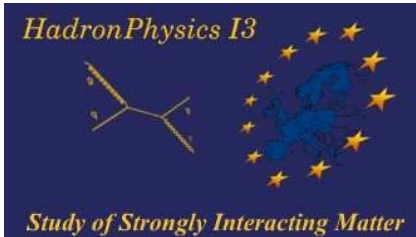




LUND
UNIVERSITY



QUARK MASS DEPENDENCE AT TWO LOOPS FOR MESON PROPERTIES

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Various ChPT: <http://www.thep.lu.se/~bijnens/chpt.html>

Alternative Title

What is Known About Low Energy Constants and Quark Mass Dependence in Chiral Perturbation Theory from the Continuum

Overview

- Chiral Perturbation Theory (ChPT, CHPT, χ PT)
- Expand in which quantities
- Two-flavour ChPT at NNLO: **one mass**
 - Calculations
 - LECs
 - Quark-mass dependence of m_π^2 , F_π
- Three-flavour ChPT at NNLO: **3-5 masses**
 - Calculations
 - What about p^6 LECs
 - Fits to data
 - Some results on quark mass dependence
- Even more flavours at NNLO (Partially Quenched)

Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

Chiral Perturbation Theory

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Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory*,
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

Power Counting

- ▶▶▶ gap in the spectrum \implies separation of scales
- ▶▶▶ with the lower degrees of freedom, build the most general effective Lagrangian
- ▶▶▶ $\infty \#$ parameters
- ▶▶▶ Where did my predictivity go ?

Power Counting

- ⇒ gap in the spectrum \implies separation of scales
 - ⇒ with the lower degrees of freedom, build the most general effective Lagrangian
 - ⇒ $\infty \#$ parameters
 - ⇒ Where did my predictivity go ?
- ⇒ Need some ordering principle: power counting

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting in momenta/masses

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Chiral Perturbation Theory

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken \implies 8 massless degrees of freedom
and interaction vanishes at zero momentum

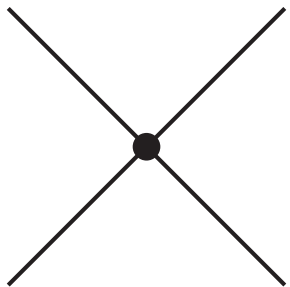
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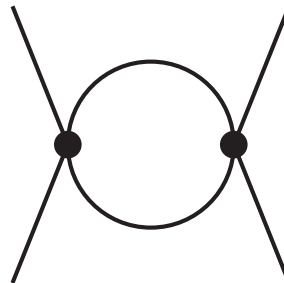
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Power counting in momenta: **Meson loops**



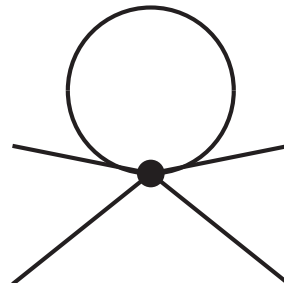
$$p^2$$



$$(p^2)^2 (1/p^2)^2 p^4 = p^4$$



$$1/p^2$$



$$(p^2) (1/p^2) p^4 = p^4$$

$$\int d^4 p$$

$$p^4$$

Chiral Perturbation Theory

Large subject:

- Steven Weinberg, Physica A96:327,1979: 1716 citations
- Juerg Gasser and Heiri Leutwyler, Nucl.Phys.B250:465,1985: 2194 citations
- Juerg Gasser and Heiri Leutwyler, Annals Phys.158:142,1984: 2153 citations
- Sum: **3618**
- K.G. Wilson, Phys. Rev. D14:2455,1974: **2961** citations
- Checked on 23/7/2007 in SPIRES

For lectures, review articles: see

<http://www.thep.lu.se/~bijmens/chpt.html>

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
- ...

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
 - Two or Three (or even more) Flavours
 - Strong interaction and couplings to external currents/densities
 - Including electromagnetism
 - Including weak nonleptonic interactions
 - Treating kaon as heavy

Only part pushed to NNLO: this talk

Lagrangians

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}.$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4} \{ \langle D_\mu U^\dagger D^\mu U \rangle + \langle \chi^\dagger U + \chi U^\dagger \rangle \},$

$$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu,$$

left and right external currents: $r(l)_\mu = v_\mu + (-)a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$

quark masses via scalar density: $s = \mathcal{M} + \dots$

$$\langle A \rangle = Tr_F (A)$$

Lagrangians

$$\begin{aligned}\mathcal{L}_4 = & L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\ & + L_3 \langle D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U \rangle + L_4 \langle D^\mu U^\dagger D_\mu U \rangle \langle \chi^\dagger U + \chi U^\dagger \rangle \\ & + L_5 \langle D^\mu U^\dagger D_\mu U (\chi^\dagger U + U^\dagger \chi) \rangle + L_6 \langle \chi^\dagger U + \chi U^\dagger \rangle^2 \\ & + L_7 \langle \chi^\dagger U - \chi U^\dagger \rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger \rangle \\ & - i L_9 \langle F_{\mu\nu}^R D^\mu U D^\nu U^\dagger + F_{\mu\nu}^L D^\mu U^\dagger D^\nu U \rangle \\ & + L_{10} \langle U^\dagger F_{\mu\nu}^R U F^{L\mu\nu} \rangle + H_1 \langle F_{\mu\nu}^R F^{R\mu\nu} + F_{\mu\nu}^L F^{L\mu\nu} \rangle + H_2 \langle \chi^\dagger \chi \rangle\end{aligned}$$

L_i : Low-energy-constants (LECs)

H_i : Values depend on definition of currents/densities

These absorb the divergences of loop diagrams: $L_i \rightarrow L_i^r$

Renormalization: order by order in the powercounting

Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	\hat{L}_i^r, \hat{H}_i^r	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2 : Weinberg 1966

p^4 : Gasser, Leutwyler 84,85

p^6 : JB, Colangelo, Ecker 99,00

- ▣ replica method \implies PQ obtained from N_F flavour
- ▣ All infinities known
- ▣ 3 flavour special case of 3+3 PQ: $\hat{L}_i^r, K_i^r \rightarrow L_i^r, C_i^r$
- ▣ 53 \rightarrow 52 [arXiv:0705.0576](https://arxiv.org/abs/0705.0576) [hep-ph]

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2} \log \frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu) \right] + \dots$$

$$M^2 = 2B\hat{m}$$

$B \neq B_0, F \neq F_0$ (two versus three-flavour)

LECs and μ

$$l_3^r(\mu)$$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i^r(\mu) - \log \frac{M_\pi^2}{\mu^2}.$$

Independent of the scale μ .

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

μ :

- m_π, m_K : chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$ large N_c arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exists
- Express orders in terms of physical masses and quantities (F_π , F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering

See e.g. Descotes-Genon talk

- I prefer physical masses
- Thresholds correct
- Chiral logs are from physical particles propagating

An example

$$m_\pi = \frac{m_0}{1 + a \frac{m_0}{f_0}}$$

$$f_\pi = \frac{f_0}{1 + b \frac{m_0}{f_0}}$$

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$$m_\pi = m_0 - a \frac{m_0^2}{f_0} + a^2 \frac{m_0^3}{f_0^2} + \dots$$

$$f_\pi = f_0 \left(1 - b \frac{m_0}{f_0} + b^2 \frac{m_0^2}{f_0^2} + \dots \right)$$

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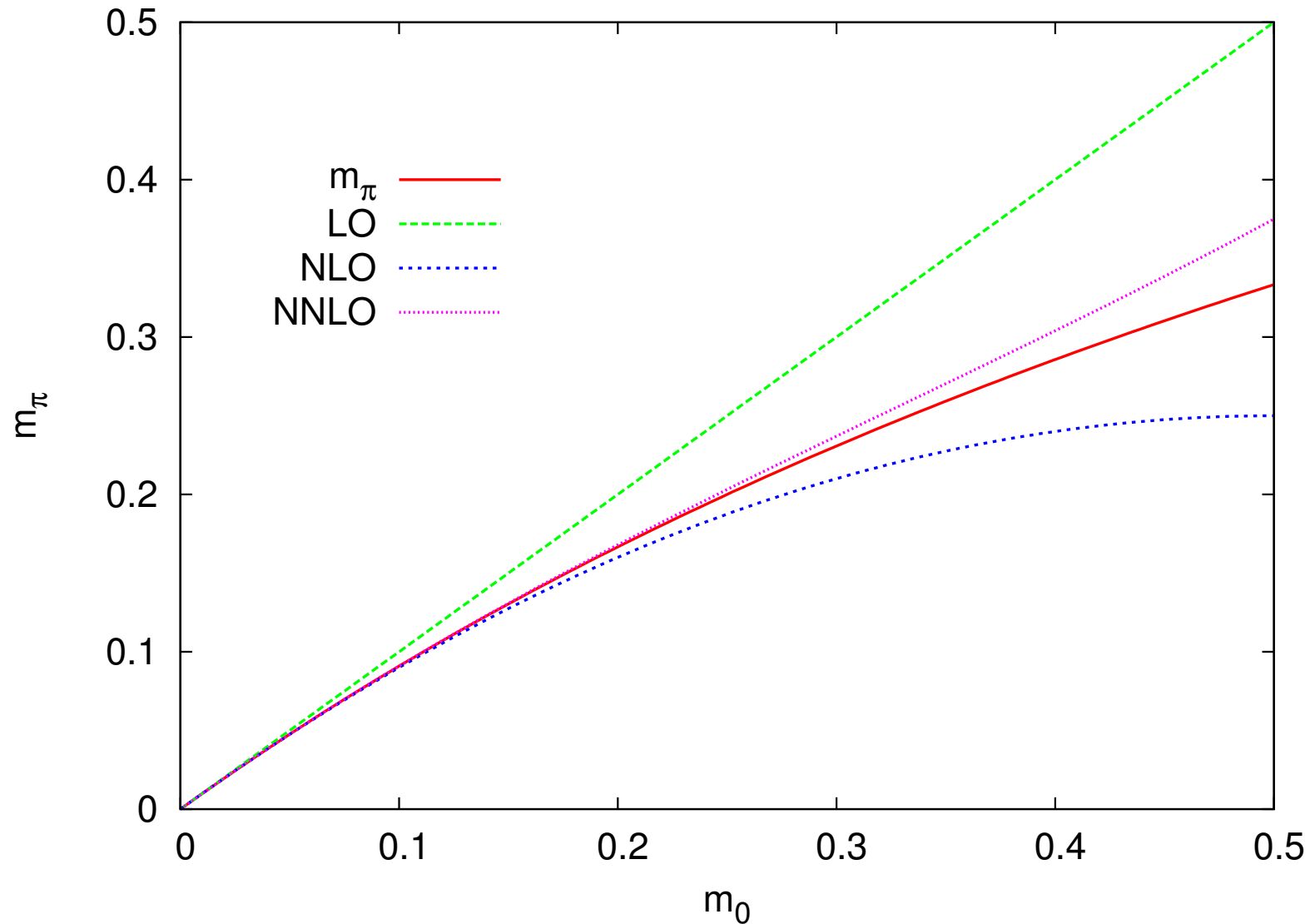
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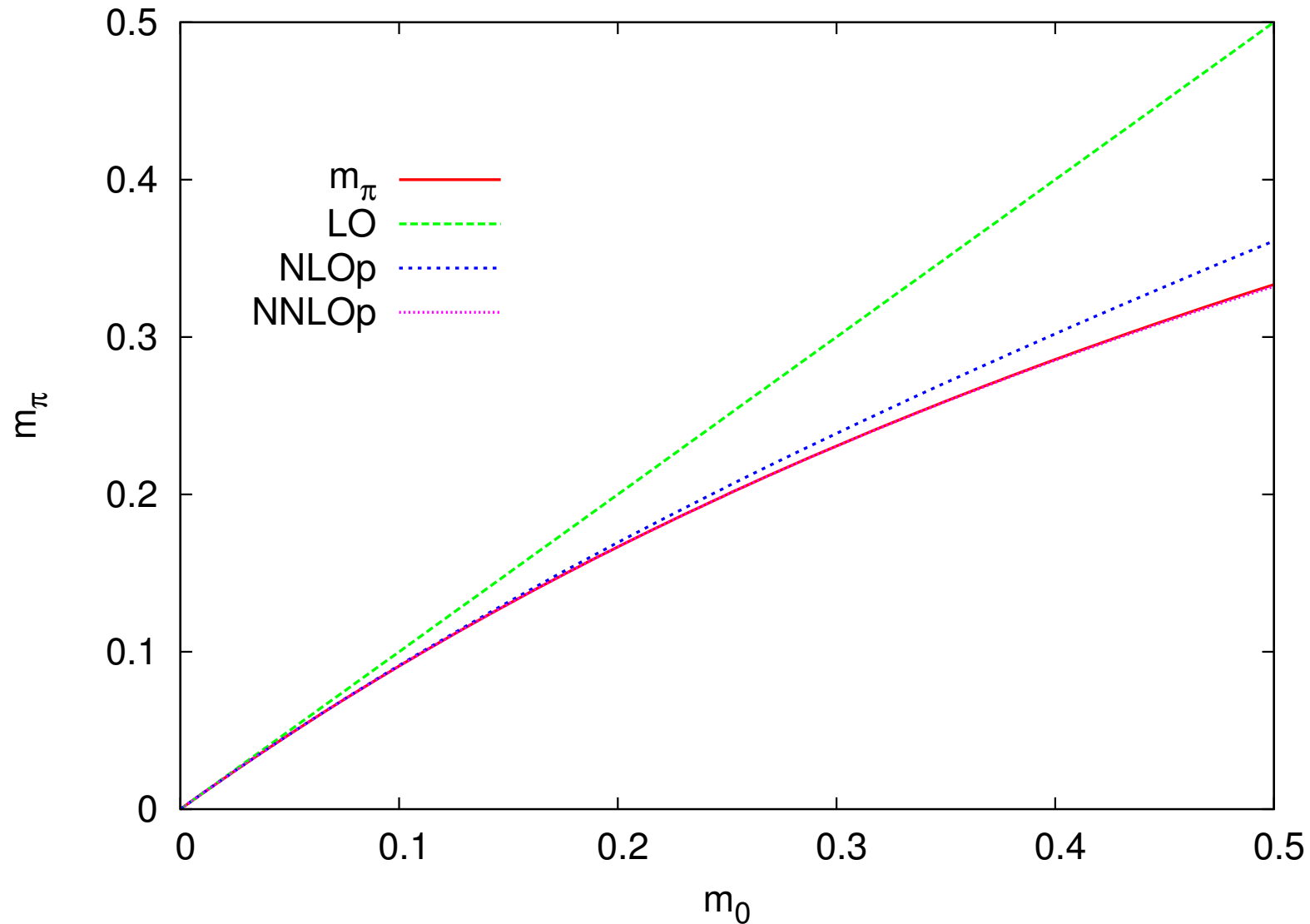
$$f_\pi = f_0 \left(1 - b \frac{m_\pi}{f_\pi} + b(2b - a) \frac{m_\pi^2}{f_\pi^2} + \dots \right)$$

