PARTIALLY QUENCHED AND THREE FLAVOURS AT TWO LOOPS

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Various ChPT: http://www.thep.lu.se/~bijnens/chpt.html
Overview

- Chiral Perturbation Theory
- Lagrangians
- What is partially quenched?
- PQChPT and some problems
- Some results of PQChPT at Two-loops
- A Two-loop Timeline
- Some comments on and results from ChPT at two loops
- Conclusions
Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so $M_\rho$ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{\partial} q_L + i\bar{q}_R \not{\partial} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$. 
\[ \langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0 \]

\( SU(3)_L \times SU(3)_R \) broken spontaneously to \( SU(3)_V \)

8 generators broken \( \implies \) 8 massless degrees of freedom and interaction vanishes at zero momentum

**Power counting in momenta:**

\[ \int d^4p \]

\[ 1/p^2 \]

\[ p^2 \]

\[ (p^2)((1/p^2)p^4) = p^4 \]

\[ (p^4) = p^4 \]
Two Loop: Lagrangians

Lagrangian Structure:

<table>
<thead>
<tr>
<th>2 flavour</th>
<th>3 flavour</th>
<th>3+3 PQChPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^2$</td>
<td>$F, B$</td>
<td>2</td>
</tr>
<tr>
<td>$p^4$</td>
<td>$l^r_i, h^r_i$</td>
<td>7+3</td>
</tr>
<tr>
<td>$p^6$</td>
<td>$c^r_i$</td>
<td>53+4</td>
</tr>
<tr>
<td></td>
<td>$F_0, B_0$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$L^r_i, H^r_i$</td>
<td>10+2</td>
</tr>
<tr>
<td></td>
<td>$C^r_i$</td>
<td>90+4</td>
</tr>
<tr>
<td></td>
<td>$F_0, B_0$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$L^r_i, H^r_i$</td>
<td>11+2</td>
</tr>
<tr>
<td></td>
<td>$K^r_i$</td>
<td>112+3</td>
</tr>
</tbody>
</table>

$p^2$: Weinberg 1966  
$p^4$: Gasser, Leutwyler 84,85  
$p^6$: JB, Colangelo, Ecker 99,00

Eurodaphne II/Euridice (PQChPT)
Constructing Lagrangians

Easy part:
- Construct a complete set of needed basic objects
- Often useful to make everything transform under H alone (conserved part)
- Construct all terms up to the order you want

Tricky part: finding a minimal basis
- Field redefinitions or equations of motion
- Partial integration
- Cayley-Hamilton identities
- Bianchi identities, $\det \chi$ is also chirally invariant

Final check: can I determine all from experiment?
Constructing Lagrangians

Infinities:
the general divergence structure can be derived using heat kernel methods and/or background field methods.

Very useful for checks on calculations
What is Partially Quenched?

In Lattice gauge theory one calculates

\[
\langle 0 | (\bar{u} \gamma_5 d)(x) (\bar{d} \gamma_5 u)(0) | 0 \rangle
\]

\[
= \int [dq][d\overline{q}][dG](\bar{u} \gamma_5 d)(x) (\bar{d} \gamma_5 u)(0) e^{i \int d^4 y \mathcal{L}_{\text{QCD}}} \frac{i \int d^4 y \mathcal{L}_{\text{QCD}}}{\int [dq][d\overline{q}][dG]\bar{d} \gamma_5 u(0) e^{i \int d^4 y \mathcal{L}_{\text{QCD}}}}
\]

for Euclidean separations \( x \)

Integrals also performed after rotation to Euclidean

(note that I use Minkowski notation throughout)
What is Partially Quenched?

\[ \int [d\bar{q}] [d\bar{q}][dG](\bar{u}\gamma_5 d)(x)(\bar{d}\gamma_5 u)(0)e^{i \int d^4 y \mathcal{L}_{\text{QCD}}} \propto \]

\[ \int [dG] e^{i \int d^4 x (-1/4)G_{\mu\nu}G^{\mu\nu}(\mathcal{D}_G^u)^{-1}(x,0)(\mathcal{D}_G^d)^{-1}(0,x)\det(\mathcal{D}_G)_{\text{QCD}}] \]
What is Partially Quenched?

