ChPT loops for the lattice: pion mass and decay constant; HVP at finite volume and $n\bar{n}$-oscillations

Johan Bijnens

Lund University

bijnens@thep.lu.se
http://thep.lu.se/~bijnens
http://thep.lu.se/~bijnens/chpt/
http://thep.lu.se/~bijnens/chiron/

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Overview

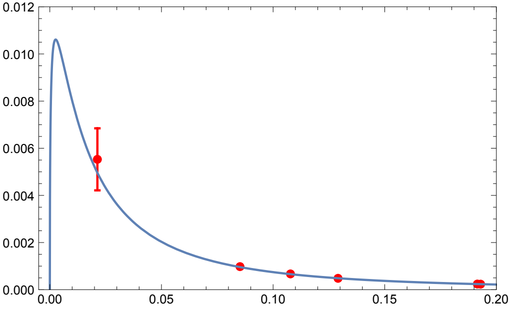
Chiral Perturbation Theory

- ChPT = Effective field theory describing the lowest order pseudo-scalar representation
- or the (pseudo) Goldstone bosons from spontaneous breaking of chiral symmetry.
- The number of degrees of freedom depend on the case we look at
- Treat π, η, K as light and pointlike with a derivative and quark-mass expansion
- Recent review of LECs:

  JB, Ecker,Ann.Rev.Nucl.Part.Sci. 64 (2014) 149 [arXiv:1405.6488]

Why

Muon: $a_\mu = (g-2)/2$ and $a_\mu^{\mathrm{LO,HVP}} = \int_0^\infty dQ^2 f\left(Q^2\right)\hat{\Pi}\left(Q^2\right)$

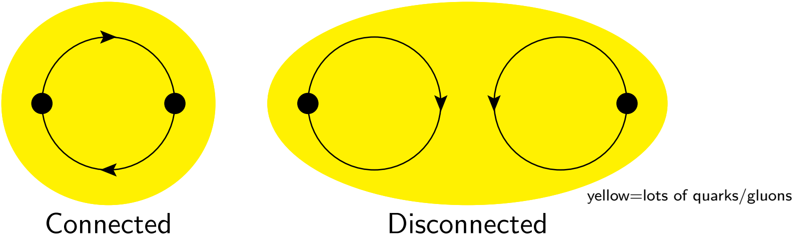


plot: $f\left(Q^2\right)\hat{\Pi}\left(Q^2\right)$ with $Q^2 = -q^2$ in GeV^2

Figure and data: Aubin, Blum, Chau, Golterman, Peris, Tu, Phys. Rev. D93 (2016) 054508 [arXiv:1512.07555]

Low energy quantity so ChPT should be useful

Two-point: Connected versus disconnected



- $\Pi^{\mu\nu}_{ab}(q) \equiv i \int d^4x e^{iq\cdot x} \langle T(j^\mu_a(x) j^{\nu\dagger}_a(0))\rangle$
- $j^\mu_{\pi^+} = \bar{d}\gamma^\mu u$
- $j^\mu_u = \bar{u}\gamma^\mu u, \quad j^\mu_d = \bar{d}\gamma^\mu d, \quad j^\mu_s = \bar{s}\gamma^\mu s$
- $j^\mu_e = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{s}\gamma^\mu s$
- ChPT p^4: Della Morte, Jüttner, JHEP 1011(2010)154 [arXiv:1009.3783]
- ChPT p^6: JB, Relefors, JHEP 1611(2016)086 [arXiv:1609.01573]

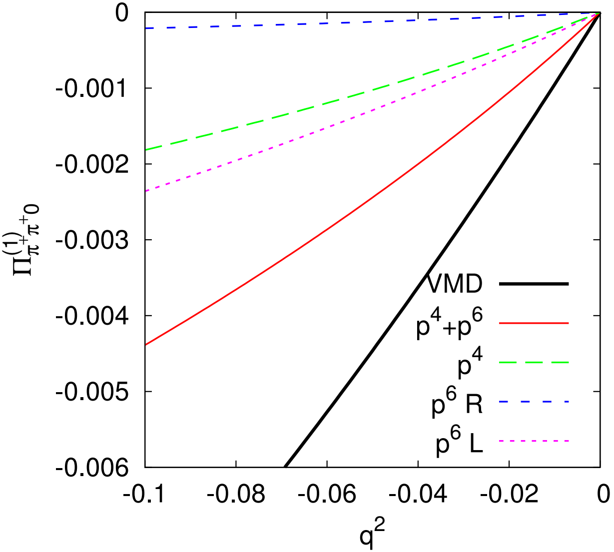
Two-point: Connected versus disconnected

