EFFECTIVE FIELD THEORY, FLAVOUR AND CP

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Overview

- Motivation : Standard Model and some bits of Flavours Physics
- Effective Field Theory
- Chiral Perturbation Theory
- some selected results from two-loop ChPT
- some selected results from nonleptonic matrix-elements

The Standard Model

The Standard Model Lagrangian has four parts:

$$\underbrace{\mathcal{L}_H(\phi)}_{\text{Higgs}} + \underbrace{\mathcal{L}_G(W, Z, G)}_{\text{Gauge}}$$

$$\underbrace{\sum_{\psi=\text{fermions}} \bar{\psi} i \not{D}\, \psi}_{\text{gauge-fermion}} + \underbrace{\sum_{\psi,\psi'=\text{fermions}} g_{\psi\psi'} \bar{\psi}\phi\psi'}_{\text{Yukawa}}$$

The Standard Model

What is tested ?

gauge-fermion	Very well tested
Higgs	Limits only, real tests coming up
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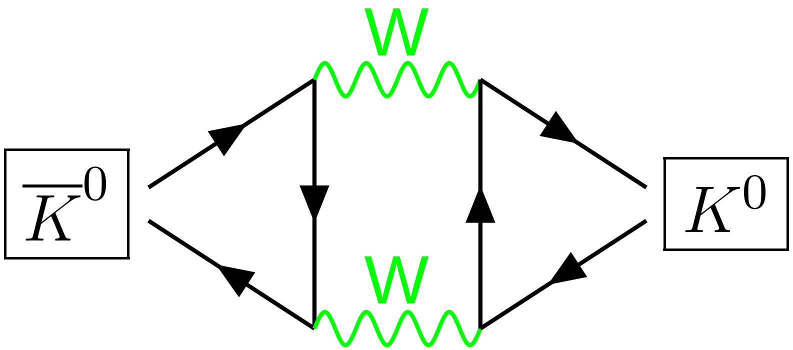
Discrete symmetries:

- C Charge Conjugation
- P Parity
- T Time Reversal

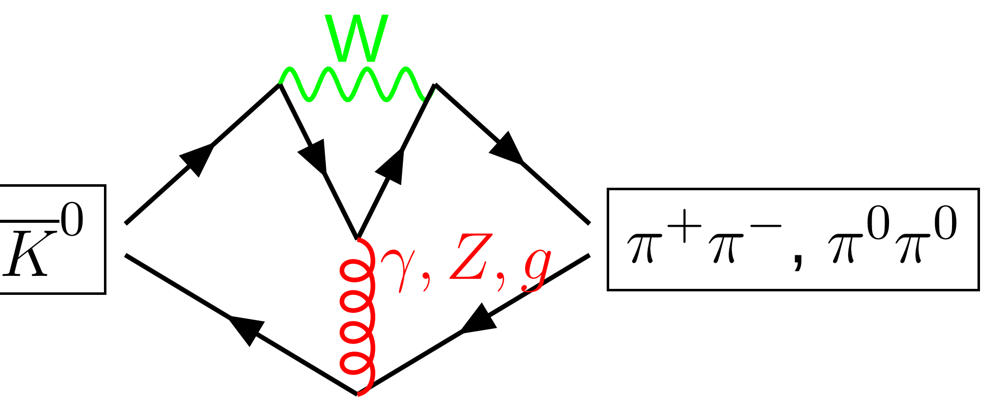
QCD and QED conserve C,P,T separately,
Weak breaks C and P, only Yukawa breaks CP
Field theory implies CPT

CP-violation: where is it measured ?

$|\varepsilon| = 2.28 \cdot 10^{-3}$
from



AND from



we get ε'

Experiment: Re $\left(\frac{\varepsilon'}{\varepsilon}\right)$

NA31	$(23.0 \pm 6.5) \times 10^{-4}$
E731	$(7.4 \pm 5.9) \times 10^{-4}$
KTeV	$(28.0 \pm 4.1) \times 10^{-4}$
NA48 97	$(18.5 \pm 7.3) \times 10^{-4}$
NA48 98	$(12.2 \pm 4.9) \times 10^{-4}$
ALL	$(19.3 \pm 2.4) \times 10^{-4}$

($\chi^2/dof = 11.1/5$)

Loops: also beyond SM

B measurements

A large program is under way to measure the same type of quantities in the bottom (and also charm) system as well

LHC will contribute here (especially for B_s)

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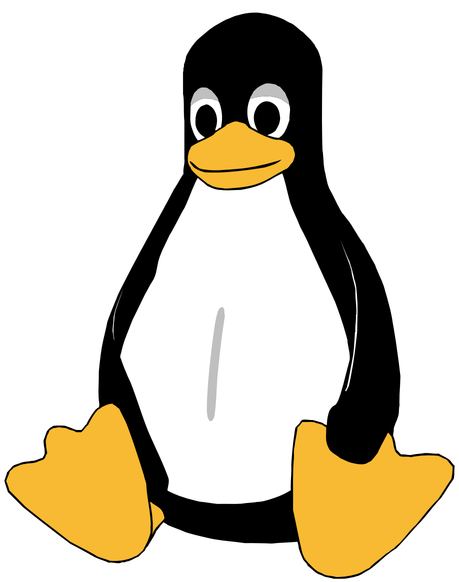

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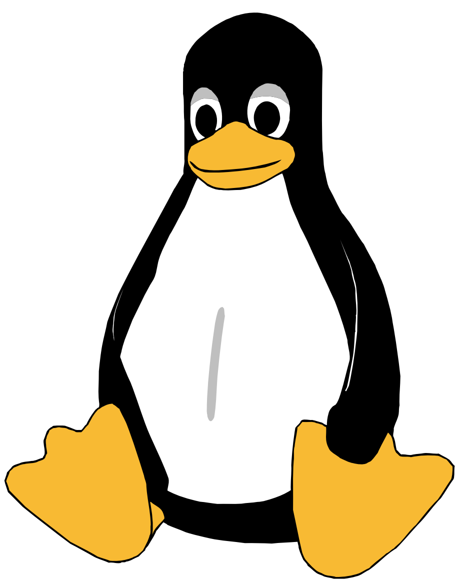
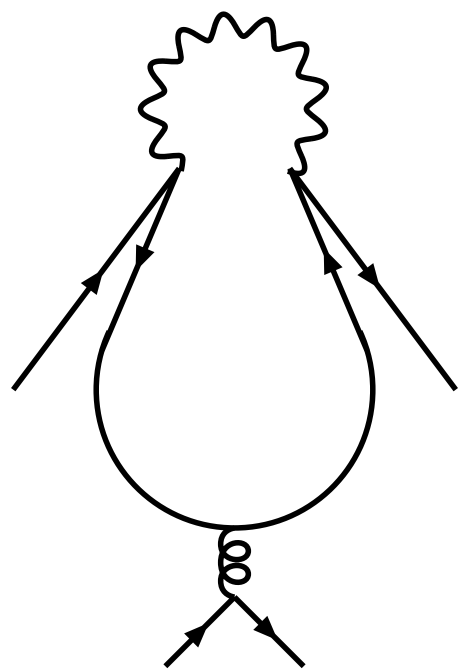
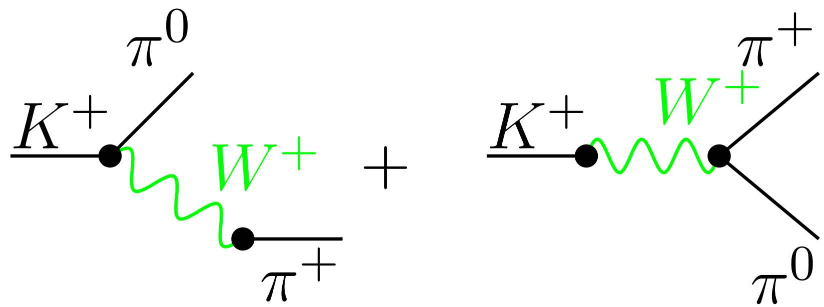




$K \rightarrow \pi\pi$ and the $\Delta I = 1/2$ Rule

$K^+ \longrightarrow \pi^+\pi^0$:



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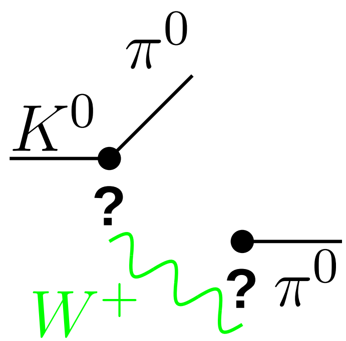

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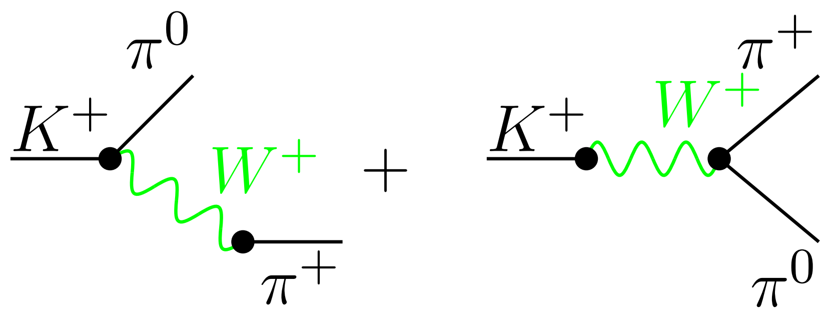

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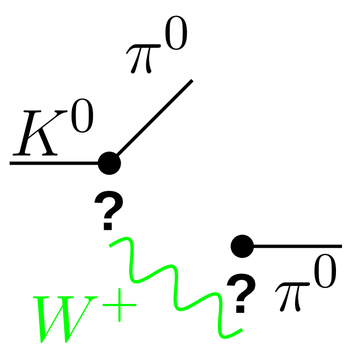

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Experimentally:

$$\Gamma(K^0 \longrightarrow \pi^0\pi^0) = \frac{1}{2}\Gamma(K_S \longrightarrow \pi^0\pi^0) = 2.3\ 10^{-12}\ \text{MeV}$$

$$\Gamma(K^+ \longrightarrow \pi^+\pi^0) = \qquad = 1.1\ 10^{-14}\ \text{MeV}$$

The zero one is the largest !!!

