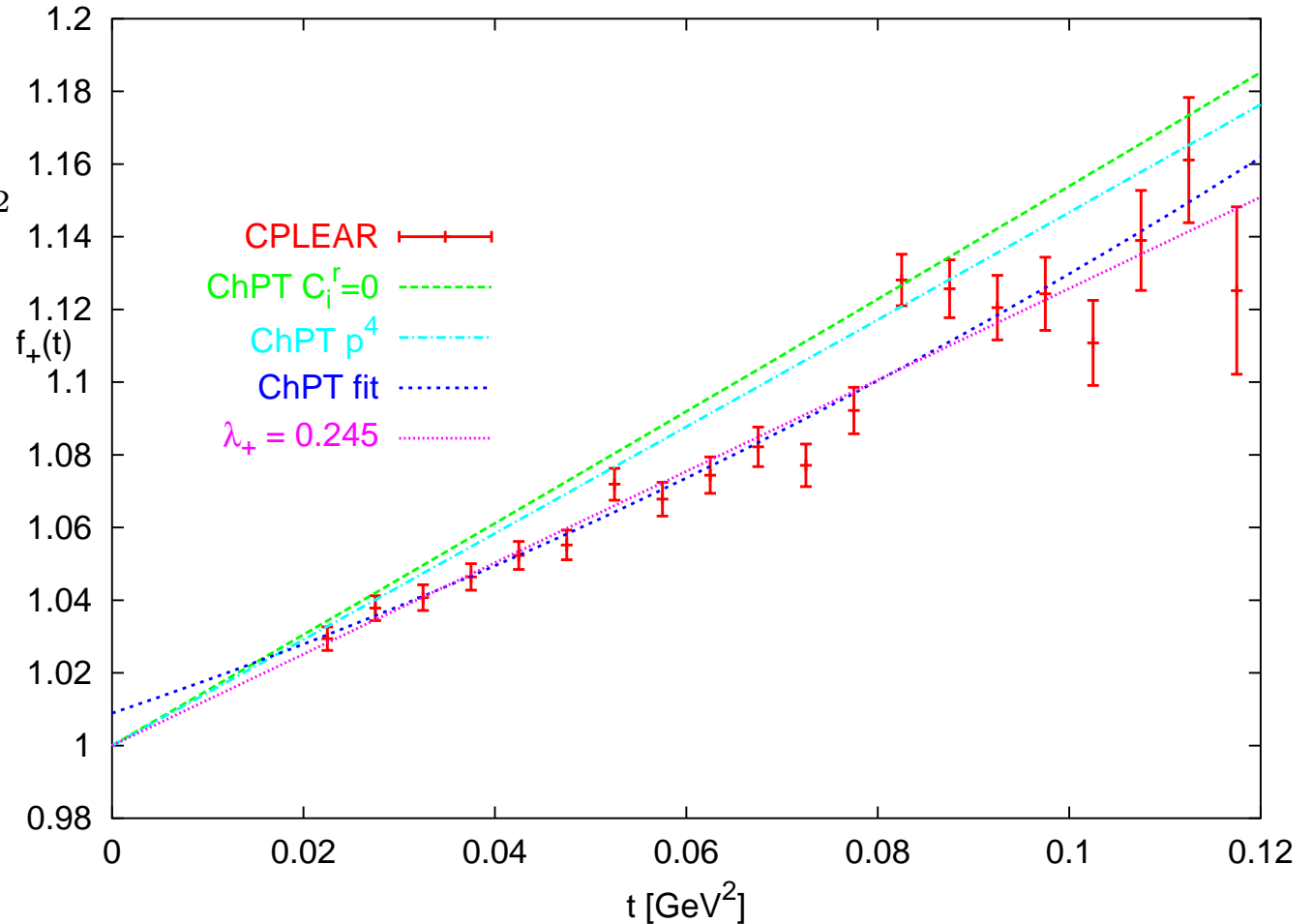
# ChPT fit to $f_+(t)$

$$\Rightarrow R_{+1}^{K\pi} = -(4.7 \pm 0.5) 10^{-5} \text{ GeV}^2$$

$$(c_+ = 3.2 \text{ GeV}^{-4})$$

$$\Rightarrow a_+ = 1.009 \pm 0.004$$

$$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$$



# $f_0(t)$

## Main Result:

$$f_0(t) = 1 - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 \\ + 8 \frac{t}{F_\pi^4} (2C_{12}^r + C_{34}^r) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) \\ - \frac{8}{F_\pi^4} t^2 C_{12}^r + \bar{\Delta}(t) + \Delta(0).$$

$\bar{\Delta}(t)$  and  $\Delta(0)$  contain **NO**  $C_i^r$  and only depend on the  $L_i^r$  at order  $p^6$