$$a = 1 \quad b = 0.5 \quad f_0 = 1$$

An example: m_0/f_0



An example: m_π / f_π



Two-loop Two-flavour

Review paper on Two-Loops: JB, hep-ph/0604043 Prog. Part.
Nucl. Phys. 58 (2007) 521

Dispersive Calculation of the nonpolynomial part in q^2, s, t, u

- Gasser-Meißner: F_V, F_S : 1991 numerical
- Knecht-Moussallam-Stern-Fuchs: $\pi\pi$: 1995 analytical
- Colangelo-Finkemeier-Urech: F_V, F_S : 1996 analytical

Two-Loop Two-flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \rightarrow \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \rightarrow \pi^+\pi^-$, F_π , m_π : 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi$, F_π , m_π : 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t)$, $F_{S\pi}$: 1998
- JB-Talavera: $\pi \rightarrow \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \rightarrow \pi^0\pi^0$, $\gamma\gamma \rightarrow \pi^+\pi^-$: 2005-2006
- m_π , F_π , F_V , F_S , $\pi\pi$: simple analytical forms
- Colangelo-(Dürr-)Haefeli: Finite volume F_π , m_π : 2005-2006

LECs

\bar{l}_1 to \bar{l}_4 : ChPT at order p^6 and the Roy equation analysis in $\pi\pi$ and F_S Colangelo, Gasser and Leutwyler, *Nucl. Phys. B* 603 (2001) 125 [hep-ph/0103088]

\bar{l}_5 and \bar{l}_6 : from F_V and $\pi \rightarrow \ell\nu\gamma$ JB,(Colangelo,)Talavera

$$\bar{l}_1 = -0.4 \pm 0.6 ,$$

$$\bar{l}_2 = 4.3 \pm 0.1 ,$$

$$\bar{l}_3 = 2.9 \pm 2.4 ,$$

$$\bar{l}_4 = 4.4 \pm 0.2 ,$$

$$\bar{l}_6 - \bar{l}_5 = 3.0 \pm 0.3 ,$$

$$\bar{l}_6 = 16.0 \pm 0.5 \pm 0.7 .$$

$l_7 \sim 5 \cdot 10^{-3}$ from π^0 - η mixing Gasser, Leutwyler 1984

LECs

Some combinations of order p^6 LECs are known as well: curvature of the scalar and vector formfactor, two more combinations from $\pi\pi$ scattering (implicit in b_5 and b_6)