- Include also singlet part of the vector current
- There are new terms in the Lagrangian
- p^4 only one more: $\langle L_{\mu\nu}\rangle\langle L^{\mu\nu}\rangle + \langle R_{\mu\nu}\rangle\langle R^{\mu\nu}\rangle$
  (drops out when subtracting $\Pi(0)$)
- $\Longrightarrow$ The pure singlet vector current does not couple to mesons until p^6
- $\Longrightarrow$ Loop diagrams involving the pure singlet vector current only appear at p^8 (implies relations)
- p^6 (no full classification, just some examples)
  $\langle D_\rho L_{\mu\nu}\rangle\langle D^\rho L^{\mu\nu}\rangle + \langle D_\rho R_{\mu\nu}\rangle\langle D^\rho R^{\mu\nu}\rangle$,
  $\langle L_{\mu\nu}\rangle\left\langle L^{\mu\nu}\chi^\dagger U\right\rangle + \langle R_{\mu\nu}\rangle\left\langle R^{\mu\nu}\chi U^\dagger\right\rangle$,...
- Results at two-loop order, unquenched isospin limit

Two-point: Connected versus disconnected

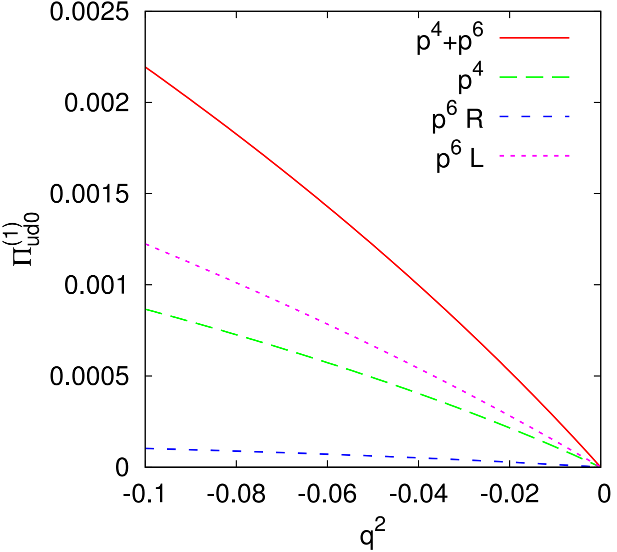
- $\Pi^{\mu\nu}_{\pi^+\pi^+}$: only connected
- $\Pi^{\mu\nu}_{ud}$: only disconnected
- $\Pi^{\mu\nu}_{uu} = \Pi^{\mu\nu}_{\pi^+\pi^+} + \Pi^{\mu\nu}_{ud}$
- $\Pi^{\mu\nu}_{ee} = \frac{5}{9}\Pi^{\mu\nu}_{\pi^+\pi^+} + \frac{1}{9}\Pi^{\mu\nu}_{ud}$
- Infinite volume (and the *ab* considered here):
  $\Pi^{\mu\nu}_{ab} = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi^{(1)}_{ab}$
- Large N_c + VMD estimate: $\Pi^{(1)}_{\pi^+\pi^+} = \dfrac{4F_\pi^2}{M_V^2 - q^2}$
- Plots on next pages are for $\Pi^{(1)}_{ab0}(q^2) = \Pi^{(1)}_{ab}(q^2) - \Pi^{(1)}_{ab}(0)$
- At p^4 the extra LEC cancels, at p^6 there are new LEC contributions, but no new ones in the loop parts