\[ \int [dq][d\bar{q}][dG](\bar{u}\gamma_5 d)(x)(\bar{d}\gamma_5 u)(0)e^{i \int d^4y \mathcal{L}_{\text{QCD}}} \propto \]

\[ \int [dG]e^{i \int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}((\mathcal{D}_G^u)^{-1}(x,0)(\mathcal{D}_G^d)^{-1}(0,x)\det(\mathcal{D}_G))_{\text{QCD}} \]

\[ \int [dG] \text{ done via importance sampling} \]
What is Partially Quenched?

\[
\int [dq][d\bar{q}][dG] (\bar{u}\gamma_5 d)(x)(\bar{d}\gamma_5 u)(0)e^{i\int d^4y \mathcal{L}_{QCD}} \propto
\]

\[
\int [dG] e^{i\int d^4x (-1/4)G_{\mu\nu}G^{\mu\nu} (\mathcal{D}_G^u)^{-1}(x, 0)(\mathcal{D}_G^d)^{-1}(0, x) \det (\mathcal{D}_G)^{QCD}}
\]

\[
\int [dG] \text{done via importance sampling}
\]

- Quenched: get distribution from \( e^{i\int d^4x (-1/4)G_{\mu\nu}G^{\mu\nu}} \) only

Unquenched: include \( \det (\mathcal{D}_G)^{QCD} \)

Very expensive

Partially quenched: \( (\mathcal{D}_G^u)^{-1}(x, 0)(\mathcal{D}_G^d)^{-1}(0, x) \)

DIFFERENT Quarks then in \( \det (\mathcal{D}_G)^{QCD} \)
What is Partially Quenched?

\[
\int [dq][d\bar{q}][dG](\bar{u}\gamma_5 d)(x)(\bar{d}\gamma_5 u)(0)e^{i \int d^4 y \mathcal{L}_{QCD}} \propto
\]

\[
\int [dG]e^{i \int d^4 x (-1/4)G_{\mu\nu}G^{\mu\nu} (\mathcal{D}_G^u)^{-1}(x,0)(\mathcal{D}_G^d)^{-1}(0,x) \det (\mathcal{D}_G)_{QCD}
\]

\[
\int [dG] \text{ done via importance sampling}
\]

- Quenched: get distribution from \( e^{i \int d^4 x (-1/4)G_{\mu\nu}G^{\mu\nu} \) only
- Unquenched: include \( \det (\mathcal{D}_G)_{QCD} \) \( \text{VERY expensive} \)
What is Partially Quenched?

\[
\int [dq][d\bar{q}][dG](\bar{u}\gamma_5 d)(x)(\bar{d}\gamma_5 u)(0)e^{i\int d^4 y \mathcal{L}_{\text{QCD}}} \propto \int \]

\[
\int [dG]e^{i\int d^4 x (-1/4)G_{\mu\nu}G^{\mu\nu}(\mathcal{D}_G^u)^{-1}(x,0)(\mathcal{D}_G^d)^{-1}(0,x)\det(\mathcal{D}_G)_{\text{QCD}}}
\]

\[
\int [dG] \text{ done via importance sampling}
\]

- Quenched: get distribution from \( e^{i\int d^4 x (-1/4)G_{\mu\nu}G^{\mu\nu}} \) only
- Unquenched: include \( \det(\mathcal{D}_G)_{\text{QCD}} \) VERY expensive
- Partially quenched: \( (\mathcal{D}_G^u)^{-1}(x,0)(\mathcal{D}_G^d)^{-1}(0,x) \) DIFFERENT Quarks then in \( \det(\mathcal{D}_G)_{\text{QCD}} \)
What is Partially Quenched?

Why do this?

