CHIRAL PERTURBATION THEORY FOR KAONS

Johan Bijnens

Lund University

Introduction

Large subject: will obviously cover only parts
CHPT + chiral symmetry already seen a lot at this meeting

- Constraints on the lattice plus other calculations:
  Hambye, de Rafael, Pakvasa, Sehgall, Savage, ...

- Rare decays: Isidori

<u>Talk</u>:

- Chiral Perturbation Theory
- Semileptonic decays
- Nonleptonic decays
  - $(K \to \pi) \leftrightarrow (K \to \pi\pi)$
  - checks of CHPT in $K \to \pi\pi\pi$
  - B_6 and the chiral limit

CHIRAL PERTURBATION THEORY

S. Weinberg, Physica 96A (19979) 327;

J. Gasser and H. Leutwyler, Ann. of Phys. 158(1984)142,

Nucl. Phys. B250(1985)465

Based on:

- Chiral Symmetry and its breakdown: $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$
- Only relevant degrees of freedom are Goldstone Bosons at low energies (i.e. only singularities)
- Analyticity, causality cluster expansion, relativity H. Leutwyler, Ann. of Phys. 235 (1994) 165

Minimal number of parameters allowed by symmetry + rest by essentially unitarity.
Current algebra possible, Effective Field Theory much simpler.
Therefore only TWO questions:

- Does the expansion in m_q, p^2 converge ? We use $m_q \sim p^2$
- If higher orders important :
  - can we estimate them ?
  - enough data to determine all needed parameters ?

SEMILEPTONIC DECAYS

$$\mathcal{L}_2 = \frac{F_0^2}{4}\text{tr}\left(D_\mu U D^\mu U^\dagger + \chi U^\dagger + U\chi^\dagger\right)$$

2 parameters: F_0, B_0 (+quark masses)

$\mathcal{L}_4$: 10+2 parameters Gasser-Leutwyler 1985

$\Rightarrow$ only 7 are relevant; 3 more in meson masses

+ Wess-Zumino term + Loops

$\mathcal{L}_6$: 90+4 parameters JB-Colangelo-Ecker 1999, Fearing-Scherer 1996

+ Two-loop diagrams + ...

General Situation:

p^2 Current Algebra : sixties

p^4 One-loop 80's, early 90's

p^6
- Estimates using dispersive and/or models: "done"
- Double Log Contributions: mostly done JB-Colangelo-Ecker 1999
- Two-flavour full calculations: Done
- Three flavour full calculations: few done, several in progress.

e^2p^2 In progress Knecht, Neufeld, Rupertsberger, Talavera

Experiment Progress from DAPHNE, NA48, BNL, KTeV, ...

K_{l2}

- Determine F_K and F_π.
- Lepton universality: $\dfrac{K \to e\nu}{K \to \mu\nu}$

Theory: p^6 done Amoros, JB, Talavera 1999,

Golowich, Kambor 1998 (F_π)

Theory:

	F_π/F_0	F_K/F_π	$(F_K/F_\pi)^{(6)}$
p^2	1	1	—
p^4	1.07	1.22	—
p^6 set A	0.96 (1.08)	1.27 (1.30)	0.05 (0.08)
p^6 set A μ=0.9 GeV	0.96 (1.10)	1.30 (1.34)	0.08 (0.12)
p^6 set B	0.90 (1.02)	1.25 (1.28)	0.035 (0.06)

full p^6 PRELIMINARY

C_i set to zero

brackets extended double log approximation

JB, Colangelo, Ecker 98

set A $10^3 L_{i=1,3} = (0.4, 1.35, -3.5)$;

set B $10^3 L_{i=1,3} = (0.6, 1.5, -3.3)$;

$10^3 L_{i=4,10} = (-0.3, 1.4, -0.2, 0.9, 6.9, 5.5)$;