Two-point: Connected versus disconnected



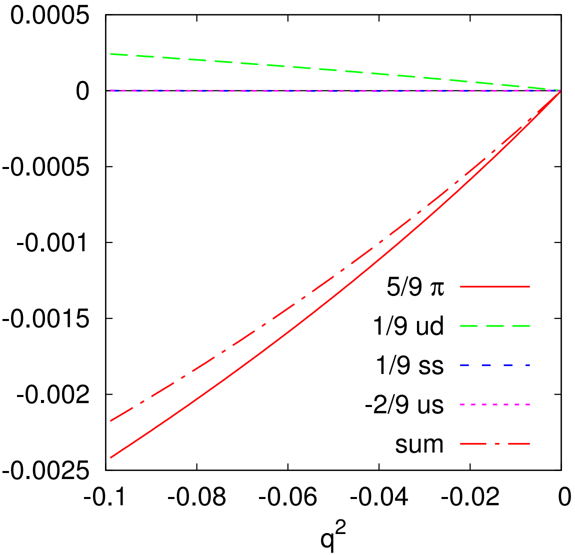
- $\boxed{\text{Connected}}$
- p^6 is large
- Due to the L_i^r loops

Two-point: Connected versus disconnected



- $\boxed{\text{Disconnected}}$
- p^6 is large
- Due to the L_i^r loops
- about $-\frac{1}{2}$ connected
- $-\frac{1}{10}$ is from $\Pi^{(1)}_{ee} = \frac{5}{9}\Pi^{(1)}_{\pi^+\pi^+} + \frac{1}{9}\Pi^{(1)}_{ud}$

Two-point: with strange, electromagnetic current



- π connected u,d
- *ud* disconnected u,d
- *ss* strange current
- *us* mixed s–u,d
- new p^6 LEC cancels
- Disconnected strange $\approx -15\%$ of total strange

JB, Relefors, LU TP 16-51 to appear

Finite volume

- One-loop calculation in finite volume done by
  Aubin et al, Phys.Rev. D88 (2013) 7, 074505 [arXiv:1307.4701]
  Aubin et al. Phys. Rev. D93 (2016) 054508 [arXiv:1512.07555]
  and found to fit lattice data well
- two-loop in partially quenched
  JB, Relefors, LU TP 16-51 to appear
- I will stay with ChPT and the p regime ($M_\pi L >> 1$)
- $1/m_\pi = 1.4$ fm
  may need to (and I will) go beyond leading $e^{-m_\pi L}$ terms
  "around the world as often as you like"
- Convergence of ChPT is given by $1/m_\rho \approx 0.25$ fm

Finite volume and Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i/L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions: $q(x^i + L) = e^{i\theta^i_q} q(x^i)$
- Then momenta are $p^i = \theta^i/L + 2\pi n^i/L$. Allows to map out momentum space on the lattice much better

  Bedaque,...
- Small note:
  - Beware what people call momentum: is θ^i/L included or not?
  - Reason: a colour singlet gauge transformation $G^S_\mu \to G^S_\mu - \partial_\mu \epsilon(x)$, $q(x) \to e^{i\epsilon(x)} q(x)$, $\epsilon(x) = -\theta^i_q x^i/L$
  - Boundary condition Twisted $\Leftrightarrow$ constant background field+periodic

Finite volume and twisting: Drawbacks

Drawbacks:

- Box: Rotation invariance $\rightarrow$ cubic invariance
- Twisting: reduces symmetry further

Consequences:

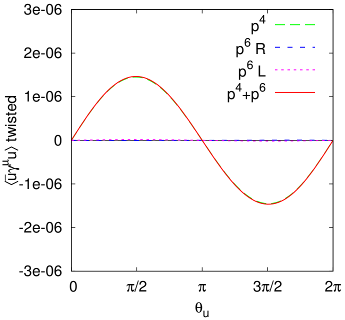
- $m^2(\vec{p}^2) = E^2 - \vec{p}^2$ is not constant
- There are typically more form-factors
- In general: quantities depend on more (all) components of the momenta
- Charge conjugation involves a change in momentum

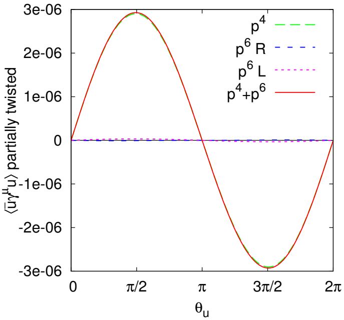
Two-point function: twisted boundary conditions