- Is not Quenched: Real QCD is continuous limit from Partially Quenched
- More handles to turn:
  - Allows more systematic studies by varying parameters
  - Sometimes allows to disentangle things from different observables
- \( \det (\mathcal{D}_G)_{QCD} \): Sea quarks
- \( (\mathcal{D}_G^{u})^{-1}(x, 0)(\mathcal{D}_G^{d})^{-1}(0, x) \): Valence Quarks
What is Partially Quenched?

Why not do this?

- It is not QCD as soon as Valence $\neq$ Sea
- not a Quantum Field Theory
- No unitarity
- No clusterdecomposition
- No CPT theorem
- No spin statistics theorem
What is Partially Quenched?

Do anyway

- Cheap computationally
- Allows extra studies
- Hope it works near real QCD case
- Is a well defined Euclidean statistical model
**ChPT and Lattice QCD**

Mesons = Quark Flow Valence + Quark Flow Sea + \cdots

Valence is easy to deal with in lattice QCD
Sea is very difficult

They can be treated separately: i.e. different quark masses
Partially Quenched ChPT (PQChPT)
Add ghost quarks: remove the unwanted free valence loops

Mesons = Quark Flow Valence + Quark Flow Valence + Quark Flow Sea + Quark Flow Ghost

Possible problem: $\text{QCD} \implies \text{ChPT}$ relies heavily on unitarity

Partially quenched: at least one dynamical sea quark

$\implies \Phi_0$ is heavy: remove from PQChPT

Symmetry group becomes $SU(n_v + n_s|n_v) \times SU(n_v + n_s|n_v)$

(approximately)
Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables.

Exceptions: baryons and Cayley-Hamilton relations.

So Luckily: can use the $n$ flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities.

Very important note: ChPT is a limit of PQChPT

$\leadsto$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

E.g. $L^r_1 = L^r_{0(3pq)}/2 + L^r_{1(3pq)}$
Subject started:
valence equal mass, 3 sea equal mass:
\[ m^2_{\pi^+} : \text{JB}, \text{Danielsson}, \text{Lähde}, \text{hep-lat/0406017} \]

Other mass combinations:
\[ F_{\pi^+} : \text{JB}, \text{Lähde}, \text{hep-lat/0501014} \]
\[ F_{\pi^+}, m^2_{\pi^+} \text{ two sea quarks: } \text{JB}, \text{Lähde}, \text{hep-lat/0506004} \]
\[ m^2_{\pi^+} : \text{JB}, \text{Danielsson}, \text{Lähde}, \text{hep-lat/0602003} \]

Neutral masses: \text{JB, Danielsson, hep-lat/0606017}

Actual Calculations:
\[ \text{heavy use of FORM Vermaseren} \]
\[ \text{use PQ without super } \Phi_0 \text{ in supersymmetric formalism} \]
\[ \text{Main problem: sheer size of the expressions} \]

Iso breaking from lattice data: \( \alpha \) and \( L \) extrapolations needed
Long Expressions

\[
\begin{align*}
\text{loops} & = 213 \\
& + 761 \\
& = 972
\end{align*}
\]

\[
\begin{align*}
A & = 2r + 1 \quad \text{R}\phantom{R} \quad R
\end{align*}
\]

\[
\begin{align*}
c & = 3 \\
& - 3 \quad \text{R}\phantom{R} \quad \text{R}\phantom{R} \quad \text{R}\phantom{R}
\end{align*}
\]

\[
\begin{align*}
A & = 8 \\
& = p + 1 \quad \text{R}\phantom{R} \quad q
\end{align*}
\]

\[
\begin{align*}
s & = 16 \\
& - 1 \quad \text{R}\phantom{R} \quad \text{R}\phantom{R}
\end{align*}
\]

\[
\begin{align*}
A & = 13 \\
& = p + 1 \quad \text{R}\phantom{R} \quad q
\end{align*}
\]

\[
\begin{align*}
A & = 32 \\
& - 3 \quad \text{R}\phantom{R} \quad \text{R}\phantom{R} \quad \text{R}\phantom{R}
\end{align*}
\]

\[
\begin{align*}
r & = 2 \\
& - 3 \quad \text{R}\phantom{R} \quad \text{R}\phantom{R} \quad \text{R}\phantom{R}
\end{align*}
\]

\[
\begin{align*}
r & = 2 \\
& - 3 \quad \text{R}\phantom{R} \quad \text{R}\phantom{R} \quad \text{R}\phantom{R}
\end{align*}
\]
plus several more pages
Why so long expressions