Weak interaction: quarks to mesons

ENERGY SCALE	FIELDS	Effective Theory
M_W	W, Z, γ, g; τ, μ, e, ν_ℓ; t, b, c, s, u, d	Standard Model
	$\Downarrow$ using OPE	
$\lesssim m_c$	γ, g; μ, e, ν_ℓ; s, d, u	QCD,QED,$\mathcal{H}_{\text{eff}}^{\lvert\Delta S\rvert=1,2}$
	$\Downarrow$???	
M_K	γ; μ, e, ν_ℓ; π, K, η	CHPT

Effective Field Theory

Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples:

- Solid state physics: conductors: neglect the empty bands above the partially filled one
- Atomic physics: Blue sky: neglect atomic structure

Power Counting

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$\Longrightarrow$ Need some ordering principle: power counting

- Taylor series expansion does not work (convergence radius is zero)
- Continuum of excitation states need to be taken into account

Why Field Theory ?

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- Many parameters (but finite number at any order)
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Advantages

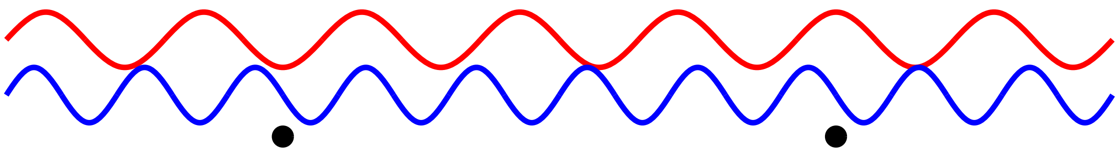
- Calculations are (relatively) simple
- It is general: model-independent
- Theory $\Longrightarrow$ errors can be estimated
- Systematic: ALL effects at a given order can be included
- Even if no convergence: classification of models often useful

Why is the sky blue ?

System: Photons of visible light and neutral atoms

Length scales: a few 1000 Å versus 1 Å

Atomic excitations suppressed by $\approx 10^{-3}$



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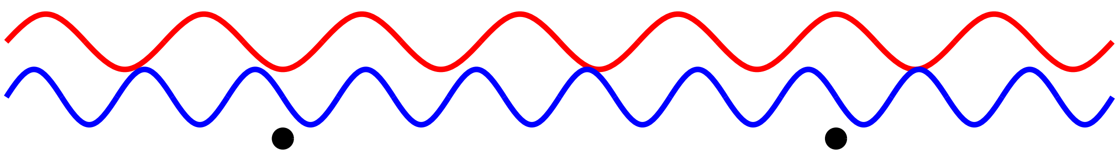

$$\mathcal{L}_A = \Phi_v^\dagger \partial_t \Phi_v + \ldots \qquad \mathcal{L}_{\gamma A} = G F_{\mu\nu}^2 \Phi_v^\dagger \Phi_v + \ldots$$

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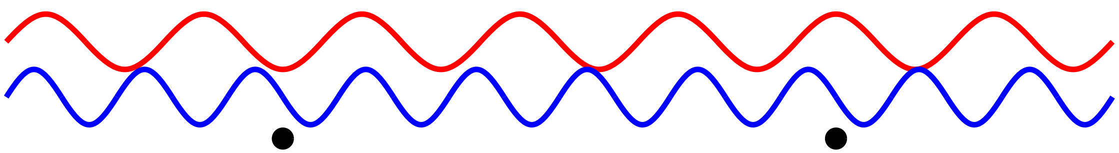

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$$\boxed{\sigma \approx G^2 E_\gamma^4}$$

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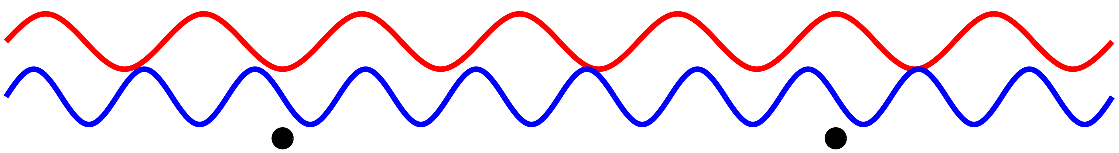

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blue light scatters a lot more than red $\left\{ \begin{array}{l} \Longrightarrow \text{red sunsets} \\ \Longrightarrow \text{blue sky} \end{array} \right.$

Higher orders suppressed by 1 Å$/\lambda_\gamma$.

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

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Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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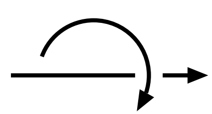
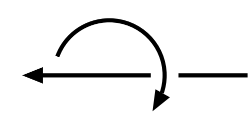
Can also see that via



$v < c,\ m_q \neq 0 \Longrightarrow$

$v = c,\ m_q = 0 \not\Longrightarrow$



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$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$

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8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
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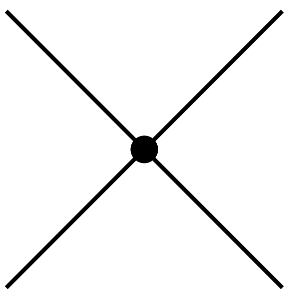
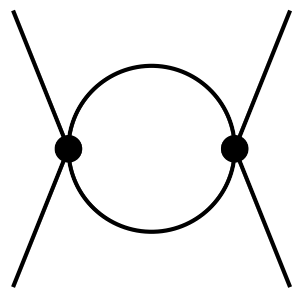
Power counting in momenta:



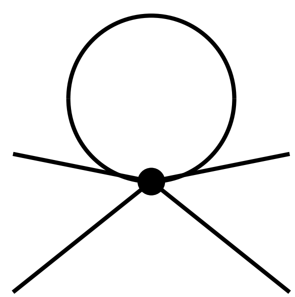
p^2



$(p^2)^2 \, (1/p^2)^2 \, p^4 = p^4$

$1/p^2$

$\int d^4p$ $\qquad p^4$



$(p^2) \, (1/p^2) \, p^4 = p^4$

Two Loop: General

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

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Note

- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour is a special case of 3+3 PQ:

  $\hat{L}_i^r, K_i^r \rightarrow L_i^r, C_i^r$

Three Flavours at Two Loop

Review paper: JB, LU TP 06-16 hep-ph/0604043

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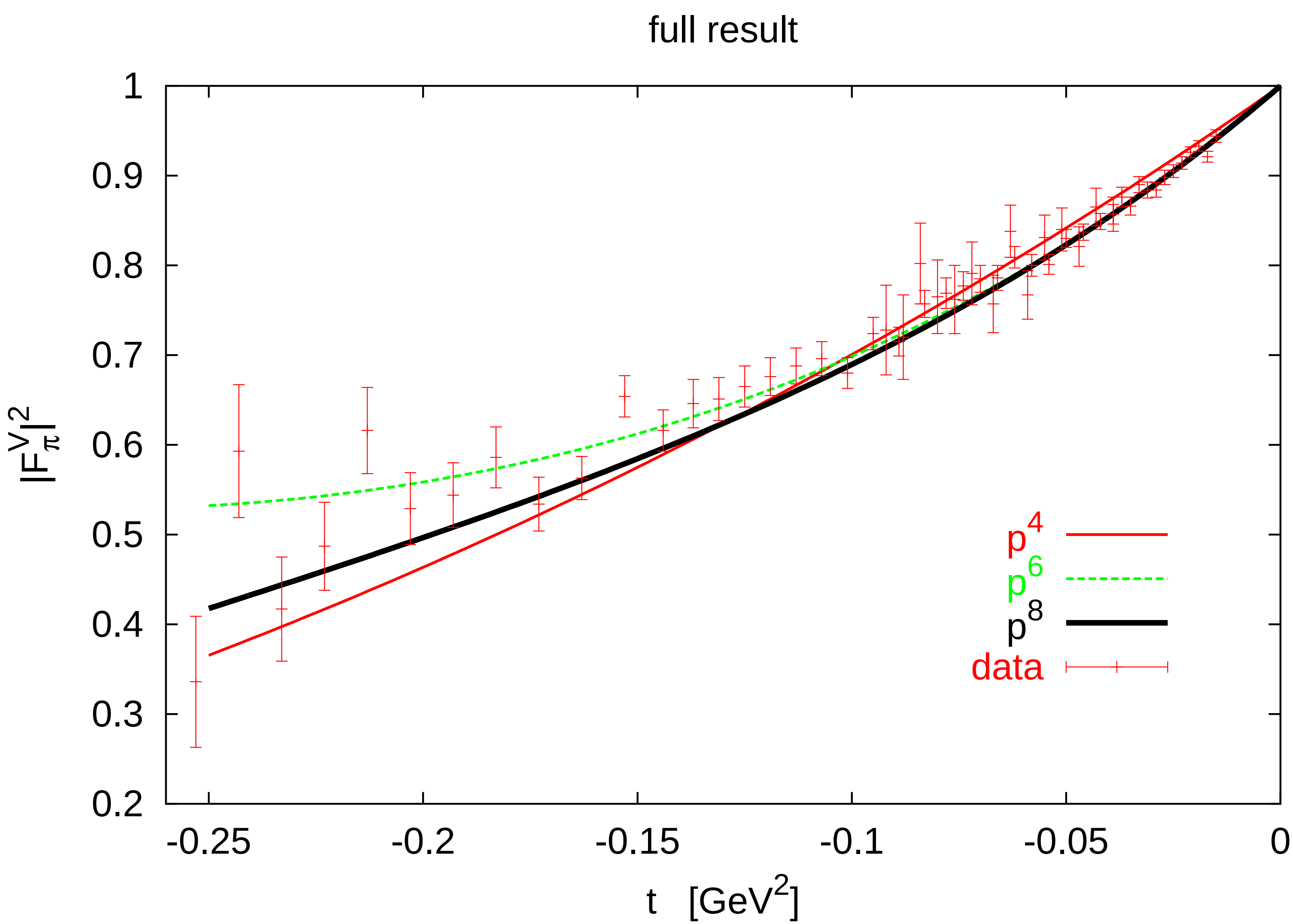
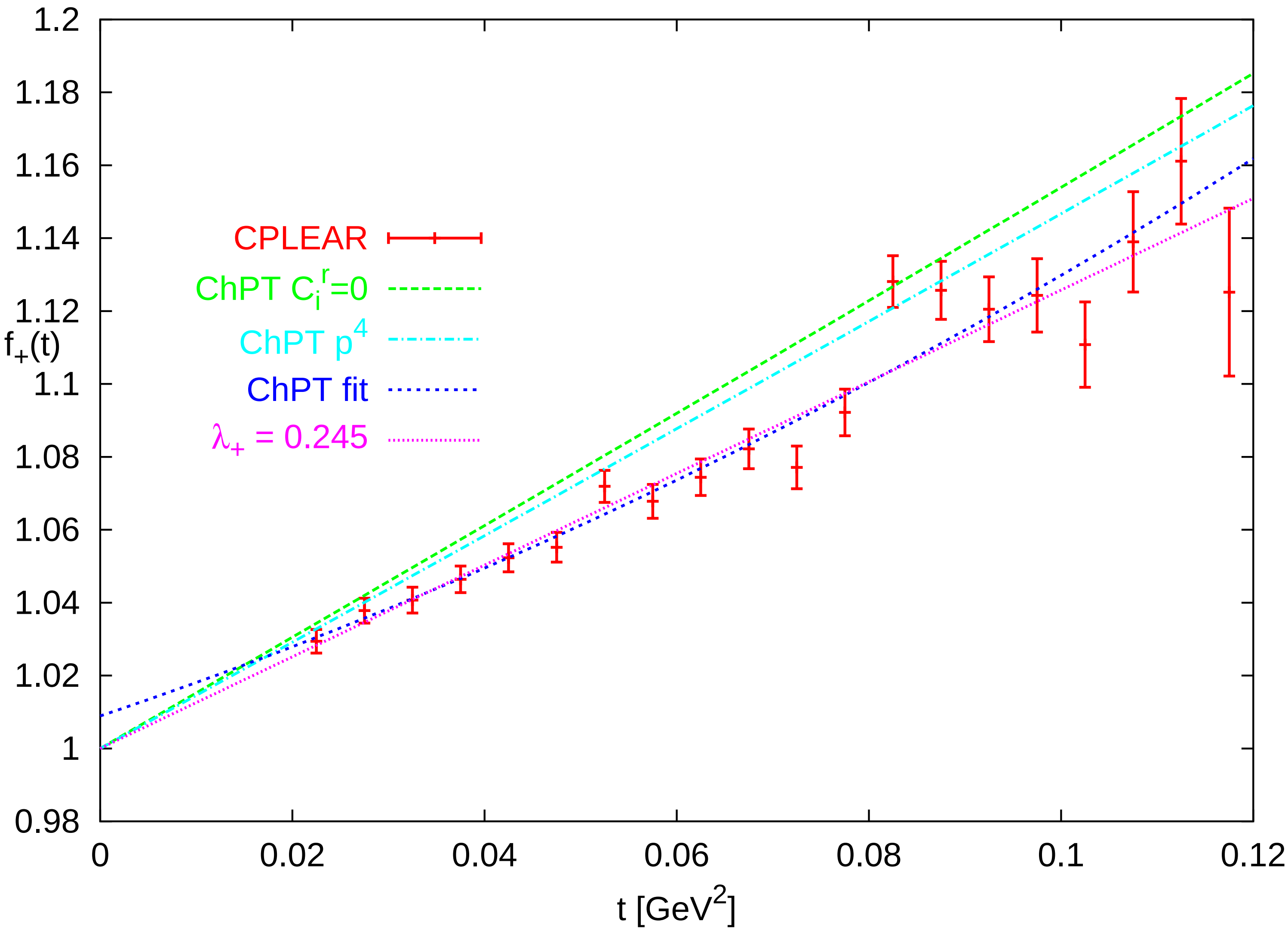
$\Pi_{VV\pi}$, $\Pi_{VV\eta}$, Π_{VVK}	Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
$\Pi_{VV\rho\omega}$	Maltman
$\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η	Kambor, Golowich; Amorós, JB, Talavera
Π_{SS}	Moussallam $\boxed{L_4^r, L_6^r}$
Π_{VVK}, Π_{AAK}, F_K, m_K	Amorós, JB, Talavera
$K_{\ell 4}$, $\langle \overline{q}q \rangle$	Amorós, JB, Talavera $\boxed{L_1^r, L_2^r, L_3^r}$
F_M, m_M, $\langle \overline{q}q \rangle$ $(m_u \neq m_d)$	Amorós, JB, Talavera $\boxed{L_{5,7,8}^r, m_u/m_d}$
$F_{V\pi}$, F_{VK^+}, F_{VK^0}	Post, Schilcher; JB, Talavera $\boxed{L_9^r}$
$K_{\ell 3}$	Post, Schilcher; JB, Talavera $\boxed{V_{us}}$
$F_{S\pi}$, F_{SK} (includes σ-terms)	JB, Dhonte $\boxed{L_4^r, L_6^r}$
$K, \pi \to \ell\nu\gamma$	Geng, Ho, Wu $\boxed{L_{10}^r}$
$\pi\pi$	JB,Dhonte,Talavera
πK	JB,Dhonte,Talavera
m_M and F_M PQChPT	JB,Danielsson,Lähde
Finite Volume	JB,Colangelo,Ghorbani,Haefeli