JB, Relefors, JHEP 05 (201)4 015 [arXiv:1402.1385]

- $\int_V \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{k^2 - m^2} \neq 0$
- $\langle \bar{u}\gamma^\mu u \rangle \neq 0$
- $j^\mu_{\pi^+} = \bar{d}\gamma^\mu u$
  satisfies $\partial_\mu \left\langle T(j^\mu_{\pi^+}(x) j^{\nu\dagger}_{\pi^+}(0)) \right\rangle = \delta^{(4)}(x) \left\langle \bar{d}\gamma^\nu d - \bar{u}\gamma^\nu u \right\rangle$
- $\Pi^{\mu\nu}_a(q) \equiv i \int d^4x e^{iq\cdot x} \left\langle T(j^\mu_a(x) j^{\nu\dagger}_a(0)) \right\rangle$
  Satisfies WT identity. $q_\mu \Pi^{\mu\nu}_{\pi^+} = \left\langle \bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d \right\rangle$
- ChPT at one-loop satisfies this
  see also Aubin et al, Phys.Rev. D88 (2013) 7, 074505 [arXiv:1307.4701]
- two-loop in partially quenched
  JB, Relefors, LU TP 16-51, to appear
  satisfies the WT identity (as it should)

$\langle \bar{u}\gamma^\mu u \rangle$





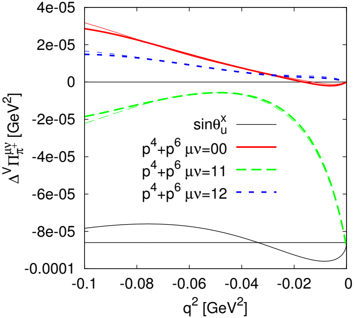
Fully twisted Partially twisted

$\theta_u = (0, \theta_u, 0, 0)$, all others untwisted

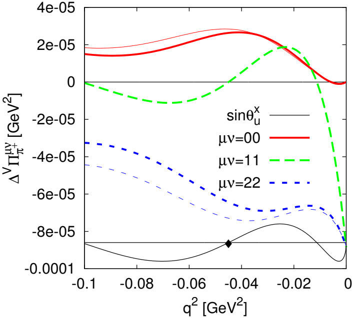
$m_\pi L = 4$ (ratio at $p^4 \equiv 2$ up to kaon loops)

For comparison: $\langle \bar{u}u \rangle^V \approx -2.4\ 10^{-5}\ \mathrm{GeV}^3$
$\langle \bar{u}u \rangle \approx -1.2\ 10^{-2}\ \mathrm{GeV}^3$

Two-point partially twisted: components



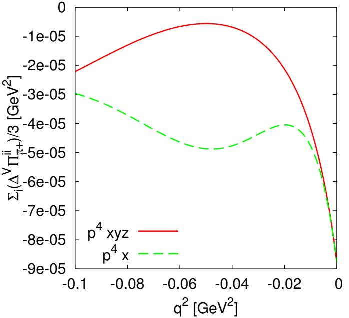
Twisting spatially symmetric



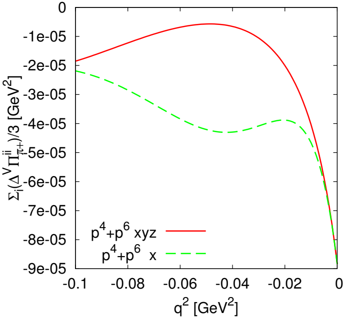
Twisting in x-direction

- Small p^6 corrections (thin lines: p^4 only)
- $m_{\pi 0} L = 4$ $m_{\pi 0} = 0.135$ GeV
- $-q^2 \Pi^{(1)}_{\mathrm{VMD}} = \frac{-4q^2 F_\pi^2}{M_V^2 - q^2} \approx$5e-3$\cdot\frac{q^2}{0.1} \Longrightarrow$ Correction at % level
- Can use the difference between different twists with same q^2 to check the finite volume corrections