- Many different quark and meson masses ($\chi_{ij}$)
- Charged propagators: $-i \, G^c_{ij}(k) = \frac{\epsilon_j}{k^2 - \chi_{ij} + i\epsilon} \quad (i \neq j)$
- Neutral propagators: $G^m_{ij}(k) = G^c_{ij}(k) \, \delta_{ij} - \frac{1}{n_{\text{sea}}} \, G^q_{ij}(k)$
Why so long expressions

Many different quark and meson masses ($\chi_{ij}$)

Charged propagators: $-i \ G^c_{ij}(k) = \frac{\epsilon_j}{k^2 - \chi_{ij} + i\varepsilon} \ (i \neq j)$

Neutral propagators: $G^n_{ij}(k) = G^c_{ij}(k) \delta_{ij} - \frac{1}{n_{\text{sea}}} \ G^q_{ij}(k)$

$-i \ G^q_{ii}(k) = \frac{R^d_i}{(k^2 - \chi_i + i\varepsilon)^2} + \frac{R^c_i}{k^2 - \chi_i + i\varepsilon} + \frac{R^\pi_{iii}}{k^2 - \chi_\pi + i\varepsilon} + \frac{R^n_{\pi ii}}{k^2 - \chi_\eta + i\varepsilon}$

$R^i_{jkl} = R^z_{i456jkl}, \quad R^d_i = R^z_{i456\pi\eta},$

$R^c_i = R^i_{4\pi\eta} + R^i_{5\pi\eta} + R^i_{6\pi\eta} - R^i_{\pi\eta\eta} - R^i_{\pi\pi\eta}$

$R^z_{ab} = \chi_a - \chi_b, \quad R^z_{abc} = \frac{\chi_a - \chi_b}{\chi_a - \chi_c}, \quad R^z_{abedf} = \frac{(\chi_a - \chi_b)(\chi_a - \chi_c)(\chi_a - \chi_d)}{(\chi_a - \chi_e)(\chi_a - \chi_f)(\chi_a - \chi_g)}$
Why so long expressions

- Many different quark and meson masses ($\chi_{ij}$)
- Charged propagators: $-i\, G^c_{ij}(k) = \frac{\epsilon_j}{k^2 - \chi_{ij} + i\epsilon} \quad (i \neq j)$
- Neutral propagators: $G^n_{ij}(k) = G^c_{ij}(k) \delta_{ij} - \frac{1}{n_{\text{sea}}} \, G^q_{ij}(k)$

\[
-i\, G^q_{ii}(k) = \frac{R^d_i}{(k^2 - \chi_i + i\epsilon)^2} + \frac{R^e_i}{k^2 - \chi_i + i\epsilon} + \frac{R^{\pi ii}_n}{k^2 - \chi_\pi + i\epsilon} + \frac{R^{\eta ii}_n}{k^2 - \chi_{\eta} + i\epsilon}
\]

\[
R^i_{jkl} = R^z_{i456jkl}, \quad R^d_i = R^z_{i456\pi\eta},
\]

\[
R^c_i = R^i_{4\pi\eta} + R^i_{5\pi\eta} + R^i_{6\pi\eta} - R^i_{\pi\eta\eta} - R^i_{\pi\pi\eta}
\]

\[
R^z_{ab} = \chi_a - \chi_b, \quad R^z_{abc} = \frac{\chi_a - \chi_b}{\chi_a - \chi_c}, \quad R^z_{abcd} = \frac{(\chi_a - \chi_b)(\chi_a - \chi_c)}{\chi_a - \chi_d}
\]

\[
R^z_{abcdefg} = \frac{(\chi_a - \chi_b)(\chi_a - \chi_c)(\chi_a - \chi_d)}{(\chi_a - \chi_e)(\chi_a - \chi_f)(\chi_a - \chi_g)}
\]

- Relations $\implies$ order of magnitude smaller
Double poles?