Pion electromagnetic form-factor



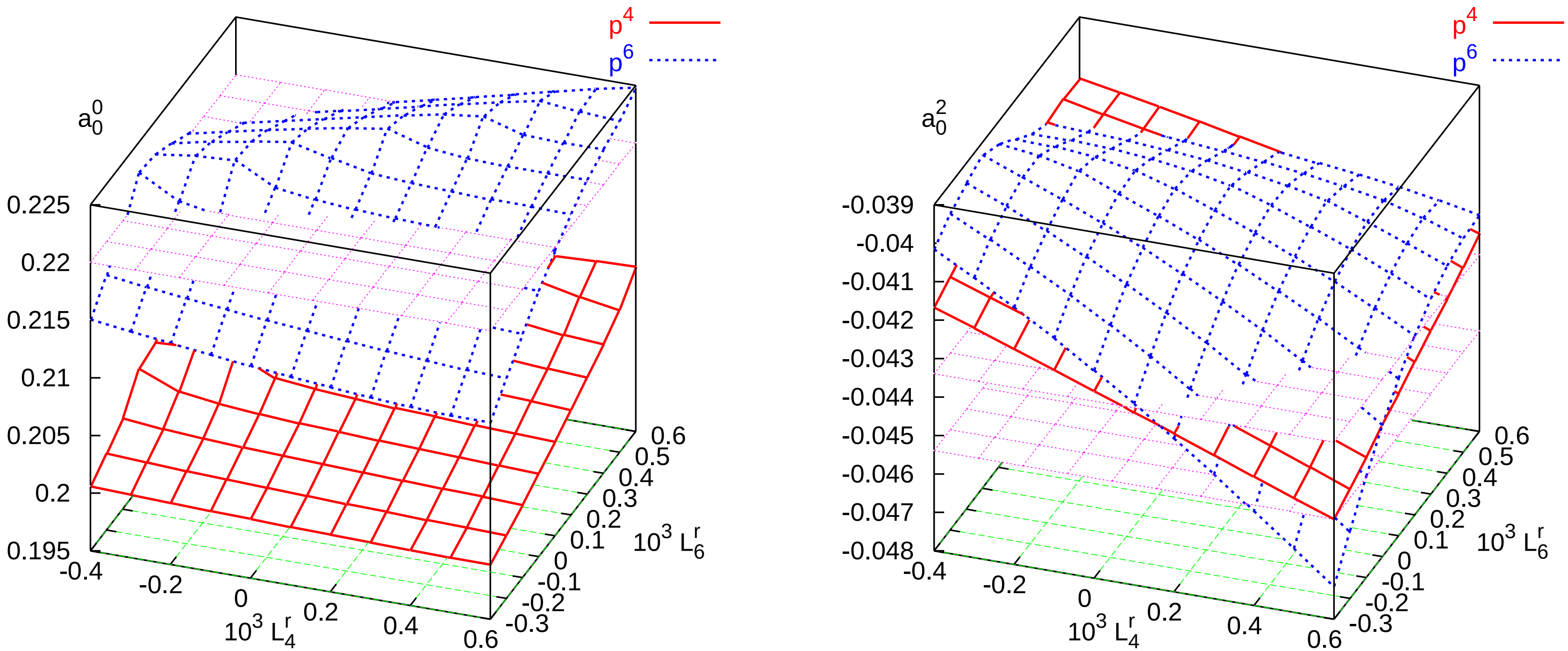
JB,Talavera

f_+ in $K^0 \to \pi^- e^+ \nu$



JB,Talavera

$\pi\pi$

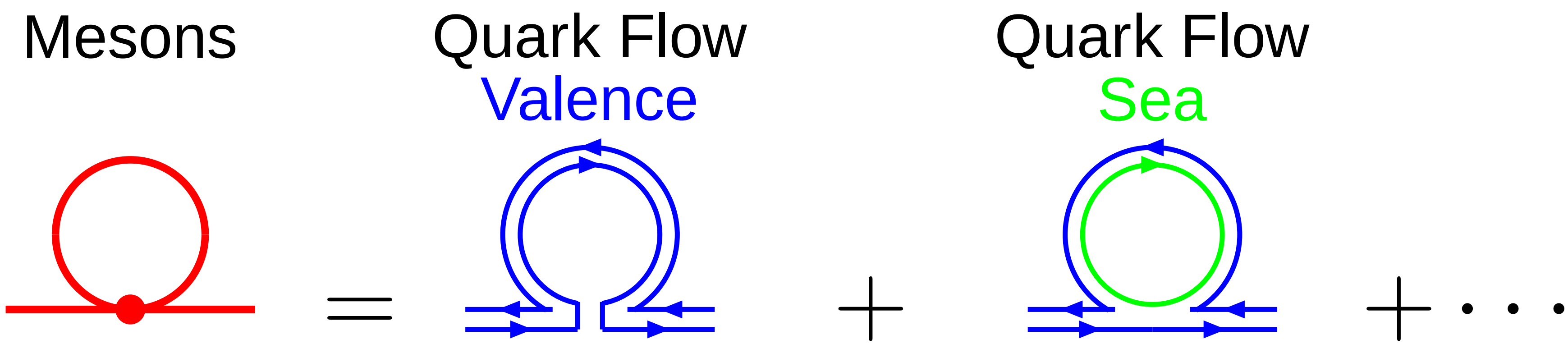


$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159$ $a_0^2 = -0.0454$ at order p^2 JB,Dhonte,Talavera

ChPT and Lattice QCD



Valence is easy to deal with in lattice QCD
Sea is very difficult

They can be treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

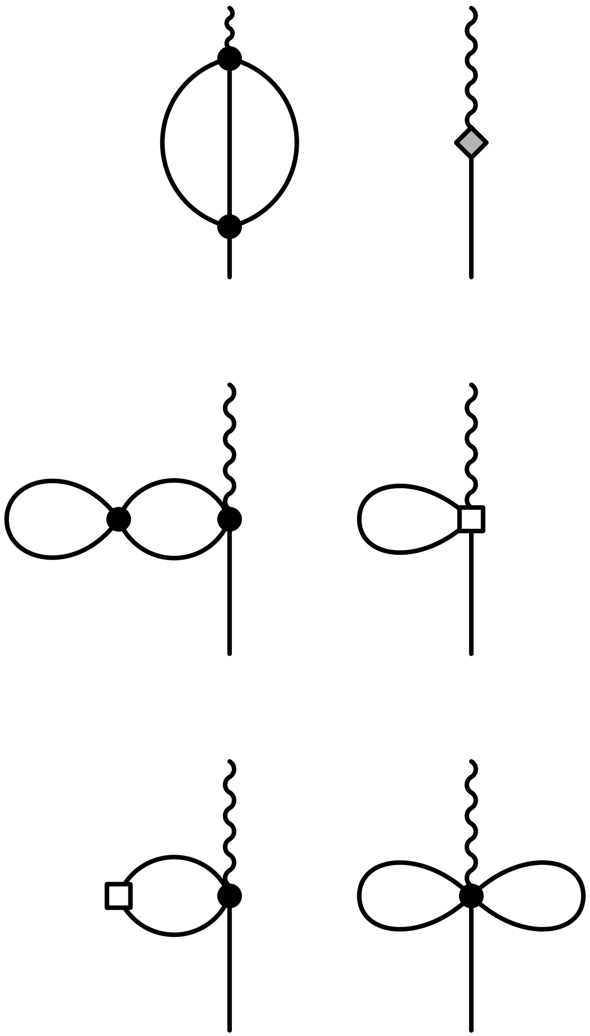
PQChPT at Two Loop

Subject started:
valence equal mass, 3 sea equal mass:
$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0406017
Other mass combinations:
F_{π^+}: JB,Lähde, hep-lat/0501014
F_{π^+}, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004
$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0602003

Actual Calculations:
- heavy use of FORM Vermaseren
- use PQ without super Φ_0 in super-symmetric formalism
- Main problem: sheer size of the expressions

Iso breaking from lattice data: a and L extrapolations needed

Long Expressions



$\Longrightarrow$

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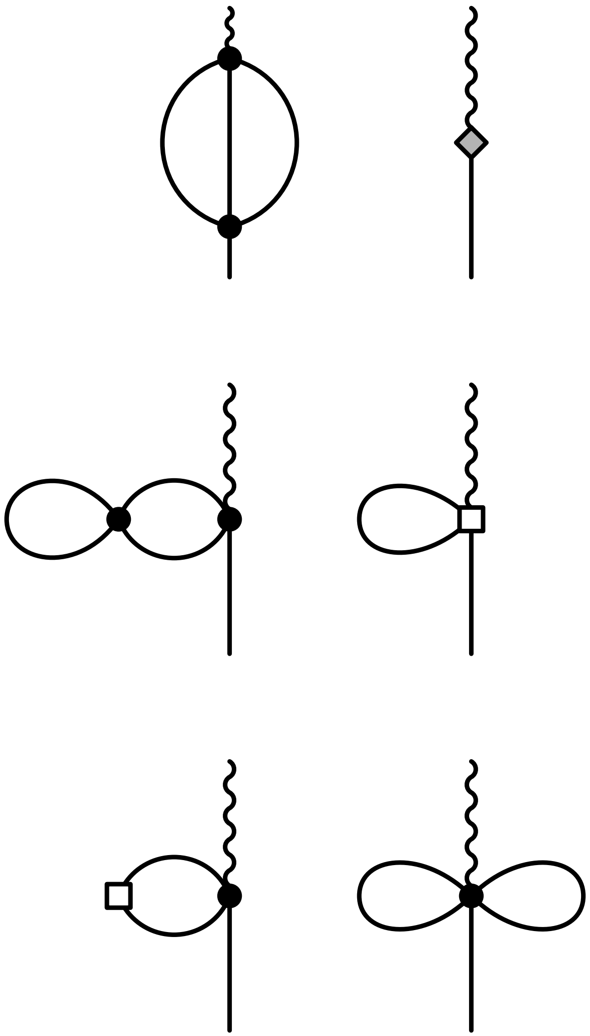