Two-point partially twisted: spatial average



p^4 only



$p^4 + p^6$

- Plotted: volume correction to $\overline{\Pi} = \dfrac{1}{3}\displaystyle\sum_{i=x,y,z} \Pi^{ii}$
- Small p^6 corrections: compare left and right
- Can use the difference between different twists with same q^2 to check the finite volume corrections

Two flavour ChPT: mass and decay constant

- First step towards finding out why hard-pion ChPT does not work at three-loops
- Lowest order: Gell-Mann, Oakes, Renner (1968)
- Chiral logarithm Langacker, Pagels (1973)
- Full NLO (and properly starting ChPT) Gasser-Leutwyler (1984)
- NNLO Buergi (1996), JB, Colangelo, Ecker, Gasser, Sainio (1996)
- NNNLO JB, Hermansson-Truedsson (2017)

Methods used

- LO and Chiral logs: current algebra
- NLO: Feynman diagrams (by hand) and direct expansion of functional integral (with REDUCE)
- NNLO: Feynman diagrams
  (with a little help from FORM)
- NNLO: Feynman diagrams purely with FORM
- Main stumbling block: integrals
  - Reduction to master integrals with REDUZE Studerus (2009)
  - Master Integrals known
    Laporta-Remiddi (1996); Melnikov, van Ritbergen (2001)
- Lots of book-keeping: FORM
- Checks:
  - All nonlocal divergences must cancel
  - Use different parametrizations of the Lagrangian
  - Agree with known leading log result JB, Carloni (2009)

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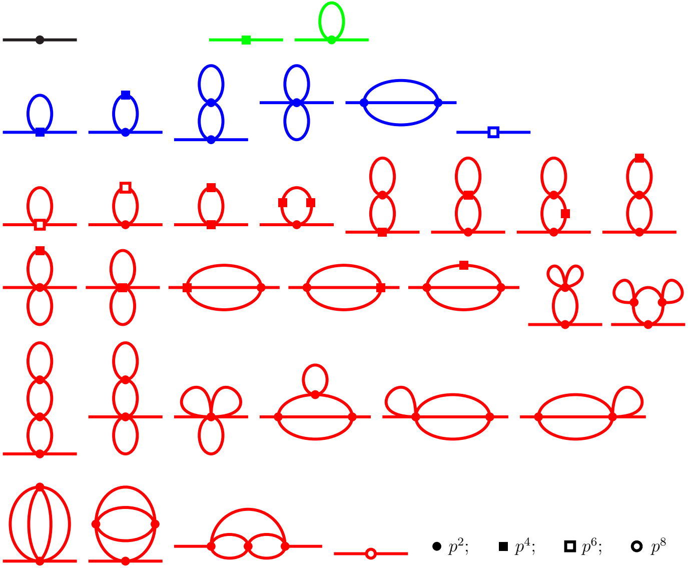
Diagrams



Results: LO or x-expansion/physical or ξ-expansion

- $x = \dfrac{M^2}{16\pi^2 F^2}, \qquad L_x = \log\dfrac{M^2}{\mu^2}, \qquad M^2 = 2B\hat{m}$

$$\frac{M_\pi^2}{M^2} = 1 + x\left(a_{11}^M L_x + a_{10}^M\right) + x^2\left(a_{22}^M L_x^2 + a_{21}^M L_x + a_{20}^M\right) + x^3\left(a_{33}^M L_x^3 + a_{32}^M L_x^2 + a_{31}^M L_x + a_{30}^M\right) + \cdots$$

$$\frac{F_\pi}{F} = 1 + x\left(a_{11}^F L_x + a_{10}^F\right) + x^2\left(a_{22}^F L_x^2 + a_{21}^F L_x + a_{20}^F\right) + x^3\left(a_{33}^F L_x^3 + a_{32}^F L_x^2 + a_{31}^F L_x + a_{30}^F\right) + \cdots$$