$\implies$

**All needed parameters can be determined experimentally**

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r].$$

# Experiment

## Form Factor comparison

### KTeV [PRD 70(2004)]

$K_{e3}^0$  quadratic fit:  $\lambda''_+ \neq 0$  @  $4\sigma$  level

$K_{\mu 3}^0$  quadratic fit:  $\lambda_0 = (13.72 \pm 1.31) 10^{-3}$

Slopes consistent for  $K_{e3}^0$  and  $K_{\mu 3}^0$

### ISTRA+ [PLB 581(2004), PLB589(2004)]

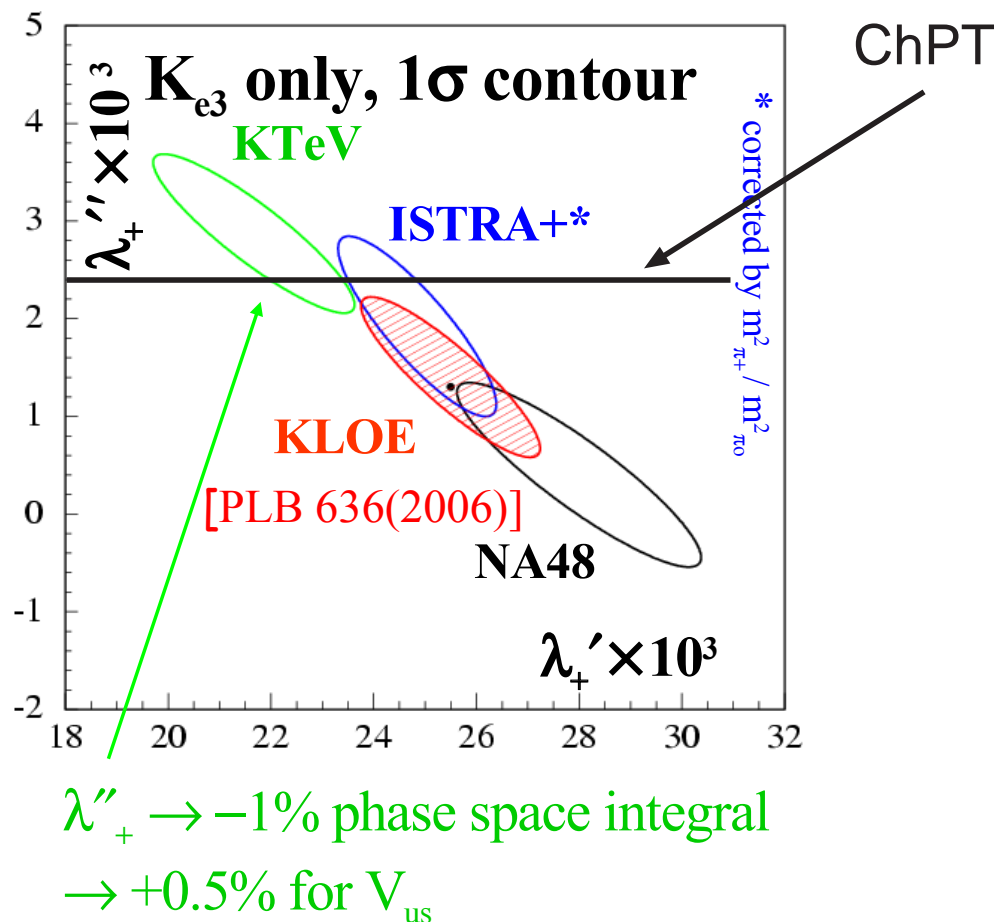
$K_{e3}^-$  quadratic fit:  $\lambda''_+ \neq 0$  @  $2\sigma$  level

$K_{\mu 3}^-$  quadratic fit:  $\lambda_0 = (17.11 \pm 2.31) 10^{-3}$

### NA48 [PLB 604(2004), HEP2005 289]

$K_{e3}^0$ : No evidence for quadratic term

$K_{\mu 3}^0$  linear fit:  $\lambda_0 = (12.0 \pm 1.7) 10^{-3}$

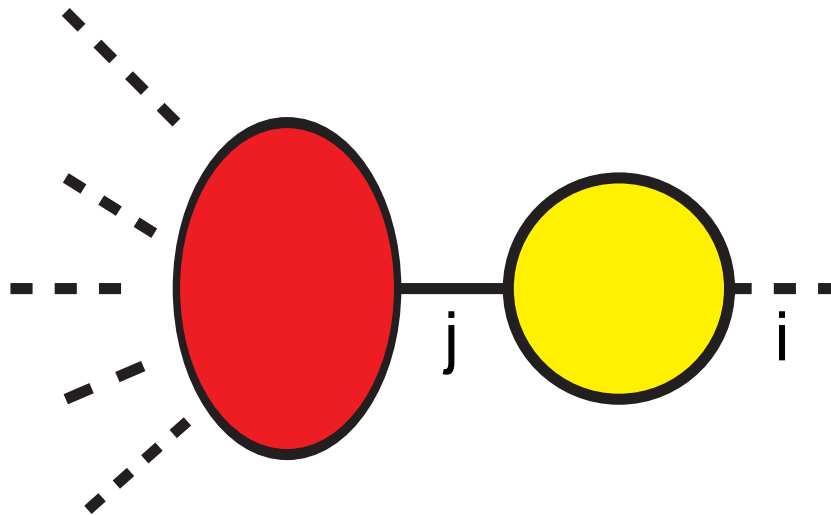


# New for isospin breaking

Take LSZ into account properly

Amoros, JB, Talavera, 2001

Matrix element:



$$\mathcal{A}_{i_1 \dots i_n} = \left( \frac{(-i)^n}{\sqrt{Z_{i_1} \dots Z_{i_n}}} \right) \prod_{i=1}^n \lim_{k_i^2 \rightarrow m_i^2} (k_i^2 - m_i^2) G_{i_1 \dots i_n}(k_1, \dots, k_n)$$

# Isospin Breaking

$$\begin{aligned}
 \mathcal{A}_3 = & \mathcal{G}_3^{(2)} + \left\{ \mathcal{G}_3^{(4)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right\} \\
 & + \left[ \mathcal{G}_3^{(6)} - \frac{1}{2} Z_{33}^{(6)} \mathcal{G}_3^{(2)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(4)} + \frac{3}{8} \left( Z_{33}^{(4)} \right)^2 \mathcal{G}_3^{(2)} \right. \\
 & + \frac{Z_{38}^{(4)} \Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_3^{(2)} - \frac{1}{2} \left( \frac{\Pi_{38}^{(4)}}{\Delta m^2} \right)^2 \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(4)} \\
 & \left. - \frac{\Pi_{38}^{(6)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{\Pi_{38}^{(4)} \Pi_{88}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{1}{2} Z_{33}^{(4)} \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right]
 \end{aligned}$$



# Isospin Breaking

$$\begin{aligned}
 \mathcal{A}_3 = & \mathcal{G}_3^{(2)} + \left\{ \mathcal{G}_3^{(4)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right\} \\
 & + \left[ \mathcal{G}_3^{(6)} - \frac{1}{2} Z_{33}^{(6)} \mathcal{G}_3^{(2)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(4)} + \frac{3}{8} \left( Z_{33}^{(4)} \right)^2 \mathcal{G}_3^{(2)} \right. \\
 & + \frac{Z_{38}^{(4)} \Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_3^{(2)} - \frac{1}{2} \left( \frac{\Pi_{38}^{(4)}}{\Delta m^2} \right)^2 \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(4)} \\
 & \left. - \frac{\Pi_{38}^{(6)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{\Pi_{38}^{(4)} \Pi_{88}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{1}{2} Z_{33}^{(4)} \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right]
 \end{aligned}$$

Compute all diagrams, produce numerical programs, ...

