CHIRAL PERTURBATION THEORY IN NEW SURROUNDINGS

Johan Bijnens

Lund University

bijnens@thep.lu.se
http://www.thep.lu.se/∼bijnens

Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

New stuff in Lund

Three extensions to Chiral Perturbation Theory:

- Hard pion Chiral Perturbation Theory (one-loop)
  - main part of the talk
- Leading logarithms to high orders (up to 6 loops)
  - a bit of an overview
- Chiral Extrapolation Formulas for Technicolor and QCDlike theories
  - Only in the extremely unlikely case of time left

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- Chiral Extrapolation Formulas for Technicolor and QCDlike theories
  - I.e. equal mass ChPT for (at two loops)
    - $SU(n) \times SU(n)/SU(n)$
    - $SU(2n)/SO(2n)$
    - $SU(2n)/Sp(2n)$
  - JB + Jie Lu, arXiv:0910.5424, arXiv:1102.0172, arXiv:1111.1886 (yesterday)

Overview

- Effective Field Theory
- Chiral Perturbation Theor(y)(ies)
- Hard Pion Chiral Perturbation Theory

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- Hard Pion Chiral Perturbation Theory
  - $K_{\ell 3}$ Flynn-Sachrajda, arXiv:0809.1229
  - $K \to \pi\pi$ JB+ Alejandro Celis, arXiv:0906.0302
  - F_π^S and F_π^V JB + Ilaria Jemos, arXiv:1011.6531 a two-loop check
  - $B, D \to \pi$ JB + Ilaria Jemos, arXiv:1006.1197
  - $B, D \to \pi, K, \eta$ JB + Ilaria Jemos, arXiv:1011.6531
  - $\chi_c(J = 0, 2) \to \pi\pi, KK, \eta\eta$ JB+Ilaria Jemos, arxiv:1109.5033
  - Some examples which do not have a chiral log prediction

Overview

- Effective Field Theory
- Chiral Perturbation Theor(y)(ies)
- Hard Pion Chiral Perturbation Theory
- Leading logarithms at high orders in the massive nonlinear sigma model and large N.
  - Mass
    JB, Carloni, arXiv:0909.5086
  - F_π, F_V, F_S, $\pi\pi$-scattering
    JB,Carloni, arXiv:1008.3499
  - Adding the anomaly (and some higher orders)
    JB, K. Kampf, S. Lanz, in preparation

Wikipedia

`http://en.wikipedia.org/wiki/Effective_field_theory`

In physics, an effective field theory is an approximate theory (usually a quantum field theory) that contains the appropriate degrees of freedom to describe physical phenomena occurring at a chosen length scale, but ignores the substructure and the degrees of freedom at shorter distances (or, equivalently, higher energies).

Effective Field Theory (EFT)

Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples:

- Solid state physics: conductors: neglect the empty bands above the partially filled one
- Atomic physics: Blue sky: neglect atomic structure

EFT: Power Counting

- gap in the spectrum $\Longrightarrow$ separation of scales
- with the lower degrees of freedom, build the most general effective Lagrangian

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  - Where did my predictivity go ?

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  - $\infty\#$ parameters
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$\Longrightarrow$ Need some ordering principle: power counting
Higher orders suppressed by powers of $1/\Lambda$

References

- A. Manohar, Effective Field Theories (Schladming lectures), hep-ph/9606222
- I. Rothstein, Lectures on Effective Field Theories (TASI lectures), hep-ph/0308266
- G. Ecker, Effective field theories, Encyclopedia of Mathematical Physics, hep-ph/0507056
- D.B. Kaplan, Five lectures on effective field theory, nucl-th/0510023
- A. Pich, Les Houches Lectures, hep-ph/9806303
- S. Scherer, Introduction to chiral perturbation theory, hep-ph/0210398
- J. Donoghue, Introduction to the Effective Field Theory Description of Gravity, gr-qc/9512024

Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

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Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory,* Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

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Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

We have 8 candidates that are light compared to the other hadrons: $\pi^0, \pi^+, \pi^-, K^+, K^-, K^0, \overline{K^0}, \eta$

