QUARK MASS DEPENDENCE AT TWO LOOPS FOR MESON PROPERTIES

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Alternative Title

What is Known About

Low Energy Constants

and

Quark Mass Dependence

in Chiral Perturbation Theory

from the Continuum

Overview

- Chiral Perturbation Theory (ChPT, CHPT, χPT)
- Expand in which quantities
- Two-flavour ChPT at NNLO: one mass
  - Calculations
  - LECs
  - Quark-mass dependence of m_π^2, F_π
- Three-flavour ChPT at NNLO: 3-5 masses
  - Calculations
  - What about p^6 LECs
  - Fits to data
  - Some results on quark mass dependence
- Even more flavours at NNLO (Partially Quenched)

Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

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Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory,* Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

Power Counting

- gap in the spectrum $\Longrightarrow$ separation of scales
- with the lower degrees of freedom, build the most general effective Lagrangian

  - $\infty\#$ parameters
  - Where did my predictivity go ?

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$\Longrightarrow$ Need some ordering principle: power counting

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting in momenta/masses

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

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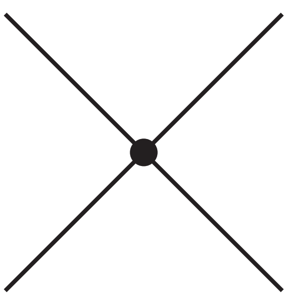
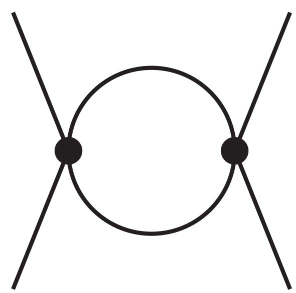
Power counting in momenta: Meson loops

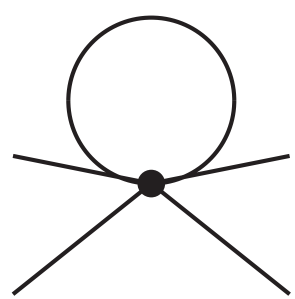


p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$



$(p^2)\,(1/p^2)\,p^4 = p^4$

$\int d^4p$ $\quad p^4$

Chiral Perturbation Theory

Large subject:

- Steven Weinberg, Physica A96:327,1979: 1716 citations
- Juerg Gasser and Heiri Leutwyler, Nucl.Phys.B250:465,1985: 2194 citations
- Juerg Gasser and Heiri Leutwyler, Annals Phys.158:142,1984: 2153 citations
- Sum: 3618
- K.G. Wilson, Phys. Rev. D14:2455,1974: 2961 citations
- Checked on 23/7/2007 in SPIRES

For lectures, review articles: see
http://www.thep.lu.se/∼bijnens/chpt.html

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
- ...

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
  - Two or Three (or even more) Flavours
  - Strong interaction and couplings to external currents/densities
  - Including electromagnetism
  - Including weak nonleptonic interactions
  - Treating kaon as heavy

Only part pushed to NNLO: this talk

Lagrangians

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta_8}{\sqrt{6}} \end{pmatrix} .$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}\,,$

$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu\,,$
left and right external currents: $r(l)_\mu = v_\mu + (-) a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$
quark masses via scalar density: $s = \mathcal{M} + \cdots$

$\langle A \rangle = Tr_F\,(A)$

Lagrangians

$$\mathcal{L}_4 = L_1\langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2\langle D_\mu U^\dagger D_\nu U\rangle\langle D^\mu U^\dagger D^\nu U\rangle$$
$$+L_3\langle D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U\rangle + L_4\langle D^\mu U^\dagger D_\mu U\rangle\langle\chi^\dagger U + \chi U^\dagger\rangle$$
$$+L_5\langle D^\mu U^\dagger D_\mu U(\chi^\dagger U + U^\dagger\chi)\rangle + L_6\langle\chi^\dagger U + \chi U^\dagger\rangle^2$$
$$+L_7\langle\chi^\dagger U - \chi U^\dagger\rangle^2 + L_8\langle\chi^\dagger U\chi^\dagger U + \chi U^\dagger\chi U^\dagger\rangle$$
$$-iL_9\langle F^R_{\mu\nu}D^\mu U D^\nu U^\dagger + F^L_{\mu\nu}D^\mu U^\dagger D^\nu U\rangle$$
$$+L_{10}\langle U^\dagger F^R_{\mu\nu}U F^{L\mu\nu}\rangle + H_1\langle F^R_{\mu\nu}F^{R\mu\nu} + F^L_{\mu\nu}F^{L\mu\nu}\rangle + H_2\langle\chi^\dagger\chi\rangle$$

L_i: Low-energy-constants (LECs)
H_i: Values depend on definition of currents/densities

These absorb the divergences of loop diagrams: $L_i \to L_i^r$
Renormalization: order by order in the powercounting

Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour special case of 3+3 PQ: $\hat{L}_i^r, K_i^r \longrightarrow L_i^r, C_i^r$
- 53 $\rightarrow$ 52 arXiv:0705.0576 [hep-ph]

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2}\log\frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu)\right] + \cdots$$

$M^2 = 2B\hat{m}$
$B \neq B_0$, $F \neq F_0$ (two versus three-flavour)

LECs and μ

$l_3^r(\mu)$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i}\, l_i^r(\mu) - \log\frac{M_\pi^2}{\mu^2}\,.$$

Independent of the scale μ.

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

μ :

- m_π, m_K: chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$ large N_c arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exists
- Express orders in terms of physical masses and quantities (F_π, F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s+t+u = 2m_\pi^2 + 2m_K^2$ in πK scattering

See e.g. Descotes-Genon talk

- I prefer physical masses
- Thresholds correct
- Chiral logs are from physical particles propagating

An example

$$m_\pi = \frac{m_0}{1 + a\frac{m0}{f0}} \qquad f_\pi = \frac{f_0}{1 + b\frac{m0}{f0}}$$

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$$m_\pi = m_0 - a\frac{m_0^2}{f_0} + a^2\frac{m_0^3}{f_0^2} + \cdots$$

$$f_\pi = f_0\left(1 - b\frac{m_0}{f_0} + b^2\frac{m_0^2}{f_0^2} + \cdots\right)$$

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An example

$$m_\pi = \frac{m_0}{1+a\frac{m0}{f0}} \qquad f_\pi = \frac{f_0}{1+b\frac{m0}{f0}}$$

$$m_\pi = m_0 - a\frac{m_0^2}{f_0} + a^2\frac{m_0^3}{f_0^2} + \cdots$$

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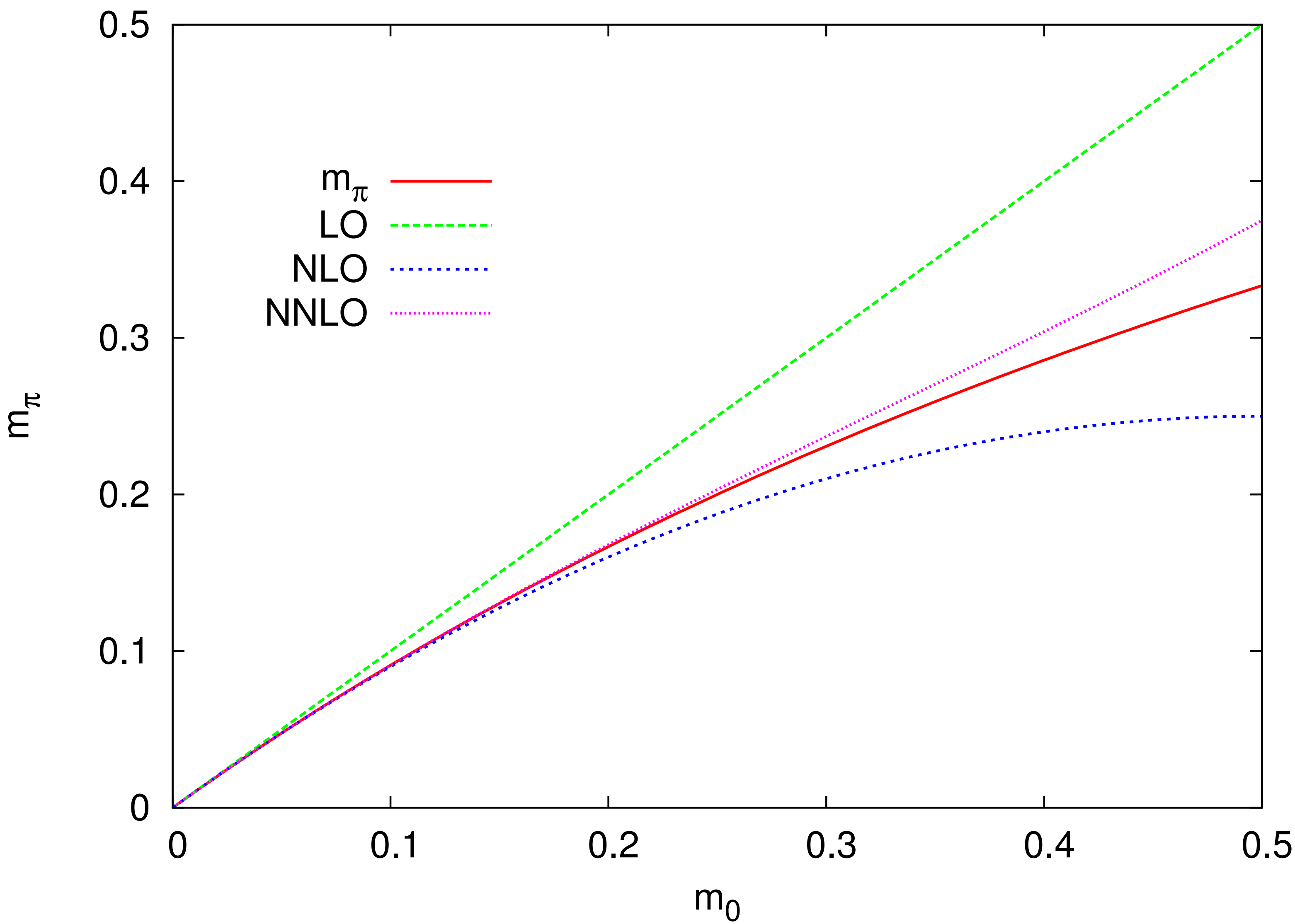
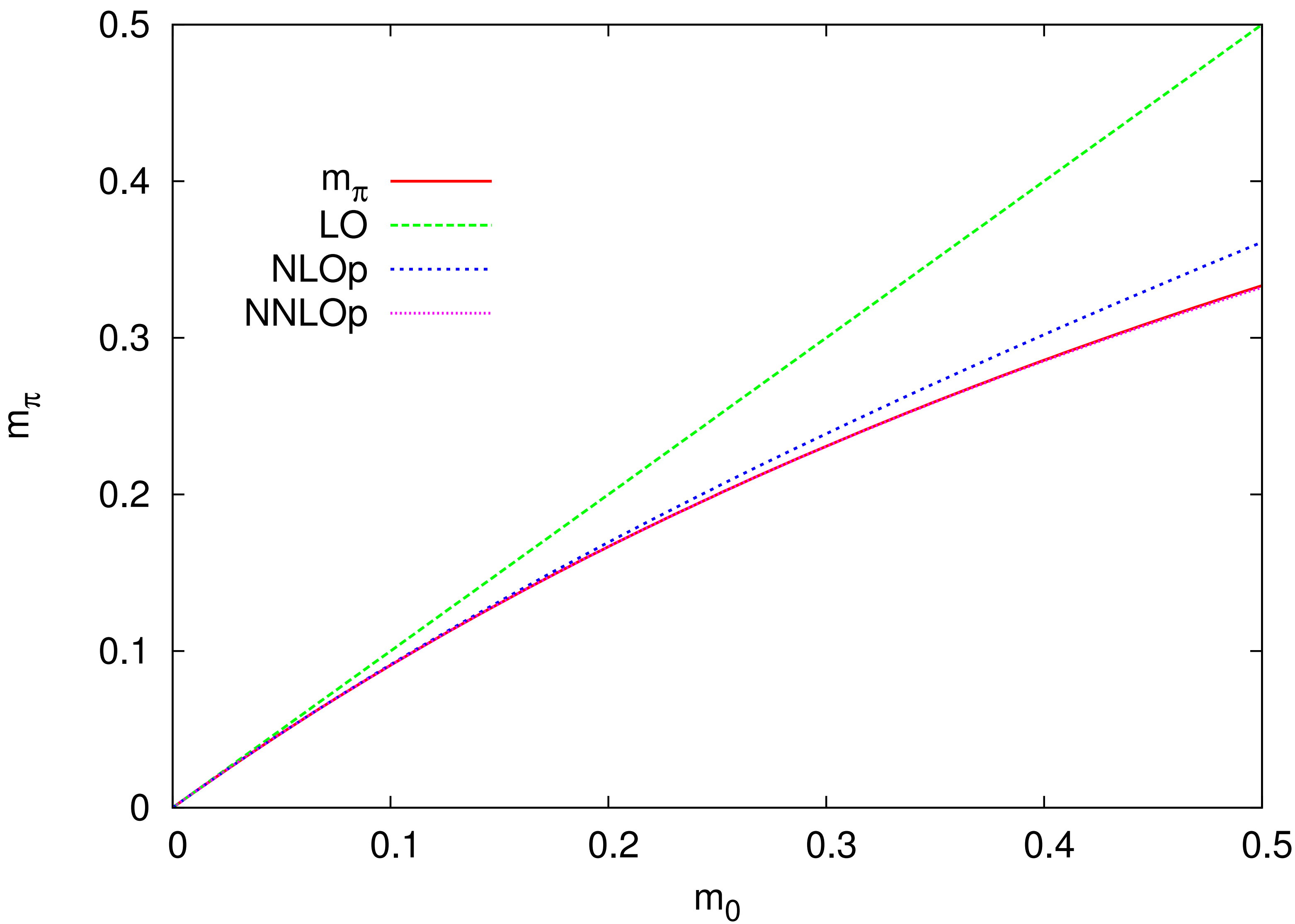
$$a = 1 \quad b = 0.5 \quad f_0 = 1$$