# Numerical results for $f_+(0)$

**VERY PRELIMINARY**

Decay	$p^2$	$p^4$	pure 2-loop	$L_i^r$ at $p^6$	$C_i^r$	total
Iso conserving calculation						
$K^0$	1	-0.02266	0.01130	0.00332	???	0.99196
$K^+$	1	-0.02276	0.01104	0.00320	???	0.99154
$m_u/m_d = 0.45$						
$K^0$	1	-0.02310	0.01131	0.00325	???	0.99146
$K^+$	1.02465	-0.01741	0.00379	0.00648	???	1.01751
ratio	1.02465	1.0311				1.0262
$m_u/m_d = 0.58$						
$K^0$	1	-0.02299	0.01124	0.00325	???	0.99150
$K^+$	1.01702	-0.01897	0.00657	0.00551	???	1.01013
ratio	1.0170	1.0215				1.0188

# Some Comments

- $K^0$  only again on  $C_{12}^r + C_{34}^r$   
Ademollo-Gatto + Callan-Treiman as in isoconserving case but  $(m_s - m_u)^2$
- Not checked yet whether  $C_i^r$  in  $K^+$  decay can be determined
- $p^6$  lowers the isospin breaking
- $p^6$  fit has  $m_u/m_d = 0.45$  and not 0.58
- $0.58 \rightarrow 0.52$  from  $p^6$  and  $0.52 \rightarrow 0.45$  from violation of Dashen's theorem
- **Very preliminary numerics:**  $p^6$  correction about  $-0.5\%$  but with changed  $m_u/m_d$ , total effect  $+0.5\%$  compared to Leutwyler-Roos

# $K_{\ell 4}$

$$K^+(p) \rightarrow \pi^+(p_+) \pi^-(p_-) \ell^+(p_\ell) \nu_\ell(p_\nu),$$

$$K^+(p) \rightarrow \pi^0(p_+) \pi^0(p_-) \ell^+(p_\ell) \nu_\ell(p_\nu),$$

$$K^0(p) \rightarrow \pi^-(p_+) \pi^0(p_-) \ell^+(p_\ell) \nu_\ell(p_\nu).$$

Kinematical variables for hadronic system:  $t, u, s_\pi, s_\ell$

$$T^{+-} = \frac{G_F}{\sqrt{2}} V_{us}^* \bar{u}(p_\nu) \gamma_\mu (1 - \gamma_5) v(p_\ell) (V^\mu - A^\mu),$$

$$V_\mu = -\frac{H}{m_K^3} \epsilon_{\mu\nu\rho\sigma} (p_\ell + p_\nu)^\nu (p_+ + p_-)^\rho (p_+ - p_-)^\sigma,$$

$$A_\mu = -\frac{i}{m_K} [(p_+ + p_-)_\mu F + (p_+ - p_-)_\mu G + (p_\ell + p_\nu)_\mu R].$$

# $K_{\ell 4}$

$$T^{+-} = \frac{T^{-0}}{\sqrt{2}} + T^{00}$$

$T^{-0}$  is anti-symmetric under  $t \leftrightarrow u$

$T^{00}$  is symmetric.

Lowest order: Weinberg:  $F = G = \frac{m_K}{\sqrt{2}F_\pi}$ ,

Order  $p^4$ : JB 1990, Riggensbach et al. 1991

Could fit data with reasonable corrections:

Determine  $L_i^r$   $i = 1, 2, 3$

Dispersive estimate of  $p^6$  corrections:

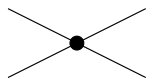
JB, Colangelo, Gasser 1994

# $K_{\ell 4}$

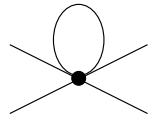
Parametrization for experiment: Amoros,JB, 1999

Full  $p^6$  calculation: Amoros, JB, Talavera 2000

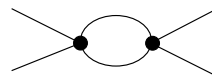
Ametller,JB,Bramon,Cornet 1993 (H only)



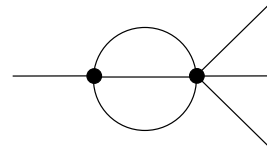
(a)



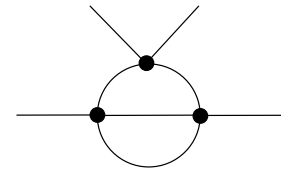
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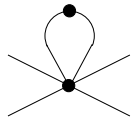
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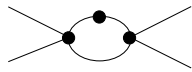
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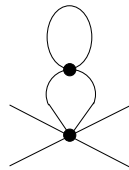
(i)



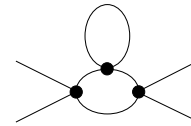
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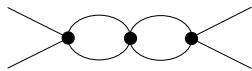
(e)



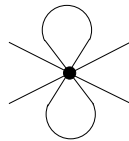
(f)



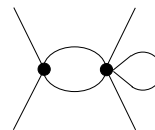
(g)



(h)

