PARTIALLY QUENCHED AND THREE FLAVOURS AT TWO LOOPS

Johan Bijnens

Lund University

bijnens@thep.lu.se

http://www.thep.lu.se/∼bijnens

Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- Chiral Perturbation Theory
- Lagrangians
- What is partially quenched?
- PQChPT and some problems
- Some results of PQChPT at Two-loops
- A Two-loop Timeline
- Some comments on and results from ChPT at two loops
- Conclusions

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\quad \mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

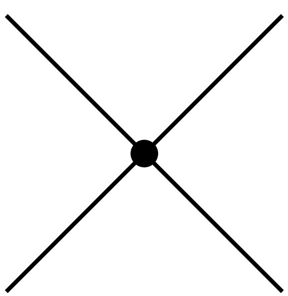
Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$

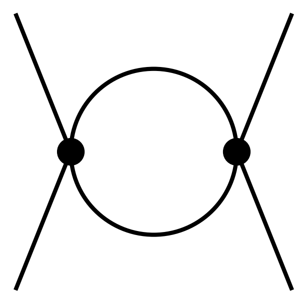
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

Power counting in momenta:

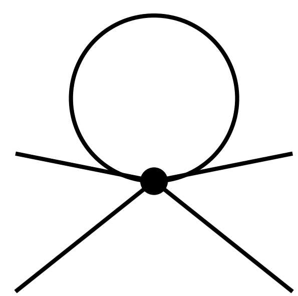


p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$



$(p^2)\,(1/p^2)\,p^4 = p^4$

$\int d^4p$ $\quad p^4$

Two Loop: Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

Eurodaphne II/Euridice (PQChPT)

Constructing Lagrangians

Easy part:

- Construct a complete set of needed basic objects
- Often useful to make everything transform under H alone (conserved part)
- Construct all terms up to the order you want

Tricky part: finding a minimal basis

- Field redefinitions or equations of motion
- Partial integration
- Cayley-Hamilton identities
- Bianchi identities, $\det\chi$ is also chirally invariant

Final check: can I determine all from experiment?

Constructing Lagrangians

Infinities:
the general divergence structure can be derived using heat kernel methods and/or background field methods.

Very useful for checks on calculations

What is Partially Quenched?

In Lattice gauge theory one calculates

$$\langle 0|(\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)|\rangle$$

$$= \frac{\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y \mathcal{L}_{\mathrm{QCD}}}}{\int [dq][d\overline{q}][dG]\overline{d}\gamma_5 u(0)e^{i\int d^4y \mathcal{L}_{\mathrm{QCD}}}}$$

for Euclidean separations x

Integrals also performed after rotation to Euclidean

(note that I use Minkowski notation throughout)

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)\,\det\,(\not{D}_G)_{\mathrm{QCD}}$$

What is Partially Quenched?

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$\boxed{\int [dG] \text{ done via importance sampling}}$

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$$\int [dG] e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}} (\not{D}^u_G)^{-1}(x,0)(\not{D}^d_G)^{-1}(0,x) \det(\not{D}_G)_{\mathrm{QCD}}$$

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- Quenched: get distribution from $e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}$ only

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$\boxed{\int [dG] \text{ done via importance sampling}}$

- Quenched: get distribution from $e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}$ only
- Unquenched: include $\det(\not{D}_G)_{\mathrm{QCD}}$ VERY expensive

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)\,\det(\not{D}_G)_{\mathrm{QCD}}$$

$\int [dG]$ done via importance sampling

- Quenched: get distribution from $e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}$ only
- Unquenched: include $\det(\not{D}_G)_{\mathrm{QCD}}$ VERY expensive
- Partially quenched: $(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)$
  DIFFERENT Quarks then in $\det(\not{D}_G)_{\mathrm{QCD}}$

What is Partially Quenched?

Why do this?

- Is not Quenched: Real QCD is continuous limit from Partially Quenched
- More handles to turn:
  - Allows more systematic studies by varying parameters
  - Sometimes allows to disentangle things from different observables
- $\det\left(\not{D}_G\right)_{\mathrm{QCD}}$: Sea quarks
- $(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)$: Valence Quarks

What is Partially Quenched?

Why not do this?

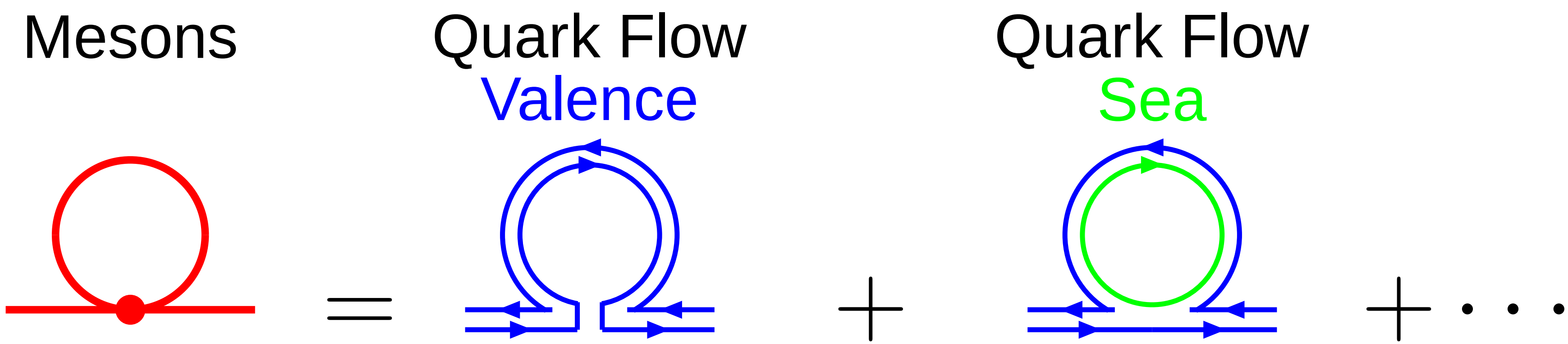
- It is not QCD as soon as Valence$\neq$Sea
- not a Quantum Field Theory
- No unitarity
- No clusterdecomposition
- No CPT theorem
- No spin statistics theorem

What is Partially Quenched?