An example: m_0/f_0



An example: m_π / f_π



Two-loop Two-flavour

Review paper on Two-Loops: JB, hep-ph/0604043 Prog. Part. Nucl. Phys. 58 (2007) 521

Dispersive Calculation of the nonpolynomial part in q^2, s, t, u

- Gasser-Meißner: F_V, F_S: 1991 numerical
- Knecht-Moussallam-Stern-Fuchs: $\pi\pi$: 1995 analytical
- Colangelo-Finkemeier-Urech: F_V, F_S: 1996 analytical

Two-Loop Two-flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \to \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \to \pi^+\pi^-$, F_π, m_π: 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi$, F_π, m_π: 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t)$, $F_{S\pi}$: 1998
- JB-Talavera: $\pi \to \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \to \pi^0\pi^0$, $\gamma\gamma \to \pi^+\pi^-$: 2005-2006
- m_π, F_π, F_V, F_S, $\pi\pi$: simple analytical forms
- Colangelo-(Dürr-)Haefeli: Finite volume F_π, m_π 2005-2006

LECs

$\bar{l}_1$ to $\bar{l}_4$: ChPT at order p^6 and the Roy equation analysis in $\pi\pi$ and F_S Colangelo, Gasser and Leutwyler, *Nucl. Phys.* B 603 (2001) 125 [hep-ph/0103088]

$\bar{l}_5$ and $\bar{l}_6$: from F_V and $\pi \to \ell\nu\gamma$ JB,(Colangelo,)Talavera

$$\begin{aligned}
\bar{l}_1 &= -0.4 \pm 0.6\,,\\
\bar{l}_2 &= 4.3 \pm 0.1\,,\\
\bar{l}_3 &= 2.9 \pm 2.4\,,\\
\bar{l}_4 &= 4.4 \pm 0.2\,,\\
\bar{l}_6 - \bar{l}_5 &= 3.0 \pm 0.3\,,\\
\bar{l}_6 &= 16.0 \pm 0.5 \pm 0.7\,.
\end{aligned}$$

$l_7 \sim 5 \cdot 10^{-3}$ from π^0-η mixing Gasser, Leutwyler 1984

LECs

Some combinations of order p^6 LECs are known as well: curvature of the scalar and vector formfactor, two more combinations from $\pi\pi$ scattering (implicit in b_5 and b_6)

Note: c_i^r for m_π, f_π, $\pi\pi$: small effect

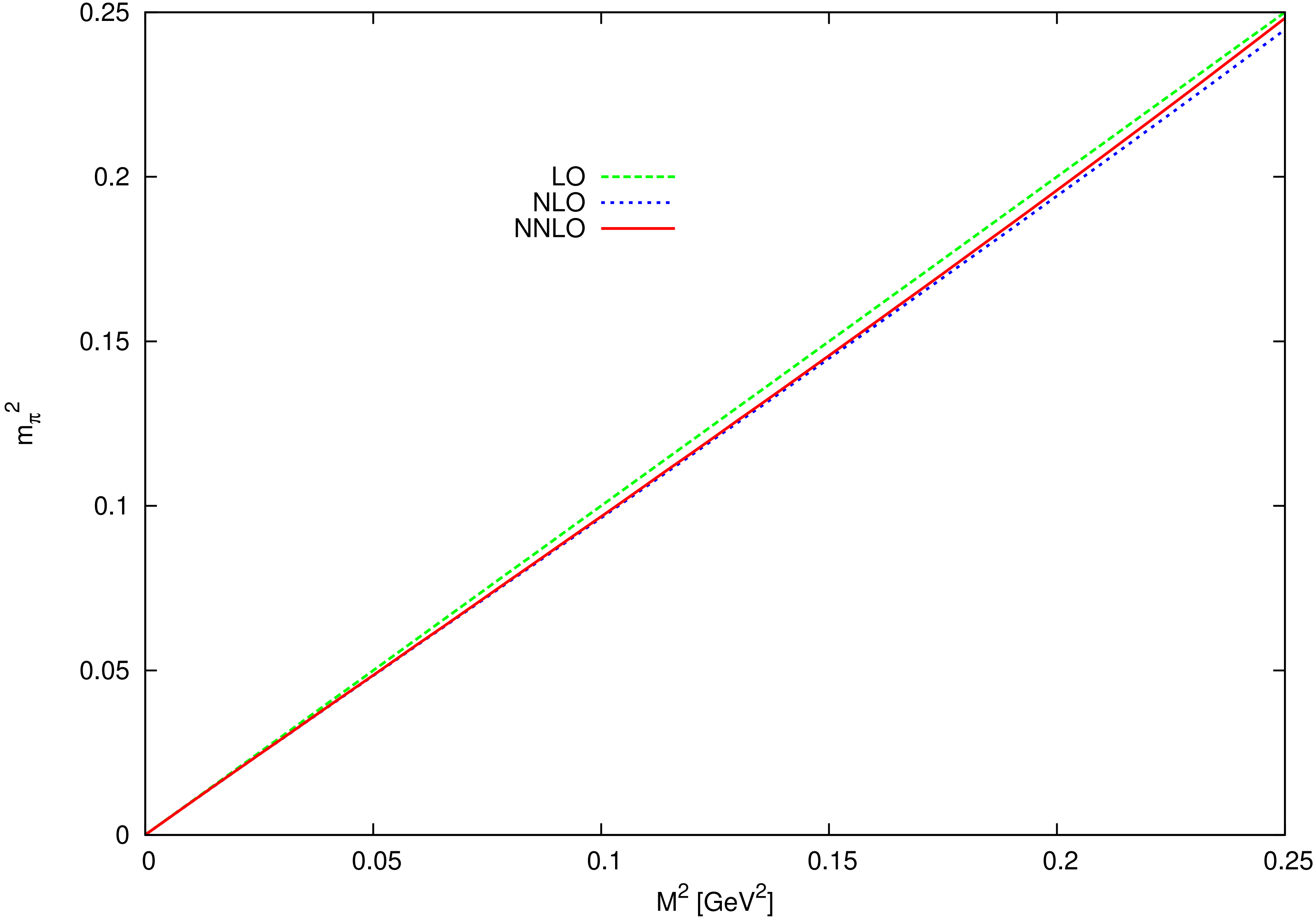
$c_i^r(770 MeV) = 0$ for plots shown

expansion in m_π^2/F_π^2 shown

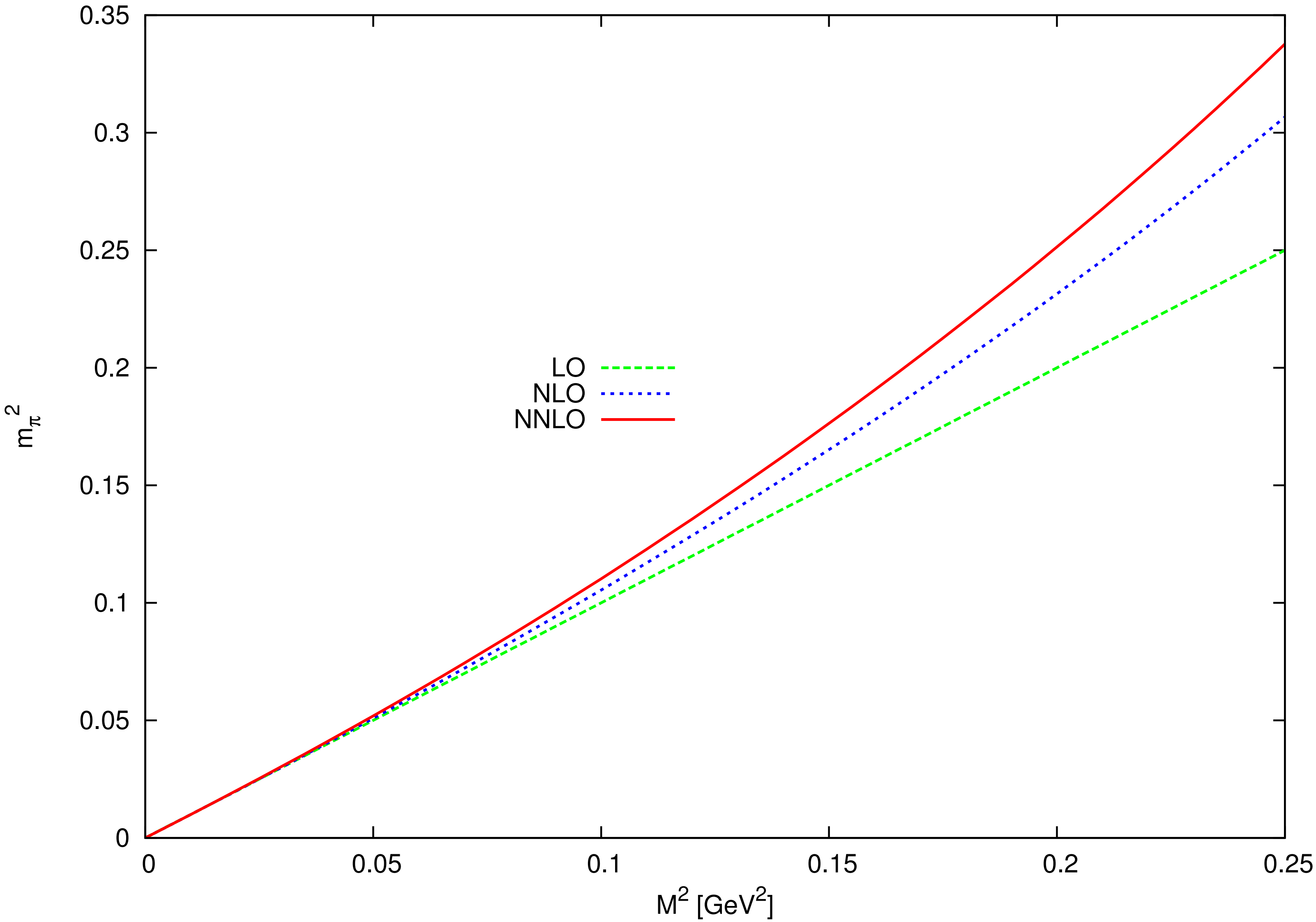
General observation:

- Obtainable from kinematical dependences: known
- Only via quark-mass dependence: poorely known

m_π^2

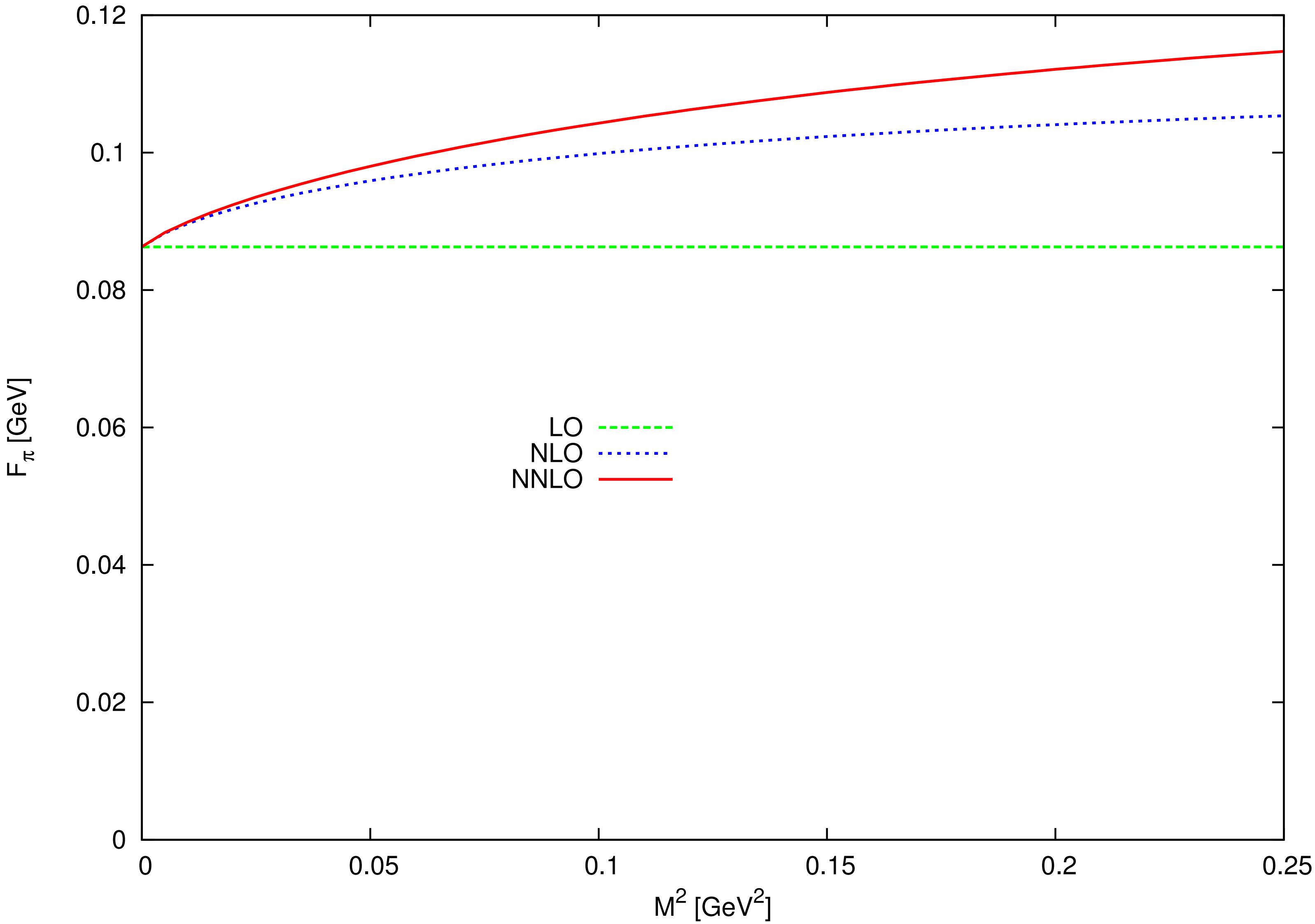


m_π^2 ($\bar{l}_3 = 0$)



0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
m_π^2
M^2 [GeV^2]
LO
NLO
NNLO

F_π



0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.05
0.1
0.15
0.2
0.25
F_π [GeV]
M^2 [GeV^2]
LO
NLO
NNLO

Two-loop Three-flavour, $\leq$2001

- $\Pi_{VV\pi}$, $\Pi_{VV\eta}$, Π_{VVK} Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
- $\Pi_{VV\rho\omega}$ Maltman
- $\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η Kambor, Golowich; Amorós, JB, Talavera
- Π_{SS} Moussallam $\boxed{L_4^r, L_6^r}$
- Π_{VVK}, Π_{AAK}, F_K, m_K Amorós, JB, Talavera
- $K_{\ell 4}$, $\langle \overline{q}q \rangle$ Amorós, JB, Talavera $\boxed{L_1^r, L_2^r, L_3^r}$
- F_M, m_M, $\langle \overline{q}q \rangle$ $(m_u \neq m_d)$ Amorós, JB, Talavera $\boxed{L_{5,7,8}^r, m_u/m_d}$

Two-loop Three-flavour, $\geq$2001

- $F_{V\pi}$, F_{VK^+}, F_{VK^0} — Post, Schilcher; JB, Talavera — L_9^r
- $K_{\ell 3}$ — Post, Schilcher; JB, Talavera — V_{us}
- $F_{S\pi}$, F_{SK} (includes σ-terms) — JB, Dhonte — L_4^r, L_6^r
- $K, \pi \to \ell\nu\gamma$ — Geng, Ho, Wu — L_{10}^r
- $\pi\pi$ — JB,Dhonte,Talavera
- πK — JB,Dhonte,Talavera
- relation l_i^r and L_i^r, C_i^r — Gasser,Haefeli,Ivanov,Schmid
- Finite volume $\langle \overline{q}q \rangle$ — JB,Ghorbani

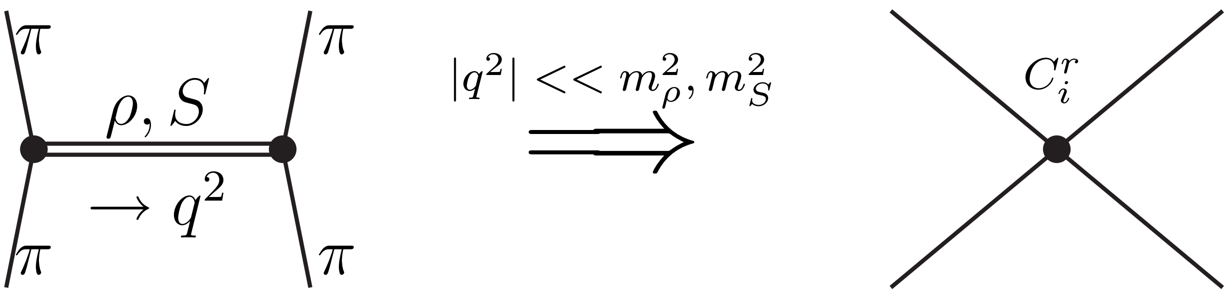
Two-loop Three-flavour

Known to be in progress

- $\eta \to 3\pi$: being written up — JB,Ghorbani
- $K_{\ell 3}$ iso: preliminary results available — JB,Ghorbani
- Finite Volume: sunsetintegrals being written up — JB,Lähde
- relation c_i^r and L_i^r, C_i^r — Gasser,Haefeli,Ivanov,Schmid

C_i^r

Most analysis use:
C_i^r from (single) resonance approximation



Motivated by large N_c: large effort goes in this

Ananthanarayan, JB, Cirigliano, Donoghue, Ecker, Gamiz, Golterman, Kaiser, Knecht, Peris, Pich, Prades, Portoles, de Rafael,. . .

C_i^r

$$
\begin{aligned}
\mathcal{L}_V &= -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}m_V^2\langle V_\mu V^\mu\rangle - \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle \\
&\quad - \frac{ig_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + f_\chi\langle V_\mu[u^\mu, \chi_-]\rangle \\
\mathcal{L}_A &= -\frac{1}{4}\langle A_{\mu\nu}A^{\mu\nu}\rangle + \frac{1}{2}m_A^2\langle A_\mu A^\mu\rangle - \frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle \\
\mathcal{L}_S &= \frac{1}{2}\langle\nabla^\mu S\nabla_\mu S - M_S^2 S^2\rangle + c_d\langle S u^\mu u_\mu\rangle + c_m\langle S\chi_+\rangle \\
\mathcal{L}_{\eta'} &= \frac{1}{2}\partial_\mu P_1\partial^\mu P_1 - \frac{1}{2}M_{\eta'}^2 P_1^2 + i\tilde{d}_m P_1\langle\chi_-\rangle\,.
\end{aligned}
$$

$f_V = 0.20$, $f_\chi = -0.025$, $g_V = 0.09$, $c_m = 42$ MeV, $c_d = 32$ MeV, $\tilde{d}_m = 20$ MeV,

$m_V = m_\rho = 0.77$ GeV, $m_A = m_{a_1} = 1.23$ GeV, $m_S = 0.98$ GeV, $m_{P_1} = 0.958$ GeV

f_V, g_V, f_χ, f_A: experiment

c_m and c_d from resonance saturation at $\mathcal{O}(p^4)$

C_i^r

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

C_i^r

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What we did about it:

- Vary resonance estimate by factor of two
- Vary the scale μ at which it applies: 600-900 MeV
- Check the estimates for the measured ones
- Again: kinematic can be had, quark-mass dependence difficult

Inputs

$K_{\ell 4}$: $F(0)$, $G(0)$, λ — E865 BNL

$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$ — em with Dashen violation