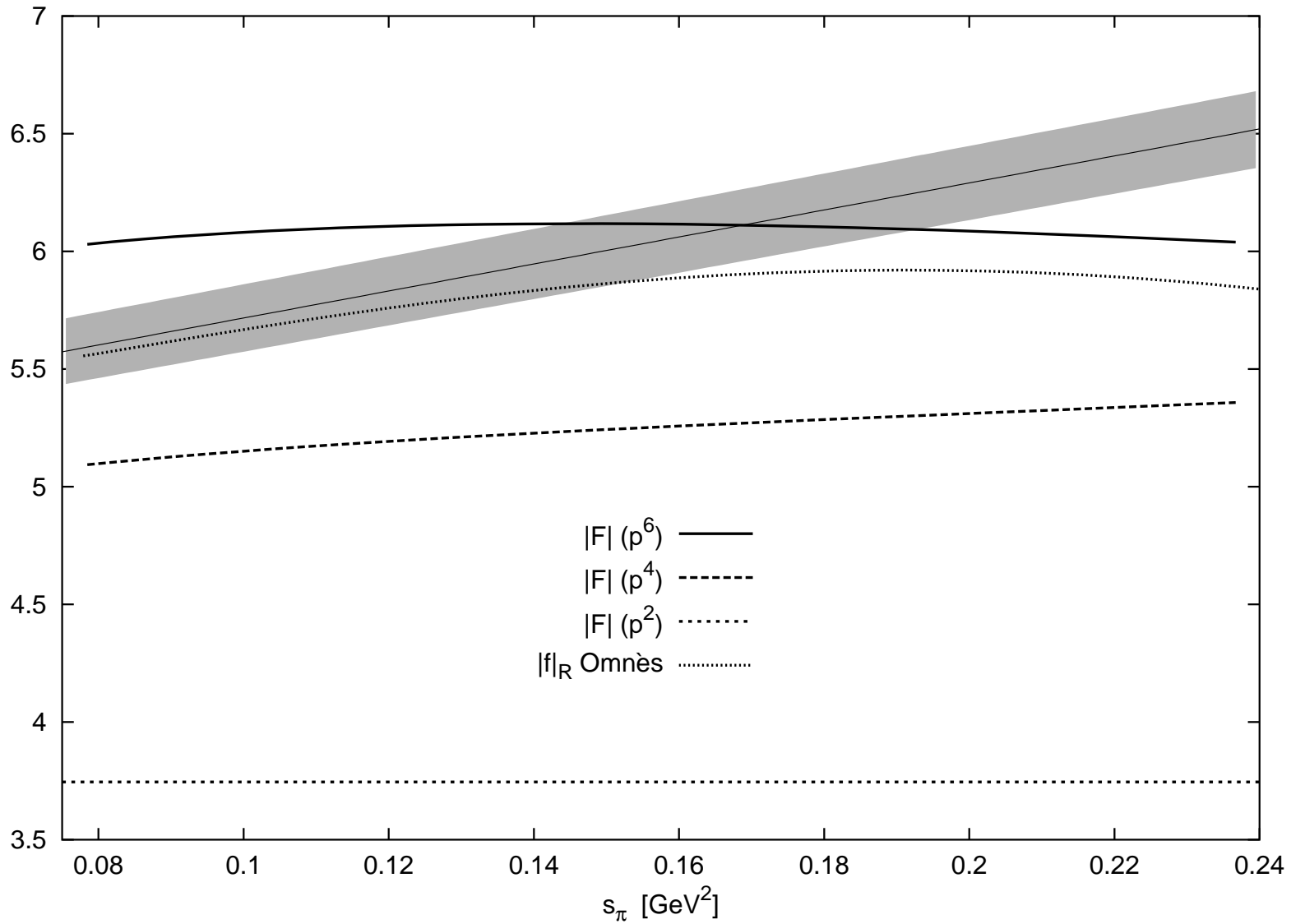


(i)