Note: c_i^r for m_π , f_π , $\pi\pi$: small effect

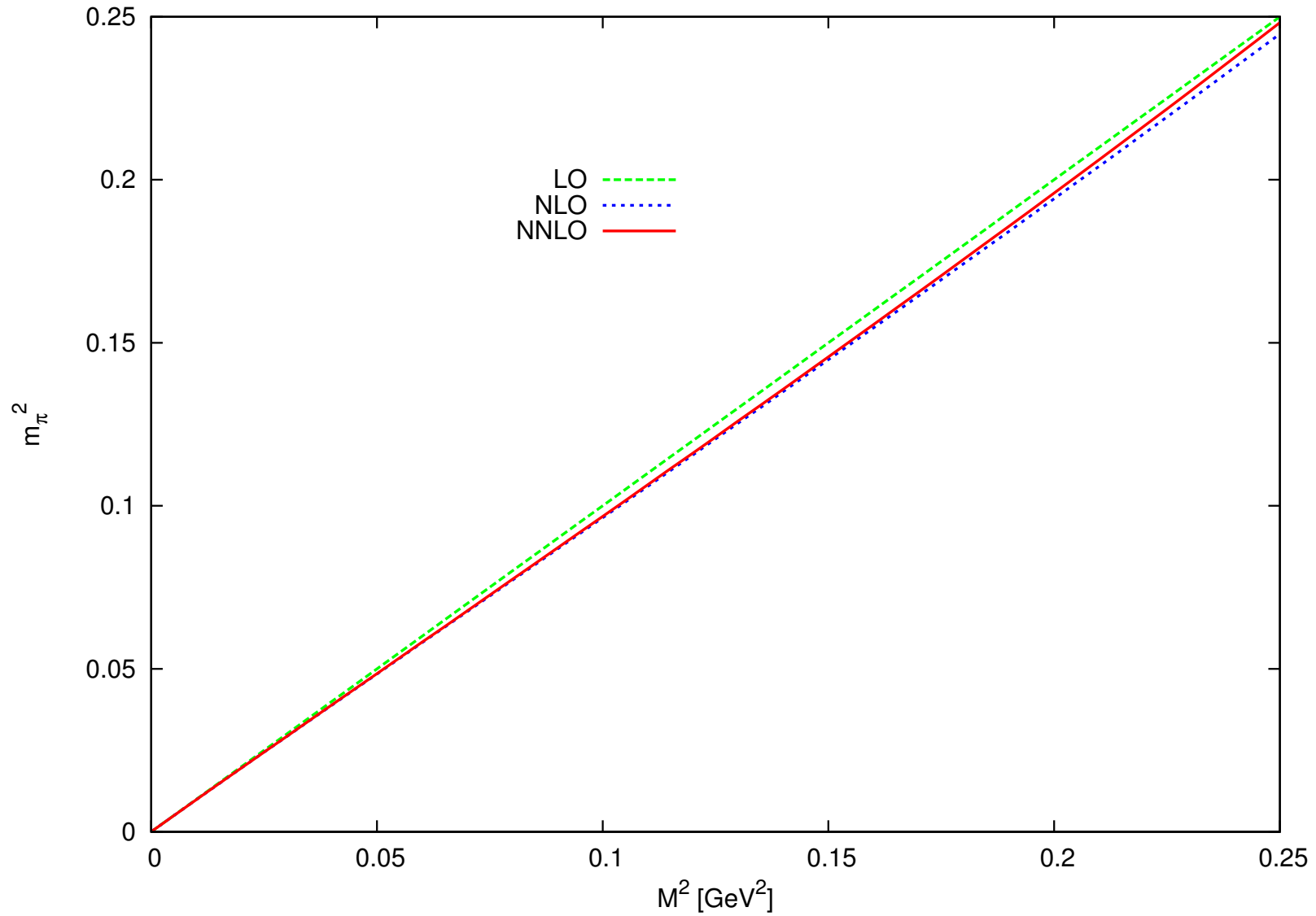
$c_i^r(770MeV) = 0$ for plots shown

expansion in m_π^2/F_π^2 shown

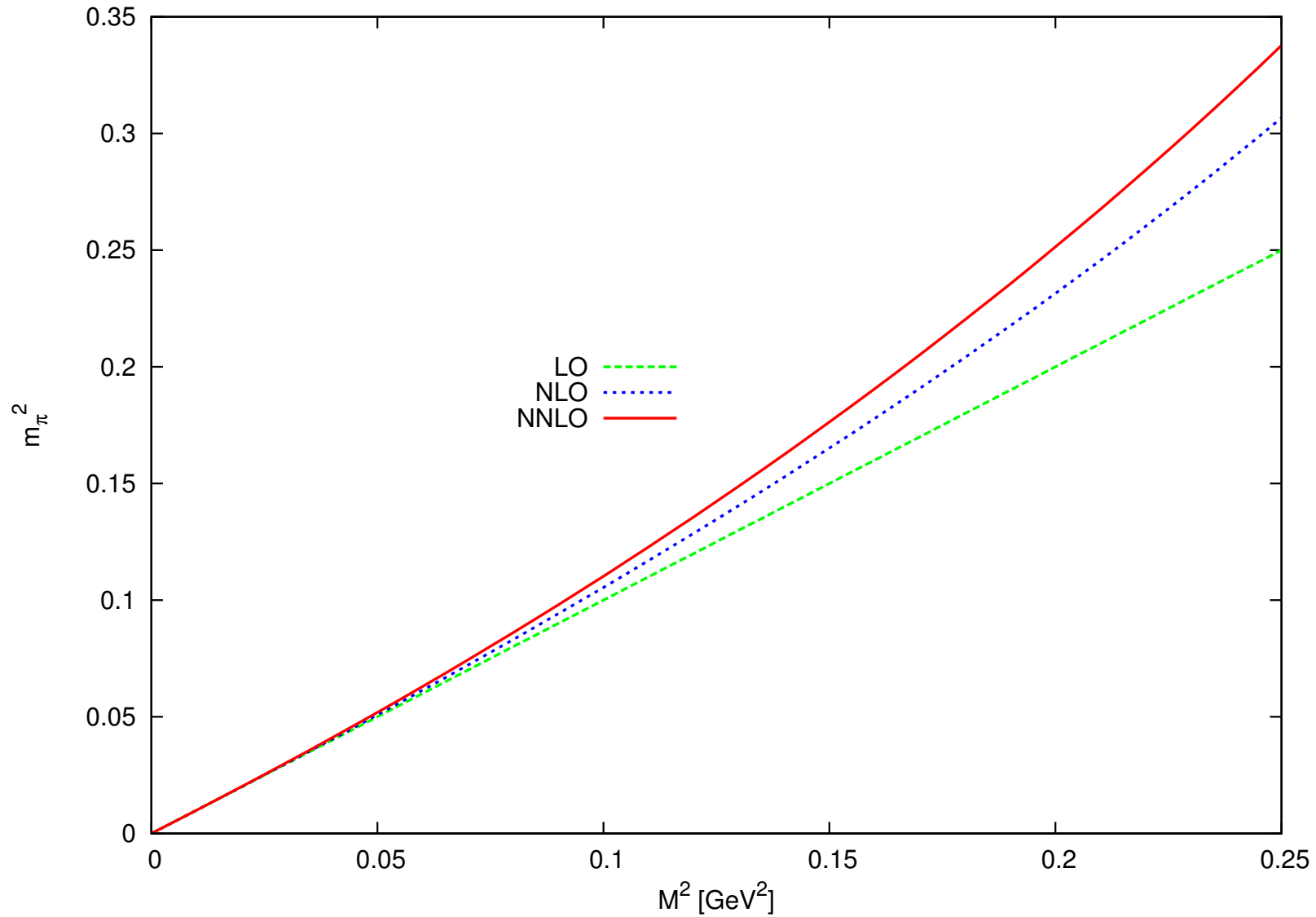
General observation:

- Obtainable from kinematical dependences: known
- Only via quark-mass dependence: poorly known

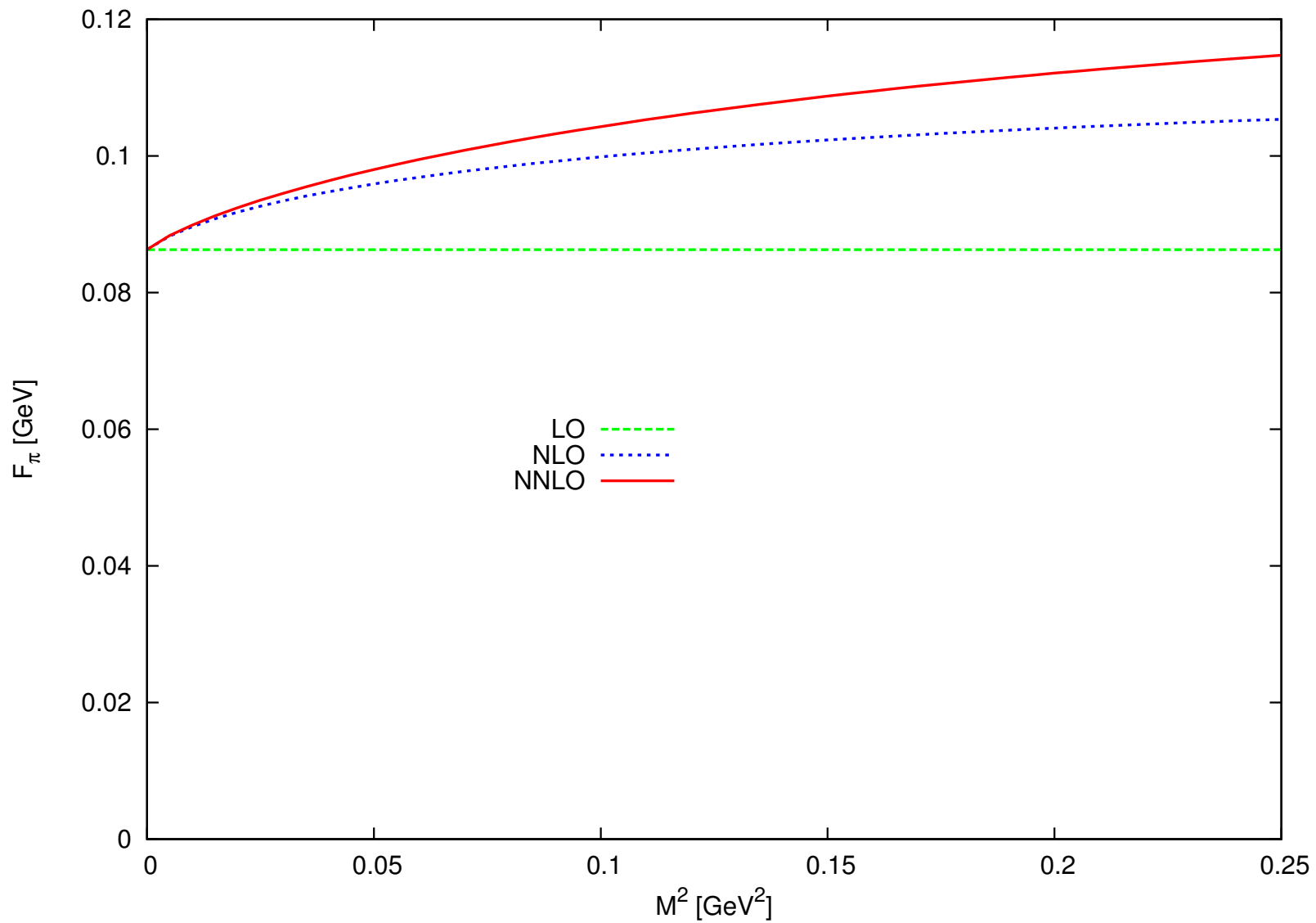
m_π^2



$$m_\pi^2 (\bar{l}_3 = 0)$$



F_π



Two-loop Three-flavour, ≤ 2001

- $\Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVK}$ Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
- $\Pi_{VV\rho\omega}$ Maltman
- $\Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta$ Kambor, Golowich; Amorós, JB, Talavera
- Π_{SS} Moussallam L_4^r, L_6^r
- $\Pi_{VVK}, \Pi_{AAK}, F_K, m_K$ Amorós, JB, Talavera
- $K_{\ell 4}, \langle \bar{q}q \rangle$ Amorós, JB, Talavera L_1^r, L_2^r, L_3^r
- $F_M, m_M, \langle \bar{q}q \rangle (m_u \neq m_d)$ Amorós, JB, Talavera $L_{5,7,8}^r, m_u/m_d$

Two-loop Three-flavour, ≥ 2001

- $F_{V\pi}, F_{VK^+}, F_{VK^0}$ Post, Schilcher; JB, Talavera L_9^r
- $K_{\ell 3}$ Post, Schilcher; JB, Talavera V_{us}
- $F_{S\pi}, F_{SK}$ (includes σ -terms) JB, Dhonte L_4^r, L_6^r
- $K, \pi \rightarrow \ell\nu\gamma$ Geng, Ho, Wu L_{10}^r
- $\pi\pi$ JB, Dhonte, Talavera
- πK JB, Dhonte, Talavera
- relation l_i^r and L_i^r, C_i^r Gasser, Haefeli, Ivanov, Schmid
- Finite volume $\langle \bar{q}q \rangle$ JB, Ghorbani

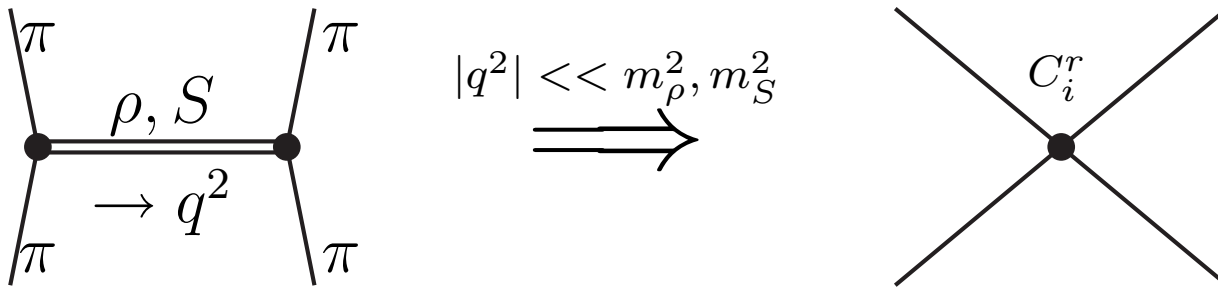
Two-loop Three-flavour

Known to be in progress

- $\eta \rightarrow 3\pi$: being written up JB,Ghorbani
- $K_{\ell 3}$ iso: preliminary results available JB,Ghorbani
- Finite Volume: sunsetintegrals being written up JB,Lähde
- relation c_i^r and L_i^r, C_i^r Gasser,Haefeli,Ivanov,Schmid

Most analysis use:

C_i^r from (single) resonance approximation



Motivated by large N_c : large effort goes in this

Ananthanarayan, JB, Cirigliano, Donoghue, Ecker, Gamiz, Golterman, Kaiser, Knecht, Peris, Pich, Prades, Portoles, de Rafael, . . .

$$\begin{aligned} \mathcal{L}_V = & -\frac{1}{4} \langle V_{\mu\nu} V^{\mu\nu} \rangle + \frac{1}{2} m_V^2 \langle V_\mu V^\mu \rangle - \frac{f_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle \\ & - \frac{ig_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle + f_\chi \langle V_\mu [u^\mu, \chi_-] \rangle \end{aligned}$$

$$\mathcal{L}_A = -\frac{1}{4} \langle A_{\mu\nu} A^{\mu\nu} \rangle + \frac{1}{2} m_A^2 \langle A_\mu A^\mu \rangle - \frac{f_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle$$

$$\mathcal{L}_S = \frac{1}{2} \langle \nabla^\mu S \nabla_\mu S - M_S^2 S^2 \rangle + c_d \langle S u^\mu u_\mu \rangle + c_m \langle S \chi_+ \rangle$$

$$\mathcal{L}_{\eta'} = \frac{1}{2} \partial_\mu P_1 \partial^\mu P_1 - \frac{1}{2} M_{\eta'}^2 P_1^2 + i\tilde{d}_m P_1 \langle \chi_- \rangle.$$

$$f_V = 0.20, \quad f_\chi = -0.025, \quad g_V = 0.09, \quad c_m = 42 \text{ MeV}, \quad c_d = 32 \text{ MeV}, \quad \tilde{d}_m = 20 \text{ MeV},$$

$$m_V = m_\rho = 0.77 \text{ GeV}, \quad m_A = m_{a_1} = 1.23 \text{ GeV}, \quad m_S = 0.98 \text{ GeV}, \quad m_{P_1} = 0.958 \text{ GeV}$$

f_V, g_V, f_χ, f_A : experiment

c_m and c_d from resonance saturation at $\mathcal{O}(p^4)$

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

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What we did about it:

- Vary resonance estimate by factor of two
- Vary the scale μ at which it applies: 600-900 MeV
- Check the estimates for the measured ones
- Again: kinematic can be had, quark-mass dependence difficult

Inputs

$K_{\ell 4}$: $F(0)$, $G(0)$, λ

E865 BNL

$m_{\pi^0}^2$, m_{η}^2 , $m_{K^+}^2$, $m_{K^0}^2$

em with Dashen violation

F_{π^+}

F_{K^+}/F_{π^+}

m_s/\hat{m}

24 (26)