F_{π^+}

F_{K^+}/F_{π^+}

$m_s/\hat{m}$ — 24 (26) — $\hat{m} = (m_u + m_d)/2$

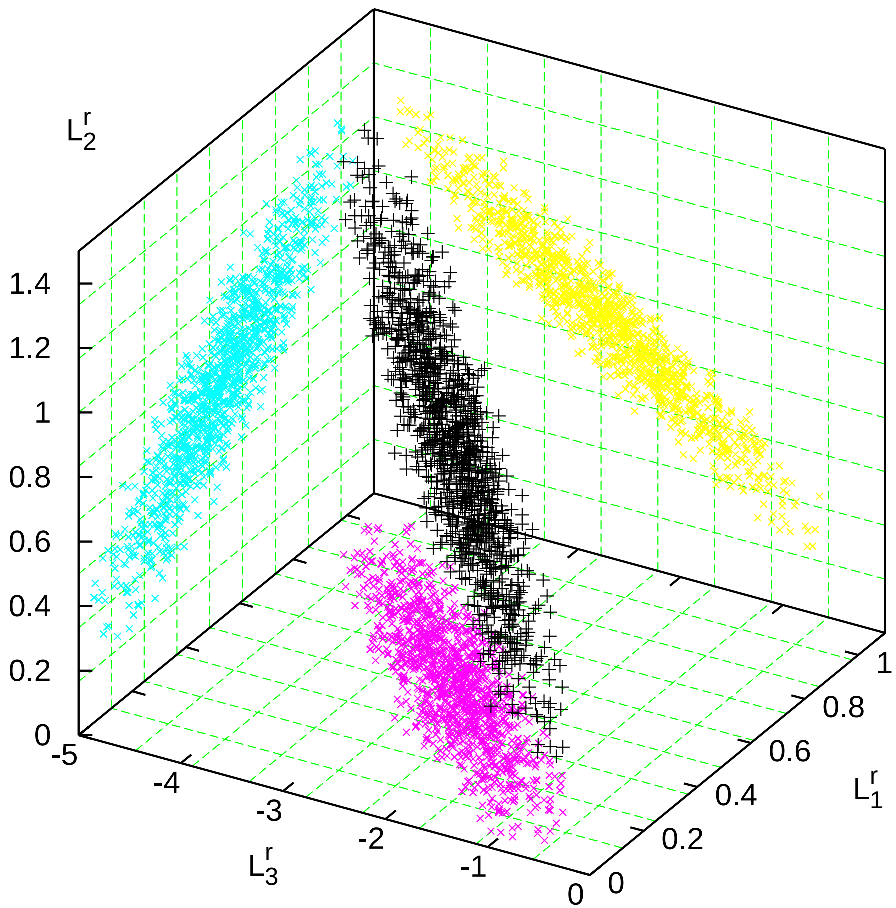
L^r_4, L^r_6

Outputs: I

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying C_i^r factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and πK thresholds

Correlations



(older fit)

$10^3\,L_1^r = 0.52 \pm 0.23$

$10^3\,L_2^r = 0.72 \pm 0.24$

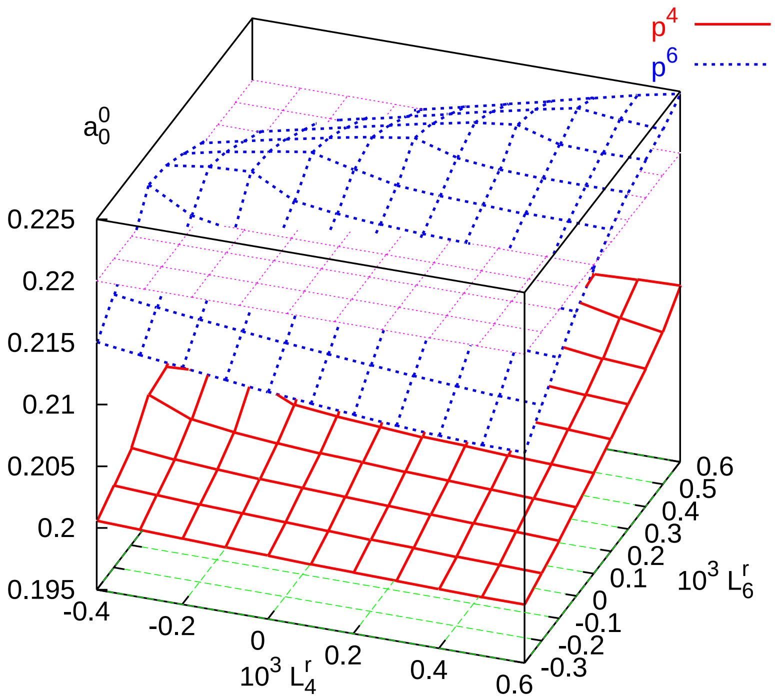
$10^3\,L_3^r = -2.70 \pm 0.99$

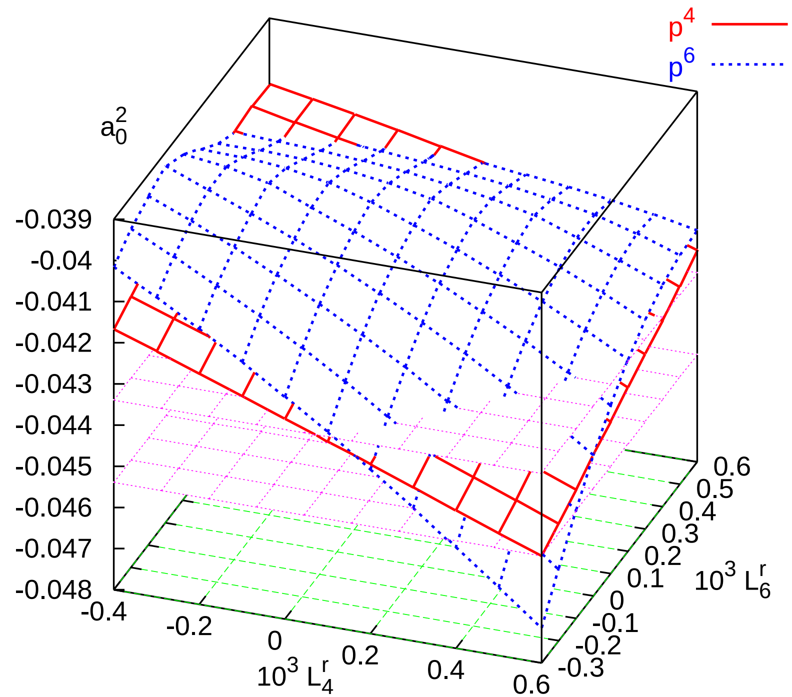
Outputs: II

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	0.009,$\equiv 0$	$-$0.138,0.009	$-$0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	0.075,$\equiv 0$	$-$0.149,0.094	$-$0.096,0.201
m_η^2: p^4, p^6	$-$0.052,0.318	0.013,$\equiv 0$	$-$0.197,0.073	$-$0.151,0.197
m_u/m_d	0.45$\pm$0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}$: p^4, p^6	0.169,0.051	0.22,$\equiv 0$	0.153,0.067	0.159,0.061

- $m_u = 0$ always very far from the fits
- F_0: pion decay constant in the chiral limit

$\pi\pi$



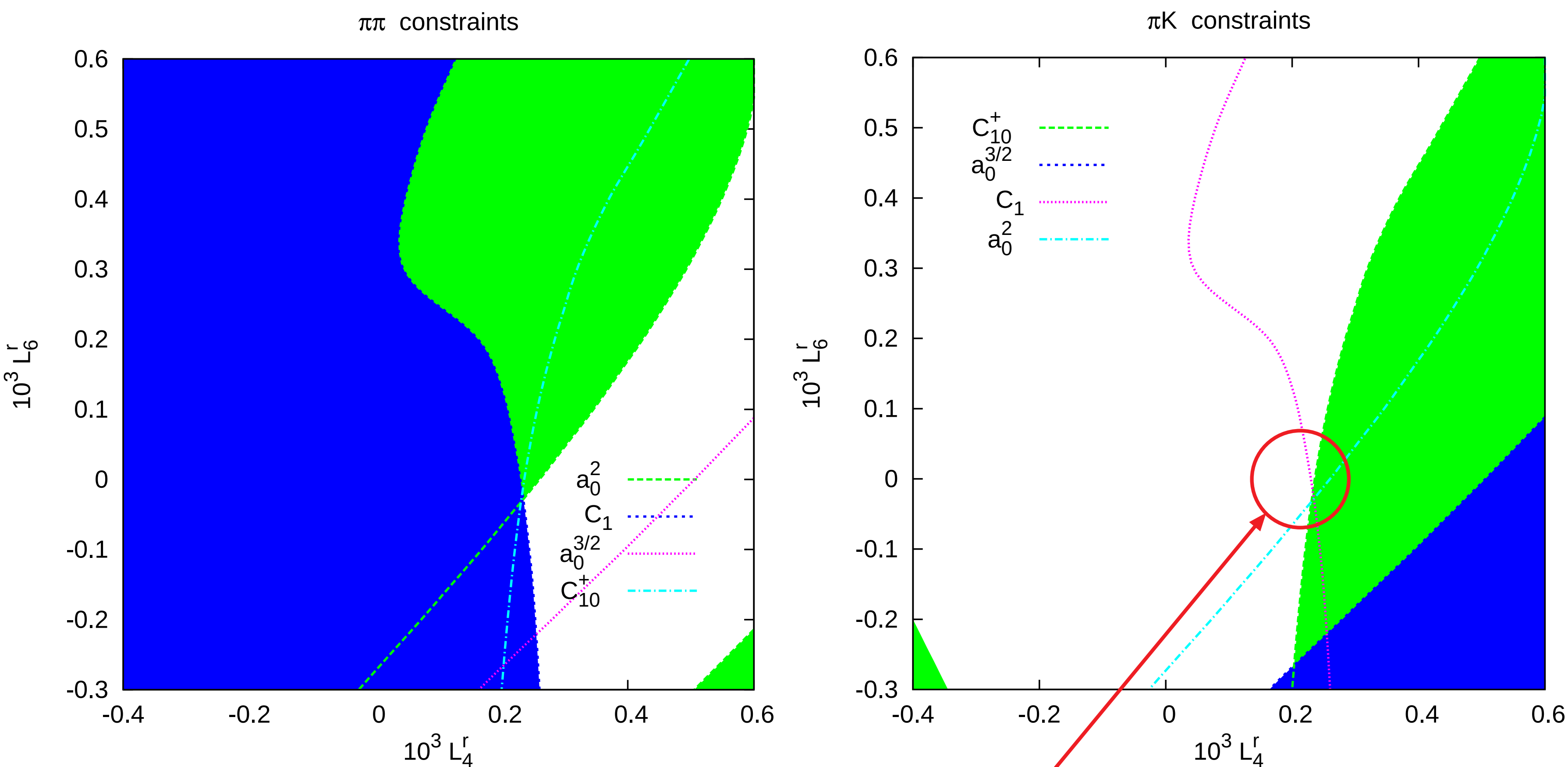


$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159\ a_0^2 = -0.0454$ at order p^2