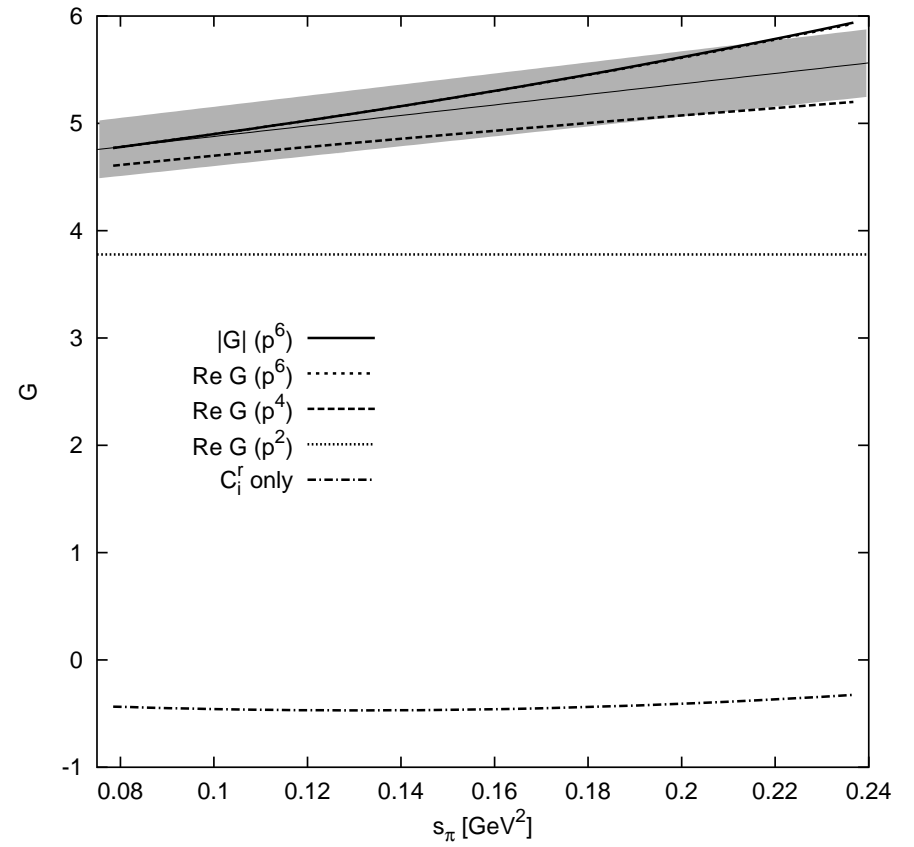
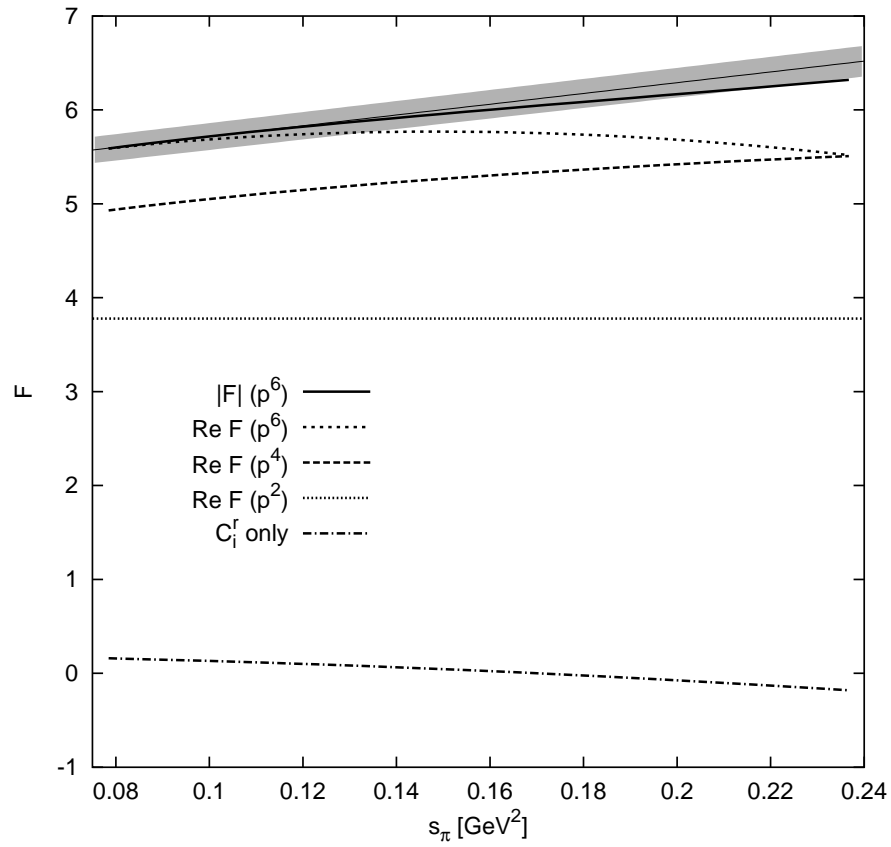


(j)

# $K_{\ell 4}$



# $K_{\ell 4}$





# General Strategy and some comments

	fit 10	same $p^4$	fit B	fit D
$10^3 L_1^r$	$0.43 \pm 0.12$	0.38	0.44	0.44
$10^3 L_2^r$	$0.73 \pm 0.12$	1.59	0.60	0.69
$10^3 L_3^r$	$-2.53 \pm 0.37$	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	$0.97 \pm 0.11$	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	$-0.31 \pm 0.14$	-0.49	-0.26	-0.28
$10^3 L_8^r$	$0.60 \pm 0.18$	1.00	0.50	0.54

- ▣ errors are very correlated
- ▣  $\mu = 770$  MeV; 550 or 1000 within errors
- ▣ varying  $C_i^r$  factor 2 about errors
- ▣  $L_4^r, L_6^r \approx -0.3, \dots, 0.6 \cdot 10^{-3}$  OK
- ▣ fit B: small corrections to pion “sigma” term, fit scalar radius
- ▣ fit D: fit  $\pi\pi$  and  $\pi K$  thresholds

# General Strategy and some comments

	fit 10	same $p^4$	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
$m_\pi^2: p^4, p^6$	0.006,0.258	0.009, $\equiv 0$	-0.138,0.009	-0.091,0.133
$m_K^2: p^4, p^6$	0.007,0.306	0.075, $\equiv 0$	-0.149,0.094	-0.096,0.201
$m_\eta^2: p^4, p^6$	-0.052,0.318	0.013, $\equiv 0$	-0.197,0.073	-0.151,0.197
$m_u/m_d$	$0.45 \pm 0.05$	0.52	0.52	0.50
$F_0$ [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}: p^4, p^6$	0.169,0.051	0.22, $\equiv 0$	0.153,0.067	0.159,0.061

- ▣▣▣▣  $m_u = 0$  always very far from the fits
- ▣▣▣▣  $F_0$ : pion decay constant in the chiral limit
- ▣▣▣▣ Looking forward to newest data and new fit

# $K_{e5}$

	branching ratio
$K^+ \rightarrow \pi^+ \pi^- \pi^0 e^+ \nu_e$	$3 \cdot 10^{-12}$
$K^+ \rightarrow \pi^0 \pi^0 \pi^0 e^+ \nu_e$	$2.5 \cdot 10^{-12}$
$K^0 \rightarrow \pi^0 \pi^0 \pi^- e^+ \nu_e$	$12 \cdot 10^{-12}$
$K^0 \rightarrow \pi^+ \pi^- \pi^- e^+ \nu_e$	$33 \cdot 10^{-12}$