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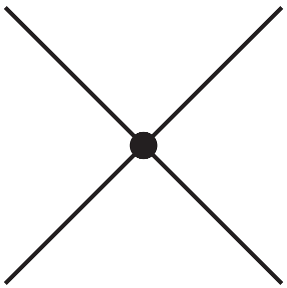
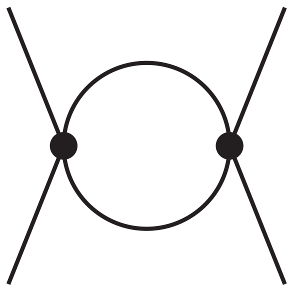
Power counting in momenta (all lines soft):

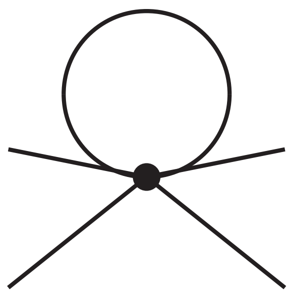


p^2



$(p^2)^2 \, (1/p^2)^2 \, p^4 = p^4$

$1/p^2$



$(p^2) \, (1/p^2) \, p^4 = p^4$

$\int d^4p$ $\quad p^4$

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
  - Two or Three (or even more) Flavours
  - Strong interaction and couplings to external currents/densities
  - Including electromagnetism
  - Including weak nonleptonic interactions
  - Treating kaon as heavy

Many similarities with strongly interacting Higgs

Hard pion ChPT?

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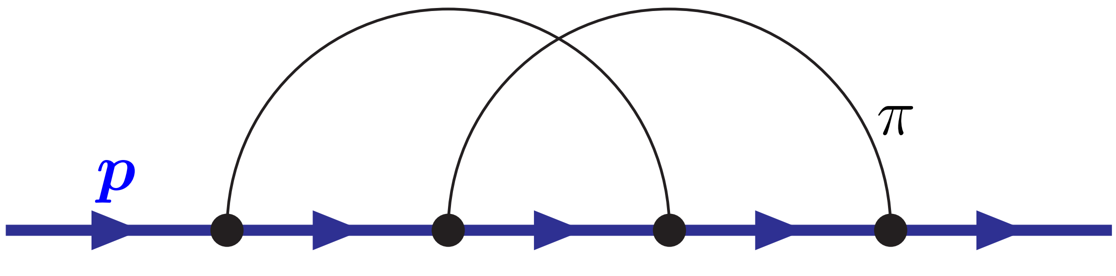
- Baryon and Heavy Meson ChPT: $p, n, \ldots\ B, B^*$ or D, D^*
  - $p = M_B v + k$
  - Everything else soft
  - Works because baryon or b or c number conserved so the non soft line is continuous



  - See talk by Fissum for some experimental tests

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  - Decay constant works: takes away all heavy momentum
  - General idea: M_p dependence can always be reabsorbed in LECs, is analytic in the other parts k.

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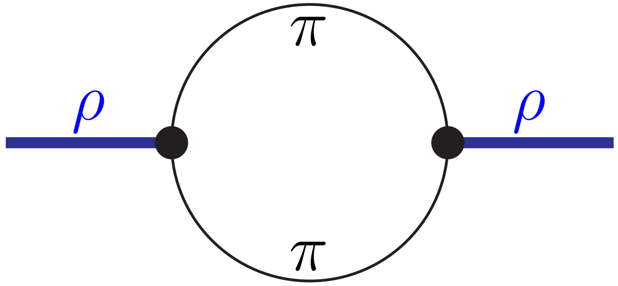

Hard pion ChPT?

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  - (Vector) Meson: $p = M_V v + k$
  - Everyone else soft or $p = M_V v + k$
  - But (Heavy) (Vector) Meson ChPT decays strongly
    - First: keep diagrams where vectors always present
    - Applied to masses and decay constants
    - Decay constant works: takes away all heavy momentum
  - *It was argued that this could be done, the nonanalytic parts of diagrams with pions at large momenta are reproduced correctly* JB-Gosdzinsky-Talavera
  - Done both in relativistic and heavy meson formalism
  - General idea: M_V dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,. . .
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
  - Works like all the previous *heavy* ChPT

Hard pion ChPT?