$\hat{m} = (m_u + m_d)/2$

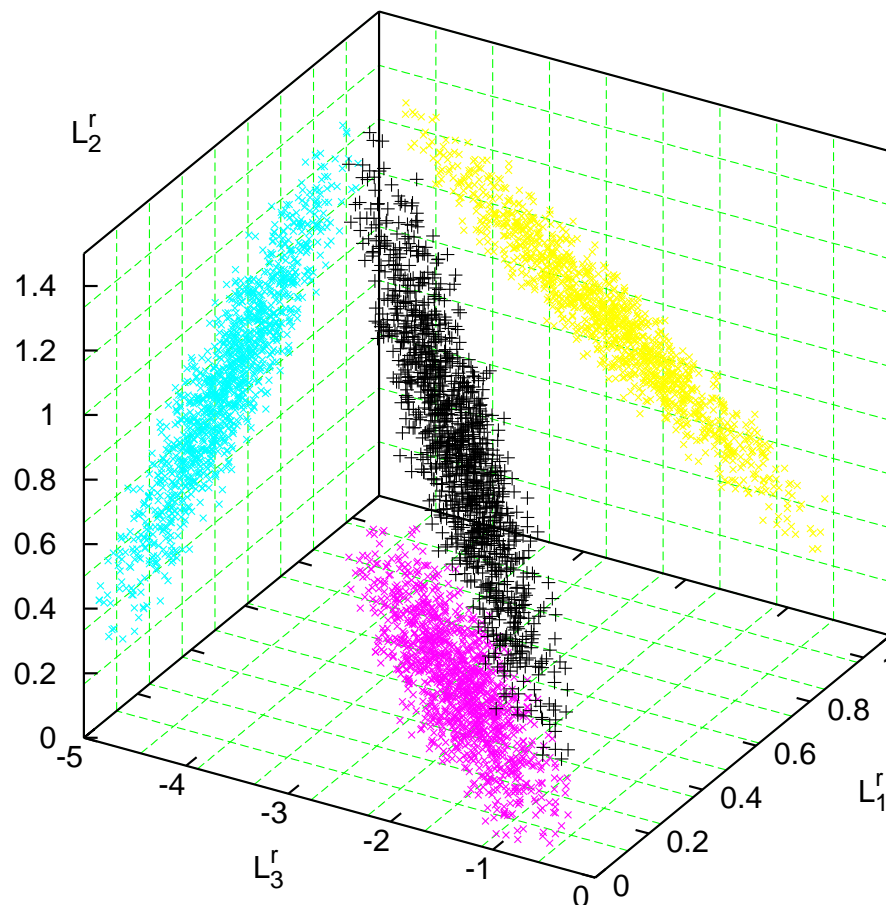
L_4^r , L_6^r

Outputs: I

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

- ▣ errors are very correlated
- ▣ $\mu = 770$ MeV; 550 or 1000 within errors
- ▣ varying C_i^r factor 2 about errors
- ▣ $L_4^r, L_6^r \approx -0.3, \dots, 0.6 \cdot 10^{-3}$ OK
- ▣ fit B: small corrections to pion “sigma” term, fit scalar radius
- ▣ fit D: fit $\pi\pi$ and πK thresholds

Correlations



(older fit)

$$10^3 L_1^r = 0.52 \pm 0.23$$

$$10^3 L_2^r = 0.72 \pm 0.24$$

$$10^3 L_3^r = -2.70 \pm 0.99$$

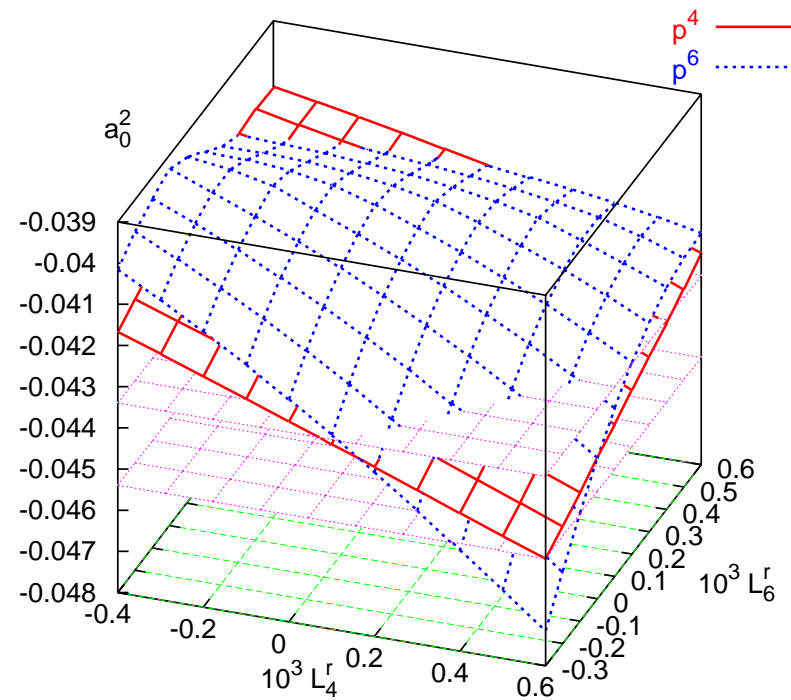
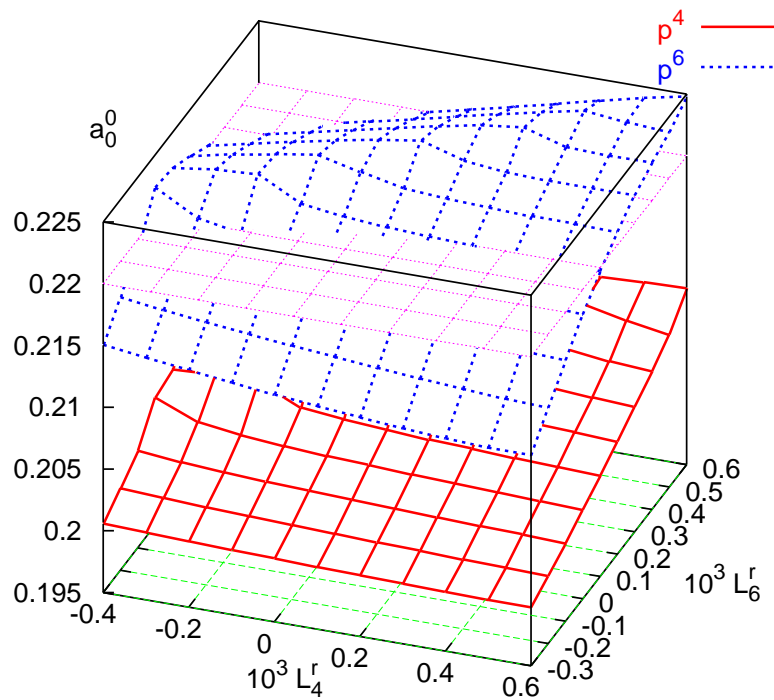
Outputs: II

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
$m_\pi^2: p^4, p^6$	0.006,0.258	0.009, $\equiv 0$	-0.138,0.009	-0.091,0.133
$m_K^2: p^4, p^6$	0.007,0.306	0.075, $\equiv 0$	-0.149,0.094	-0.096,0.201
$m_\eta^2: p^4, p^6$	-0.052,0.318	0.013, $\equiv 0$	-0.197,0.073	-0.151,0.197
m_u/m_d	0.45 ± 0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}: p^4, p^6$	0.169,0.051	0.22, $\equiv 0$	0.153,0.067	0.159,0.061