S. Blaser, Phys.Lett.B345:287-290,1995 hep-ph/9410368

# $K_{\ell 2 \gamma}$

$$K^+(p) \rightarrow l^+(p_l) \nu_l(p_\nu) \gamma(q) \quad [K_{\ell 2 \gamma}]$$

$$T = -i G_F e V_{us}^* \epsilon_\mu^* \{ F_K L^\mu - H^{\mu\nu} l_\nu \}$$

$$L^\mu = m_l \bar{u}(p_\nu) (1 + \gamma_5) \left( \frac{2p^\mu}{2pq} - \frac{2p_l^\mu + \not{q} \gamma^\mu}{2p_l q} \right) v(p_l)$$

$$l^\mu = \bar{u}(p_\nu) \gamma^\mu (1 - \gamma_5) v(p_l)$$

$$H^{\mu\nu} = iV(W^2) \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta - A(W^2) (qW g^{\mu\nu} - W^\mu q^\nu)$$

$$W^\mu = (p - q)^\mu = (p_l + p_\nu)^\mu.$$

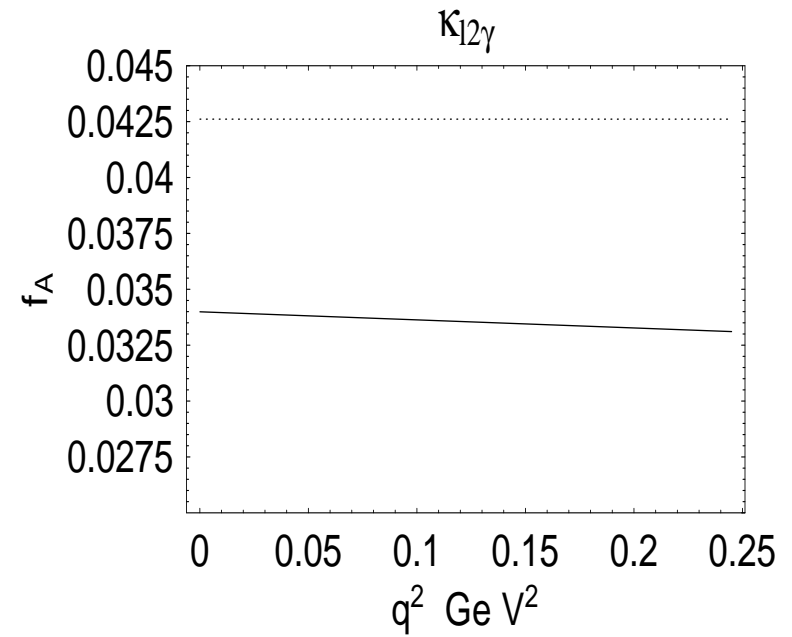
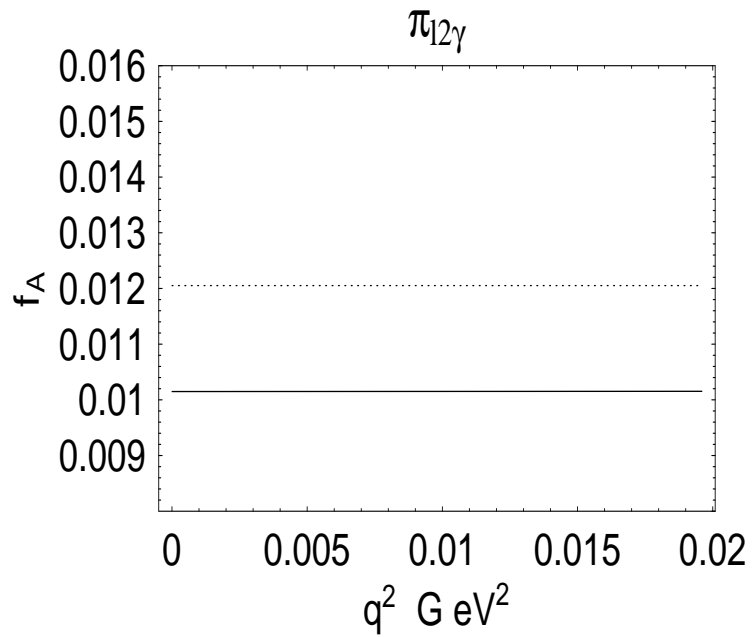
$L_\mu$ : IB or inner Bremsstrahlung part

$V$  and  $A$ : SD or structure dependent part, starts at  $p^4$

$V$ : anomaly at  $p^4$ , known to  $p^6$ : Ametller, JB, Bramon, Cornet 1993

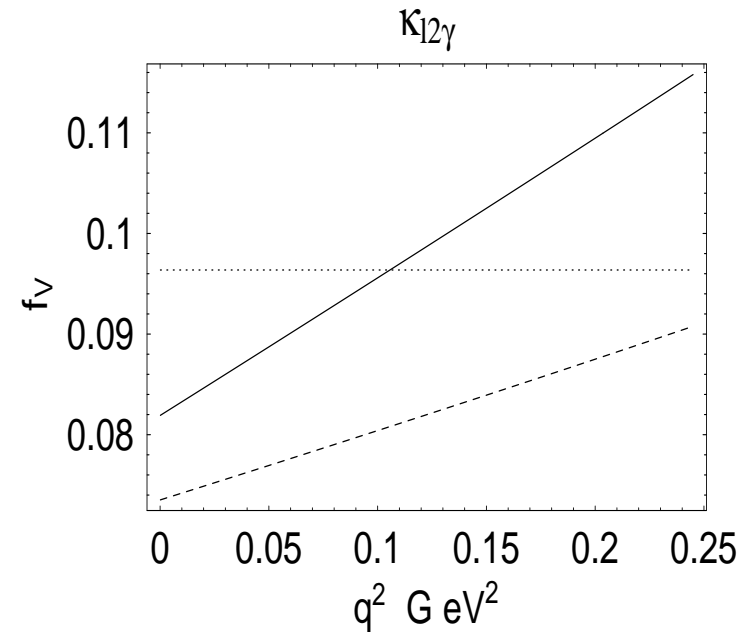
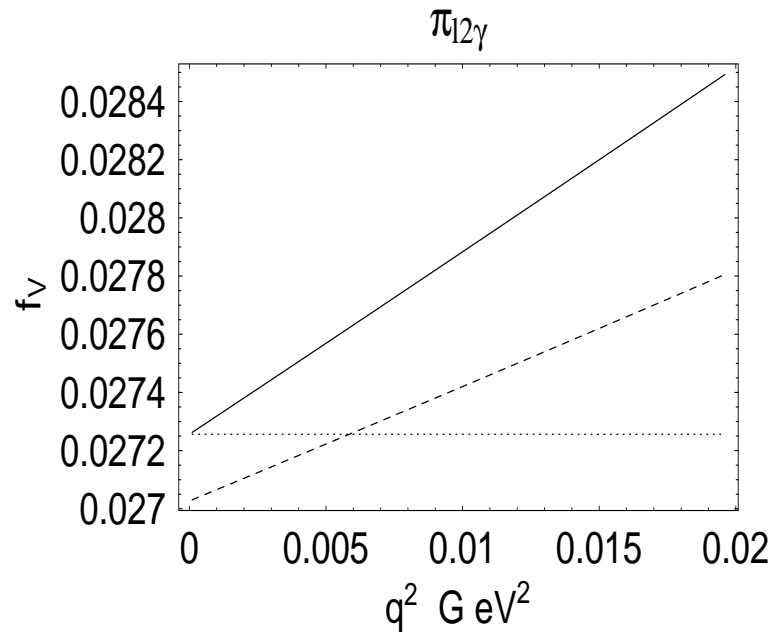
$A$ :  $p^4$  JB, Ecker, Gasser 1993,  $p^6$  Geng, Ho, Wu 2004