$\mu = 0.77$ GeV

$\boxed{K_{l2\gamma}}$

2 formfactors: $A(W^2)$

$V(W^2)$ ANOMALY

to p^4: $A = -\dfrac{4}{F}(L_9^r + L_{10}^r) = \dfrac{-0.030}{m_K}$

Donoghue et al 1989, JB, Ecker, Gasser 1993

$V = -\dfrac{1}{8\pi^2 F} = \dfrac{-0.067}{m_K}$

to p^6: V : 10-20% correction in relevant phase space

W^2-dependence Ametller et al 1993

A : 25% correction in $\pi \to \ell\nu\gamma$

some W^2-dependence JB, Talavera, 1997

Test of $V - A$ of W-couplings

$\Rightarrow$ measure W^2 dependence and improve $A - V$:

$m_K|A+V| = 0.105 \pm 0.008$ $\qquad$ $m_K|A-V| \leq 0.35$

$\boxed{K_{l2\ell\ell}}$

3 axial : p^4 JB, Ecker, Gasser 1993

1 vector : p^6 Ametller et al 1993

final states: $e^+\nu\mu^+\mu^-$, $e^+\nu e^+e^-$, $\mu^+\nu\mu^+\mu^-$, $\mu^+\nu e^+e^-$

$e^+\nu$ modes : large enhancement over Bremsstrahlung

New limit on $K^+ \to e^+\nu\mu^+\mu^- < 5 \cdot 10^{-7}$ (BNL E787)

Consistent with data + upper limits in all channels

$\boxed{K_{l3}}$ Main source for V_{us}

$K^{+,0}(p) \to \pi^{0,-}(p')\ell^+\nu$

$\langle\pi|V_\mu^{4-i5}|H\rangle = \frac{1}{\sqrt{2}}\left[(p+p')_\mu f_+(t) + (p-p')_\mu f_-(t)\right]$

exp: $f_+(t) = f_+(0)\left[1+\lambda_+\frac{t}{m_\pi^2}\right]$

$$f_0(t) = f_+(t) + \frac{t f_-(t)}{m_K^2-m_\pi^2} = f_+(0)\left[1+\lambda_0\frac{t}{m_\pi^2}\right]$$

p^4 Gasser, Leutwyler 85

p^6 JB, Ecker, Colangelo 98

	p^2	p^4	Ext. double log	
f_+	1	-0.0023	$+(-0.005 \to 0.004)$	Good !
	$(K^0 \to \pi^- e^+\nu)$			
λ_+		0.031	$+(-0.006 \to -0.0044)$	
		exp.:	0.029 ± 0.002	
λ_0		0.017	$+(0.003 \to 0.009)$	
		exp.:	0.025 ± 0.006 $K^0_{\mu3}$	
			0.006 ± 0.007 $K^+_{\mu3}$	
λ'_+	quadratic slope		$(0.0002 \to 0.0003)$	
λ'_0	quadratic slope		$(0.0001 \to 0.0002)$	

$\boxed{K_{l3\gamma}}$ JB,Ecker,Gasser 93

10 formfactors

$\Rightarrow$ complicated interplay between tree level, counter-terms and loops, final change small

e.g. $K^+_{e3\gamma}$ with $E_\gamma \geq 30$ MeV

$$\theta_{\ell\gamma} \geq 20^o$$

$$\text{tree } 2.8\ 10^{-4} \quad \xrightarrow{L_i} \quad 3.2\ 10^{-4} \quad \xrightarrow{\text{loops}} \quad 3.0\ 10^{-4}$$

Experiments:

Leber et al. PLB369(96)69 (NA31)

$K^0_{e3\gamma}\ :\ 3.61 \pm 0.14 \pm 0.21\ 10^{-3}$

prediction: $3.6 \longrightarrow 4.0 \longrightarrow 3.8\ 10^{-3}$

Bender et al. PLB418(98)411 (NA48)