Think quark lines and add gluons everywhere

Full

Quenched

So no resummation at the quark level:
naively a double pole

Same follows from inverting the lowest order kinetic terms
Usual resummation from 1PI

\[ G_{ij}^n = G_{ij}^0 + G_{ik}^0 (-i) \Sigma_{kl} G_{lj}^0 + G_{ik}^0 (-i) \Sigma_{kl} G_{lm}^0 (-i) \Sigma_{mn} G_{nj}^0 + \cdots \]

\[ = G^0 (1 + i \Sigma G^0)^{-1} \]

can be done if \( \Sigma \) and \( G^0 \) diagonal: usual resummation

\[ \frac{1}{p^2 - m^2} \rightarrow \frac{1}{p^2 - m^2 + \Sigma(p^2)} \]

so works in the charged or off-diagonal or \( \bar{q}q' \) sector
PQChPT at Two Loop

Use lowest order mass squared: $\chi_i = 2B_0m_i = m_M^{2(0)}$

Remember: $\chi_i \approx 0.3 \text{ GeV}^2 \approx (550 \text{ MeV})^2 \sim \text{border ChPT}$

Relative corrections: mass$^2$  

decay constant
Neutral Masses

Full resummation done by JB, Danielsson to show how to get at the full propagator from one-particle-irreducible diagrams.
Earlier work: Sharpe-Shoresh-Bernard-Golterman
Double poles remain also after resummation.

Actually useful: residue of double pole allows to get at all LECs needed for the neutral masses
The eta mass from PQChPT

Expand around the double pole term

\[ G_{ij}^m = \frac{-iZD}{(p^2 - M_{ch}^2)^2} \]

Measure on the lattice via (Sharpe-Shoresh)

\[ R_0(t) = \frac{\langle \pi_{ii}(t, \vec{p} = 0)\pi_{jj}(x = 0) \rangle}{\langle \pi_{ij}(t, \vec{p} = 0)\pi_{ji}(x = 0) \rangle} \]

\[ R_0(t \to \infty) = \frac{Dt}{2M_{ij}} \]
The eta mass from PQChPT

Expand around the double pole term

\[ G_{ij}^m = \frac{-i Z D}{(p^2 - M_{ch}^2)^2} + \cdots . \]

Measure on the lattice via (Sharpe-Shoresh)

\[ R_0(t) \equiv \frac{\langle \pi_{ii}(t, \vec{p} = 0) \pi_{jj}(x = 0) \rangle}{\langle \pi_{ij}(t, \vec{p} = 0) \pi_{ji}(x = 0) \rangle} \quad R_0(t \to \infty) = \frac{D t}{2 M_{ij}} , \]

From \( D \) can get \( L_7^r \) (Sharpe-Shoresh) and has been extended\n\[ \text{JB,Danielsson} \] to NNLO as well that all LECs relevant for \( m_{\eta}^2 \) can be had from \( D \).
A ChPT two-loop timeline: Prehistory

Review paper on Two-Loops: JB, LU TP 06-16
hep-ph/0604043

Dispersive Calculation of the nonpolynomial part in $q^2, s, t, u$

- Gasser-Meißner: $F_V, F_S$: 1991 numerical
- Knecht-Moussallam-Stern-Fuchs: $\pi\pi$: 1995 analytical
- Colangelo-Finkemeier-Urech: $F_V, F_S$: 1996 analytical

Roughly Eurodaphne I
A ChPT two-loop timeline: One mass

Two-Loop Two-Flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \rightarrow \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \rightarrow \pi^+\pi^-, F_\pi, m_\pi$: 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi, F_\pi, m_\pi$: 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t), F_{S\pi}$: 1998
- JB-Talavera: $\pi \rightarrow \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \rightarrow \pi^0\pi^0, \gamma\gamma \rightarrow \pi^+\pi^-$: 2005-2006