# $K_{\ell 2\gamma}$



From Geng, Ho, Wu 2004

# $K_{\ell 2\gamma}$



From Geng, Ho, Wu 2004

dotted:  $p^4$

solid  $p^6 C_i^W$  from VMD, dashed  $p^6 C_i^W$  from CQM

# $K_{\ell 3\gamma}$

$p^2$ : Fearing, Fischbach, Smith 1970 IB only

$p^4$ : JB, Ecker, Gasser, 1993

$p^6$ : Axial form-factors fully known

$p^6$ : Vector form-factors: approximately known

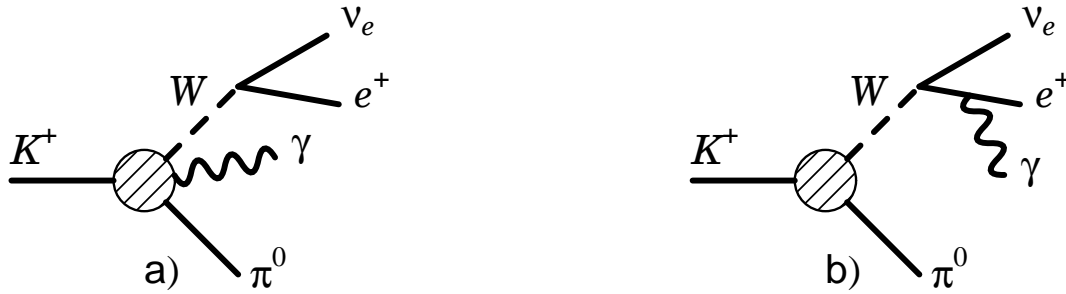
Gasser, Kubis, Paver, Verbeni hep-ph/0412130:  $K_{Le\nu\gamma}$

Müller, Kubis, Meißner hep-ph/0607151: T-odd correlations

Kubis, Müller, Gasser, Schmid hep-ph/0611366:  $K_{e\nu\gamma}^+$

Approximately known: structure functions smooth cuts:  $p$ -wave or far away: approximate by polynomials

# $K_{\ell 3\gamma}$



Remainder is from [Kubis et al. 2006](#)

$$\begin{aligned}
 T(K_{e3\gamma}^+) &= \frac{G_F}{\sqrt{2}} e V_{us}^* \epsilon^\mu(q)^* \left[ (V_{\mu\nu} - A_{\mu\nu}) \bar{u}(p_\nu) \gamma^\nu (1 - \gamma_5) v(p_e) \right. \\
 &\quad \left. + \frac{F_\nu}{2p_e q} \bar{u}(p_\nu) \gamma^\nu (1 - \gamma_5) (m_e - \not{p}_e - \not{q}) \gamma_\mu v(p_e) \right]
 \end{aligned}$$



# $K_{\ell 3 \gamma}$

$$A_{\mu\nu} = \frac{i}{\sqrt{2}} \left[ \epsilon_{\mu\nu\rho\sigma} (A_1 p'^{\rho} q^{\sigma} + A_2 q^{\rho} W^{\sigma}) + \epsilon_{\mu\lambda\rho\sigma} p'^{\lambda} q^{\rho} W^{\sigma} \left( \frac{A_3}{M_K^2 - W^2} W_{\nu} + A_4 p'_{\nu} \right) \right],$$

$$V_{\mu\nu} = V_{\mu\nu}^{IB} + V_{\mu\nu}^{SD}$$

$V_{\mu\nu}^{SD}$  has again 4 structure function  $V_i$

$V_{\mu\nu}^{IB}$ : IB part, mainly determined by Low's theorem and from the  $K_{\ell 3}$  form-factors

$$R(E_{\gamma}^{\text{cut}}, \theta_{e\gamma}^{\text{cut}}) = \frac{\Gamma(K_{e3\gamma}^{\pm}, E_{\gamma}^* > E_{\gamma}^{\text{cut}}, \theta_{e\gamma}^* > \theta_{e\gamma}^{\text{cut}})}{\Gamma(K_{e3}^{\pm})},$$