▣▣▣▣ $m_u = 0$ always very far from the fits

▣▣▣▣ F_0 : pion decay constant in the chiral limit

$\pi\pi$

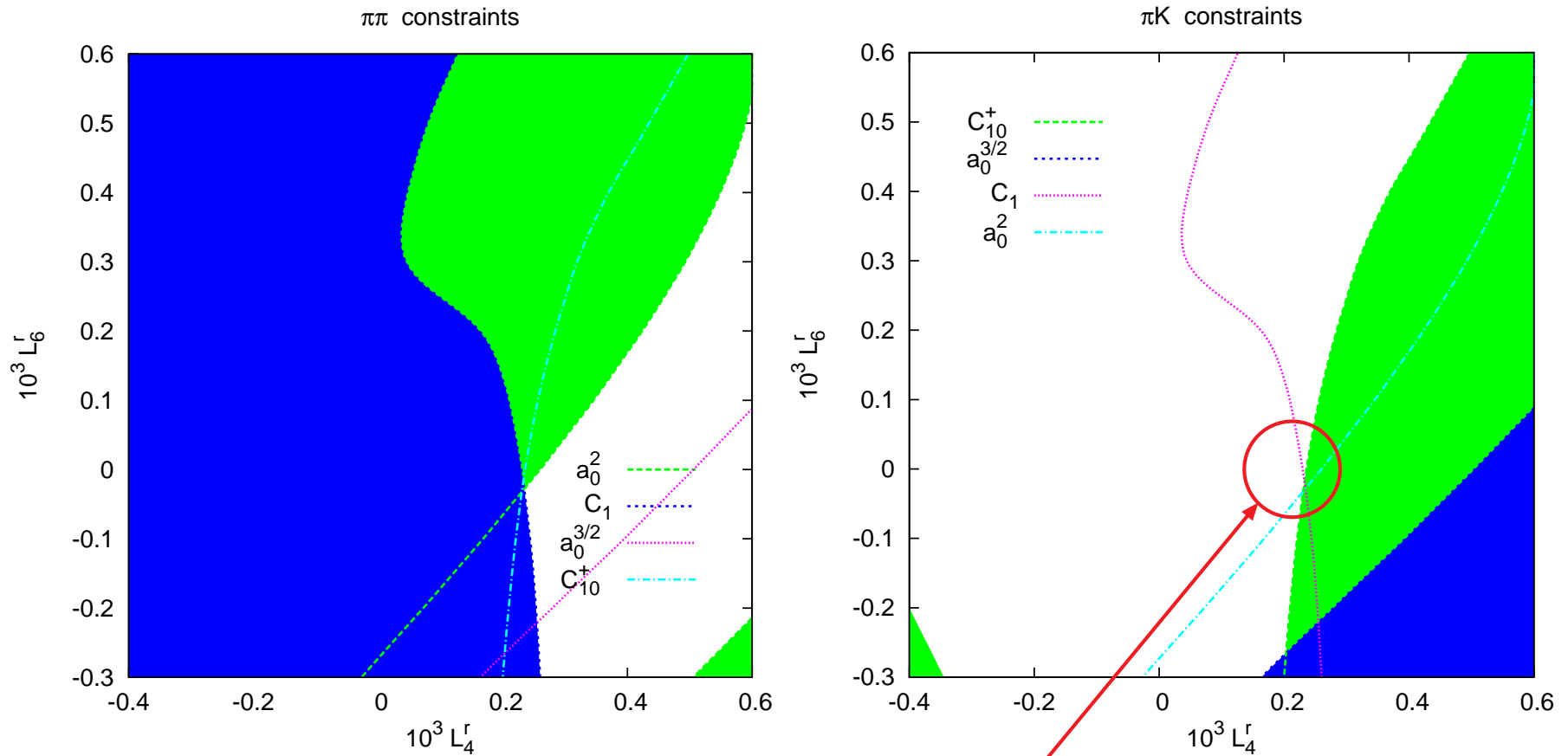


$$a_0^0 = 0.220 \pm 0.005, a_0^2 = -0.0444 \pm 0.0010$$

Colangelo, Gasser, Leutwyler

$$a_0^0 = 0.159 \quad a_0^2 = -0.0454 \text{ at order } p^2$$

$\pi\pi$ and πK



preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

General fitting needs more work and systematic studies

Quark mass dependences

Updates of plots in

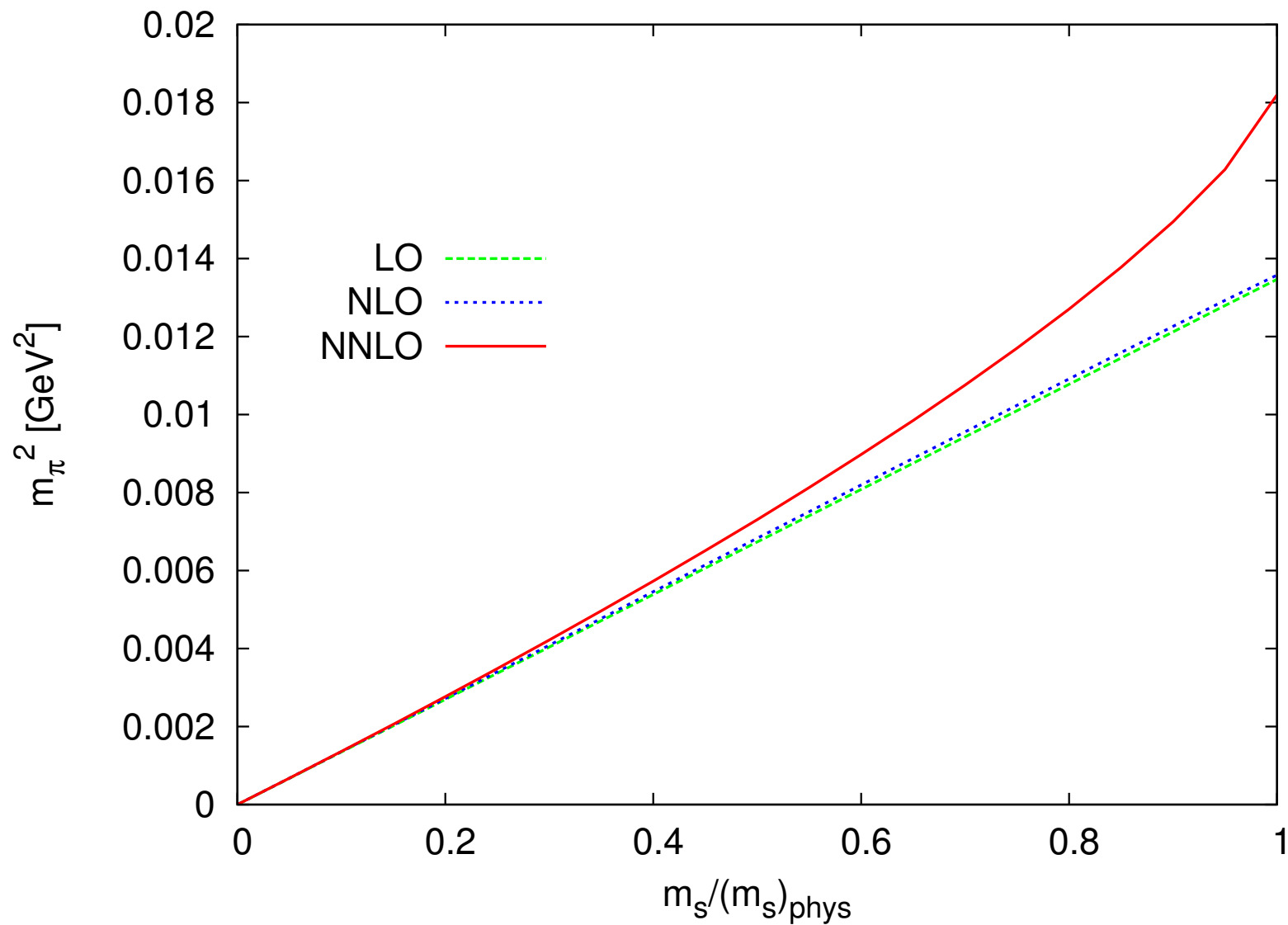
Amorós, JB and Talavera, hep-ph/0003258, Nucl. Phys. B585 (2000) 293

Some new ones

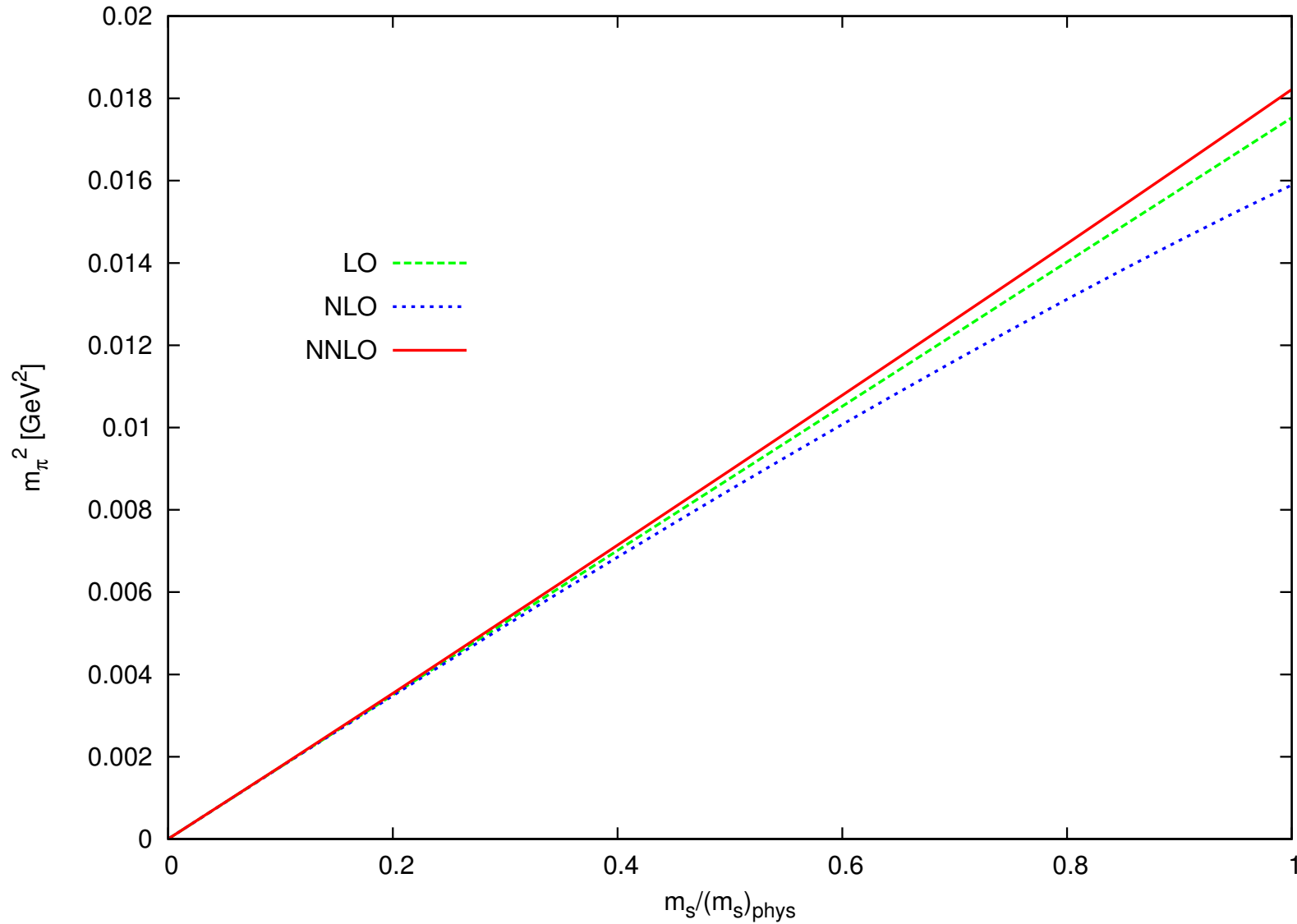
Procedure: calculate a consistent set of $m_\pi, m_K, m_\eta, f_\pi$ with the given input values (done iteratively)

- vary $m_s / (m_s)_{phys}$, keep $m_s / \hat{m} = 24$
 $m_\pi^2, m_K^2, F_\pi, F_K$
- vary $m_s / (m_s)_{phys}$ keep \hat{m} fixed
 m_π^2, F_π
- vary m_π , keep m_K fixed
 $f_+(0)$: the formfactor in $K_{\ell 3}$ decays
 $f_+(0), f_+(0) / (m_K^2 - m_\pi^2)^2$