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  - $p = M_K v + k$
  - First: only keep diagrams where Kaon goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,. . .
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
- Flynn-Sachrajda argued $K_{\ell 3}$ also for q^2 away from q^2_{max}.
- JB-Celis Argument generalizes to other processes with hard/fast pions and applied to $K \to \pi\pi$
- JB Jemos $B, D \to D, \pi, K, \eta$ vector formfactors, charmonium decays and a two-loop check
- General idea: heavy/fast dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

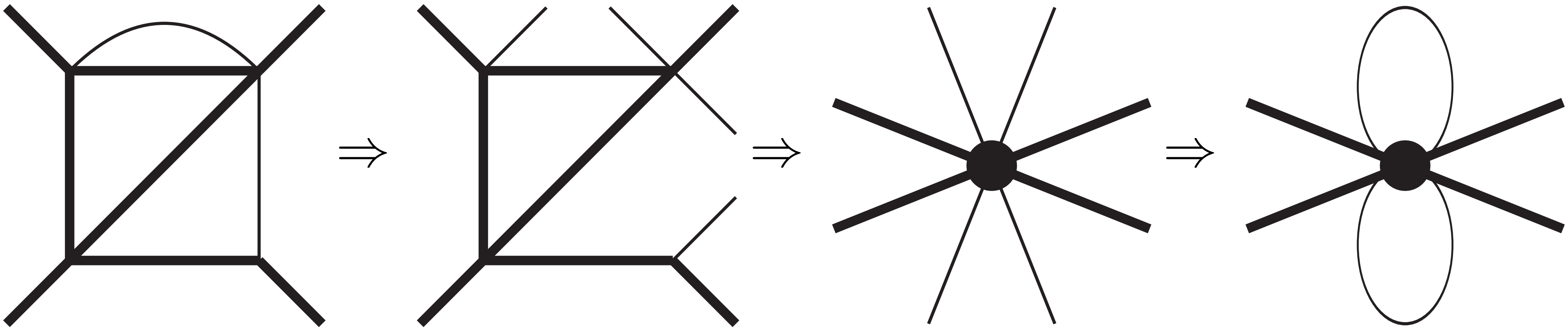
- Nothing prevents hard pions to be in the states α or β
- So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence

Hard pion ChPT?

Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta (Weinberg's folklore theorem)
- Envisage this effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included.

Hard pion ChPT?



This procedure works at one loop level, matching at tree level, nonanalytic dependence at one loop:

- Toy models and vector meson ChPT JB, Gosdzinsky, Talavera
- Recent work on relativistic baryon ChPT Gegelia, Scherer et al.
- Extra terms kept in many of our calculations: a one-loop check
- Some two-loop checks

Hard pion ChPT?

- This effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included: possibly an infinite number of terms
- If symmetries present, Lagrangian should respect them
- but my powercounting is gone

Hard pion ChPT?

- This effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included: possibly an infinite number of terms
- If symmetries present, Lagrangian should respect them
- In some cases we can prove that up to a certain order in the expansion in light masses, not momenta, matrix elements of higher order operators are reducible to those of lowest order.
- Lagrangian should be complete in *neighbourhood* of original process
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses
  Crucial part of the argument

The main technical trick

- For getting soft singularities in an integral we need the meson close to on-shell
- This only happens in an area of order m^4
- So typically $\int d^4p\, 1/(p^2-m^2) \sim m^4/m^2$ but if $\partial_\mu\phi$ on that propagator we get an extra factor of m.
- So extra derivatives are only at same order if they hit hard lines
- and then they are part of the hard part which can be expanded around

$K \to 2\pi$ in $SU(2)$ ChPT

Add $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ Roessl

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\pi\pi} &= \frac{F^2}{4}\left(\langle u_\mu u^\mu\rangle + \langle \chi_+\rangle\right), \\
\mathcal{L}^{(1)}_{\pi K} &= \nabla_\mu K^\dagger \nabla^\mu K - \overline{M}^2_K K^\dagger K\,, \\
\mathcal{L}^{(2)}_{\pi K} &= A_1 \langle u_\mu u^\mu\rangle K^\dagger K + A_2 \langle u^\mu u^\nu\rangle \nabla_\mu K^\dagger \nabla_\nu K + A_3 K^\dagger \chi_+ K + \cdots
\end{aligned}
$$