Do anyway

- Cheap computationally
- Allows extra studies
- Hope it works near real QCD case
- Is a well defined Euclidean statistical model

ChPT and Lattice QCD



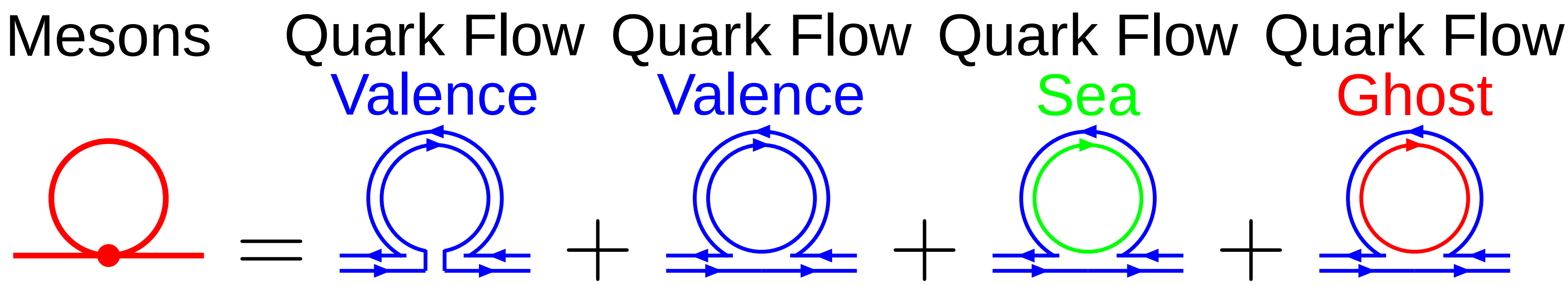
Valence is easy to deal with in lattice QCD
Sea is very difficult

They can be treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

PQChPT at Two Loops: General

Add ghost quarks: remove the unwanted free valence loops



Possible problem: QCD $\Longrightarrow$ ChPT relies heavily on unitarity

Partially quenched: at least one dynamical sea quark
$\Longrightarrow$ Φ_0 is heavy: remove from PQChPT

Symmetry group becomes $SU(n_v + n_s|n_v) \times SU(n_v + n_s|n_v)$ (approximately)

PQChPT at Two Loops: General

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT
$\Longrightarrow$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

E.g. $L_1^r = L_0^{r(3pq)}/2 + L_1^{r(3pq)}$

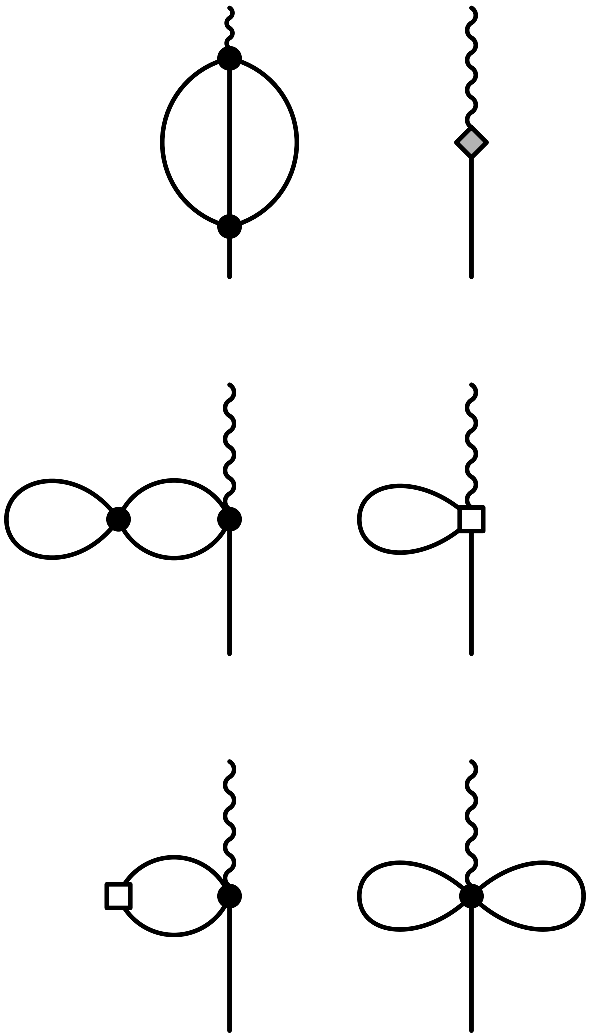
PQChPT at Two Loop

Subject started:
valence equal mass, 3 sea equal mass:
$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0406017
Other mass combinations:
F_{π^+}: JB,Lähde, hep-lat/0501014
F_{π^+}, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004
$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0602003
Neutral masses: JB,Danielsson, hep-lat/0606017

Actual Calculations:
- ➠ heavy use of FORM Vermaseren
- ➠ use PQ without super Φ_0 in super-symmetric formalism
- ➠ Main problem: sheer size of the expressions

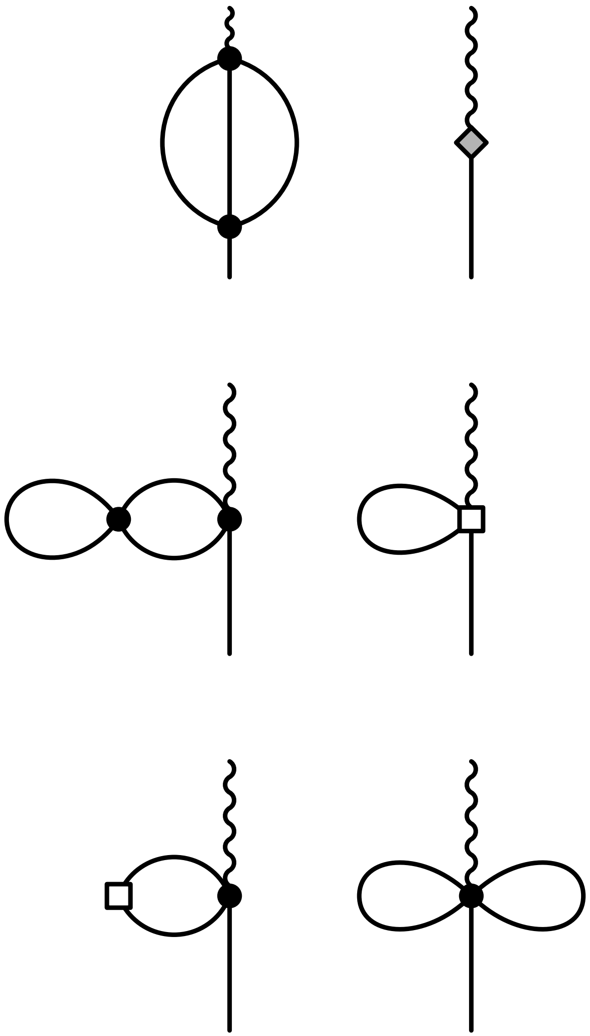
Iso breaking from lattice data: a and L extrapolations needed