$K^0_{\mu3\gamma}\ :\ 0.56 \pm 0.05 \pm 0.05\ 10^{-3}$

prediction: $0.52 \longrightarrow 0.59 \longrightarrow 0.56\ 10^{-3}$

Notice $F_K/F_\pi \approx 1.22$ so these are good tests of p^4

$\boxed{K_{l4}}$

4 formfactors : F, G, H, R, R only in $K_{\mu 4}$

Experiment: Rosselet et al. 77 $\quad K^+ \to \pi^+\pi^- e^+\nu$

Makof et al. 93 $\quad K_L \to \pi^\pm\pi^0 e^\mp\nu$

$$F = F(0)\left(1 + \lambda(s_{\pi\pi}/(4m_\pi^2) - 1)\right)$$
$$G = G(0)\left(1 + \lambda(s_{\pi\pi}/(4m_\pi^2) - 1)\right)$$
$$H = H(0)\left(1 + \lambda(s_{\pi\pi}/(4m_\pi^2) - 1)\right)$$

$$\left.\begin{array}{l} \lambda = 0.08 \pm 0.02 \\ F(0) = 5.59 \pm 0.14 \\ G(0) = 4.77 \pm 0.27 \end{array}\right\} \text{main input for } L_1, L_2, L_3$$

$H(0) = -2.7 \pm 0.7$ Check on anomaly

Theory:

F, G : p^4 JB 90, Riggenbach et al. 90

$$\left.\begin{array}{l} F, G : \text{ dispersive} \\ R : \; p^4 \end{array}\right\} \text{JB, Colangelo, Gasser 94}$$

F, G : Ext. doub. log. JB, Colangelo, Ecker 98

H : p^6 Amettler et al. 93

H : good agreement

	$F(0)$	$\lambda F(0)$	$G(0)$
p^2	3.74	—	3.74
p^4	set B; fit to L_1, L_2, L_3		
dispersive	set A; fit to L_1, L_2, L_3		
p^6 set A	0.86	0.38	-0.15
p^6 set A $\mu = 0.9\ GeV$	1.13	0.51	-0.20
p^6 set B	0.75	0.17	-0.04
exp	5.59 ± 0.14	0.45 ± 0.11	4.77 ± 0.27

p^6 is ext. double log.
similar to dispersive, large corrections to $F(0)$ and slope there too

Full calculation is in progress $\Longrightarrow$ refit L_1, L_2, L_3
Measure $\pi\pi$ scattering here, known to p^6 in CHPT, main uncertainty is L_1, L_2, L_3
Note: useful parametrization to fit data + extract $\pi\pi$ phase, Amoros, JB 99 in addition to Pais-Treiman method.

Nonleptonic Decays

Lowest order: 3 parameters G_8, G_{27}, G_8'

$$\mathcal{L} = -\frac{3G_F}{5\sqrt{2}} V_{ud} V_{us}^* \{ G_8 \mathrm{tr}(\lambda_{32} u^\dagger u_\mu u^\mu u) + G_8' \mathrm{tr}(\lambda_{32} u^\dagger \chi_+ u) + G_{27} t^{ijkl} \mathrm{tr}(\lambda_{ij} u^\dagger u_\mu u^\mu u) \mathrm{tr}(\lambda_{kl} u^\dagger u_\mu u^\mu u) \}$$

Note: weak mass term contributes to processes with photons at lowest order and otherwise at NLO

JB, Pallante, Prades 98

NLO: about 30 parameters: Ecker, Kambor, Wyler 93

$$(K \to \pi) \longleftrightarrow (H \to \pi\pi)$$

JB, Pallante, Prades 98

$K \to \pi\pi$: 7 E_i $+G_8, G_{27}, G_8'$
6 D_i

$K \to \pi$: well defined off-shell as a Green Function of pseudoscalar currents
4 E_i $+G_8, G_{27}, G_8'$
5 D_i

So almost all can be gotten from off-shell $K \to \pi$

$\Longrightarrow$ so in lattice (or elsewhere) allows to easier calculate several of the constants : determine reliability of factorization, models etc. in more than just decay rates.