Add a spurion for the weak interaction $\Delta I = 1/2$, $\Delta I = 3/2$

JB,Celis

$$t^{ij}_k \longrightarrow t^{i'j'}_{k'} = t^{ij}_k (g_L)_{k'}{}^k (g_L^\dagger)_i{}^{i'} (g_L^\dagger)_j{}^{j'}$$

$$t^i_{1/2} \longrightarrow t^{i'}_{1/2} = t^i_{1/2} (g_L^\dagger)_i{}^{i'}.$$

$K \to 2\pi$ in $SU(2)$ ChPT

The $\Delta I = 1/2$ terms: $\tau_{1/2} = t_{1/2} u^\dagger$

$$
\begin{aligned}
\mathcal{L}_{1/2} \;=\;& iE_1\,\tau_{1/2}K + E_2\,\tau_{1/2}u^\mu\nabla_\mu K + iE_3\langle u_\mu u^\mu\rangle\tau_{1/2}K \\
& + iE_4\tau_{1/2}\chi_+K + iE_5\langle\chi_+\rangle\tau_{1/2}K + E_6\tau_{1/2}\chi_-K \\
& + E_7\langle\chi_-\rangle\tau_{1/2}K + iE_8\langle u_\mu u_\nu\rangle\tau_{1/2}\nabla^\mu\nabla^\nu K + \cdots + h.c.\,.
\end{aligned}
$$

Note: higher order terms kept in both $\mathcal{L}_{1/2}$ and $\mathcal{L}_{\pi K}^{(2)}$ to check the arguments

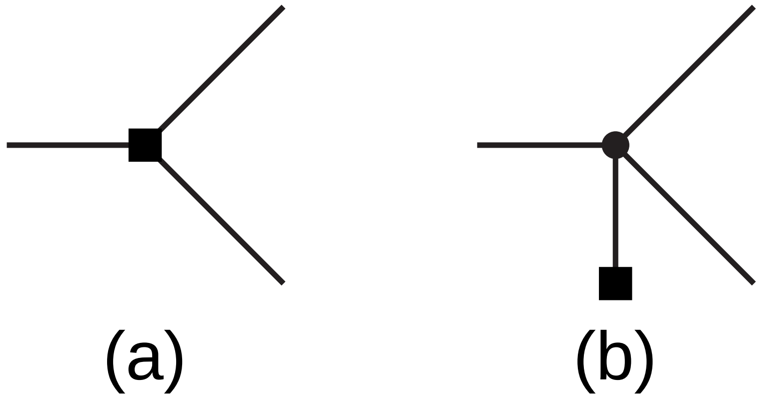
Using partial integration,. . . :

$\langle\pi(p_1)\pi(p_2)|O|K(p_K)\rangle =$

$f(\overline{M}_K^2)\langle\pi(p_1)\pi(p_2)|\tau_{1/2}K|K(p_K)\rangle + \lambda M^2 + \mathcal{O}(M^4)$

O any operator in $\mathcal{L}_{1/2}$ or with more derivatives. Similar for $\mathcal{L}_{3/2}$