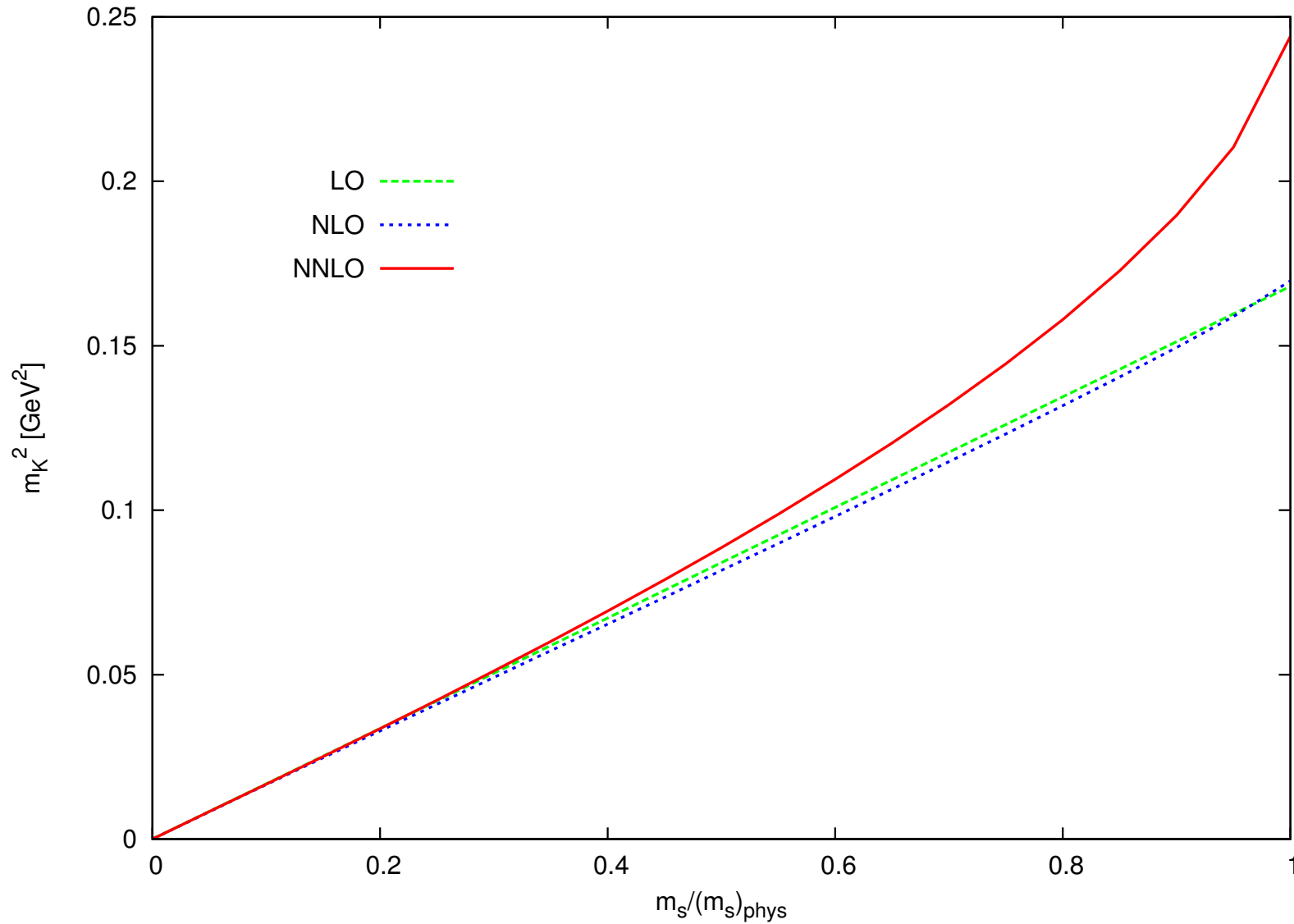
m_π^2 fit 10



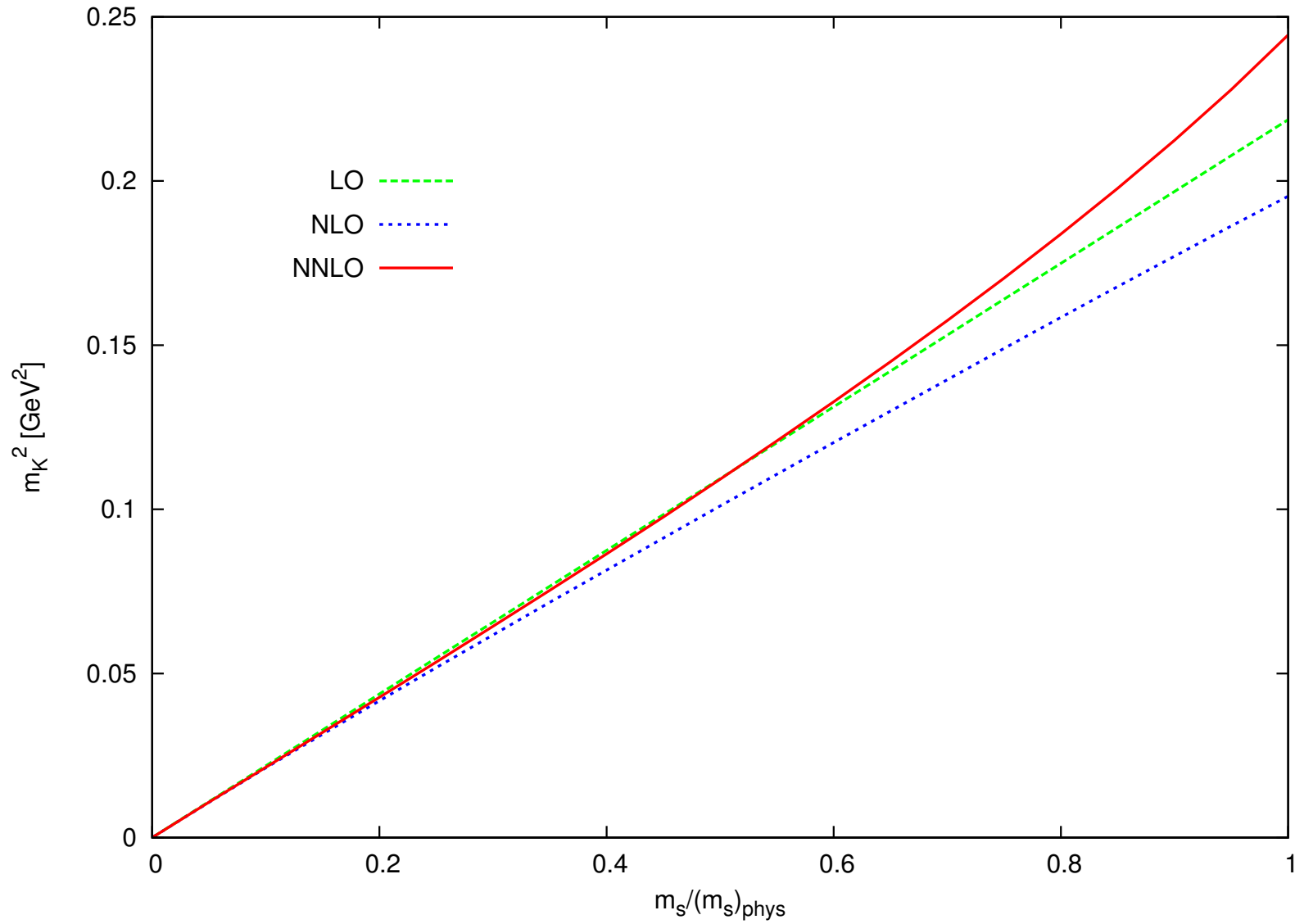
m_π^2 fit D



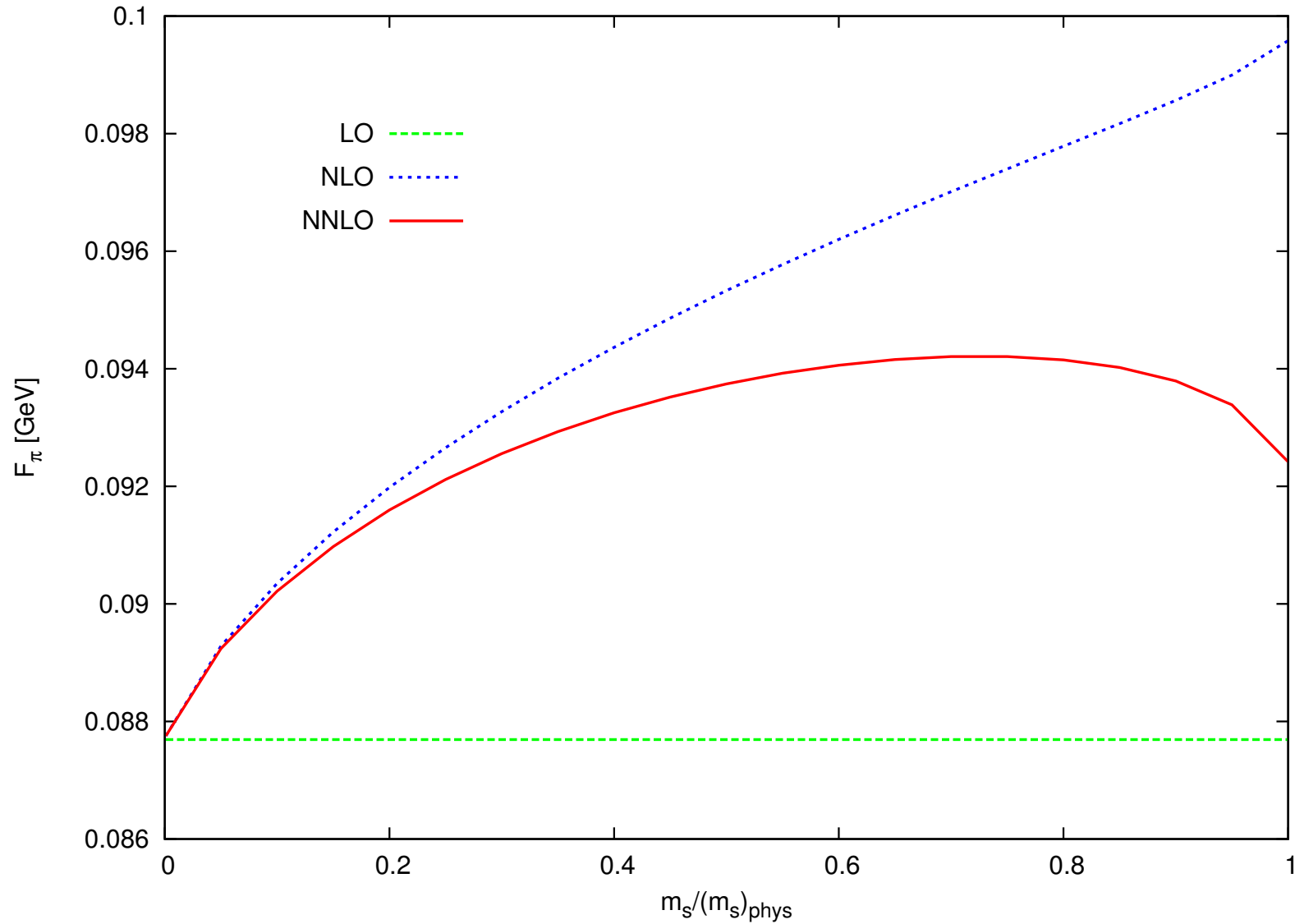
m_K^2 fit 10



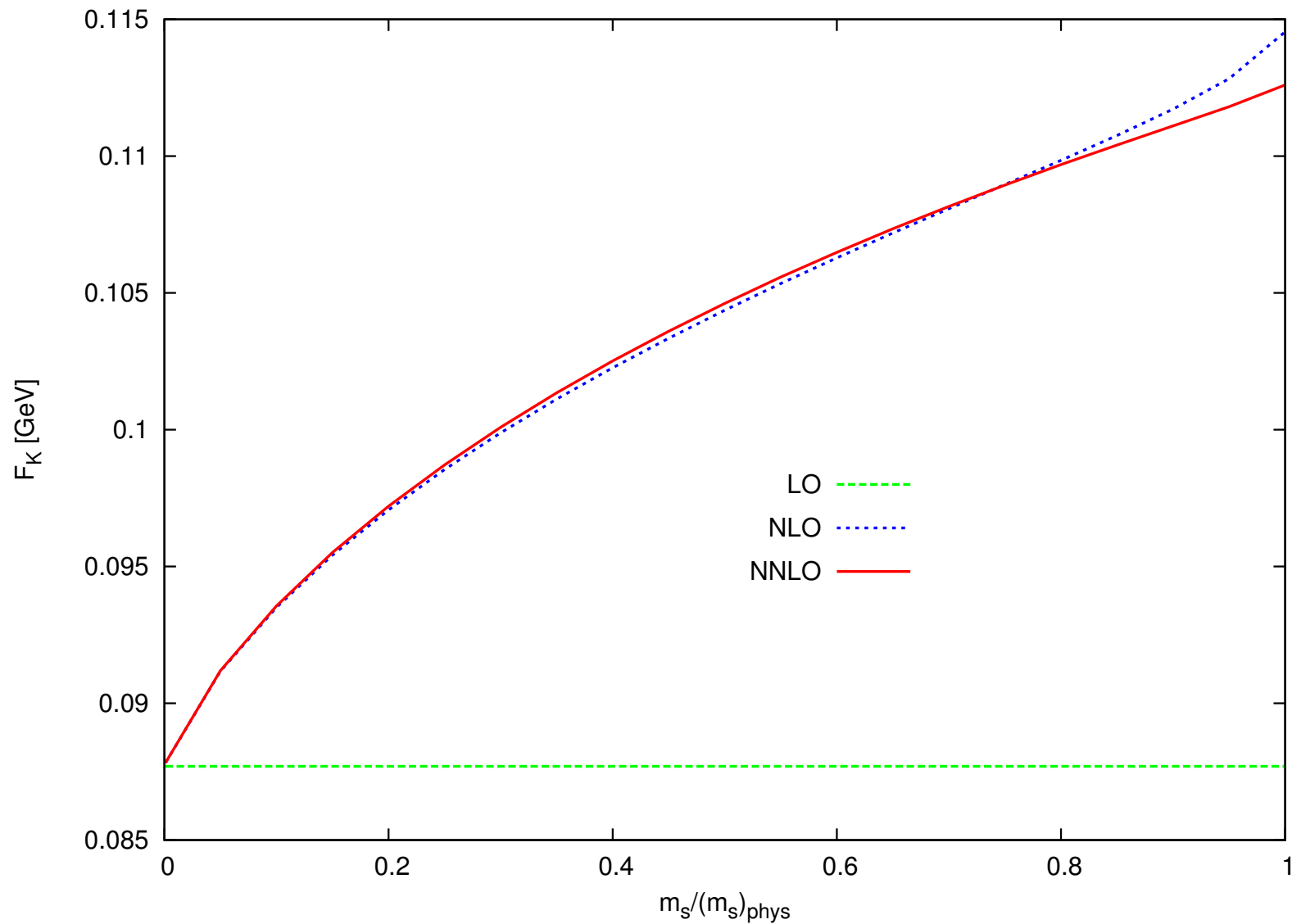
m_K^2 fit D



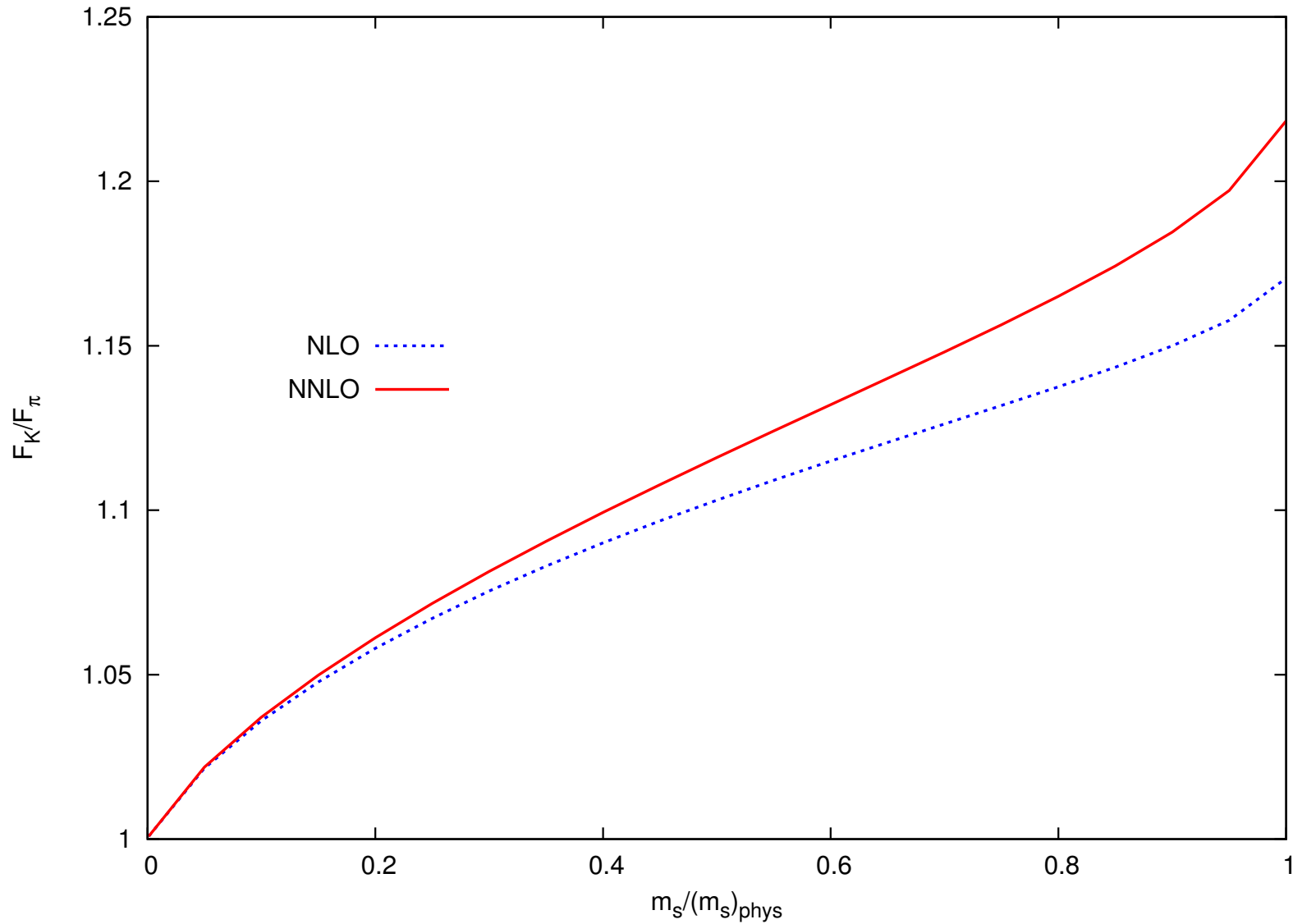
F_π fit 10



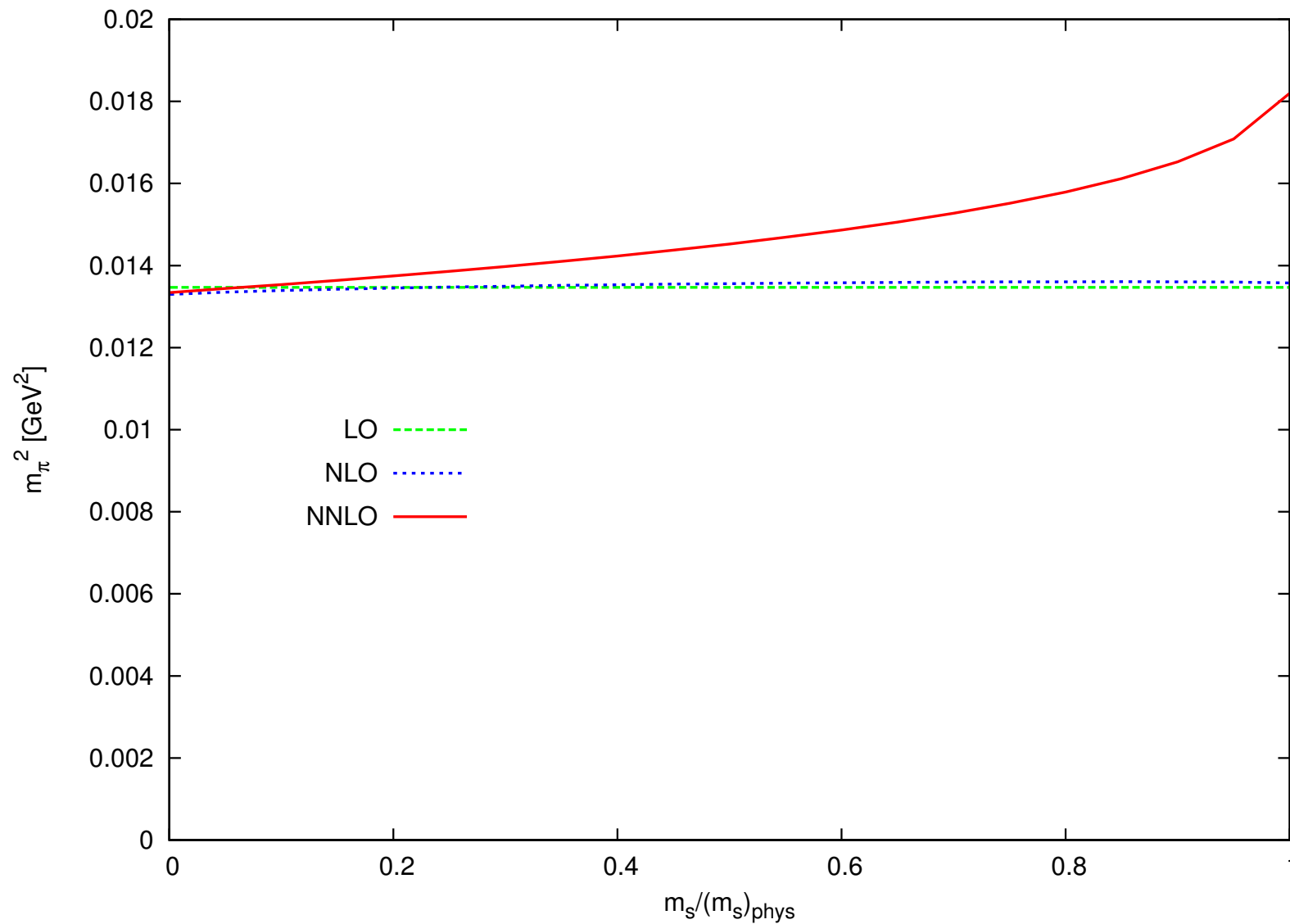
F_K fit 10



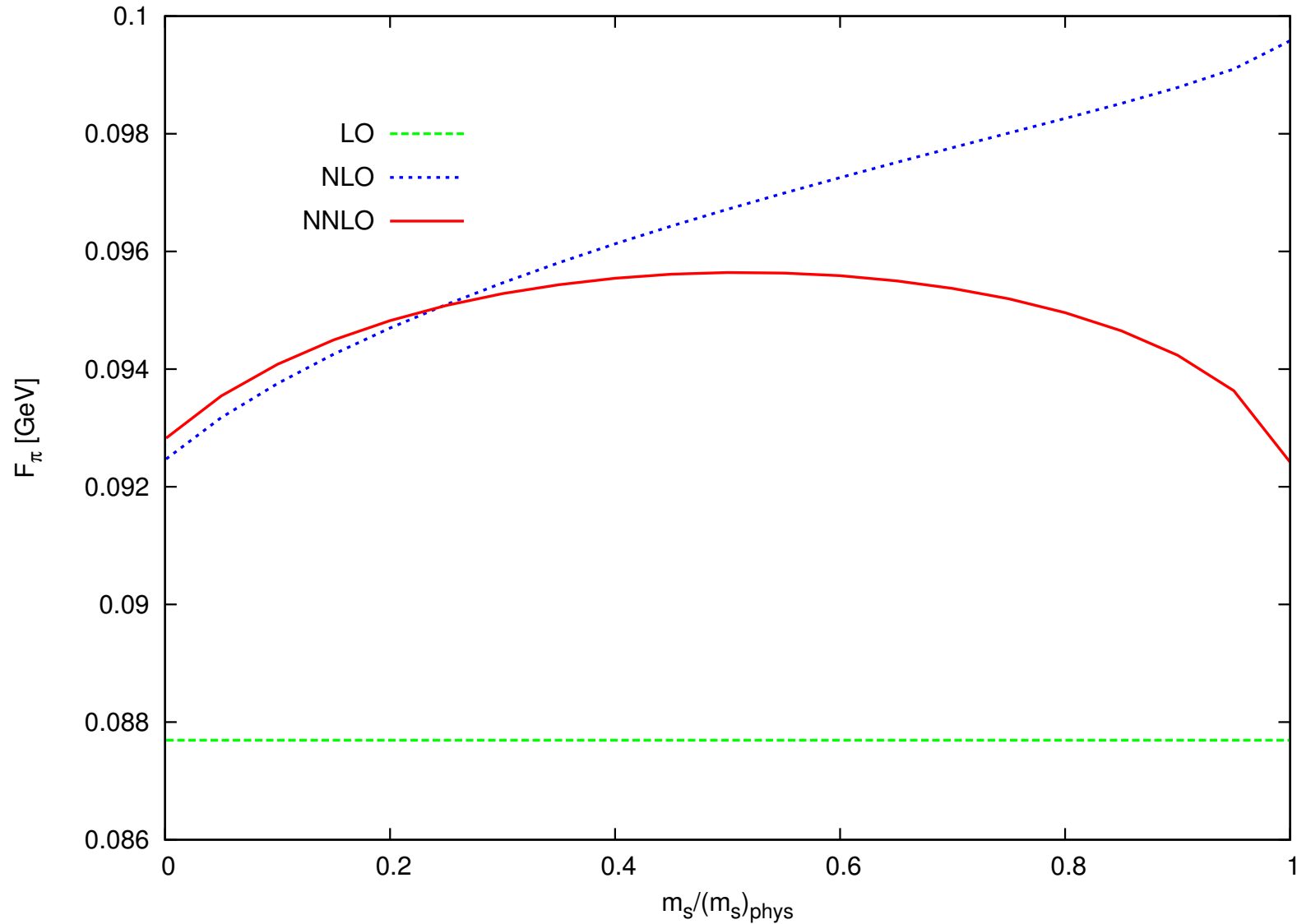
F_K / F_π fit 10



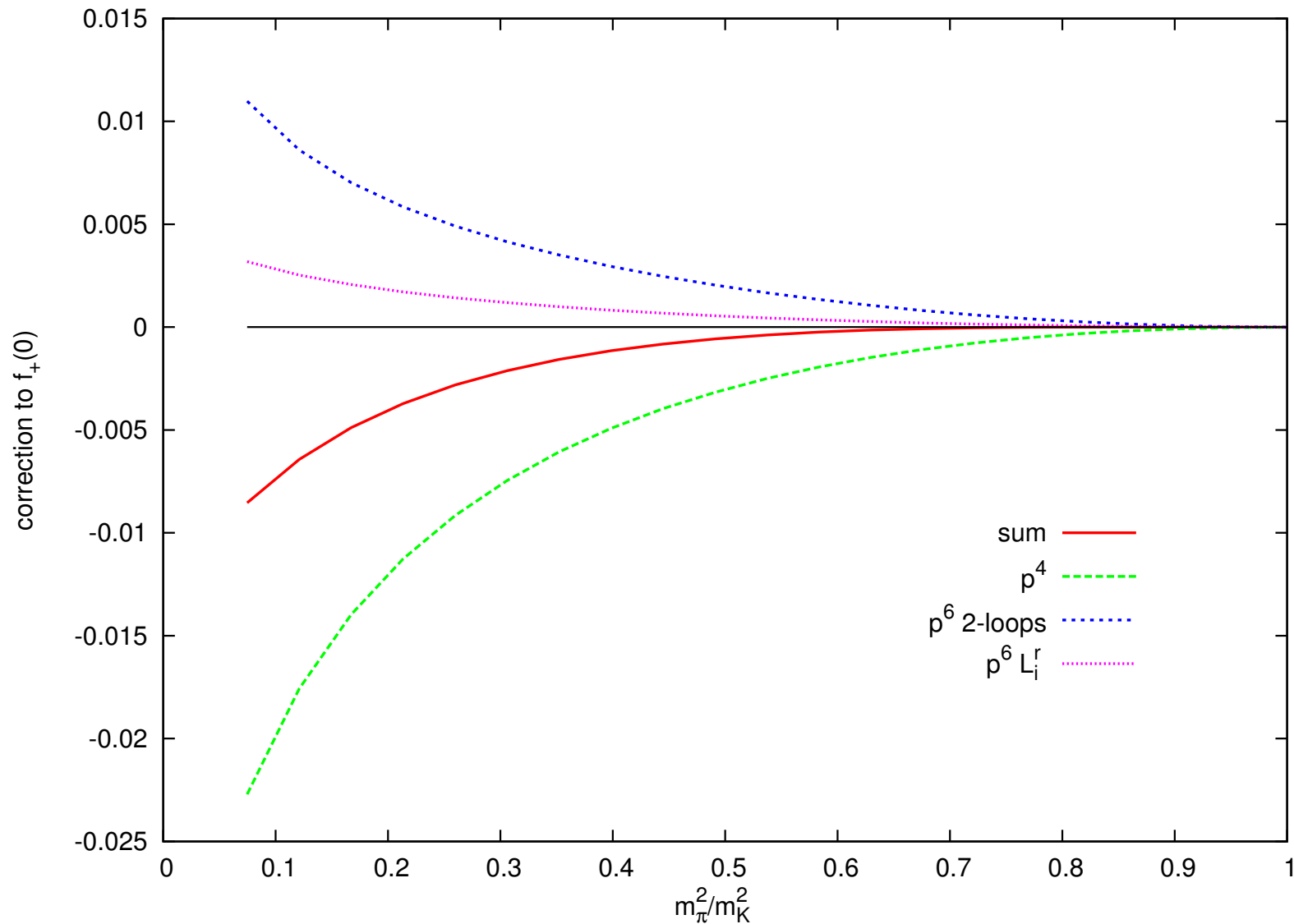
m_π^2 fit 10, fixed \hat{m}



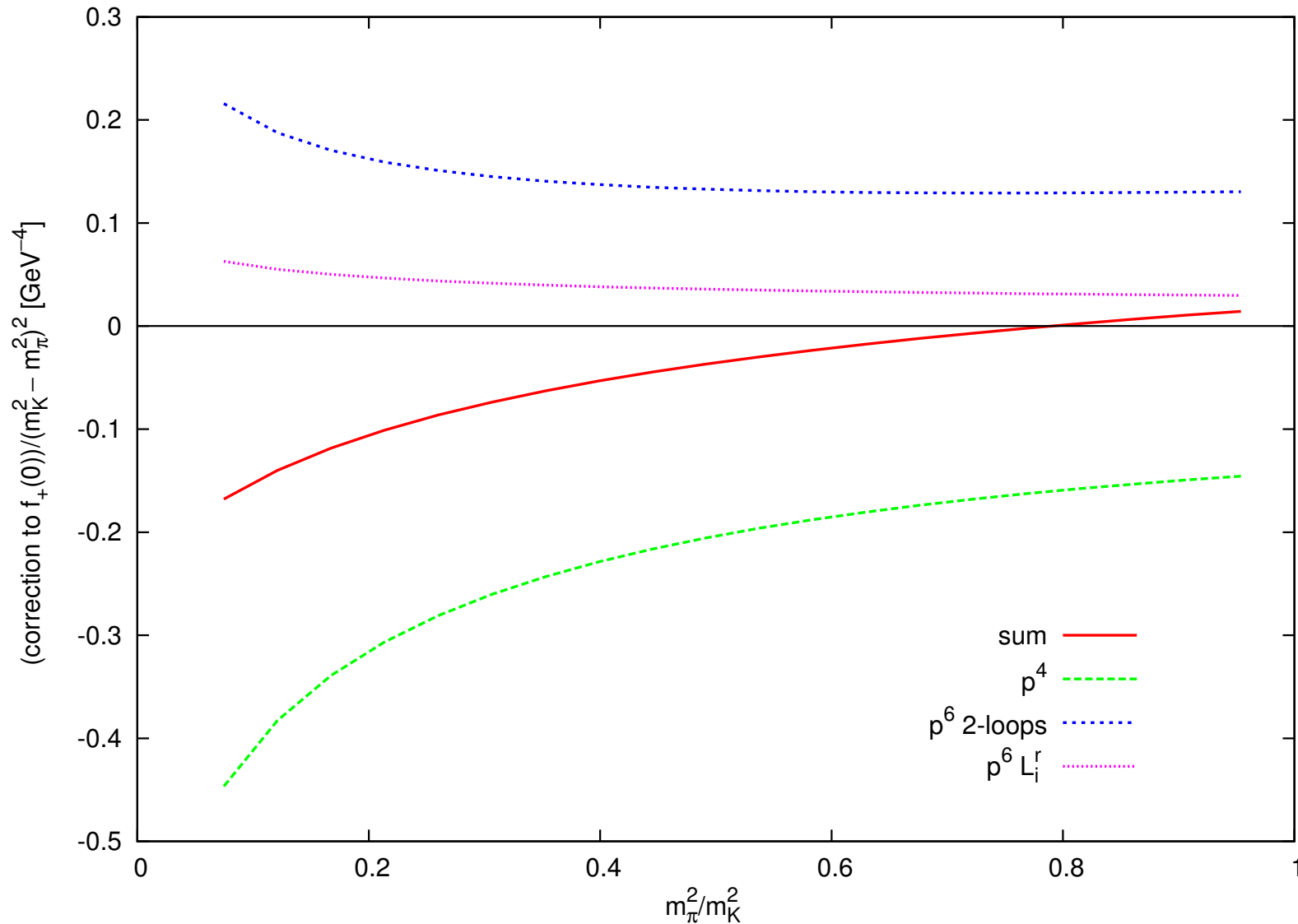
F_π fit 10, fixed \hat{m}



$f_+(0)$ fit 10, fixed m_K



$(f_+(0)) / (m_K^2 - m_\pi^2)^2$ fit 10, fixed m_K



$f_0(t)$ in $K_{\ell 3}$

Main Result: JB, Talavera

$$\begin{aligned} f_0(t) = & 1 - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 \\ & + 8 \frac{t}{F_\pi^4} (2C_{12}^r + C_{34}^r) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) \\ & - \frac{8}{F_\pi^4} t^2 C_{12}^r + \bar{\Delta}(t) + \Delta(0). \end{aligned}$$

$\bar{\Delta}(t)$ and $\Delta(0)$ contain **NO** C_i^r and only depend on the L_i^r at order p^6

\implies

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r].$$

\geq 3-flavour: PQChPT

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT
 \implies LECs from ChPT are linear combinations of LECs of PQChPT with the **same** number of sea quarks.