Many uncertainties drop out

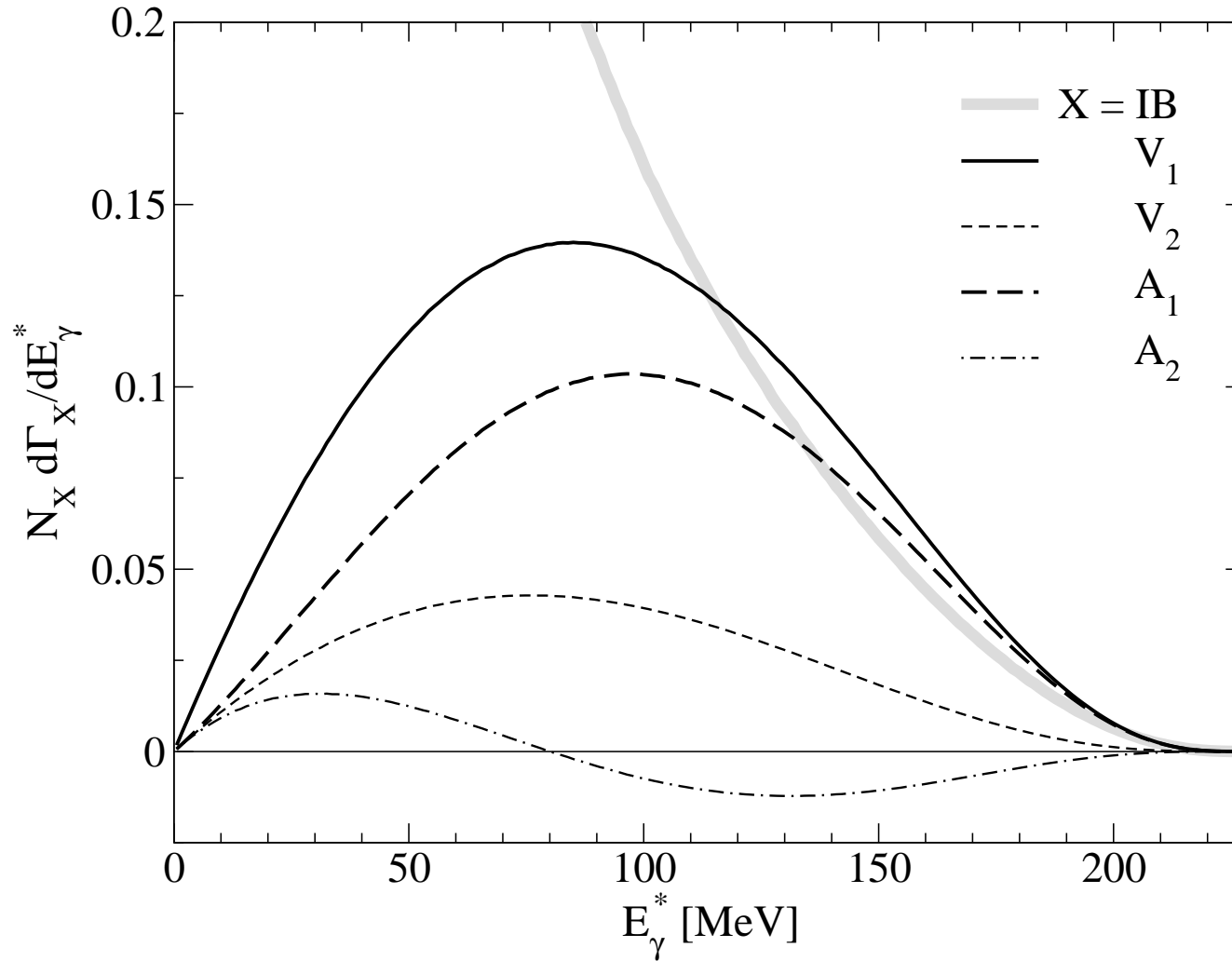
# $K_{\ell 3\gamma}$

$$R(\bar{\lambda}_+, \bar{\lambda}_+'' ) = R(1, 0) \left\{ 1 + c_1 (\bar{\lambda}_+ - 1) + c_2 (\bar{\lambda}_+ - 1)^2 + c_3 \bar{\lambda}_+'' + \dots \right\}$$

$R^{\text{IB}}$  accordingly (with expansion coefficients  $c_i^{\text{IB}}$ )

$E_\gamma^{\text{cut}}$	$\theta_{e\gamma}^{\text{cut}}$	$R^{\text{IB}} \cdot 10^2$	$R \cdot 10^2$	$c_1 \cdot 10^3$	$c_2 \cdot 10^4$	$c_3 \cdot 10^4$
30 MeV	20°	0.640	0.633 ± 0.002	12.5 ± 0.4	-5.4 ± 0.3	16.9 ± 0.4
30 MeV	10°	0.925	0.918 ± 0.002	11.1 ± 0.3	-4.7 ± 0.2	15.0 ± 0.3
10 MeV	20°	1.211	1.204 ± 0.002	7.5 ± 0.2	-3.2 ± 0.2	10.1 ± 0.2
10 MeV	10°	1.792	1.785 ± 0.002	6.7 ± 0.2	-2.8 ± 0.1	9.0 ± 0.1
10 MeV	26° – 53°	0.554	0.553 ± 0.001	5.7 ± 0.1	-2.4 ± 0.1	7.5 ± 0.1

# $K_{\ell 3\gamma}$



$$\frac{d\Gamma}{dE_\gamma^*} = \frac{d\Gamma_{\text{IB}}}{dE_\gamma^*} + \sum_{i=1}^4 \left( \langle V_i \rangle \frac{d\Gamma_{V_i}}{dE_\gamma^*} + \langle A_i \rangle \frac{d\Gamma_{A_i}}{dE_\gamma^*} \right) + \mathcal{O}(|T^{\text{SD}}|^2, \Delta V_i, \Delta A_i)$$

# Anomaly

The anomaly including the sign can be tested in many of the decays I talked about, they are all known to  $p^6$ .

# Conclusions

Chiral Perturbation Theory has played and will play a major role in Kaon physics and especially in the decays talked about here.

## Recent results:

- Precise predictions for  $K_{e3}$ : isospin breaking at  $p^6$
- Radiative decay: Precise predictions for the ratio  $R$  and predictions for the dependence on form factors