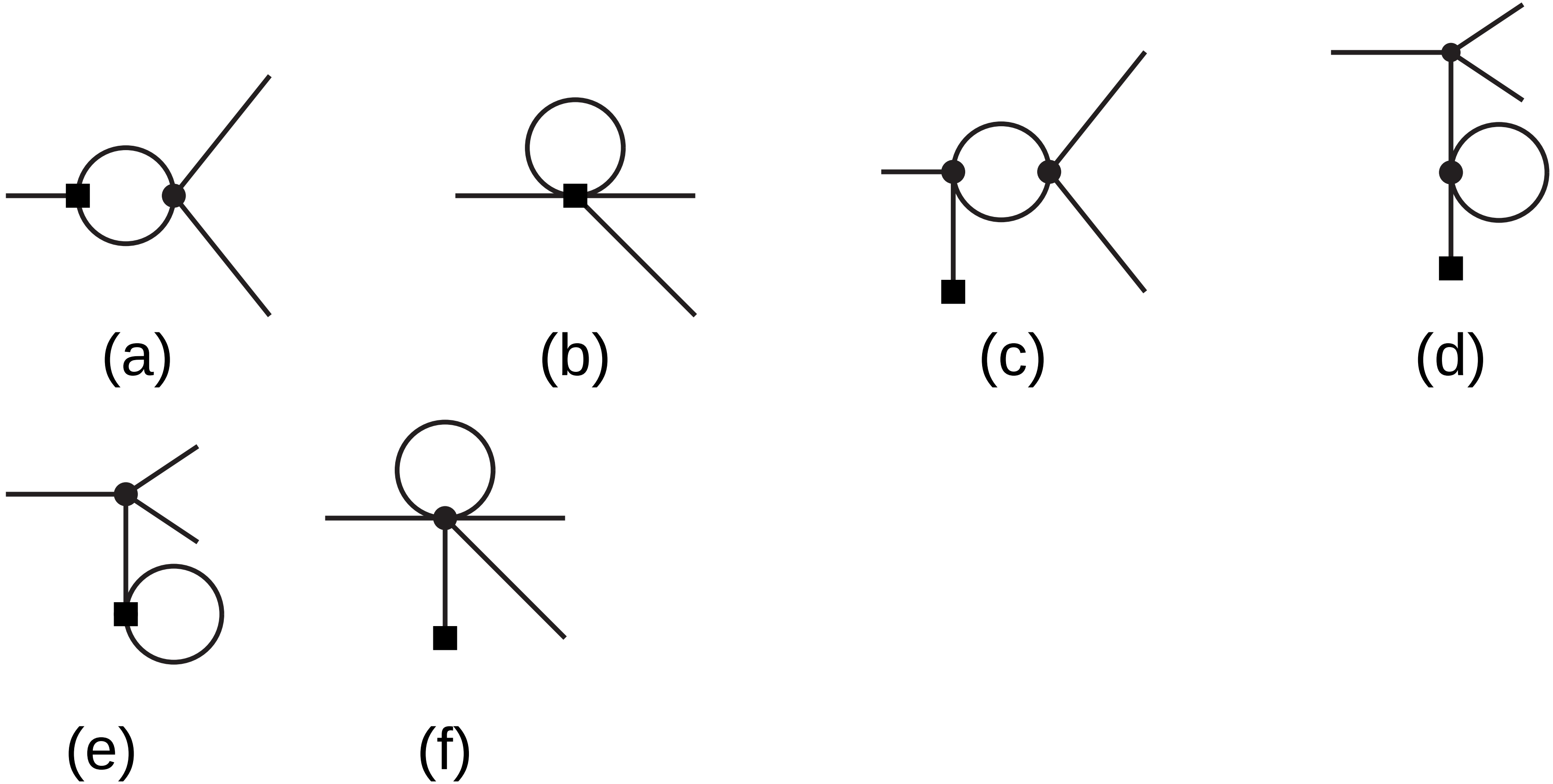
$K \to \pi\pi$: Tree level



$$A_0^{LO} = \frac{\sqrt{3}i}{2F^2}\left[-\frac{1}{2}E_1 + (E_2 - 4E_3)\,\overline{M}_K^2 + 2E_8\overline{M}_K^4 + A_1E_1\right]$$

$$A_2^{LO} = \sqrt{\frac{3}{2}}\frac{i}{F^2}\left[(-2D_1 + D_2)\,\overline{M}_K^2\right]$$

$K \to \pi\pi$: One loop



$K \to \pi\pi$: One loop

Diagram	A_0	A_2
Z	$-\frac{2F^2}{3}A_0^{LO}$	$-\frac{2F^2}{3}A_2^{LO}$
(a)	$\sqrt{3}i\left(-\frac{1}{3}E_1+\frac{2}{3}E_2\overline{M}_K^2\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{2}{3}D_2\overline{M}_K^2\right)$
(b)	$\sqrt{3}i\left(-\frac{5}{96}E_1-\left(\frac{7}{48}E_2+\frac{25}{12}E_3\right)\overline{M}_K^2+\frac{25}{24}E_8\overline{M}_K^4\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{61}{12}D_1+\frac{77}{24}D_2\right)\overline{M}_K^2$
(e)	$\sqrt{3}i\frac{3}{16}A_1E_1$	
(f)	$\sqrt{3}i\left(\frac{1}{8}E_1+\frac{1}{3}A_1E_1\right)$	

The coefficients of $\overline{A}(M^2)/F^4$ in the contributions to A_0 and A_2. Z denotes the part from wave-function renormalization.