Long Expressions



$\Longrightarrow$

Long Expressions



$\Longrightarrow$

$$
\begin{aligned}
\delta^{(6)22}_{\rm loops} = {} & \pi_{16}\, L^r_0\, \left[4/9\,\chi_\eta\chi_4 - 1/2\,\chi_1\chi_3 + \chi_{13}^2 - 13/3\,\bar\chi_1\chi_{13} - 35/18\,\bar\chi_2\right] \,-\, 2\,\pi_{16}\, L^r_1\,\chi_{13}^2 \\
- {} & \pi_{16}\, L^r_2\, \left[11/3\,\chi_\eta\chi_4 + \chi_{13}^2 + 13/3\,\bar\chi_2\right] + \pi_{16}\, L^r_3\, \left[4/9\,\chi_\eta\chi_4 - 7/12\,\chi_1\chi_3 + 11/6\,\chi_{13}^2 - 17/6\,\bar\chi_1\chi_{13} - 43/36\,\bar\chi_2\right] \\
+ {} & \pi_{16}^2\, \left[-15/64\,\chi_\eta\chi_4 - 59/384\,\chi_1\chi_3 + 65/384\,\chi_{13}^2 - 1/2\,\bar\chi_1\chi_{13} - 43/128\,\bar\chi_2\right] \,-\, 48\, L^r_4 L^r_5\,\bar\chi_1\chi_{13} \,-\, 72\, L^{r2}_4\,\bar\chi_1^2 \\
- {} & 8\, L^{r2}_5\,\chi_{13}^2 \,+\, \bar A(\chi_p)\,\pi_{16}\, \left[-1/24\,\chi_p + 1/48\,\bar\chi_1 - 1/8\,\bar\chi_1\, R^p_{q\eta} + 1/16\,\bar\chi_1\, R^c_p - 1/48\, R^p_{q\eta}\,\chi_p - 1/16\, R^p_{q\eta}\,\chi_q\right. \\
+ {} & \left. 1/48\, R^\eta_{pp}\,\chi_\eta + 1/16\, R^c_p\,\chi_{13}\right] \,+\, \bar A(\chi_p)\, L^r_0\, \left[8/3\, R^p_{q\eta}\,\chi_p + 2/3\, R^c_p\,\chi_p + 2/3\, R^d_p\right] \,+\, \bar A(\chi_p)\, L^r_3\, \left[2/3\, R^p_{q\eta}\,\chi_p\right. \\
+ {} & \left. 5/3\, R^c_p\,\chi_p + 5/3\, R^d_p\right] \,+\, \bar A(\chi_p)\, L^r_4\, \left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0} - 2\,\bar\chi_1\, R^p_{q\eta} + 3\,\bar\chi_1\, R^c_p\right] \,+\, \bar A(\chi_p)\, L^r_5\, \left[-2/3\,\bar\chi^{pp}_{\eta\eta 1} - R^p_{q\eta}\,\chi_p\right. \\
+ {} & \left. 1/3\, R^p_{q\eta}\,\chi_q + 1/2\, R^c_p\,\chi_p - 1/6\, R^c_p\,\chi_q\right] \,+\, \bar A(\chi_p)^2\, \left[1/16 + 1/72\,(R^p_{q\eta})^2 - 1/72\, R^p_{q\eta} R^c_p + 1/288\,(R^c_p)^2\right] \\
+ {} & \bar A(\chi_p)\bar A(\chi_{ps})\, \left[-1/36\, R^p_{q\eta} - 5/72\, R^p_{s\eta} + 7/144\, R^c_p\right] \,-\, \bar A(\chi_p)\bar A(\chi_{qs})\, \left[1/36\, R^p_{q\eta} + 1/24\, R^p_{s\eta} + 1/48\, R^c_p\right] \\
+ {} & \bar A(\chi_p)\bar A(\chi_\eta)\, \left[-1/72\, R^p_{q\eta} R^v_{\eta 13} + 1/144\, R^c_p R^v_{\eta 13}\right] \,+\, 1/8\,\bar A(\chi_p)\bar A(\chi_{13}) \,+\, 1/12\,\bar A(\chi_p)\bar A(\chi_{46})\, R^\eta_{pp} \\
+ {} & \bar A(\chi_p)\bar B(\chi_p,\chi_p;0)\, \left[1/4\,\chi_p - 1/18\, R^p_{q\eta} R^c_p\,\chi_p - 1/72\, R^p_{q\eta} R^d_p + 1/18\,(R^c_p)^2\,\chi_p + 1/144\, R^c_p R^d_p\right] \\
+ {} & \bar A(\chi_p)\bar B(\chi_p,\chi_\eta;0)\, \left[1/18\, R^\eta_{pp} R^c_p\,\chi_p - 1/18\, R^\eta_{13} R^c_p\,\chi_p\right] \,+\, \bar A(\chi_p)\bar B(\chi_q,\chi_q;0)\, \left[-1/72\, R^p_{q\eta} R^d_q + 1/144\, R^c_p R^d_q\right] \\
- {} & 1/12\,\bar A(\chi_p)\bar B(\chi_{ps},\chi_{ps};0)\, R^p_{s\eta}\,\chi_{ps} \,-\, 1/18\,\bar A(\chi_p)\bar B(\chi_1,\chi_3;0)\, R^q_{p\eta} R^c_p\,\chi_p \\
+ {} & 1/18\,\bar A(\chi_p)\bar C(\chi_p,\chi_p,\chi_p;0)\, R^c_p R^d_p\,\chi_p \,+\, \bar A(\chi_p;\varepsilon)\,\pi_{16}\, \left[1/8\,\bar\chi_1 R^p_{q\eta} - 1/16\bar\chi_1\, R^c_p - 1/16\, R^c_p\,\chi_p - 1/16\, R^d_p\right] \\
+ {} & \bar A(\chi_{ps})\,\pi_{16}\, \left[1/16\,\chi_{ps} - 3/16\,\chi_{qs} - 3/16\,\bar\chi_1\right] \,-\, 2\,\bar A(\chi_{ps})\, L^r_0\,\chi_{ps} \,-\, 5\,\bar A(\chi_{ps})\, L^r_3\,\chi_{ps} \,-\, 3\,\bar A(\chi_{ps})\, L^r_4\,\bar\chi_1 \\
+ {} & \bar A(\chi_{ps})\, L^r_5\,\chi_{13} \,+\, \bar A(\chi_{ps})\bar A(\chi_\eta)\, \left[7/144\, R^\eta_{pp} - 5/72\, R^\eta_{ps} - 1/48\, R^\eta_{qq} + 5/72\, R^\eta_{qs} - 1/36\, R^\eta_{13}\right] \\
+ {} & \bar A(\chi_{ps})\bar B(\chi_p,\chi_p;0)\, \left[1/24\, R^p_{s\eta}\,\chi_p - 5/24\, R^p_{s\eta}\,\chi_{ps}\right] \,+\, \bar A(\chi_{ps})\bar B(\chi_p,\chi_\eta;0)\, \left[-1/18\, R^\eta_{ps} R^z_{qp\eta}\,\chi_p\right. \\
- {} & \left. 1/9\, R^\eta_{ps} R^z_{qp\eta}\,\chi_{ps}\right] \,-\, 1/48\,\bar A(\chi_{ps})\bar B(\chi_q,\chi_q;0)\, R^d_q \,+\, 1/18\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0)\, R^q_{s\eta}\,\chi_s \\
+ {} & 1/9\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0,k)\, R^q_{s\eta} \,+\, 3/16\,\bar A(\chi_{ps};\varepsilon)\,\pi_{16}\, \left[\chi_s + \bar\chi_1\right] \,-\, 1/8\,\bar A(\chi_{p4})^2 \,-\, 1/8\,\bar A(\chi_{p4})\bar A(\chi_{p6}) \\
+ {} & 1/8\,\bar A(\chi_{p4})\bar A(\chi_{q6}) \,-\, 1/32\,\bar A(\chi_{p6})^2 \,+\, \bar A(\chi_\eta)\,\pi_{16}\, \left[1/16\,\bar\chi_1\, R^v_{\eta 13} - 1/48\, R^v_{\eta 13}\,\chi_\eta + 1/16\, R^v_{\eta 13}\,\chi_{13}\right] \\
+ {} & \bar A(\chi_\eta)\, L^r_0\, \left[4R^\eta_{13}\,\chi_\eta + 2/3\, R^v_{\eta 13}\,\chi_\eta\right] \,-\, 8\,\bar A(\chi_\eta)\, L^r_1\,\chi_\eta \,-\, 2\,\bar A(\chi_\eta)\, L^r_2\,\chi_\eta \,+\, \bar A(\chi_\eta)\, L^r_3\, \left[4R^\eta_{13}\,\chi_\eta + 5/3\, R^v_{\eta 13}\chi_\eta\right] \\