Roughly Eurodaphne II
A ChPT two-loop timeline: $\geq 3$ mass

Two-Loops Three flavours

- $\Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVK}$ Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
- $\Pi_{VV\rho\omega}$ Maltman
- $\Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta$ Kambor, Golowich; Amorós, JB, Talavera
- $\Pi_{SS}$ Moussallam $L^r_4, L^r_6$
- $\Pi_{VVK}, \Pi_{AAK}, F_K, m_K$ Amorós, JB, Talavera
- $K_{\ell 4}, \langle \bar{q}q \rangle$ Amorós, JB, Talavera $L^r_1, L^r_2, L^r_3$
- $F_M, m_M, \langle \bar{q}q \rangle (m_u \neq m_d)$ Amorós, JB, Talavera $L^r_{5,7,8}, m_u/m_d$

Roughly Eurodaphne II
A ChPT two-loop timeline: $\geq 3$ mass

Two-Loops Three flavours

- $F_{V\pi}$, $F_{VK^+}$, $F_{VK^0}$
  - Post, Schilcher; JB, Talavera
  - $L_9$

- $K_{\ell3}$
  - Post, Schilcher; JB, Talavera
  - $V_{us}$

- $F_{S\pi}$, $F_{SK}$ (includes $\sigma$-terms)
  - JB, Dhonte
  - $L_4^r$, $L_6^r$

- $K, \pi \rightarrow \ell\nu\gamma$
  - Geng, Ho, Wu
  - $L_{10}^r$

- $\pi\pi$  
  - JB, Dhonte, Talavera

- $\pi K$  
  - JB, Dhonte, Talavera

Roughly Euridice
A ChPT two-loop timeline: \( \geq 3 \) mass

Two-Loops Three flavours

Coming:

- \( \eta \rightarrow 3\pi \): status finite for 2/3 of collaboration
- \( K_{\ell 3} \) iso: status: diagrams programmed, more time per day needed
- Finite Volume: being thought about
- Finite Volume Two-Flavour: Colangelo-Dürr-Haefeli

Flavianet
A two-loop timeline: \[ \Rightarrow \Rightarrow \Rightarrow 3 \text{ mass} \]

see first part of talk
General Strategy and some comments

- Find enough inputs from experiment
- $C^r_i$: Ecker’s talk
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not $\sigma$)
  - quark masses: which scalars? unrealistically large estimates
- in $p^6$ physical or lowest order masses: thresholds in right place requires physical
General Strategy and some comments

Inputs:
- $K_{l4}$: $F(0)$, $G(0)$, $\lambda$
- $m_{\pi^0}^2$, $m_\eta^2$, $m_{K^+}^2$, $m_{K^0}^2$
- $F_{\pi^+}$
- $F_{K^+}/F_{\pi^+}$
- $m_s/\hat{m}$
- $L_4^r$, $L_6^r$
- $C_i^r$ from single resonance approximation

$E865$ BNL
em with Dashen violation

$\hat{m} = (m_u + m_d)/2$

24 (26)
## General Strategy and some comments

<table>
<thead>
<tr>
<th></th>
<th>fit 10</th>
<th>same $p^4$</th>
<th>fit B</th>
<th>fit D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3 L_1^r$</td>
<td>0.43 ± 0.12</td>
<td>0.38</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>$10^3 L_2^r$</td>
<td>0.73 ± 0.12</td>
<td>1.59</td>
<td>0.60</td>
<td>0.69</td>
</tr>
<tr>
<td>$10^3 L_3^r$</td>
<td>-2.53 ± 0.37</td>
<td>-2.91</td>
<td>-2.31</td>
<td>-2.33</td>
</tr>
<tr>
<td>$10^3 L_4^r$</td>
<td>$\equiv$ 0</td>
<td>$\equiv$ 0</td>
<td>$\equiv$ 0.5</td>
<td>$\equiv$ 0.2</td>
</tr>
<tr>
<td>$10^3 L_5^r$</td>
<td>0.97 ± 0.11</td>
<td>1.46</td>
<td>0.82</td>
<td>0.88</td>
</tr>
<tr>
<td>$10^3 L_6^r$</td>
<td>$\equiv$ 0</td>
<td>$\equiv$ 0</td>
<td>$\equiv$ 0.1</td>
<td>$\equiv$ 0</td>
</tr>
<tr>
<td>$10^3 L_7^r$</td>
<td>-0.31 ± 0.14</td>
<td>-0.49</td>
<td>-0.26</td>
<td>-0.28</td>
</tr>
<tr>
<td>$10^3 L_8^r$</td>
<td>0.60 ± 0.18</td>
<td>1.00</td>
<td>0.50</td>
<td>0.54</td>
</tr>
</tbody>
</table>

- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying $C_i^r$ factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6 \times 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and $\pi K$ thresholds
**General Strategy and some comments**

<table>
<thead>
<tr>
<th></th>
<th>fit t 10</th>
<th>same $p^4$</th>
<th>fit t B</th>
<th>fit t D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2B_0\hat{m}/m_\pi^2$</td>
<td>0.736</td>
<td>0.991</td>
<td>1.129</td>
<td>0.958</td>
</tr>
<tr>
<td>$m_\pi^2: p^4, p^6$</td>
<td>0.006, 0.258</td>
<td>0.009, $\equiv 0$</td>
<td>$-0.138, 0.009$</td>
<td>$-0.091, 0.133$</td>
</tr>
<tr>
<td>$m_K^2: p^4, p^6$</td>
<td>0.007, 0.306</td>
<td>0.075, $\equiv 0$</td>
<td>$-0.149, 0.094$</td>
<td>$-0.096, 0.201$</td>
</tr>
<tr>
<td>$m_\eta^2: p^4, p^6$</td>
<td>$-0.052, 0.318$</td>
<td>0.013, $\equiv 0$</td>
<td>$-0.197, 0.073$</td>
<td>$-0.151, 0.197$</td>
</tr>
<tr>
<td>$m_u/m_d$</td>
<td>0.45 ± 0.05</td>
<td>0.52</td>
<td>0.52</td>
<td>0.50</td>
</tr>
<tr>
<td>$F_0$ [MeV]</td>
<td>87.7</td>
<td>81.1</td>
<td>70.4</td>
<td>80.4</td>
</tr>
<tr>
<td>$F_K/F_\pi: p^4, p^6$</td>
<td>0.169, 0.051</td>
<td>0.22, $\equiv 0$</td>
<td>0.153, 0.067</td>
<td>0.159, 0.061</td>
</tr>
</tbody>
</table>

⚠️ $m_u = 0$ always very far from the fits

⚠️ $F_0$: pion decay constant in the chiral limit
\[ a_0^0 = 0.220 \pm 0.005, \quad a_0^2 = -0.0444 \pm 0.0010 \]

Colangelo, Gasser, Leutwyler

\[ a_0^0 = 0.159, \quad a_0^2 = -0.0454 \text{ at order } p^2 \]
\[ \pi \pi \text{ and } \pi K \]

Preferred region: fit D: \(10^3 L_4^r \approx 0.2, \ 10^3 L_6^r \approx 0.0\)

General fitting needs more work and systematic studies
Conclusions

- 3 flavour ChPT at 2 loops
  - many calculations done
  - things seem to work but convergence is fairly slow
  - “kinematical” and “vector” $C_i^r$ seem to be OK
  - $L_4^r, L_6^r$ nonzero but reasonable for large $N_c$
  - $\eta \to 3\pi$, isobreaking in $K_{\ell 3}$: parts done

- PQChPT at 2 loops: subject just beginning
  - $\eta \to 3\pi$: isospin breaking part needed to $p^6$, compare with dispersive (see $K_{\ell 4}$), status: infinities have canceled

- Need to redo fit with all newer data (but then many people not in this audience still use old GL 1985 value of LECs)