$$\text{E.g. } L_1^r = L_0^{r(3pq)} / 2 + L_1^{r(3pq)}$$

PQChPT

One-loop: Bernard, Golterman, Sharpe, Shoresh, Pallante,...

with electromagnetism: JB, Danielsson, hep-lat/0610127

Two loops:

$m_{\pi^+}^2$ **simplest mass case:** JB, Danielsson, Lähde, hep-lat/0406017

F_{π^+} : JB, Lähde, hep-lat/0501014

F_{π^+} , $m_{\pi^+}^2$ **two sea quarks:** JB, Lähde, hep-lat/0506004

$m_{\pi^+}^2$: JB, Danielsson, Lähde, hep-lat/0602003

Neutral masses: JB, Danielsson, hep-lat/0606017

Lattice data: a and L extrapolations needed

Programs available from me (Fortran)

Formulas: <http://www.thep.lu.se/~bijmens/chpt.html>

Wishing list

General:

- Quark-mass dependences everywhere
- Not only fits but also continuum infinite volume results at a given quark-mass (so we can also fit ourselves for studying other inputs)
- More use of the existing two-loop calculations
- Analytical NNLO only: not really fewer parameters
- Only more in LECs: p^4 scattering LECs also in masses/decay constants
- I.e. l_1^r, l_2^r ($n_F = 2$), L_1^r, L_2^r, L_3^r ($n_F = 3$), $\hat{L}_i^r, i = 0, 1, 2, 3$ (PQChPT)

Wishing list: Two-flavour

(with input from Gasser et al.)

- \bar{l}_3 and errors
- \bar{l}_4 : from $F_\pi(m_q)$ and scalar radius: can lattice check this relation