+ {} & \bar A(\chi_\eta)\, L^r_4\, \left[4\,\chi_\eta + \bar\chi_1\, R^v_{\eta 13}\right] \,-\, \bar A(\chi_\eta)\, L^r_5\, \left[1/6\, R^\eta_{pp}\,\chi_q + R^\eta_{13}\,\chi_{13} + 1/6\, R^v_{\eta 13}\,\chi_\eta\right] \,+\, 1/288\,\bar A(\chi_\eta)^2\,(R^v_{\eta 13})^2 \\
+ {} & 1/12\,\bar A(\chi_\eta)\bar A(\chi_{46})\, R^v_{\eta 13} \,+\, \bar A(\chi_\eta)\bar B(\chi_p,\chi_p;0)\, \left[-1/36\,\bar\chi^{pp\eta}_{\eta\eta 1} - 1/18\, R^p_{q\eta} R^\eta_{pp}\,\chi_p + 1/18\, R^\eta_{pp} R^c_p\,\chi_p\right. \\
+ {} & \left. 1/144\, R^d_p R^v_{\eta 13}\right] \,+\, \bar A(\chi_\eta)\bar B(\chi_p,\chi_\eta;0)\, \left[-1/18\,\bar\chi^{\eta p\eta}_{p\eta 1} + 1/18\,\bar\chi^{\eta p\eta}_{q\eta 1} + 1/18\,(R^\eta_{pp})^2 R^z_{qp\eta}\,\chi_p\right] \\
- {} & 1/12\,\bar A(\chi_\eta)\bar B(\chi_{ps},\chi_{ps};0)\, R^\eta_{ps}\,\chi_{ps} \,-\, \bar A(\chi_\eta)\bar B(\chi_\eta,\chi_\eta;0)\, \left[1/216\, R^v_{\eta 13}\,\chi_4 + 1/27\, R^v_{\eta 13}\,\chi_6\right] \\
- {} & 1/18\,\bar A(\chi_\eta)\bar B(\chi_1,\chi_3;0)\, R^1_{\eta\eta} R^3_{\eta\eta}\,\chi_\eta \,+\, 1/18\,\bar A(\chi_\eta)\bar C(\chi_p,\chi_p,\chi_p;0)\, R^\eta_{pp} R^d_p\,\chi_p \,+\, \bar A(\chi_\eta;\varepsilon)\,\pi_{16}\, \left[1/8\,\chi_\eta\right. \\
- {} & \left. 1/16\,\bar\chi_1\, R^v_{\eta 13} - 1/8\, R^\eta_{13}\,\chi_\eta - 1/16\, R^v_{\eta 13}\,\chi_\eta\right] \,+\, \bar A(\chi_1)\bar A(\chi_3)\, \left[-1/72\, R^p_{q\eta} R^c_q + 1/36\, R^1_{3\eta} R^3_{1\eta} + 1/144\, R^c_1 R^c_3\right] \\
- {} & 4\,\bar A(\chi_{13})\, L^r_1\,\chi_{13} \,-\, 10\,\bar A(\chi_{13})\, L^r_2\,\chi_{13} \,+\, 1/8\,\bar A(\chi_{13})^2 \,-\, 1/2\,\bar A(\chi_{13})\bar B(\chi_1,\chi_3;0,k) \\
+ {} & 1/4\,\bar A(\chi_{13};\varepsilon)\,\pi_{16}\,\chi_{13} \,+\, 1/4\,\bar A(\chi_{14})\bar A(\chi_{34}) \,+\, 1/16\,\bar A(\chi_{16})\bar A(\chi_{36}) \,-\, 24\,\bar A(\chi_4)\, L^r_1\,\chi_4 \,-\, 6\,\bar A(\chi_4)\, L^r_2\,\chi_4 \\
+ {} & 12\,\bar A(\chi_4)\, L^r_4\,\chi_4 \,+\, 1/12\,\bar A(\chi_4)\bar B(\chi_p,\chi_p;0)\,(R^p_{4\eta})^2\,\chi_4 \,+\, 1/6\,\bar A(\chi_4)\bar B(\chi_p,\chi_\eta;0)\, \left[R^p_{4\eta} R^\eta_{p4}\,\chi_4 - R^p_{4\eta} R^\eta_{q4}\,\chi_4\right] \\
- {} & 1/24\,\bar A(\chi_4)\bar B(\chi_\eta,\chi_\eta;0)\, R^v_{\eta 13}\,\chi_4 \,-\, 1/6\,\bar A(\chi_4)\bar B(\chi_1,\chi_3;0)\, R^1_{4\eta} R^3_{4\eta}\,\chi_4 \,+\, 3/8\,\bar A(\chi_4;\varepsilon)\,\pi_{16}\,\chi_4 \\
- {} & 32\,\bar A(\chi_{46})\, L^r_1\,\chi_{46} \,-\, 8\,\bar A(\chi_{46})\, L^r_2\,\chi_{46} \,+\, 16\,\bar A(\chi_{46})\, L^r_4\,\chi_{46} \,+\, \bar A(\chi_{46})\bar B(\chi_p,\chi_p;0)\, \left[1/9\,\chi_{46} + 1/12\, R^\eta_{pp}\,\chi_p\right. \\
+ {} & \left. 1/36\, R^\eta_{pp}\,\chi_4 + 1/9\, R^\eta_{p4}\,\chi_6\right] \,+\, \bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0)\, \left[-1/18\, R^\eta_{pp}\,\chi_4 - 1/9\, R^\eta_{p4}\,\chi_6 + 1/9\, R^\eta_{q4}\,\chi_6 + 1/18\, R^\eta_{13}\,\chi_4\right] \\
- {} & 1/6\,\bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0,k)\, \left[R^\eta_{pp} - R^\eta_{13}\right] \,+\, 1/9\,\bar A(\chi_{46})\bar B(\chi_\eta,\chi_\eta;0)\, R^v_{\eta 13}\,\chi_{46} \,-\, \bar A(\chi_{46})\bar B(\chi_1,\chi_3;0)\, \left[2/9\,\chi_{46}\right. \\
+ {} & \left. 1/9\, R^\eta_{p4}\,\chi_6 + 1/18\, R^\eta_{13}\,\chi_4\right] \,-\, 1/6\,\bar A(\chi_{46})\bar B(\chi_1,\chi_3;0,k)\, R^\eta_{13} \,+\, 1/2\,\bar A(\chi_{46};\varepsilon)\,\pi_{16}\,\chi_{46} \\
+ {} & \bar B(\chi_p,\chi_p;0)\,\pi_{16}\, \left[1/16\,\bar\chi_1\, R^d_p + 1/96\, R^d_p\,\chi_p + 1/32\, R^d_p\,\chi_q\right] \,+\, 2/3\,\bar B(\chi_p,\chi_p;0)\, L^r_0\, R^d_p\,\chi_p \\
+ {} & 5/3\,\bar B(\chi_p,\chi_p;0)\, L^r_3\, R^d_p\,\chi_p \,+\, \bar B(\chi_p,\chi_p;0)\, L^r_4\, \left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0}\chi_p - 4\,\bar\chi_1 R^p_{q\eta}\,\chi_p + 4\,\bar\chi_1 R^c_p\,\chi_p + 3\,\bar\chi_1 R^d_p\right] \\
+ {} & \bar B(\chi_p,\chi_p;0)\, L^r_5\, \left[-2/3\bar\chi^{pp}_{\eta\eta 1}\chi_p - 4/3\, R^p_{q\eta}\,\chi_p^2 + 4/3\, R^c_p\,\chi_p^2 + 1/2\, R^d_p\,\chi_p - 1/6\, R^d_p\,\chi_q\right] \\
+ {} & \bar B(\chi_p,\chi_p;0)\, L^r_6\, \left[4\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 1} + 8\,\bar\chi_1 R^p_{q\eta}\,\chi_p - 8\,\bar\chi_1 R^c_p\,\chi_p\right] \,+\, 4\,\bar B(\chi_p,\chi_p;0)\, L^r_7\,(R^d_p)^2 \\
+ {} & \bar B(\chi_p,\chi_p;0)\, L^r_8\, \left[4/3\,\bar\chi^{pp}_{\eta\eta 2} + 8/3\, R^p_{q\eta}\,\chi_p^2 - 8/3\, R^c_p\,\chi_p^2\right] \,+\, \bar B(\chi_p,\chi_p;0)^2\, \left[-1/18\, R^p_{q\eta} R^d_p\,\chi_p + 1/18\, R^c_p R^d_p\,\chi_p\right. \\
+ {} & \left. 1/288\,(R^d_p)^2\right] \,+\, 1/18\,\bar B(\chi_p,\chi_p;0)\bar B(\chi_p,\chi_\eta;0)\, \left[R^\eta_{pp} R^d_p\,\chi_p - R^\eta_{13} R^d_p\,\chi_p\right]
\end{aligned}
$$