- $\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$
- $K\pi$ intermediate state does not contribute, but did for Flynn-Sachrajda

$K \to \pi\pi$: One-loop

$$A_0^{NLO} = A_0^{LO}\left(1 + \frac{3}{8F^2}\overline{A}(M^2)\right) + \lambda_0 M^2 + \mathcal{O}(M^4)\,,$$

$$A_2^{NLO} = A_2^{LO}\left(1 + \frac{15}{8F^2}\overline{A}(M^2)\right) + \lambda_2 M^2 + \mathcal{O}(M^4)\,.$$

$K \to \pi\pi$: One-loop

$$A_0^{NLO} = A_0^{LO}\left(1+\frac{3}{8F^2}\overline{A}(M^2)\right)+\lambda_0 M^2+\mathcal{O}(M^4)\,,$$
$$A_2^{NLO} = A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathcal{O}(M^4)\,.$$

Match with three flavour $SU(3)$ calculation Kambor, Missimer, Wyler; JB, Pallante, Prades

$$A_0^{(3)LO} = -\frac{i\sqrt{6}CF_0^4}{\overline{F}_K F^2}\left(G_8+\frac{1}{9}G_{27}\right)\overline{M}_K^2\,,\qquad A_2^{(3)LO} = -\frac{i10\sqrt{3}CF_0^4}{9\overline{F}_K F^2}G_{27}\overline{M}_K^2\,,$$

When using $F_\pi = F\left(1+\frac{1}{F^2}\overline{A}(M^2)+\frac{M^2}{F^2}l_4^r\right)\,,\ F_K = \overline{F}_K\left(1+\frac{3}{8F^2}\overline{A}(M^2)+\cdots\right)\,,$

logarithms at one-loop agree with above

Hard Pion ChPT: A two-loop check

- Similar arguments to JB-Celis, Flynn-Sachrajda work for the pion vector and scalar formfactor JB-Jemos
- Therefore at any t the chiral log correction must go like the one-loop calculation.
- But note the one-loop log chiral log is with $t >> m_\pi^2$
- Predicts

$$F_V(t, M^2) = F_V(t, 0)\left(1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

$$F_S(t, M^2) = F_S(t, 0)\left(1 - \frac{5}{2}\frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

- Note that $F_{V,S}(t, 0)$ is now a coupling constant and can be complex

A two-loop check

Full two-loop ChPT JB,Colangelo,Talavera, expand in $t >> m_\pi^2$:

$$F_V(t,M^2) = F_V(t,0)\left(1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

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with

$$F_V(t,0) = 1 + \frac{t}{16\pi^2 F^2}\left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6}\ln\frac{t}{\mu^2}\right)$$

$$F_S(t,0) = 1 + \frac{t}{16\pi^2 F^2}\left(1 + 16\pi^2 l_4^r + i\pi - \ln\frac{t}{\mu^2}\right)$$

- The needed coupling constants are complex
- Both calculations have two-loop diagrams with overlapping divergences
- The chiral logs should be valid for any t where a pointlike interaction is a valid approximation

Electromagnetic formfactors

$$F_V^\pi(s) = F_V^{\pi\chi}(s)\left(1+\frac{1}{F^2}\overline{A}(m_\pi^2)+\frac{1}{2F^2}\overline{A}(m_K^2)+\mathcal{O}(m_L^2)\right),$$

$$F_V^K(s) = F_V^{K\chi}(s)\left(1+\frac{1}{2F^2}\overline{A}(m_\pi^2)+\frac{1}{F^2}\overline{A}(m_K^2)+\mathcal{O}(m_L^2)\right).$$

$B, D \to \pi, K, \eta$

$$\left\langle P_f(p_f) \left| \overline{q}_i \gamma_\mu q_f \right| P_i(p_i) \right\rangle = (p_i + p_f)_\mu f_+(q^2) + (p_i - p_f)_\mu f_-(q^2)$$
$$f_{+B\to M}(t) = f^\chi_{+B\to M}(t) F_{B\to M}$$
$$f_{-B\to M}(t) = f^\chi_{-B\to M}(t) F_{B\to M}$$

- $F_{B\to M}$ is always the same for f_+, f_- and f_0
- This is not heavy quark symmetry: not valid at endpoint and valid also for $K \to \pi$.
- Not like Low's theorem, not only dependence on external legs
- Turned out to be a consequence of LEET: only one form-factor in this limit.