- a_0^2 accurately predicted in terms of scalar radius: can lattice check this
- Isospin breaking in $\pi\pi$ scattering (important for CP violation in $K \rightarrow \pi\pi$)
- $\bar{l}_5 - \bar{l}_6$ Needed for $\pi \rightarrow \ell\nu\gamma$, can be had from Π_{AA}

Wishing list: ≥ 3 -flavour

- Ideas on how to make all those calculations usable for you
- Three and more flavour: typically slow numerically
- large N_c suppressed couplings: i.e. m_s dependence of m_π, F_π
- L_4^r, L_6^r
- *sigma* terms and scalar radii

Conclusions and final comments

- Lots of analytical work done in ChPT
- Use the correct ChPT
 - 2-flavour for varying \hat{m} and possible for $N_f = 2$ and $N_f = 2 + 1$ at fixed m_s (but have different LECs)
 - otherwise 3-flavour
 - the various partially quenched versions
- Remember at which order in ChPT you compare things

Conclusions and final comments

- Lots of analytical work done in ChPT
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 - otherwise 3-flavour
 - the various partially quenched versions
- Remember at which order in ChPT you compare things
- Seen a lot of lattice work, looking forward to seeing more
- LECs in many talks: Boyle, Matsufuru, Urbach, Kuramashi and many parallel session talks