plus several more pages

Why so long expressions

- Many different quark and meson masses (χ_{ij})
- Charged propagators: $-i\,G^c_{ij}(k) = \frac{\epsilon_j}{k^2-\chi_{ij}+i\varepsilon} \quad (i \neq j)$
- Neutral propagators: $G^n_{ij}(k) = G^c_{ij}(k)\,\delta_{ij} - \frac{1}{n_{\mathrm{sea}}}\,G^q_{ij}(k)$

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$$R^i_{jkl} = R^z_{i456jkl}, \qquad R^d_i = R^z_{i456\pi\eta},$$

$$R^c_i = R^i_{4\pi\eta} + R^i_{5\pi\eta} + R^i_{6\pi\eta} - R^i_{\pi\eta\eta} - R^i_{\pi\pi\eta}$$

$$R^z_{ab} = \chi_a - \chi_b, \qquad R^z_{abc} = \frac{\chi_a-\chi_b}{\chi_a-\chi_c}, \qquad R^z_{abcd} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)}{\chi_a-\chi_d}$$

$$R^z_{abcdefg} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)(\chi_a-\chi_d)}{(\chi_a-\chi_e)(\chi_a-\chi_f)(\chi_a-\chi_g)}$$

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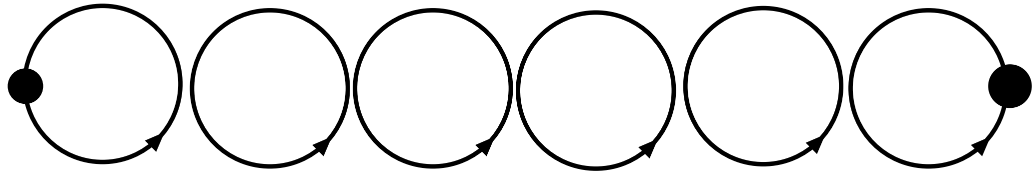
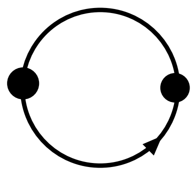
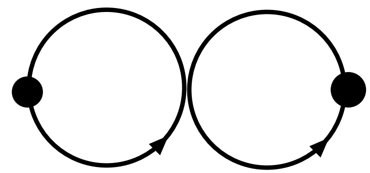
- Relations $\Longrightarrow$ order of magnitude smaller

Double poles ?

Think quark lines and add gluons everywhere

Full 

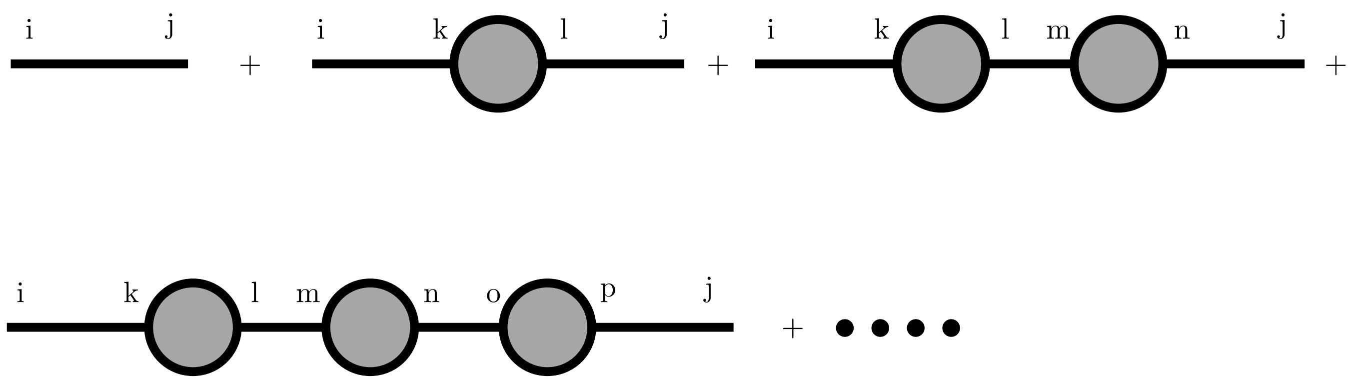
Quenched  

So no resummation at the quark level:

naively a double pole

Same follows from inverting the lowest order kinetic terms

Usual resummation from 1PI



$$G^n_{ij} = G^0_{ij} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lj} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lm}(-i)\Sigma_{mn}G^0_{nj} + \cdots$$
$$= G^0(1 + i\Sigma G^0)^{-1}$$

can be done if Σ and G^0 diagonal: usual resummation

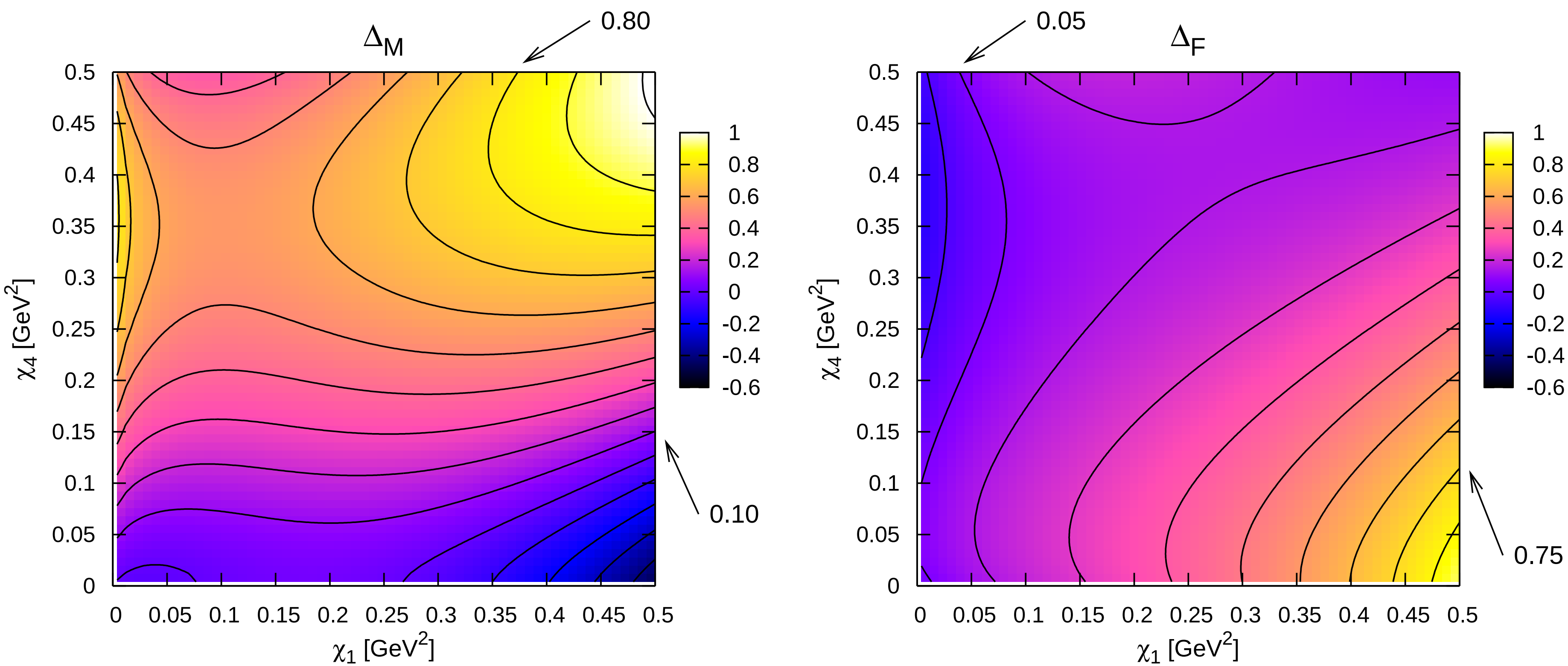
$$\frac{1}{p^2 - m^2} \to \frac{1}{p^2 - m^2 + \Sigma(p^2)}$$

so works in the charged or off-diagonal or $\overline{q}q'$ sector

PQChPT at Two Loop

Use lowest order mass squared: $\chi_i = 2B_0 m_i = m_M^{2(0)}$

Remember: $\chi_i \approx 0.3\ GeV^2 \approx (550\ MeV)^2 \sim$ border ChPT



Relative corrections: mass^2 decay constant

Neutral Masses

Full resummation done by JB,Danielsson to show how to get at the full propagator from one-particle-irreducible diagrams.
Earlier work: Sharpe-Shoresh-Bernard-Golterman
Double poles remain also after resummation.

Actually useful: residue of double pole allows to get at all LECs needed for the neutral masses

The eta mass from PQChPT

Expand around the double pole term

$$G^n_{ij} = \frac{-i\mathcal{Z}\mathcal{D}}{(p^2 - M^2_{ch})^2} + \cdots \, .$$

Measure on the lattice via (Sharpe-Shoresh)

$$R_0(t) \equiv \frac{\langle \pi_{ii}(t, \vec{p} = 0)\pi_{jj}(x = 0)\rangle}{\langle \pi_{ij}(t, \vec{p} = 0)\pi_{ji}(x = 0)\rangle} \qquad R_0(t \to \infty) = \frac{\mathcal{D}t}{2M_{ij}} \, ,$$

The eta mass from PQChPT

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From $\mathcal{D}$ can get L^r_7 (Sharpe-Shoresh) and has been extended JB,Danielsson to NNLO as well that all LECs relevant for m^2_η can be had from $\mathcal{D}$.

A ChPT two-loop timeline: Prehistory

Review paper on Two-Loops: JB, LU TP 06-16
hep-ph/0604043

Dispersive Calculation of the nonpolynomial part in q^2, s, t, u

- Gasser-Meißner: F_V, F_S: 1991 numerical
- Knecht-Moussallam-Stern-Fuchs: $\pi\pi$: 1995 analytical
- Colangelo-Finkemeier-Urech: F_V, F_S: 1996 analytical

Roughly Eurodaphne I

A ChPT two-loop timeline: One mass

Two-Loop Two-Flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \to \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \to \pi^+\pi^-$, F_π, m_π: 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi$, F_π, m_π: 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t)$, $F_{S\pi}$: 1998
- JB-Talavera: $\pi \to \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \to \pi^0\pi^0$, $\gamma\gamma \to \pi^+\pi^-$: 2005-2006