- $\xi = \dfrac{M_\pi^2}{16\pi^2 F_\pi^2}, \qquad L_\pi = \log\dfrac{M_\pi^2}{\mu^2}$
- $$\frac{M^2}{M_\pi^2} = 1 + \xi\left(b_{11}^M L_\pi + b_{10}^M\right) + \xi^2\left(b_{22}^M L_\pi^2 + b_{21}^M L_\pi + b_{20}^M\right) + \xi^3\left(b_{33}^M L_\pi^3 + b_{32}^M L_\pi^2 + b_{31}^M L_\pi + b_{30}^M\right) + \cdots$$
- $$\frac{F}{F_\pi} = 1 + \xi\left(b_{11}^F L_\pi + b_{10}^F\right) + \xi^2\left(b_{22}^F L_\pi^2 + b_{21}^F L_\pi + b_{20}^F\right) + \xi^3\left(b_{33}^F L_\pi^3 + b_{32}^F L_\pi^2 + b_{31}^F L_\pi + b_{30}^F\right) + \cdots$$

Results $\quad \tilde{l}_i = 16\pi^2 l_i^r,\ \tilde{c}_i = (16\pi^2)^2 c_i^r$

a^M_{11}	$\frac{1}{2}$
a^M_{10}	$2\tilde{l}_3$
a^M_{22}	$\frac{17}{8}$
a^M_{21}	$-3\tilde{l}_3 - 8\tilde{l}_2 - 14\tilde{l}_1 - \frac{49}{12}$
a^M_{20}	$64\tilde{c}_{18} + 32\tilde{c}_{17} + 96\tilde{c}_{11} + 48\tilde{c}_{10} - 16\tilde{c}_9 - 32\tilde{c}_8 - 16\tilde{c}_7$ $-32\tilde{c}_6 + \tilde{l}_3 + 2\tilde{l}_2 + \tilde{l}_1 + \frac{193}{96}$
a^M_{33}	$\frac{103}{24}$
a^M_{32}	$\frac{23}{2}\tilde{l}_3 - 11\tilde{l}_2 - 38\tilde{l}_1 - \frac{91}{24}$
a^M_{31}	$-416\tilde{c}_{18} - 208\tilde{c}_{17} - 32\tilde{c}_{16} + 96\tilde{c}_{14} + 8\tilde{c}_{13} - 48\tilde{c}_{12}$ $-384\tilde{c}_{11} - 192\tilde{c}_{10} + 72\tilde{c}_9 + 144\tilde{c}_8 + 72\tilde{c}_7 + 64\tilde{c}_6 - 8\tilde{c}_5$ $-56\tilde{c}_4 + 16\tilde{c}_3 + 32\tilde{c}_2 - 96\tilde{c}_1 - 8\tilde{l}_3^2 - 48\tilde{l}_3\tilde{l}_2 - 84\tilde{l}_3\tilde{l}_1$ $-\frac{88}{3}\tilde{l}_3 - \frac{231}{10}\tilde{l}_2 - \frac{69}{5}\tilde{l}_1 - \frac{74971}{8640}$
a^M_{30}	contains free p^8 LECs (and a lot more terms)

Results: comments

- Similar tables for a_i^F, b_i^M, b_i^F
- Coefficients depend on scale μ, but whole expression is μ-independent
- Can be rewritten in terms of scales in the logarithm rather than in terms of LECs à la FLAG
- Leading log: a number
- NLL: depends on l_i^r
- NNLL: depends on c_i^r
- For the mass all needed c_i^r can be had from mass, decay-constant and $\pi\pi$ parameters fitted to two-loop or p^6 (i.e. $r_M, r_F, r_1, \ldots, r_6$).
- For decay need one more (busy checking if it can be had)

Results: numerics preliminary

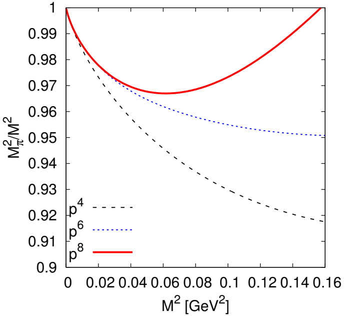
ij	a^M_{ij}	b^M_{ij}	a^F_{ij}	b^F_{ij}
10	$+0.00282$	-0.00282	$+1.09436$	-1.09436
11	$+0.5$	-0.5	-1	$+1$
20	$+1.65296$	-1.65771	-0.04734	-1.15001
21	$+2.4573$	-3.29038	-1.90577	$+4.13885$
22	$+2.125$	-0.625	-1.25	-0.25
30	$+0.39527$	-6.7854	-244.499	242.236
31	-3.75977	$+4.32719$	-19.0601	32.1315
32	$+17.1476$	$+0.62039$	-9.39462	-6.77511
33	$+4.29167$	$+5.14583$	-3.45833	-0.41666