$\pi\pi$ and πK



preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

General fitting needs more work and systematic studies

Quark mass dependences

Updates of plots in

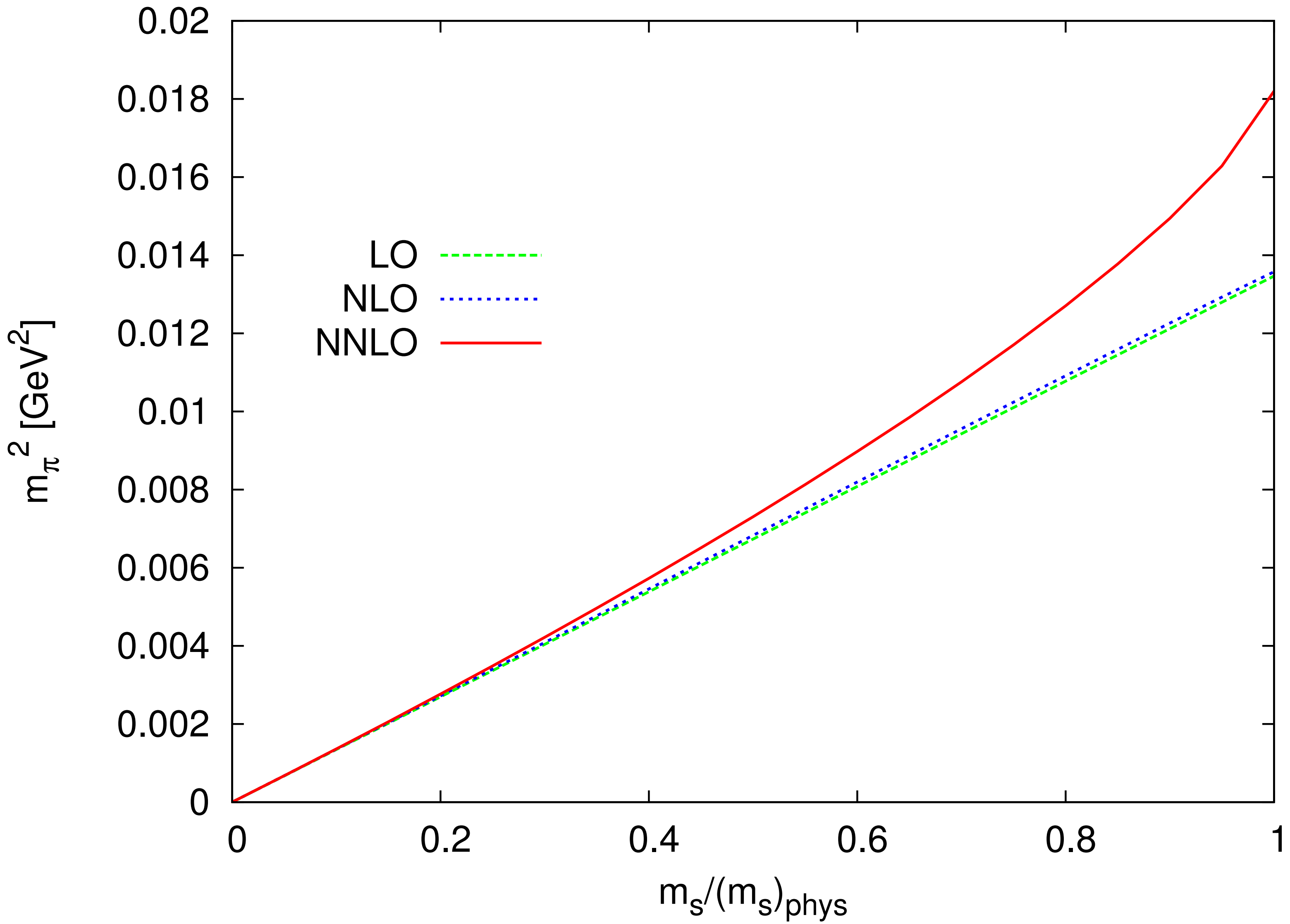
Amorós, JB and Talavera, hep-ph/0003258, Nucl. Phys. B585 (2000) 293

Some new ones

Procedure: calculate a consistent set of $m_\pi, m_K, m_\eta, f_\pi$ with the given input values (done iteratively)

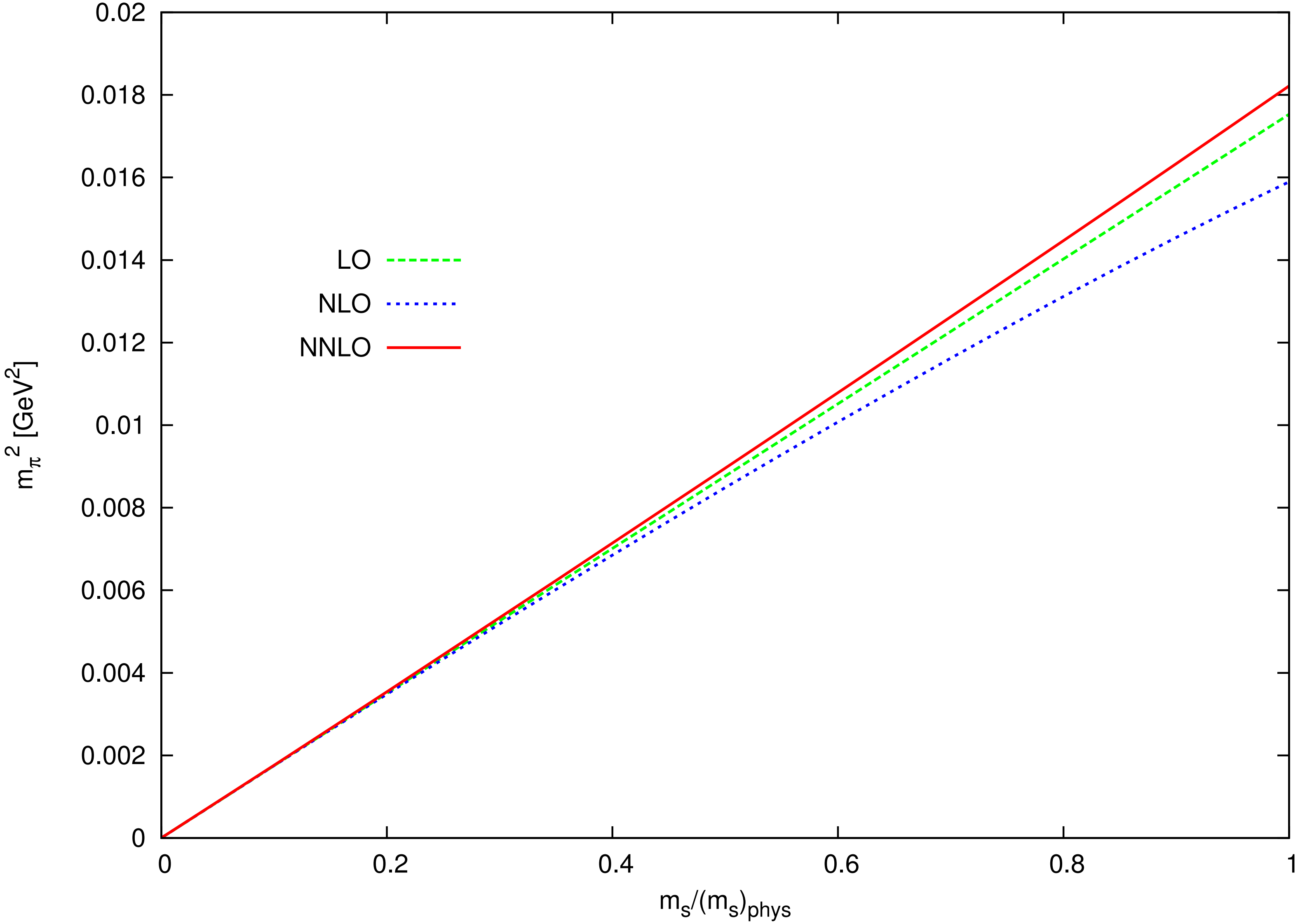
- vary $m_s/(m_s)_{phys}$, keep $m_s/\hat{m} = 24$
  m_π^2, m_K^2, F_π, F_K
- vary $m_s/(m_s)_{phys}$ keep $\hat{m}$ fixed
  m_π^2, F_π
- vary m_π, keep m_K fixed
  $f_+(0)$: the formfactor in $K_{\ell 3}$ decays
  $f_+(0)$,$f_+(0)/\left(m_K^2 - m_\pi^2\right)^2$

m_π^2 fit 10



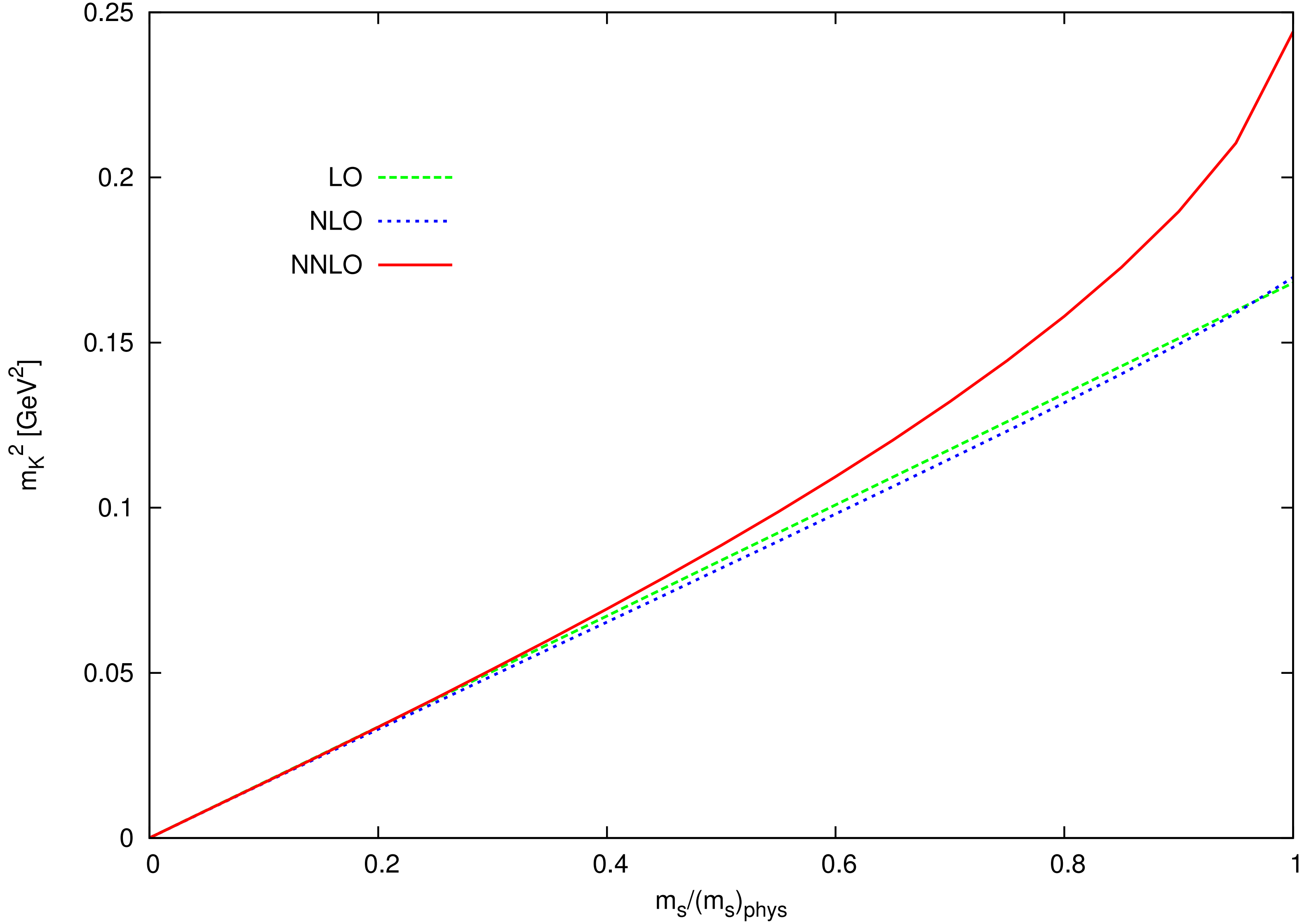
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.2
0.4
0.6
0.8
1
m_π^2 [GeV^2]
m_s/(m_s)_phys
LO
NLO
NNLO