Tests in $K \to \pi\pi$ and $K \to \pi\pi\pi$

Kambor, Missimer and Wyler 90 and 91

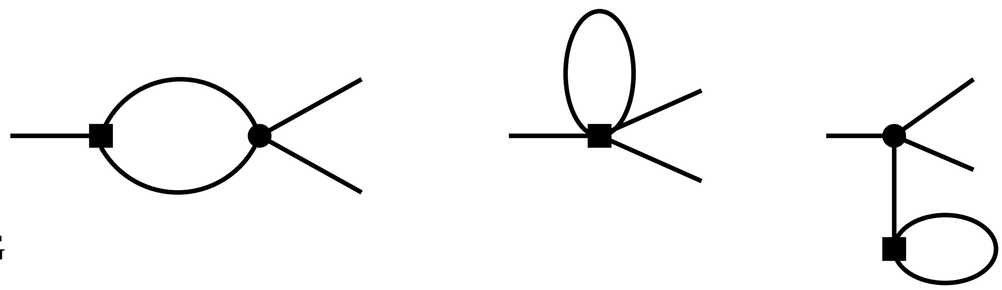
Kambor et al. PRL 68(92)1818

D'Ambrosio, Isidori 96

known to p^4

■ WEAK

● STRONG



$\#$ parameters :p^2 :2 (1) G_8, G_{27}
p^4 :7(3) (-2 that cannot be disentangled from G_8, G_{27})

$\#$ observables: After isospin

$K \to 2\pi$:	2(1)	
$K \to 3\pi$:	2(1)+1	constant in Dalitz plot
	3(1)+3	linear
	5(1)	quadratic

Phases +i up to linear in dalitz plot

$\Delta I = 1/2$ only

$K \to \pi\pi$ always used as input

variable	p^2	p^4	experiment
α_1	74	input	91.71 ± 0.32
β_1	-16.5	input	-25.68 ± 0.27
ζ_1		-0.47 ± 0.18	-0.47 ± 0.15
ξ_1		-1.58 ± 0.19	-1.51 ± 0.30
α_3	-4.1	input	-7.36 ± 0.47
β_3	-1.0	input	-2.42 ± 0.41
γ_3	1.8	input	2.26 ± 0.23
$\xi_3 6$	0.92 ± 0.030	-0.12 ± 0.17	
ξ_3'		-0.033 ± 0.077	-0.21 ± 0.51
ζ_3		-0.0011 ± 0.006	-0.21 ± 0.08

(relations indicated using colours)

TEST THE RELATIONS DIRECTLY !!

Satisfactory agreement with data
(new CPLEAR data decrease errors somewhat)

CP-violation : very difficult : the strong phases needed to interfere are too small. D'Ambrosio, Isidori 96

E.g. $\delta_2 - \delta_1$ in $K_L \to \pi^+\pi^-\pi^0$ is -0.083,
present exp. -0.33 ± 0.29

Asymmetries expected of about 10^{-6}; improve limits

B_6 and the chiral limit

$$B_6 \equiv \frac{\langle \text{out}|Q_6|\text{in}\rangle}{\langle \text{out}|Q_6|\text{in}\rangle_{\text{factorized}}}$$

depends on well defined denominator

Not True for B_6

Factorizable contains the scalar radius which is infinite in full chiral limit.

$$G_8|_{Q_6\text{fact}} = -\frac{80C_6(\mu)B_0^2(\mu)}{3F_0^2}\left[L_5^r(\nu) - \frac{3}{256\pi^2}\{2\ln\frac{m_L}{\nu} + 1\}\right]$$

ν CHPT scale, m_L meson mass, $G_{8fact} \longrightarrow \infty$ for $m_L \to 0$

Nonfactorizable has same divergence,
cancels in sum (small μ)

$$G_8|_{Q_6\text{full}} = -\frac{80C_6(\mu)B_0^2(\mu)}{3F_0^2}\left[L_5^r(\nu) - \frac{3}{256\pi^2}\{2\ln\frac{\mu}{\nu} + \frac{5}{18}\}\right]$$

Be careful in defining B factors !!

m_L	B_6 large N_c in fact	B_6 fact with divergence
50 MeV	0.76	0.16
150 MeV	0.76	0.26
300 MeV	0.76	0.44
500 MeV	0.76	0.86

Conclusions

♣ Semileptonic Decays :

- CHPT doing fine, progress on p^6

♣ Nonleptonic Decays :

- $K \to 3\pi$ Check Relations $\Longrightarrow$ know how good other predictions are
- $K \to 2\pi$ Useful in helping calculations
- Rare Decays : Many successes, see other talks.