$\Longrightarrow$

$$
\begin{aligned}
\delta^{(6)22}_{\rm loops} =\;& \pi_{16}\,L^r_0\,[4/9\,\chi_\eta\chi_4 - 1/2\,\chi_1\chi_3 + \chi_{13}^2 - 13/3\,\bar\chi_1\chi_{13} - 35/18\,\bar\chi_2] \;-\; 2\,\pi_{16}\,L^r_1\,\chi_{13}^2 \\
-\;& \pi_{16}\,L^r_2\,[11/3\,\chi_\eta\chi_4 + \chi_{13}^2 + 13/3\,\bar\chi_2] + \pi_{16}\,L^r_3\,[4/9\,\chi_\eta\chi_4 - 7/12\,\chi_1\chi_3 + 11/6\,\chi_{13}^2 - 17/6\,\bar\chi_1\chi_{13} - 43/36\,\bar\chi_2] \\
+\;& \pi_{16}^2\,[-15/64\,\chi_\eta\chi_4 - 59/384\,\chi_1\chi_3 + 65/384\,\chi_{13}^2 - 1/2\,\bar\chi_1\chi_{13} - 43/128\,\bar\chi_2] \;-\; 48\,L^r_4L^r_5\,\bar\chi_1\chi_{13} \;-\; 72\,L^{r2}_4\,\bar\chi_1^2 \\
-\;& 8\,L^{r2}_5\,\chi_{13}^2 \;+\; \bar A(\chi_p)\,\pi_{16}\,[-1/24\,\chi_p + 1/48\,\bar\chi_1 - 1/8\,\bar\chi_1\,R^p_{q\eta} + 1/16\,\bar\chi_1\,R^c_p - 1/48\,R^p_{q\eta}\,\chi_p - 1/16\,R^p_{q\eta}\,\chi_q \\
+\;& 1/48\,R^\eta_{pp}\,\chi_\eta + 1/16\,R^c_p\,\chi_{13}] \;+\; \bar A(\chi_p)\,L^r_0\,[8/3\,R^p_{q\eta}\,\chi_p + 2/3\,R^c_p\,\chi_p + 2/3\,R^d_p] \;+\; \bar A(\chi_p)\,L^r_3\,[2/3\,R^p_{q\eta}\,\chi_p \\
+\;& 5/3\,R^c_p\,\chi_p + 5/3\,R^d_p] \;+\; \bar A(\chi_p)\,L^r_4\,[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta0} - 2\,\bar\chi_1\,R^p_{q\eta} + 3\,\bar\chi_1\,R^c_p] \;+\; \bar A(\chi_p)\,L^r_5\,[-2/3\,\bar\chi^{pp}_{\eta\eta1} - R^p_{q\eta}\,\chi_p \\
+\;& 1/3\,R^p_{q\eta}\,\chi_q + 1/2\,R^c_p\,\chi_p - 1/6\,R^c_p\,\chi_q] \;+\; \bar A(\chi_p)^2\,[1/16 + 1/72\,(R^p_{q\eta})^2 - 1/72\,R^p_{q\eta}R^c_p + 1/288\,(R^c_p)^2] \\
+\;& \bar A(\chi_p)\bar A(\chi_{ps})\,[-1/36\,R^p_{q\eta} - 5/72\,R^p_{s\eta} + 7/144\,R^c_p] \;-\; \bar A(\chi_p)\bar A(\chi_{qs})\,[1/36\,R^p_{q\eta} + 1/24\,R^p_{s\eta} + 1/48\,R^c_p] \\
+\;& \bar A(\chi_p)\bar A(\chi_\eta)\,[-1/72\,R^p_{q\eta}R^v_{\eta13} + 1/144\,R^c_pR^v_{\eta13}] \;+\; 1/8\,\bar A(\chi_p)\bar A(\chi_{13}) \;+\; 1/12\,\bar A(\chi_p)\bar A(\chi_{46})\,R^\eta_{pp} \\
+\;& \bar A(\chi_p)\bar B(\chi_p,\chi_p;0)\,[1/4\,\chi_p - 1/18\,R^p_{q\eta}R^c_p\,\chi_p - 1/72\,R^p_{q\eta}R^d_p + 1/18\,(R^c_p)^2\,\chi_p + 1/144\,R^c_pR^d_p] \\
+\;& \bar A(\chi_p)\bar B(\chi_p,\chi_\eta;0)\,[1/18\,R^\eta_{pp}R^c_p\,\chi_p - 1/18\,R^\eta_{13}R^c_p\,\chi_p] \;+\; \bar A(\chi_p)\bar B(\chi_q,\chi_q;0)\,[-1/72\,R^p_{q\eta}R^d_q + 1/144\,R^c_pR^d_q] \\
-\;& 1/12\,\bar A(\chi_p)\bar B(\chi_{ps},\chi_{ps};0)\,R^p_{s\eta}\,\chi_{ps} \;-\; 1/18\,\bar A(\chi_p)\bar B(\chi_1,\chi_3;0)\,R^q_{p\eta}R^c_p\,\chi_p \\
+\;& 1/18\,\bar A(\chi_p)\bar C(\chi_p,\chi_p,\chi_p;0)\,R^c_pR^d_p\,\chi_p \;+\; \bar A(\chi_p;\varepsilon)\,\pi_{16}\,[1/8\,\bar\chi_1R^p_{q\eta} - 1/16\bar\chi_1\,R^c_p - 1/16\,R^c_p\,\chi_p - 1/16\,R^d_p] \\
+\;& \bar A(\chi_{ps})\,\pi_{16}\,[1/16\,\chi_{ps} - 3/16\,\chi_{qs} - 3/16\,\bar\chi_1] \;-\; 2\,\bar A(\chi_{ps})\,L^r_0\,\chi_{ps} \;-\; 5\,\bar A(\chi_{ps})\,L^r_3\,\chi_{ps} \;-\; 3\,\bar A(\chi_{ps})\,L^r_4\,\bar\chi_1 \\
+\;& \bar A(\chi_{ps})\,L^r_5\,\chi_{13} \;+\; \bar A(\chi_{ps})\bar A(\chi_\eta)\,[7/144\,R^\eta_{pp} - 5/72\,R^\eta_{ps} - 1/48\,R^\eta_{qq} + 5/72\,R^\eta_{qs} - 1/36\,R^\eta_{13}] \\
+\;& \bar A(\chi_{ps})\bar B(\chi_p,\chi_p;0)\,[1/24\,R^p_{s\eta}\,\chi_p - 5/24\,R^p_{s\eta}\,\chi_{ps}] \;+\; \bar A(\chi_{ps})\bar B(\chi_p,\chi_\eta;0)\,[-1/18\,R^\eta_{ps}R^z_{qp\eta}\,\chi_p \\
-\;& 1/9\,R^\eta_{ps}R^z_{qp\eta}\,\chi_{ps}] \;-\; 1/48\,\bar A(\chi_{ps})\bar B(\chi_q,\chi_q;0)\,R^d_q \;+\; 1/18\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0)\,R^q_{s\eta}\,\chi_s \\
+\;& 1/9\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0,k)\,R^q_{s\eta} \;+\; 3/16\,\bar A(\chi_{ps};\varepsilon)\,\pi_{16}\,[\chi_s + \bar\chi_1] \;-\; 1/8\,\bar A(\chi_{p4})^2 \;-\; 1/8\,\bar A(\chi_{p4})\bar A(\chi_{p6}) \\
+\;& 1/8\,\bar A(\chi_{p4})\bar A(\chi_{q6}) \;-\; 1/32\,\bar A(\chi_{p6})^2 \;+\; \bar A(\chi_\eta)\,\pi_{16}\,[1/16\,\bar\chi_1\,R^v_{\eta13} - 1/48\,R^v_{\eta13}\,\chi_\eta + 1/16\,R^v_{\eta13}\,\chi_{13}] \\
+\;& \bar A(\chi_\eta)\,L^r_0\,[4R^\eta_{13}\,\chi_\eta + 2/3\,R^v_{\eta13}\,\chi_\eta] \;-\; 8\,\bar A(\chi_\eta)\,L^r_1\,\chi_\eta \;-\; 2\,\bar A(\chi_\eta)\,L^r_2\,\chi_\eta \;+\; \bar A(\chi_\eta)\,L^r_3\,[4R^\eta_{13}\,\chi_\eta + 5/3\,R^v_{\eta13}\chi_\eta] \\