Roughly Eurodaphne II

A ChPT two-loop timeline: $\geq$3 mass

Two-Loops Three flavours

- $\Pi_{VV\pi}$, $\Pi_{VV\eta}$, Π_{VVK} Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
- $\Pi_{VV\rho\omega}$ Maltman
- $\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η Kambor, Golowich; Amorós, JB, Talavera
- Π_{SS} Moussallam $\boxed{L_4^r, L_6^r}$
- Π_{VVK}, Π_{AAK}, F_K, m_K Amorós, JB, Talavera
- $K_{\ell 4}$, $\langle \overline{q}q \rangle$ Amorós, JB, Talavera $\boxed{L_1^r, L_2^r, L_3^r}$
- F_M, m_M, $\langle \overline{q}q \rangle$ $(m_u \neq m_d)$ Amorós, JB, Talavera $\boxed{L_{5,7,8}^r, m_u/m_d}$

Roughly Eurodaphne II

A ChPT two-loop timeline: $\geq$3 mass

Two-Loops Three flavours

- $F_{V\pi}$, F_{VK^+}, F_{VK^0} — Post, Schilcher; JB, Talavera $\boxed{L_9^r}$
- $K_{\ell 3}$ — Post, Schilcher; JB, Talavera $\boxed{V_{us}}$
- $F_{S\pi}$, F_{SK} (includes σ-terms) — JB, Dhonte $\boxed{L_4^r, L_6^r}$
- $K, \pi \to \ell\nu\gamma$ — Geng, Ho, Wu $\boxed{L_{10}^r}$
- $\pi\pi$ — JB,Dhonte,Talavera
- πK — JB,Dhonte,Talavera

Roughly Euridice

A ChPT two-loop timeline: $\geq$3 mass

Two-Loops Three flavours
Coming:

- $\eta \to 3\pi$: status finite for 2/3 of collaboration
- $K_{\ell 3}$ iso: status: diagrams programmed, more time per day needed
- Finite Volume: being thought about
- Finite Volume Two-Flavour: Colangelo-Dürr-Haefeli

Flavianet

A two-loop timeline: $>>>$3 mass

see first part of talk

General Strategy and some comments

- Find enough inputs from experiment
- C_i^r: Ecker's talk
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not σ)
  - quark masses: which scalars? unrealistically large estimates
- in p^6 physical or lowest order masses: thresholds in right place requires physical

General Strategy and some comments

Inputs:

$K_{\ell 4}$: $F(0)$, $G(0)$, λ E865 BNL

$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$ em with Dashen violation

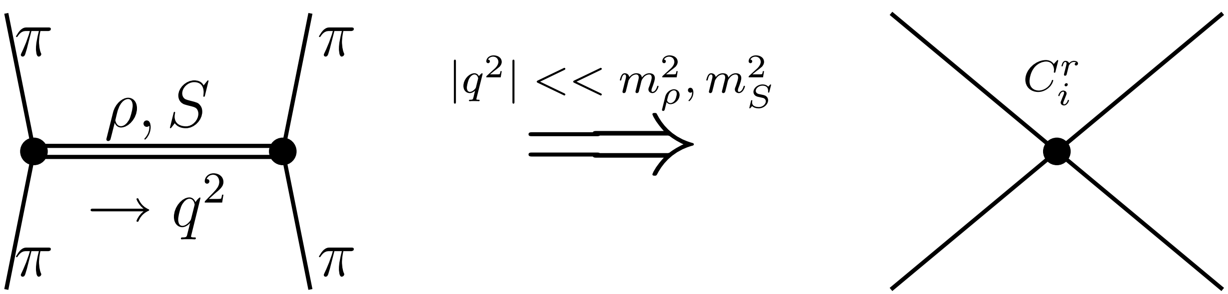
F_{π^+}

F_{K^+}/F_{π^+}

$m_s/\hat{m}$ 24 (26) $\hat{m} = (m_u + m_d)/2$

L^r_4, L^r_6

C^r_i from single resonance approximation



General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

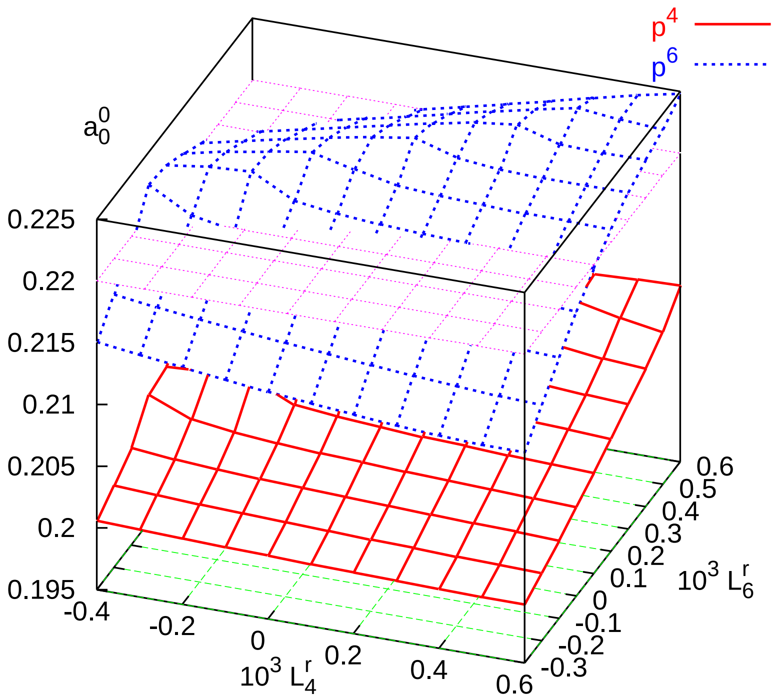
- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying C_i^r factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and πK thresholds

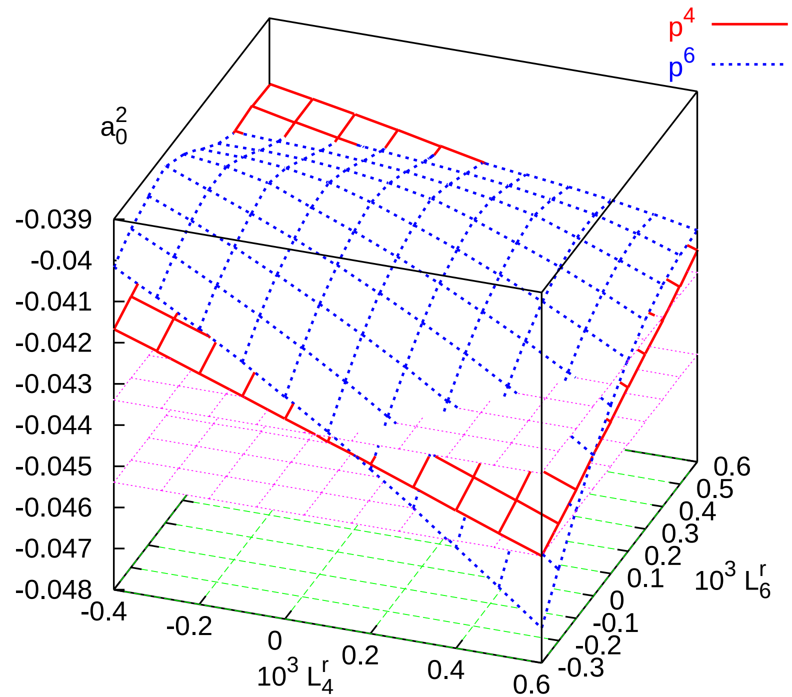
General Strategy and some comments

	fi t 10	same p^4	fi t B	fi t D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	0.009,$\equiv 0$	$-$0.138,0.009	$-$0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	0.075,$\equiv 0$	$-$0.149,0.094	$-$0.096,0.201
m_η^2: p^4, p^6	$-$0.052,0.318	0.013,$\equiv 0$	$-$0.197,0.073	$-$0.151,0.197
m_u/m_d	0.45$\pm$0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}$: p^4, p^6	0.169,0.051	0.22,$\equiv 0$	0.153,0.067	0.159,0.061