m_π^2 fit D

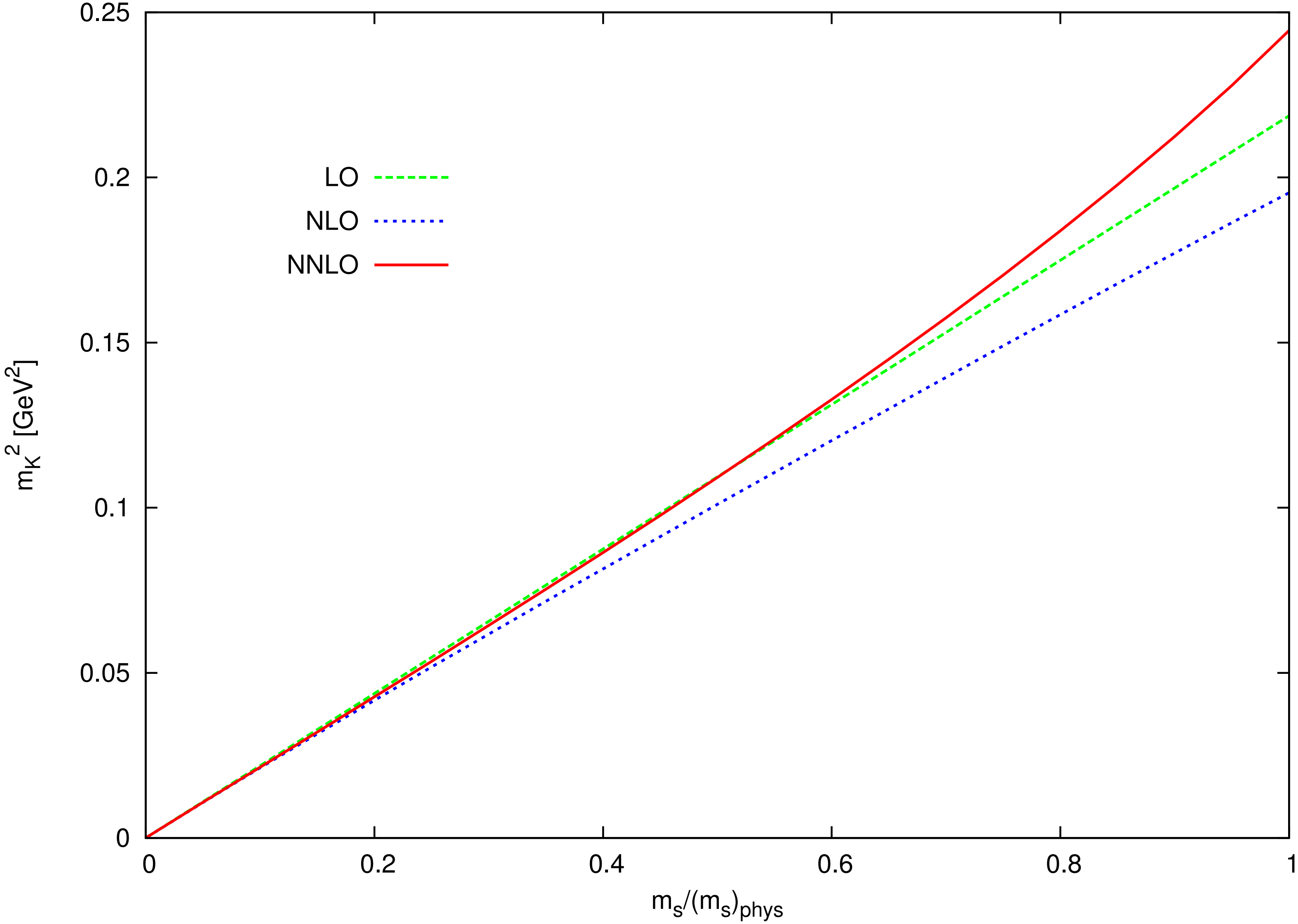


0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.2
0.4
0.6
0.8
1
LO
NLO
NNLO
m_π^2 [GeV^2]
m_s/(m_s)_phys