+\;& \bar A(\chi_\eta)\,L^r_4\,[4\,\chi_\eta + \bar\chi_1\,R^v_{\eta13}] \;-\; \bar A(\chi_\eta)\,L^r_5\,[1/6\,R^\eta_{pp}\,\chi_q + R^\eta_{13}\,\chi_{13} + 1/6\,R^v_{\eta13}\,\chi_\eta] \;+\; 1/288\,\bar A(\chi_\eta)^2\,(R^v_{\eta13})^2 \\
+\;& 1/12\,\bar A(\chi_\eta)\bar A(\chi_{46})\,R^v_{\eta13} \;+\; \bar A(\chi_\eta)\bar B(\chi_p,\chi_p;0)\,[-1/36\,\bar\chi^{pp\eta}_{\eta\eta1} - 1/18\,R^p_{q\eta}R^\eta_{pp}\,\chi_p + 1/18\,R^\eta_{pp}R^c_p\,\chi_p \\
+\;& 1/144\,R^d_pR^v_{\eta13}] \;+\; \bar A(\chi_\eta)\bar B(\chi_p,\chi_\eta;0)\,[-1/18\,\bar\chi^{\eta p\eta}_{p\eta1} + 1/18\,\bar\chi^{\eta p\eta}_{q\eta1} + 1/18\,(R^\eta_{pp})^2R^z_{qp\eta}\,\chi_p] \\
-\;& 1/12\,\bar A(\chi_\eta)\bar B(\chi_{ps},\chi_{ps};0)\,R^\eta_{ps}\,\chi_{ps} \;-\; \bar A(\chi_\eta)\bar B(\chi_\eta,\chi_\eta;0)\,[1/216\,R^v_{\eta13}\,\chi_4 + 1/27\,R^v_{\eta13}\,\chi_6] \\
-\;& 1/18\,\bar A(\chi_\eta)\bar B(\chi_1,\chi_3;0)\,R^1_{\eta\eta}R^3_{\eta\eta}\,\chi_\eta \;+\; 1/18\,\bar A(\chi_\eta)\bar C(\chi_p,\chi_p,\chi_p;0)\,R^\eta_{pp}R^d_p\,\chi_p \;+\; \bar A(\chi_\eta;\varepsilon)\,\pi_{16}\,[1/8\,\chi_\eta \\
-\;& 1/16\,\bar\chi_1\,R^v_{\eta13} - 1/8\,R^\eta_{13}\,\chi_\eta - 1/16\,R^v_{\eta13}\,\chi_\eta] \;+\; \bar A(\chi_1)\bar A(\chi_3)\,[-1/72\,R^p_{q\eta}R^c_q + 1/36\,R^1_{3\eta}R^3_{1\eta} + 1/144\,R^c_1R^c_3] \\
-\;& 4\,\bar A(\chi_{13})\,L^r_1\,\chi_{13} \;-\; 10\,\bar A(\chi_{13})\,L^r_2\,\chi_{13} \;+\; 1/8\,\bar A(\chi_{13})^2 \;-\; 1/2\,\bar A(\chi_{13})\bar B(\chi_1,\chi_3;0,k) \\
+\;& 1/4\,\bar A(\chi_{13};\varepsilon)\,\pi_{16}\,\chi_{13} \;+\; 1/4\,\bar A(\chi_{14})\bar A(\chi_{34}) \;+\; 1/16\,\bar A(\chi_{16})\bar A(\chi_{36}) \;-\; 24\,\bar A(\chi_4)\,L^r_1\,\chi_4 \;-\; 6\,\bar A(\chi_4)\,L^r_2\,\chi_4 \\
+\;& 12\,\bar A(\chi_4)\,L^r_4\,\chi_4 \;+\; 1/12\,\bar A(\chi_4)\bar B(\chi_p,\chi_p;0)\,(R^p_{4\eta})^2\,\chi_4 \;+\; 1/6\,\bar A(\chi_4)\bar B(\chi_p,\chi_\eta;0)\,[R^p_{4\eta}R^\eta_{p4}\,\chi_4 - R^p_{4\eta}R^\eta_{q4}\,\chi_4] \\
-\;& 1/24\,\bar A(\chi_4)\bar B(\chi_\eta,\chi_\eta;0)\,R^v_{\eta13}\,\chi_4 \;-\; 1/6\,\bar A(\chi_4)\bar B(\chi_1,\chi_3;0)\,R^1_{4\eta}R^3_{4\eta}\,\chi_4 \;+\; 3/8\,\bar A(\chi_4;\varepsilon)\,\pi_{16}\,\chi_4 \\
-\;& 32\,\bar A(\chi_{46})\,L^r_1\,\chi_{46} \;-\; 8\,\bar A(\chi_{46})\,L^r_2\,\chi_{46} \;+\; 16\,\bar A(\chi_{46})\,L^r_4\,\chi_{46} \;+\; \bar A(\chi_{46})\bar B(\chi_p,\chi_p;0)\,[1/9\,\chi_{46} + 1/12\,R^\eta_{pp}\,\chi_p \\
+\;& 1/36\,R^\eta_{pp}\,\chi_4 + 1/9\,R^\eta_{p4}\,\chi_6] \;+\; \bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0)\,[-1/18\,R^\eta_{pp}\,\chi_4 - 1/9\,R^\eta_{p4}\,\chi_6 + 1/9\,R^\eta_{q4}\,\chi_6 + 1/18\,R^\eta_{13}\,\chi_4] \\
-\;& 1/6\,\bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0,k)\,[R^\eta_{pp} - R^\eta_{13}] \;+\; 1/9\,\bar A(\chi_{46})\bar B(\chi_\eta,\chi_\eta;0)\,R^v_{\eta13}\,\chi_{46} \;-\; \bar A(\chi_{46})\bar B(\chi_1,\chi_3;0)\,[2/9\,\chi_{46} \\
+\;& 1/9\,R^\eta_{p4}\,\chi_6 + 1/18\,R^\eta_{13}\,\chi_4] \;-\; 1/6\,\bar A(\chi_{46})\bar B(\chi_1,\chi_3;0,k)\,R^\eta_{13} \;+\; 1/2\,\bar A(\chi_{46};\varepsilon)\,\pi_{16}\,\chi_{46} \\
+\;& \bar B(\chi_p,\chi_p;0)\,\pi_{16}\,[1/16\,\bar\chi_1\,R^d_p + 1/96\,R^d_p\,\chi_p + 1/32\,R^d_p\,\chi_q] \;+\; 2/3\,\bar B(\chi_p,\chi_p;0)\,L^r_0\,R^d_p\,\chi_p \\
+\;& 5/3\,\bar B(\chi_p,\chi_p;0)\,L^r_3\,R^d_p\,\chi_p \;+\; \bar B(\chi_p,\chi_p;0)\,L^r_4\,[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta0}\chi_p - 4\,\bar\chi_1R^p_{q\eta}\,\chi_p + 4\,\bar\chi_1R^c_p\,\chi_p + 3\,\bar\chi_1R^d_p] \\
+\;& \bar B(\chi_p,\chi_p;0)\,L^r_5\,[-2/3\bar\chi^{pp}_{\eta\eta1}\chi_p - 4/3\,R^p_{q\eta}\,\chi_p^2 + 4/3\,R^c_p\,\chi_p^2 + 1/2\,R^d_p\,\chi_p - 1/6\,R^d_p\,\chi_q] \\
+\;& \bar B(\chi_p,\chi_p;0)\,L^r_6\,[4\,\bar\chi_1\bar\chi^{pp}_{\eta\eta1} + 8\,\bar\chi_1R^p_{q\eta}\,\chi_p - 8\,\bar\chi_1R^c_p\,\chi_p] \;+\; 4\,\bar B(\chi_p,\chi_p;0)\,L^r_7\,(R^d_p)^2 \\
+\;& \bar B(\chi_p,\chi_p;0)\,L^r_8\,[4/3\,\bar\chi^{pp}_{\eta\eta2} + 8/3\,R^p_{q\eta}\,\chi_p^2 - 8/3\,R^c_p\,\chi_p^2] \;+\; \bar B(\chi_p,\chi_p;0)^2\,[-1/18\,R^p_{q\eta}R^d_p\,\chi_p + 1/18\,R^c_pR^d_p\,\chi_p \\
+\;& 1/288\,(R^d_p)^2] \;+\; 1/18\,\bar B(\chi_p,\chi_p;0)\bar B(\chi_p,\chi_\eta;0)\,[R^\eta_{pp}R^d_p\,\chi_p - R^\eta_{13}R^d_p\,\chi_p]
\end{aligned}
$$