- $m_u = 0$ always very far from the fi ts
- F_0: pion decay constant in the chiral limit

$\pi\pi$



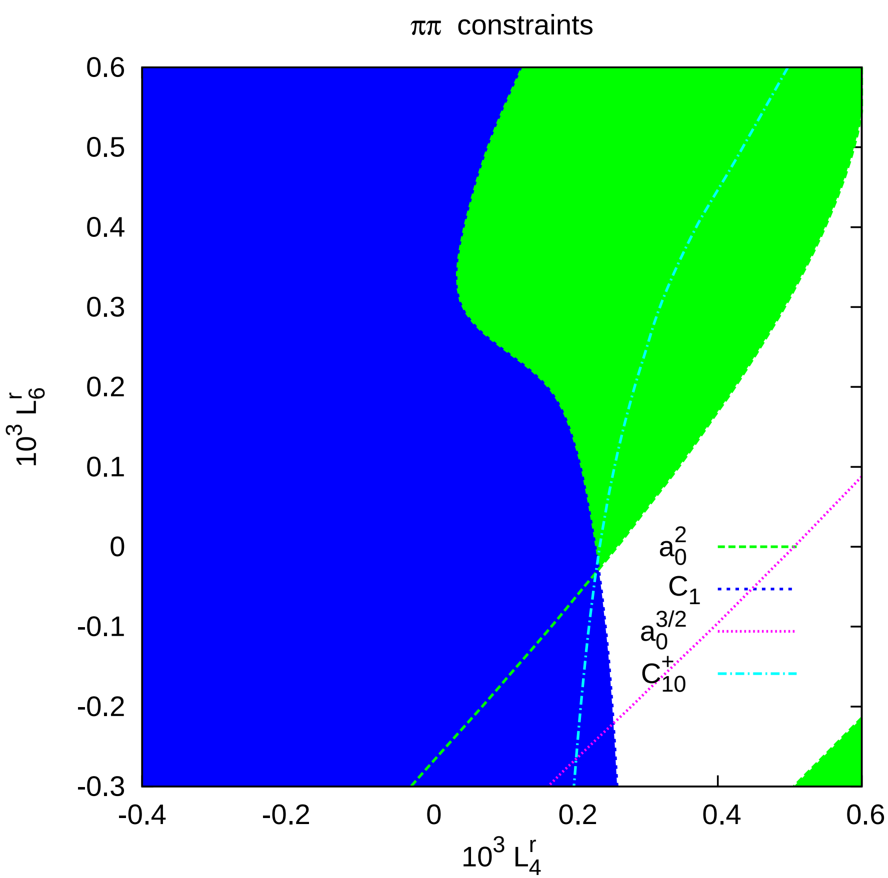


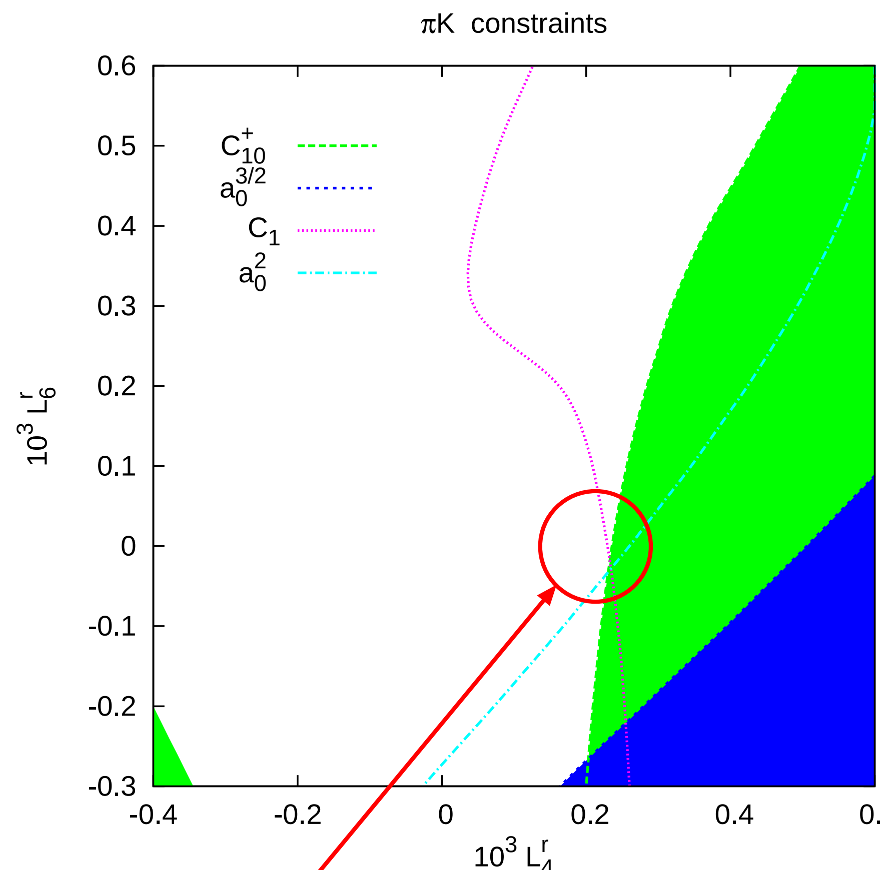
$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159$ $a_0^2 = -0.0454$ at order p^2

$\pi\pi$ and πK





preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

General fitting needs more work and systematic studies

Conclusions

- 3 flavour ChPT at 2 loops
  - many calculations done
  - things seem to work but convergence is fairly slow
  - “kinematical” and “vector” C_i^r seem to be OK
  - L_4^r, L_6^r nonzero but reasonable for large N_c
  - $\eta \to 3\pi$, isobreaking in $K_{\ell 3}$: parts done
- PQChPT at 2 loops: subject just beginning
- $\eta \to 3\pi$: isospin breaking part needed to p^6, compare with dispersive (see $K_{\ell 4}$), status: infinities have canceled
- Need to redo fit with all newer data (but then many people not in this audience still use old GL 1985 value of LECs)