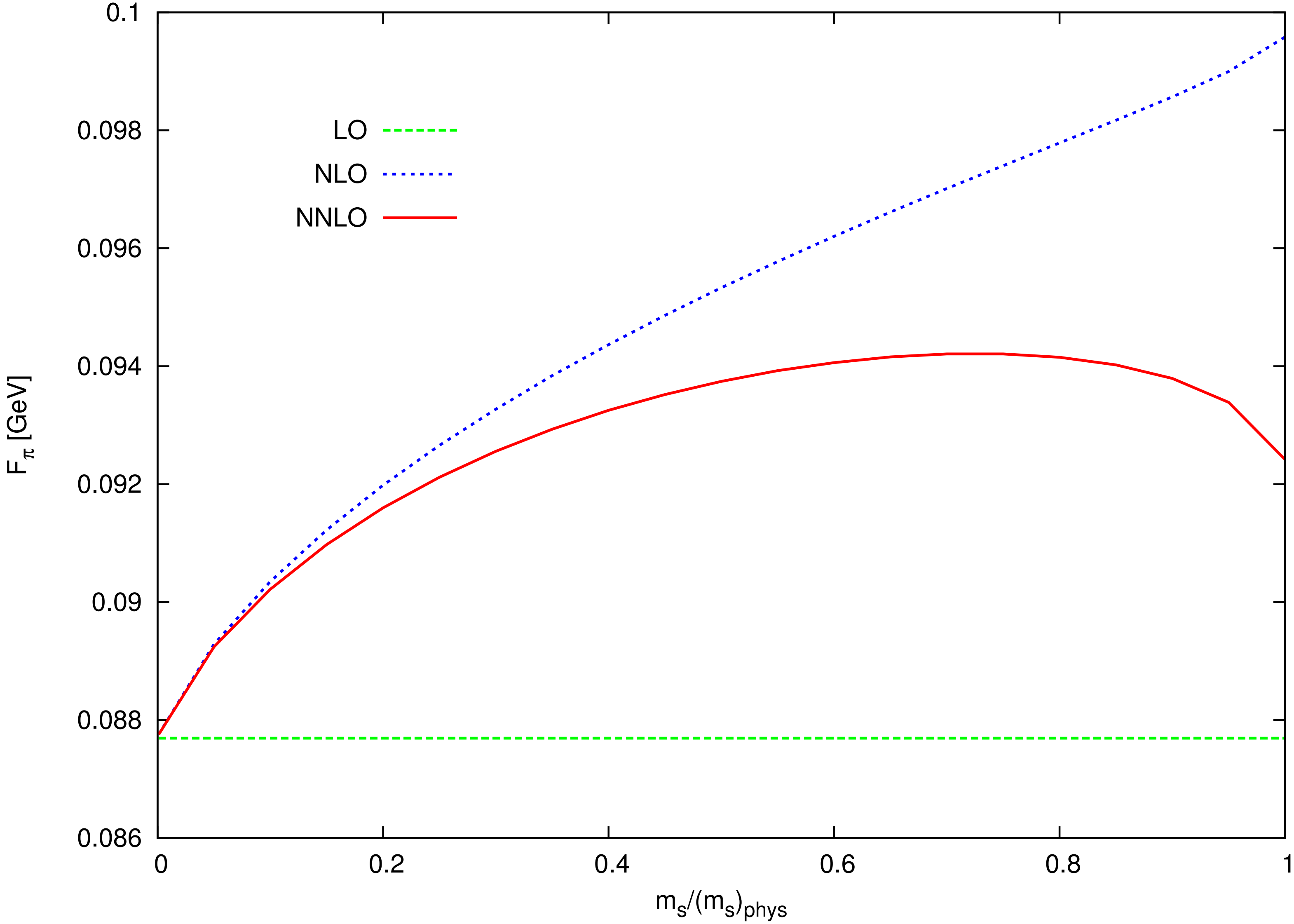
m_K^2 fit 10



m_K^2 fit D

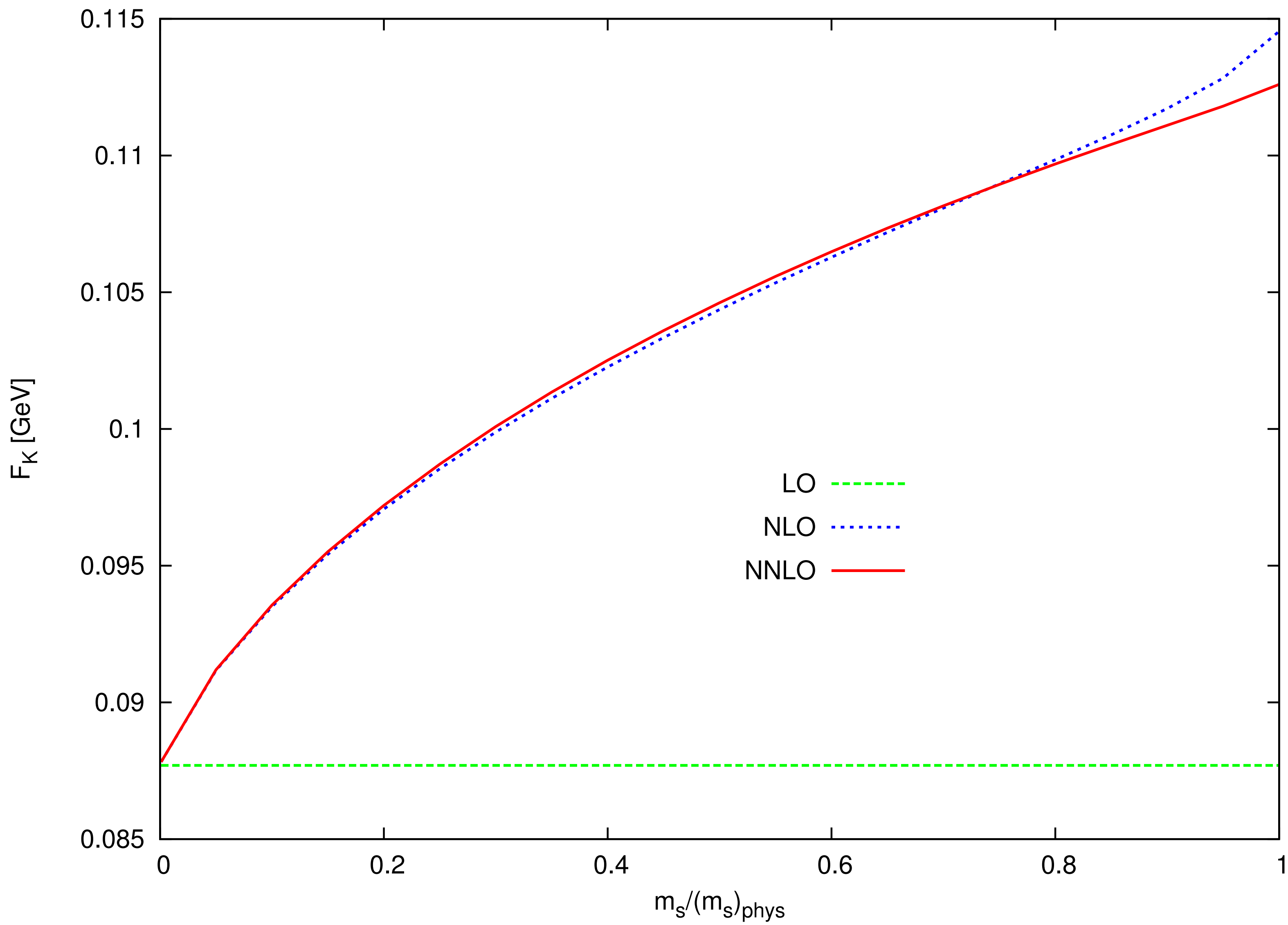


F_π fit 10



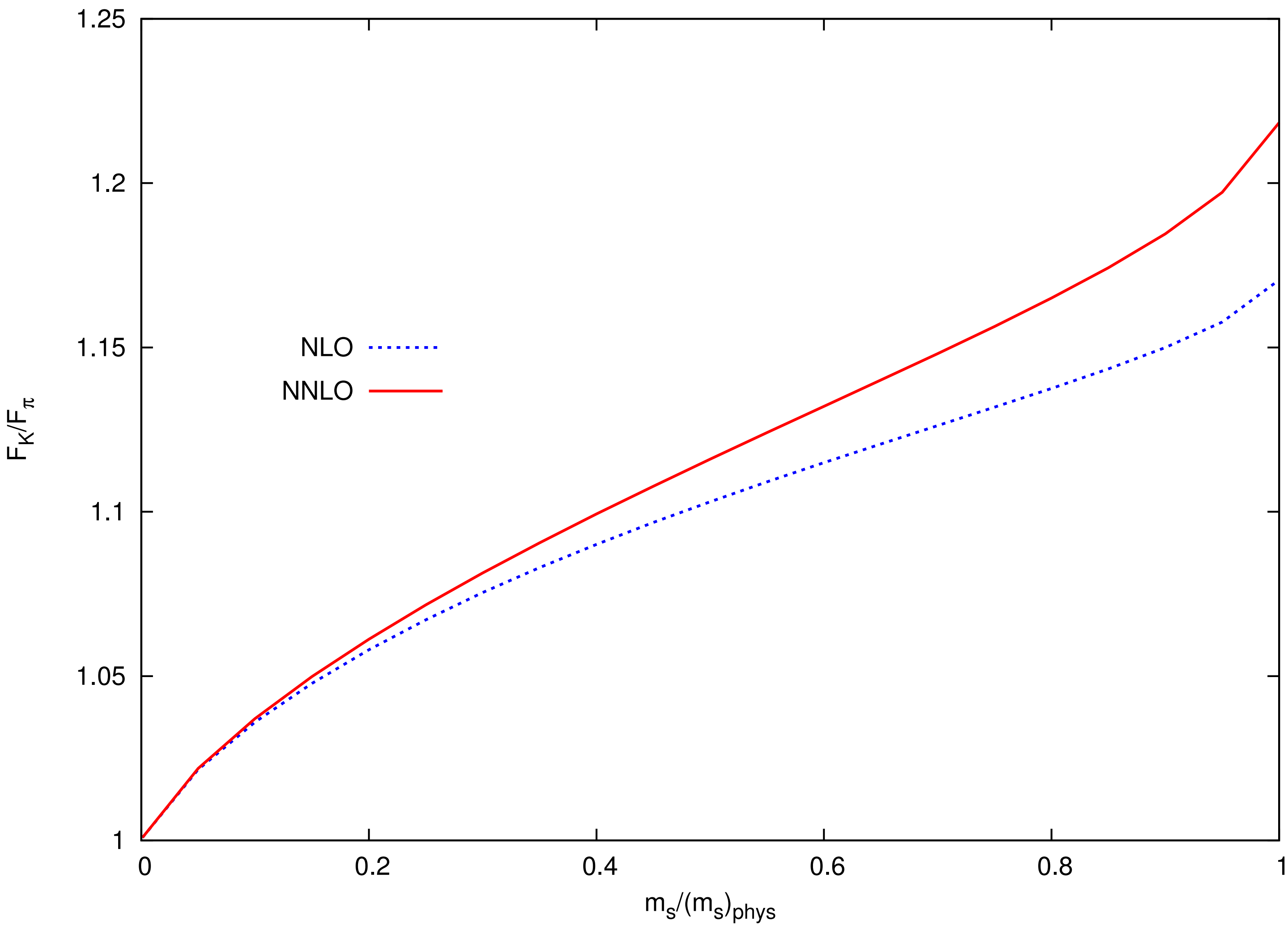
0.1
0.098
0.096
0.094
0.092
0.09
0.088
0.086
0
0.2
0.4
0.6
0.8
1
LO
NLO
NNLO
F_π [GeV]
m_s/(m_s)_phys

F_K fit 10

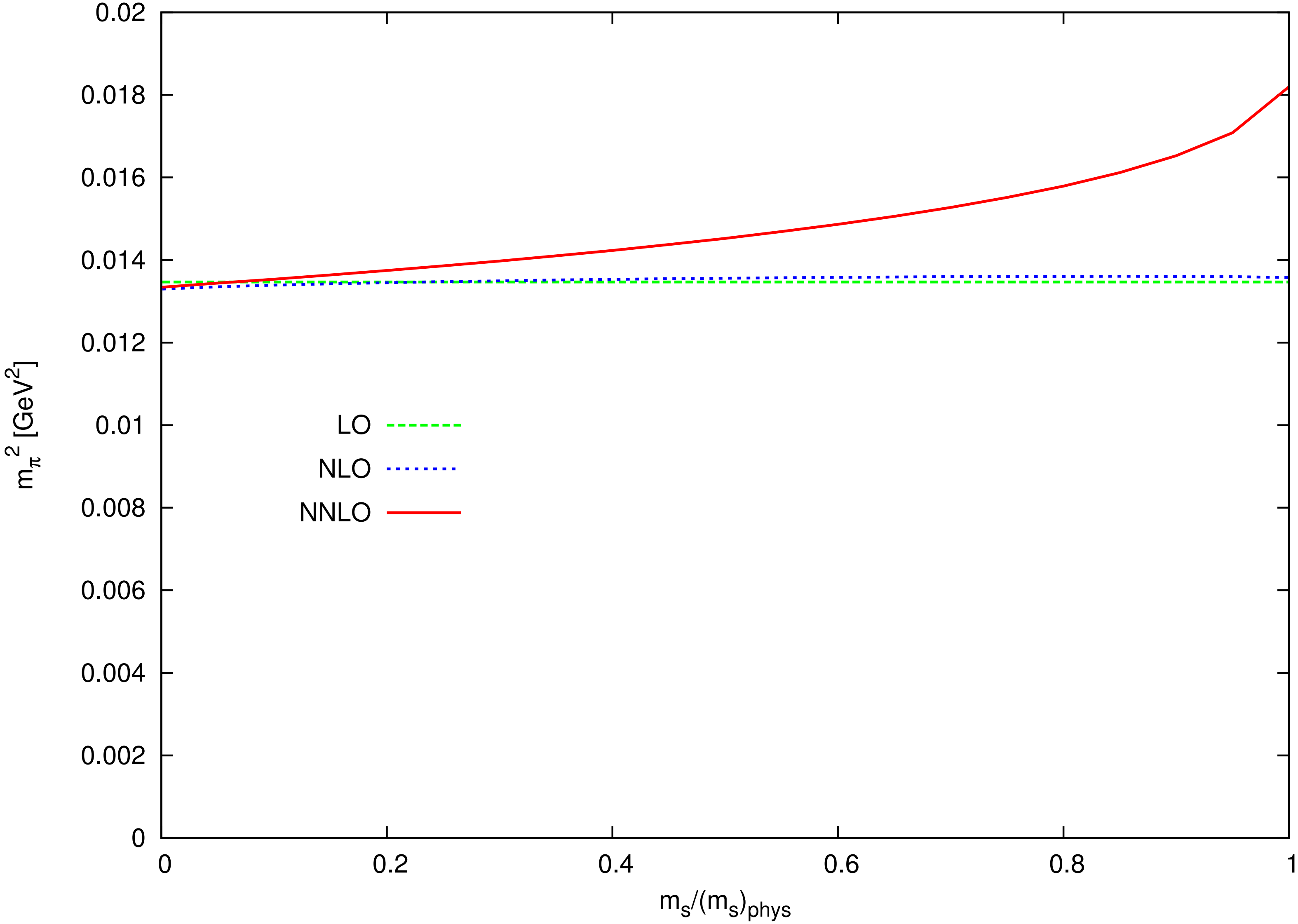


0.115
0.11
0.105
0.1
0.095
0.09
0.085
F_K [GeV]
0
0.2
0.4
0.6
0.8
1
m_s/(m_s)_phys
LO
NLO
NNLO