plus several more pages

PQChPT at Two Loop

Use lowest order mass squared: $\chi_i = 2B_0 m_i = m_M^{2(0)}$

Remember: $\chi_i \approx 0.3\ GeV^2 \approx (550\ MeV)^2 \sim$ border ChPT

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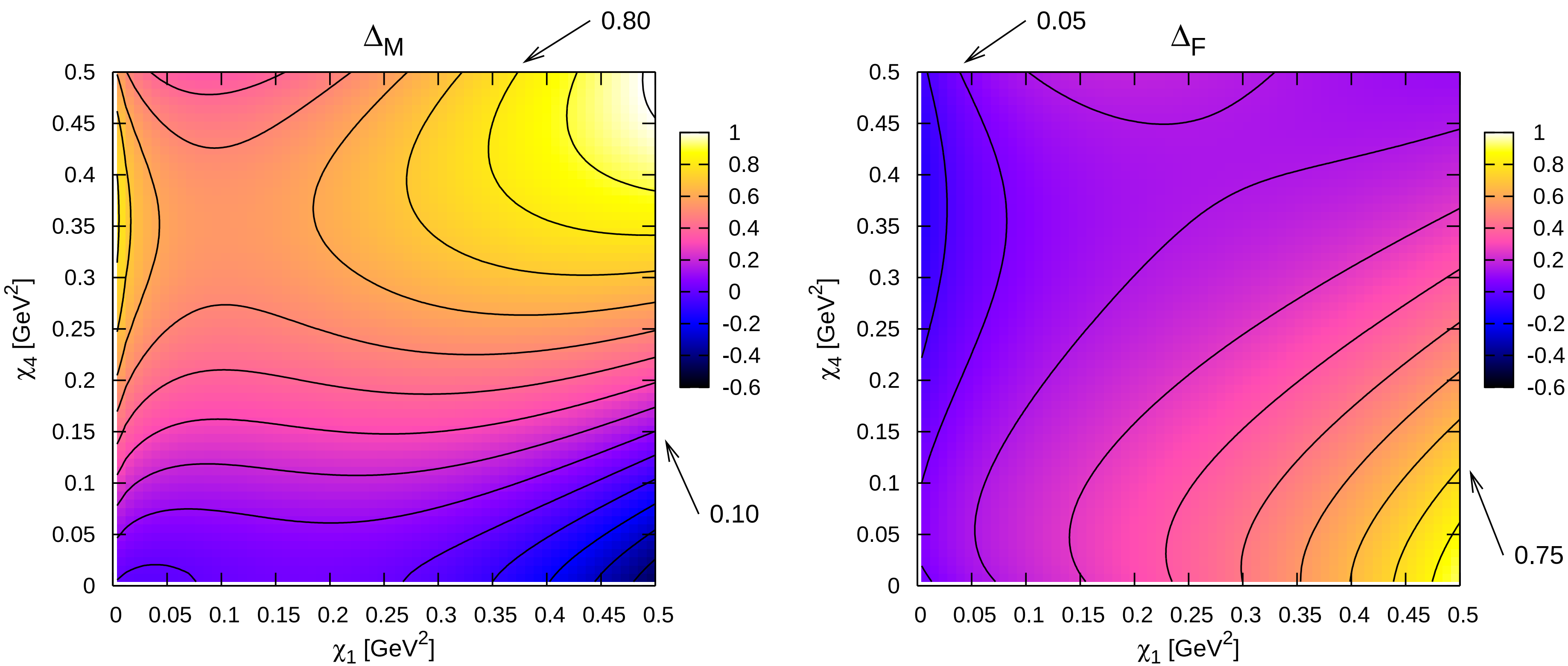


Relative corrections: mass^2 decay constant

ε'/ε and $\Delta I = 1/2$

Problem: Three regimes in momenta/distances

- long-distance, low Q^2:
- intermediate Q^2:
- Short-distance, high Q^2:

ε'/ε and $\Delta I = 1/2$

Problem: Three regimes in momenta/distances

- long-distance, low Q^2: ChPT
- intermediate Q^2:
- Short-distance, high Q^2: QCD

ε'/ε and $\Delta I = 1/2$

Problem: Three regimes in momenta/distances

- long-distance, low Q^2: ChPT
- intermediate Q^2: Models, large N_c
- Short-distance, high Q^2: QCD

Matching between the various parts

ε'/ε and $\Delta I = 1/2$

Problem: Three regimes in momenta/distances

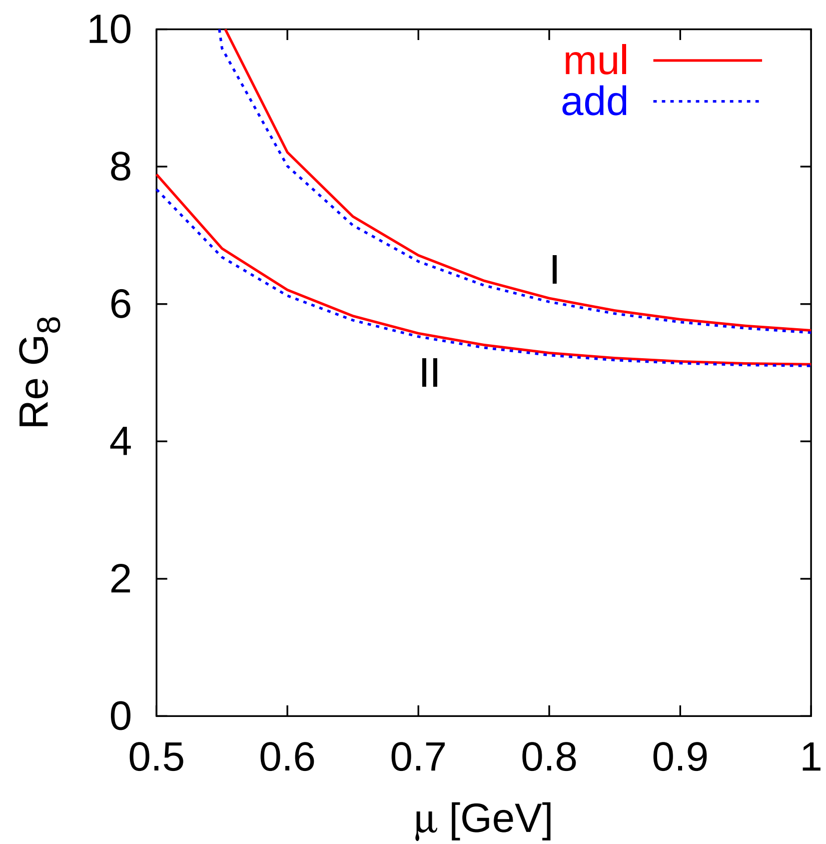
- long-distance, low Q^2: ChPT
- intermediate Q^2: Models, large N_c
- Short-distance, high Q^2: QCD