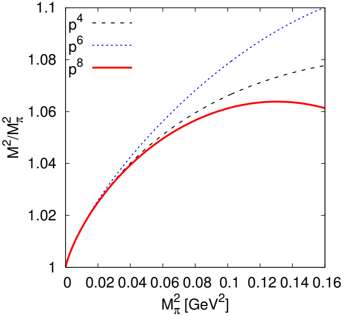
Note the large coefficients in the decay constant

Pion mass

$F_\pi = 92.2$ MeV, $F = F_\pi/1.037$, $\bar{l}_1 = -0.4$, $\bar{l}_2 = 4.3$, $\bar{l}_3 = 3.41$, $\bar{l}_4 = 4.51$,

r_i from JB et al 1997, other $c_i^r = 0$, $\mu = 0.77$ GeV

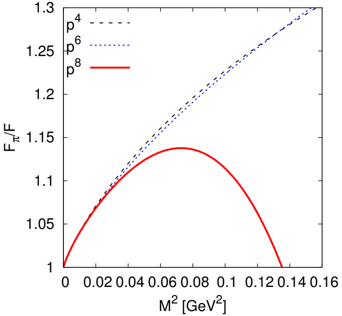




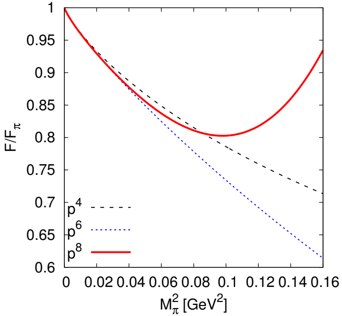
x-expansion (F-fixed) ξ-expansion (F_π fixed)

ξ-expansion converges notably better

Pion decay constant



x-expansion (F-fixed)

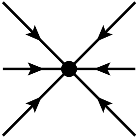


ξ-expansion (F_π fixed)

- ξ-expansion converges better
- Large p^6 due to the "240" in a_{30}^F and b_{30}^F

$n\bar{n}$ oscillations

- some GUT models have neutron-anti-neutron oscillations but no proton decay
- Limits:
  - 8.6 10^7s from free neutrons (ILL)
  - 2.7 10^8s from oxygen nuclei (super-K)
    ($\bar{n}$ mass inside nuclei very different from n-mass)
  - Possible ESS experiment improvement by up to 10^3
- Effective dimension 9 operator: "*uuduud*"



- Classification of quark operators and RGE to two loops:
  Buchoff, Wagman, Phys. Rev D93(2016)016005 [arXiv:1506.00647]
  and earlier papers in there
- E. Kofoed, Master thesis LU TP 16-62; JB, Kofoed, in preparation

Operators

- 14 operators
- Chiral representations under $SU(2)_L \times SU(2)_R$:

Chiral	#operators	Chiral	#operators
$(1_L, 3_R)$	3: Q_1, Q_2, Q_3	$(3_L, 1_R)$	3
$(5_L, 3_R)$	3: Q_5, Q_6, Q_7	$(3_L, 5_R)$	3
$(1_L, 7_R)$	1: Q_4	$(7_L, 1_R)$	1

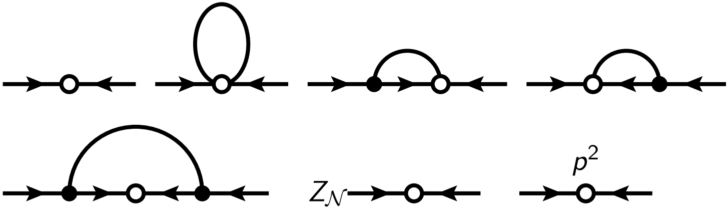
- $n\bar{n}$ is $\Delta I = 1$ so bottom line needs isospin breaking
- For the others the different operators are different elements within the same representation
- Use heavy baryon formalism with a baryon ($\mathcal{N}$) and anti-baryon doublet ($\mathcal{N}^c$) with same velocity v
- These correspond two widely separated areas (v and $-v$) in the relativistic field: so no double counting