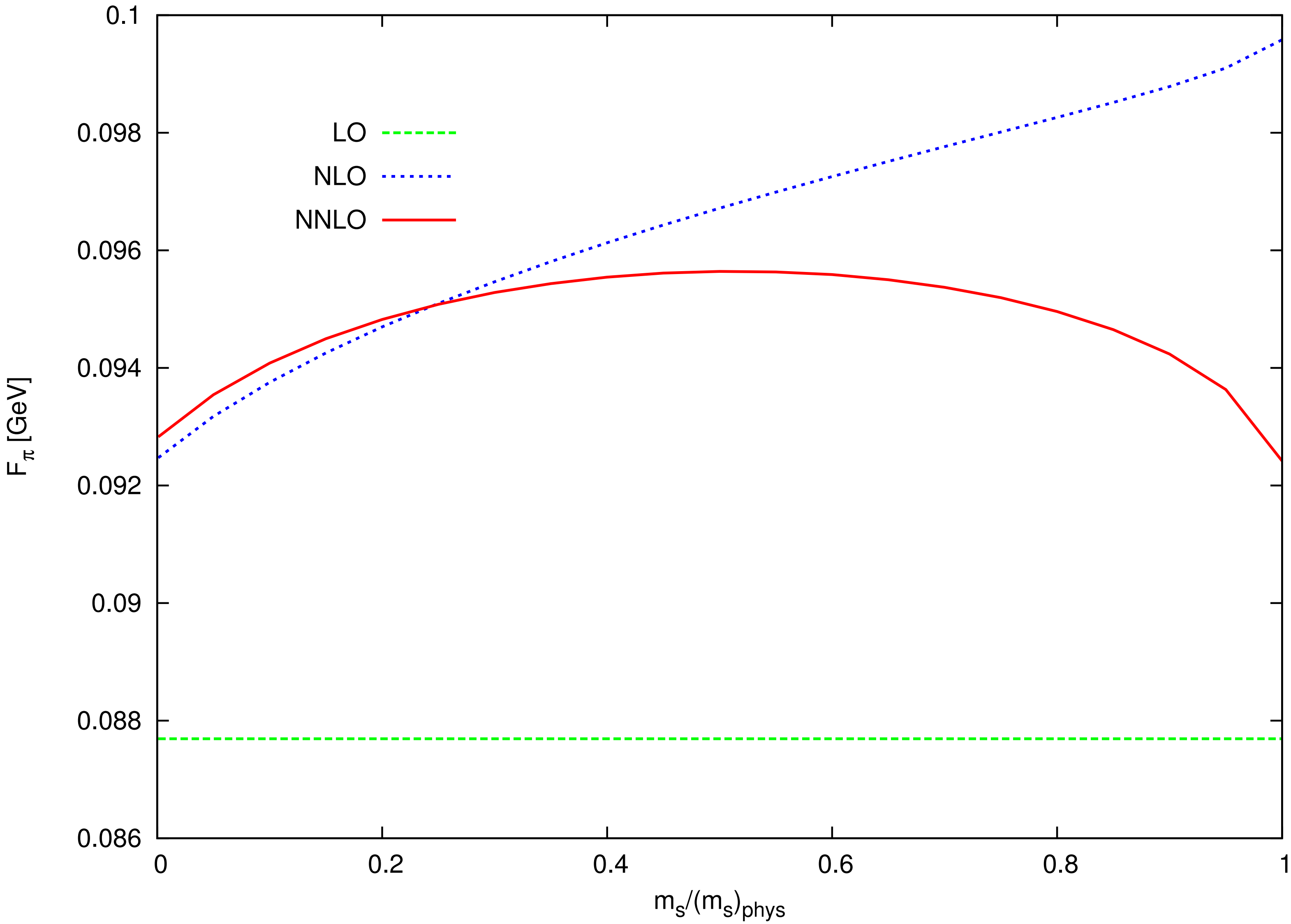
F_K/F_π fit 10



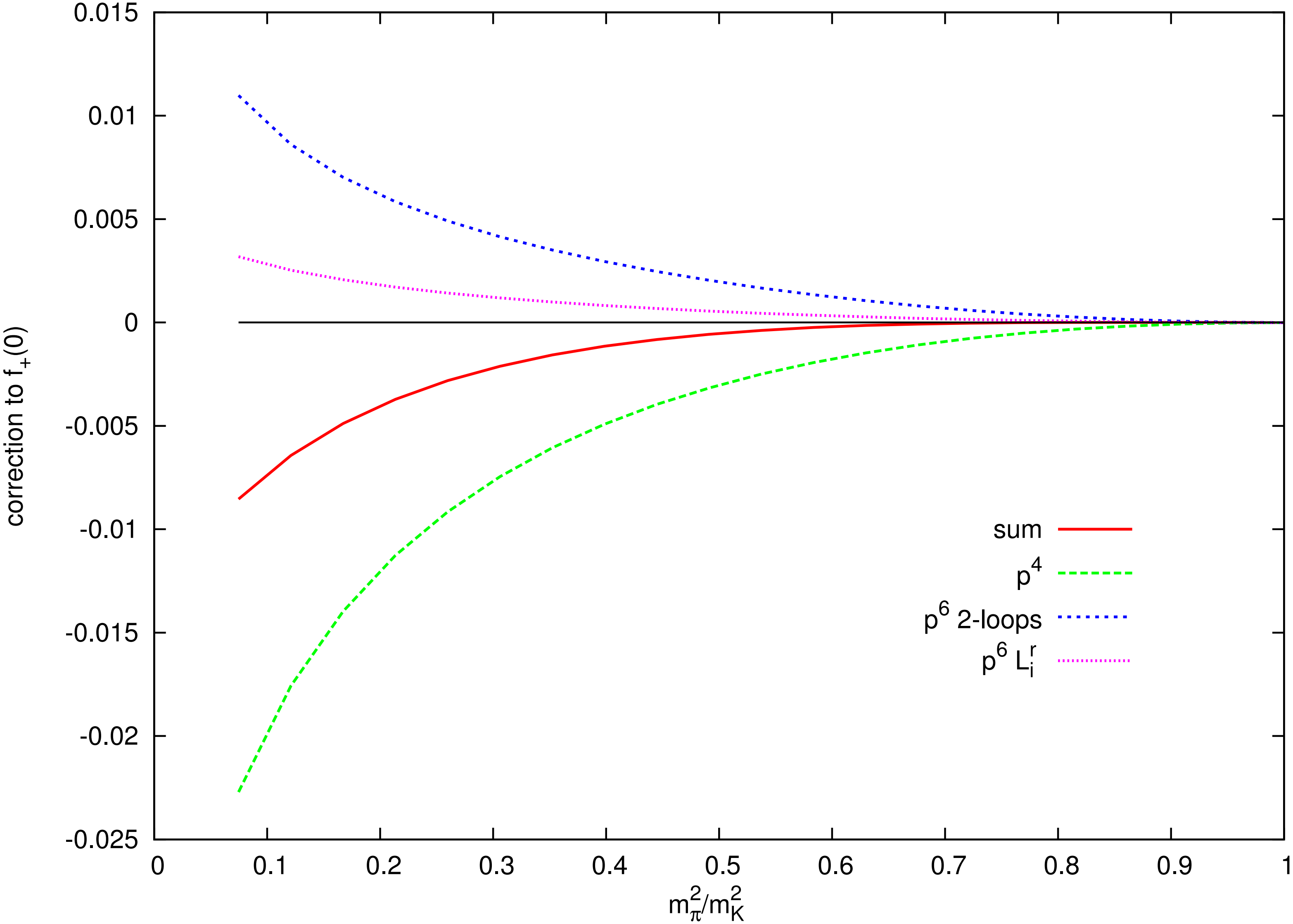
m_π^2 fit 10, fixed $\hat{m}$



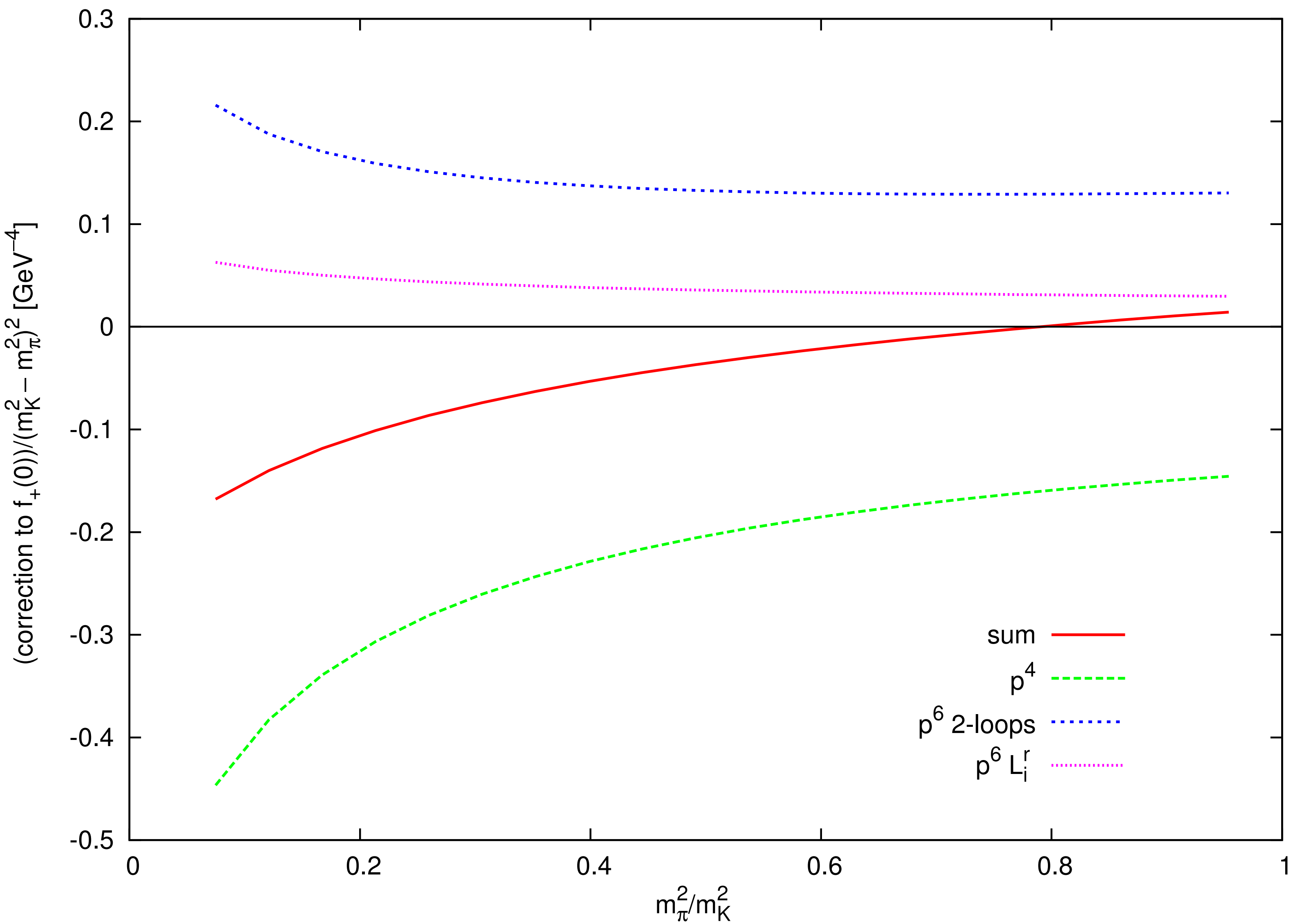
F_π fit 10, fixed $\hat{m}$



$f_+(0)$ fit 10, fixed m_K



$\left(f_+(0)\right) / \left(m_K^2 - m_\pi^2\right)^2$ fit 10, fixed m_K



$f_0(t)$ in $K_{\ell 3}$

Main Result: JB,Talavera

$$
\begin{aligned}
f_0(t) \;=\; & 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 \\
& + 8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
& - \frac{8}{F_\pi^4} t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6

$\Longrightarrow$

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.$$

≥ 3-flavour: PQChPT

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT
$\Longrightarrow$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

E.g. $L_1^r = L_0^{r(3pq)}/2 + L_1^{r(3pq)}$

PQChPT

One-loop: Bernard, Golterman, Sharpe, Shoresh, Pallante,. . .

with electromagnetism: JB,Danielsson, hep-lat/0610127

Two loops:

$m^2_{\pi^+}$ simplest mass case: JB,Danielsson,Lähde, hep-lat/0406017

F_{π^+}: JB,Lähde, hep-lat/0501014

F_{π^+}, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004

$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0602003

Neutral masses: JB,Danielsson, hep-lat/0606017

Lattice data: a and L extrapolations needed

Programs available from me (Fortran)

Formulas: `http://www.thep.lu.se/∼bijnens/chpt.html`

Wishing list

General:

- Quark-mass dependences everywhere
- Not only fits but also continuum infinite volume results at a given quark-mass (so we can also fit ourselves for studying other inputs)
- More use of the existing two-loop calculations
- Analytical NNLO only: not really fewer parameters
- Only more in LECs: p^4 *scattering* LECs also in masses/decay constants
- I.e. l_1^r, l_2^r ($n_F = 2$), L_1^r, L_2^r, L_3^r ($n_F = 3$), $\hat{L}_i^r$, $i = 0, 1, 2, 3$ (PQChPT)

Wishing list: Two-flavour

(with input from Gasser et al.)

- $\bar{l}_3$ and errors
- $\bar{l}_4$: from $F_\pi(m_q)$ and scalar radius: can lattice check this relation
- a_0^2 accurately predicted in terms of scalar radius: can lattice check this
- Isospin breaking in $\pi\pi$ scattering (important for CP violation in $K \to \pi\pi$)
- $\bar{l}_5 - \bar{l}_6$ Needed for $\pi \to \ell\nu\gamma$, can be had from Π_{AA}

Wishing list: ≥ 3-flavour

- Ideas on how to make all those calculations usable for you
- Three and more flavour: typically slow numerically
- large N_c suppressed couplings: i.e. m_s dependence of m_π, F_π
- L_4^r, L_6^r
- $sigma$ terms and scalar radii

Conclusions and final comments

- Lots of analytical work done in ChPT
- Use the correct ChPT
  - 2-flavour for varying $\hat{m}$ and possible for $N_f = 2$ and $N_f = 2 + 1$ at fixed m_s (but have different LECs)
  - otherwise 3-flavour
  - the various partially quenched versions
- Remember at which order in ChPT you compare things

Conclusions and final comments

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  - otherwise 3-flavour
  - the various partially quenched versions
- Remember at which order in ChPT you compare things
- Seen a lot of lattice work, looking forward to seeing more
- LECs in many talks: Boyle, Matsufuru, Urbach, Kuramashi and many parallel session talks