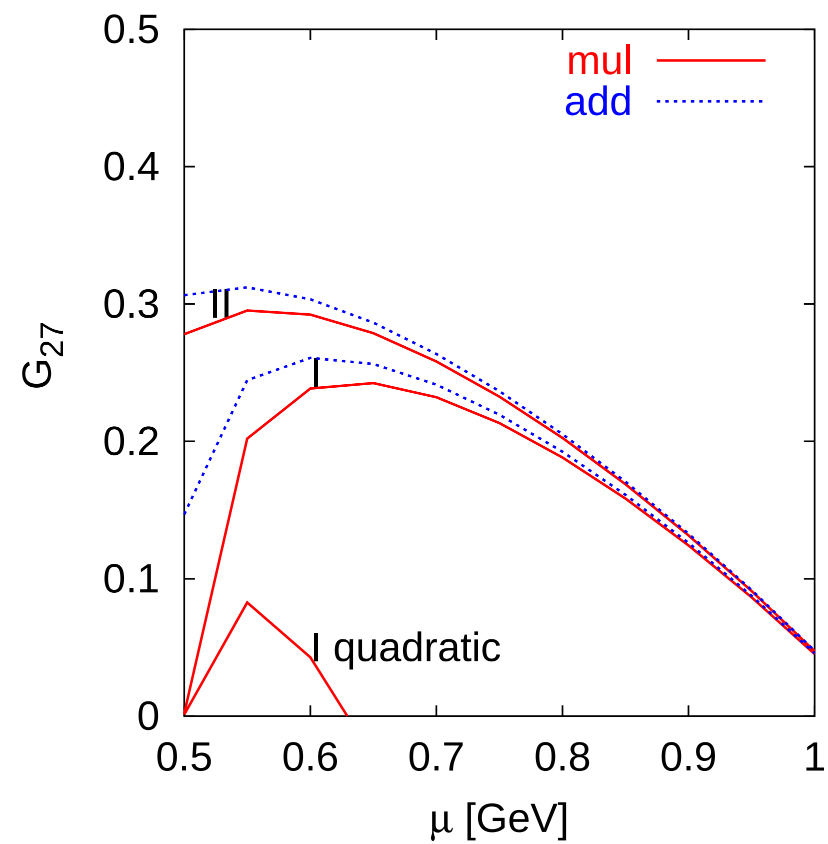
Matching between the various parts

JB,Prades,Gamiz,Lipartia

- Models: First ENJL, later ladder resummation model
- Matching: X-boson method

ε'/ε and $\Delta I = 1/2$





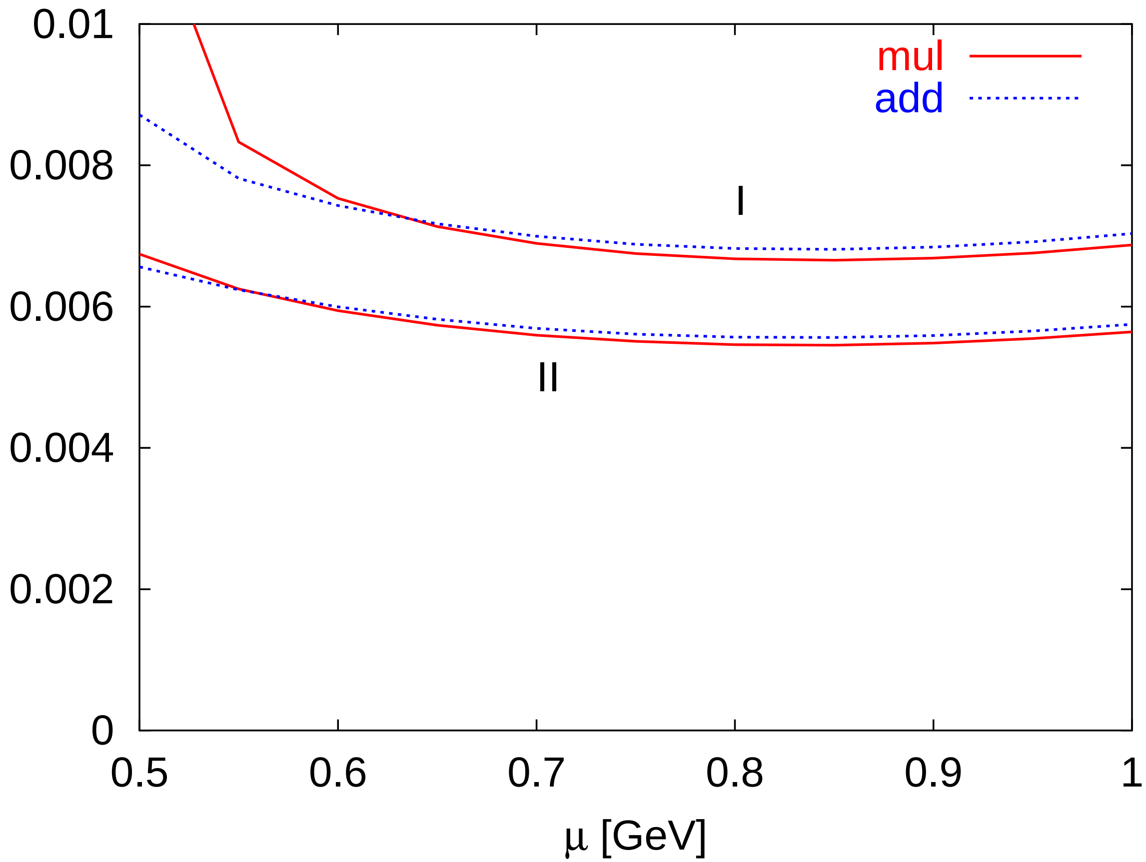
Re$G_8(\mu)$ and $G_{27}(\mu)$ (μ: matching scale hadronic-QCD)

Experiment: Re $G_8 \approx 6.24 \quad G_{27} \approx 0.48$

Large N_c limit: Re $G_8 = G_{27} = 1$

JB,Prades 2000

ε'/ε and $\Delta I = 1/2$

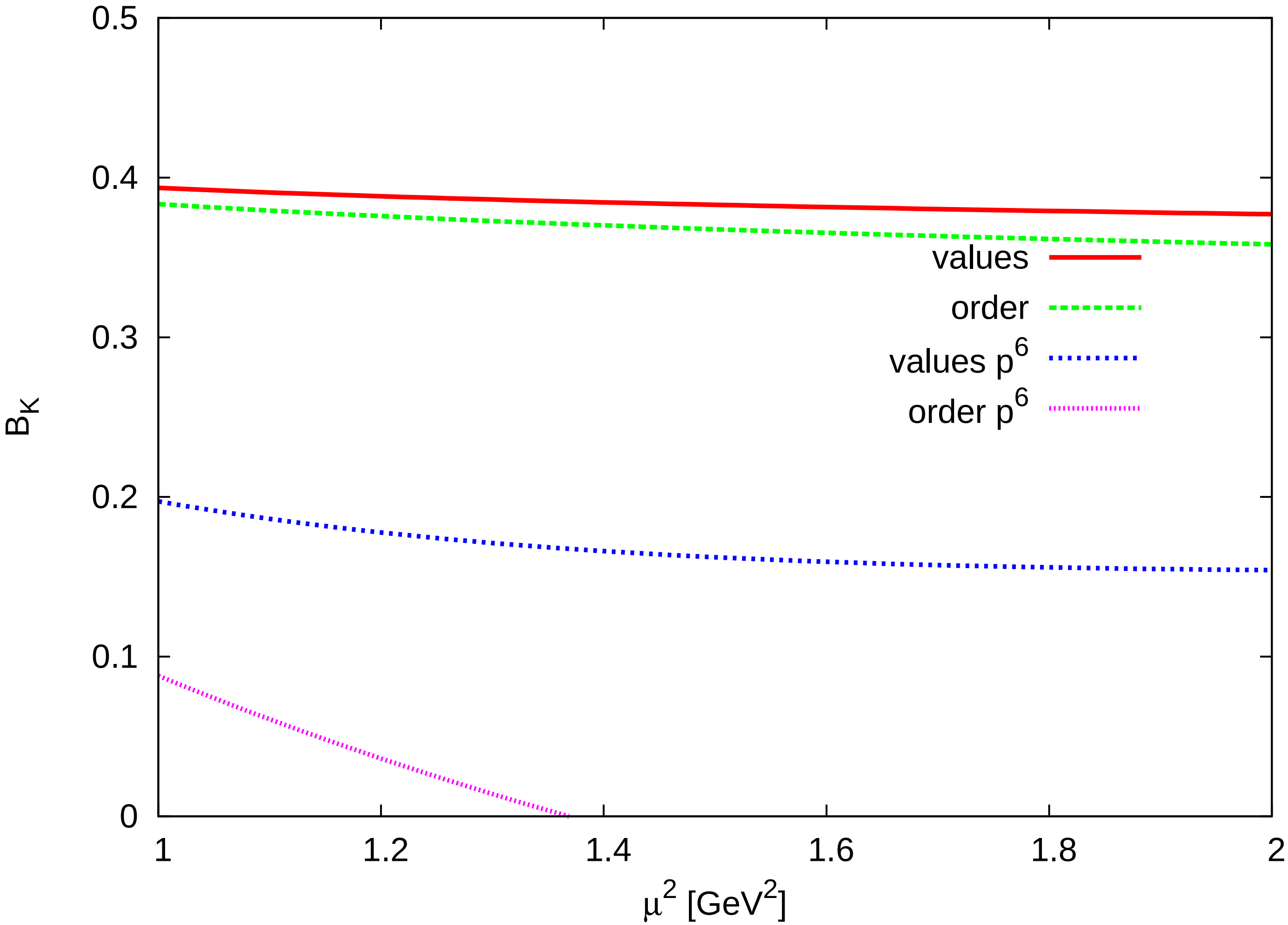


JB,Prades 2000

$\varepsilon'_K/\varepsilon_K$ in the chiral limit with experimental ε_K put in

$$\left|\frac{\varepsilon'}{\varepsilon}\right|_{\mathcal{O}(p^2)} = (6 \pm 3) \cdot 10^-3 \qquad \left|\frac{\varepsilon'}{\varepsilon}\right|_{LO} = (3.4 \pm 1.8) \cdot 10^-3$$

B_K in improved model



$$\hat{B}_K = 0.38 \pm 0.15$$

JB,Gamiz,Prades
2006

Conclusions

- At low-energies: pure ChPT alive and well and active area
- Weak decays and CP violation: painfully slow progress
- connections with lattice QCD growing
- Effective field theory also used for strongly interacting Higgs sector
- Connection few-body hadronic physics $\Leftrightarrow$ multihadron physics (fragmentation etc) remains open question

Dealing with many parameters

Inputs:

$K_{\ell 4}$: $F(0)$, $G(0)$, λ — E865 BNL

$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$ — em with Dashen violation

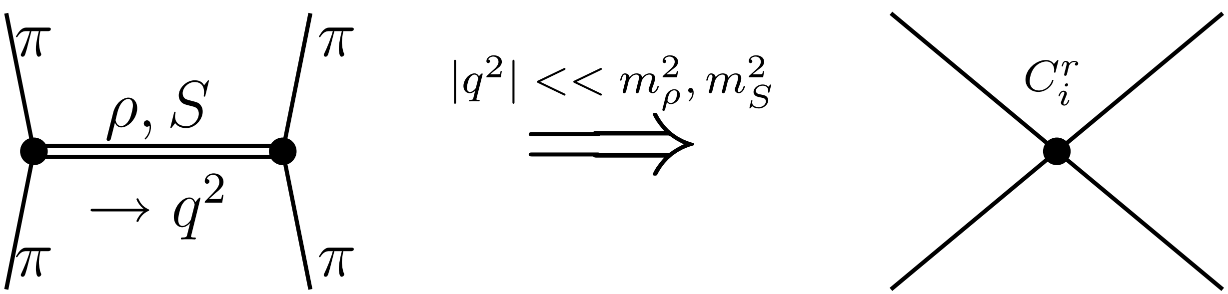
F_{π^+}

F_{K^+}/F_{π^+}

$m_s/\hat{m}$ — 24 (26) — $\hat{m} = (m_u + m_d)/2$

L^r_4, L^r_6

C^r_i from single resonance approximation



Conclusions

- 3 flavour ChPT at 2 loops
  - many calculations done
  - things seem to work but convergence is fairly slow
  - “kinematical” and “vector” C_i^r seem to be OK
  - L_4^r, L_6^r nonzero but reasonable for large N_c
  - $\eta \to 3\pi$, isobreaking in $K_{\ell 3}$: parts done
- PQChPT at 2 loops: subject just beginning
- $\eta \to 3\pi$: isospin breaking part needed to p^6, compare with dispersive (see $K_{\ell 4}$)
- $K \to 3\pi$: CP violating observables, higher orders for Cabibbo proposal