ChPT terms

- $\mathcal{L} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \overline{\mathcal{N}}\left(iv\cdot D + g_A u\cdot S\right)\mathcal{N}$
  $+\overline{\mathcal{N}^c}\tau^2\left(iv\cdot D + g_A u\cdot S\right)\tau^2\mathcal{N}^c + \text{higher orders}$
- $\mathcal{N} = \begin{pmatrix} p \\ n \end{pmatrix} \to h\mathcal{N}, \qquad \overline{\mathcal{N}^c} = \left(\overline{p^c}\ \overline{n^c}\right) \to \overline{\mathcal{N}^c}h^T$
- $h(g_L, g_R, u)$: $SU(2)_V$ compensator chiral transformation
- Spurions for each quark $n\overline{n}$ operator:
  - $(1_L, 3_R)$: two $SU(2)_R$ doublet indices
  - $(5_L, 3_R)$: four $SU(2)_L$ and two $SU(2)_L$ doublet indices
  - $(1_L, 7_R)$: six $SU(2)_R$ double indices
  - plus parity conjugates

ChPT terms and diagrams

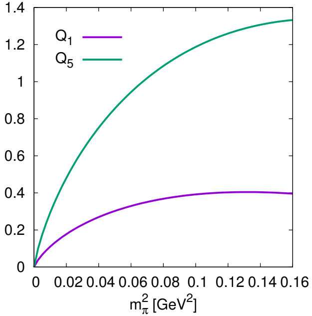
- p^0:
  - $(1_L, 3_R)$: $\left(u\overline{\mathcal{N}^c}\right)_{i_L}\left(u\mathcal{N}\right)_{j_L}$
  - $(5_L, 3_R)$: $\left(u^\dagger\overline{\mathcal{N}^c}\right)_{i_L}\left(u^\dagger\mathcal{N}\right)_{j_L}\left(U\tau^2\right)_{k_R l_L}\left(U\tau^2\right)_{m_R n_L}$
  - $(7_L, 1_R)$: none (first one at p^2)
- p^1: none that directly contribute (but in loops from p^3)
- p^2: many (at least 20 each for $(3_L, 1_R)$ and $(3_L, 5_R)$)

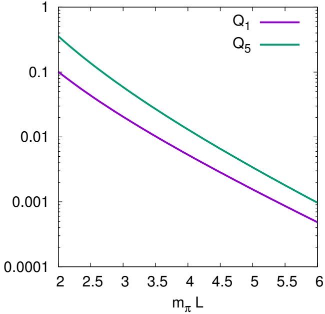


Results (Preliminary)

- Q_1, Q_2, Q_3 same factor from loops (isospin)
- Q_5, Q_6, Q_7 same factor
  (Conjecture: due to projection on $I = 1$ subspace)
- Q_1, Q_2, Q_3:
  $1 + \frac{M_\pi^2}{16\pi^2 F_\pi^2}\left(\left(-\frac{3}{2}g_A^2 - 1\right)\log\frac{M_\pi^2}{\mu^2} - 1\right) +$ order p^2 LECs
- Q_5, Q_6, Q_7:
  $1 + \frac{M_\pi^2}{16\pi^2 F_\pi^2}\left(\left(-\frac{3}{2}g_A^2 - 7\right)\log\frac{M_\pi^2}{\mu^2} - 1\right) +$ order p^2 LECs
- Also done at finite volume

Results (Preliminary)





Relative correction from loops

Relative correction from loops (absolute value)

Conclusions

Showed you results for:

- HVP: ChPT at two-loops including partially quenched
  - Connected versus disconnected at two-loops
  - Connected: twisting and finite volume at two-loops
- Two flavour ChPT correction at three loops for the pion mass and decay constant
- Two flavour ChPT correction at one-loop for $n\bar{n}$-oscillations
- Be careful: ChPT must exactly correspond to your lattice calculation
- Programs available (for published ones) via CHIRON
  Those for this talk: sometime later this year (I hope) (or ask me)