$B, D \to \pi, K, \eta$

$$F_{K\to\pi} = 1 + \frac{3}{8F^2}\overline{A}(m_\pi^2) \qquad (2-\text{flavour})$$

$$F_{B\to\pi} = 1 + \left(\frac{3}{8}+\frac{9}{8}g^2\right)\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4}+\frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24}+\frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},$$

$$F_{B\to K} = 1 + \frac{9}{8}g^2\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{2}+\frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{6}+\frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},$$

$$F_{B\to\eta} = 1 + \left(\frac{3}{8}+\frac{9}{8}g^2\right)\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4}+\frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24}+\frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},$$

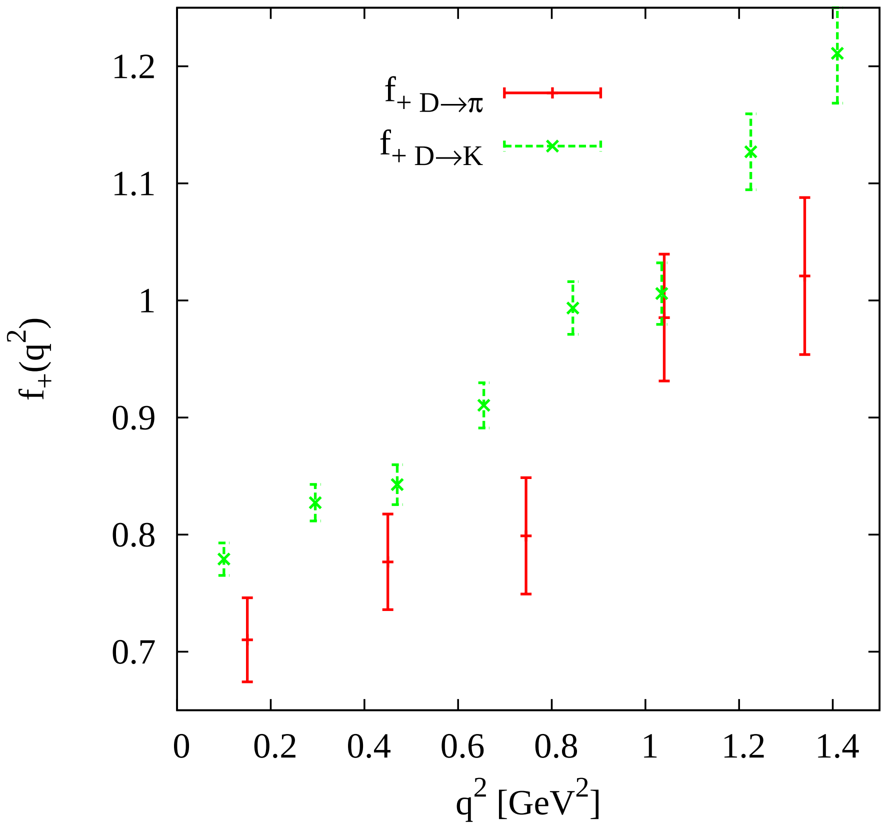
$$F_{B_s\to K} = 1 + \frac{3}{8}\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4}+\frac{3}{2}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24}+\frac{1}{2}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},$$

$$F_{B_s\to\eta} = 1 + \left(\frac{1}{2}+\frac{3}{2}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{6}+\frac{1}{2}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2}.$$

$F_{B_s\to\pi}$ vanishes due to the possible flavour quantum numbers.

Experimental check

CLEO data on $f_+(q^2)|V_{cq}|$ for $D \to \pi$ and $D \to K$ with $|V_{cd}| = 0.2253$, $|V_{cs}| = 0.9743$



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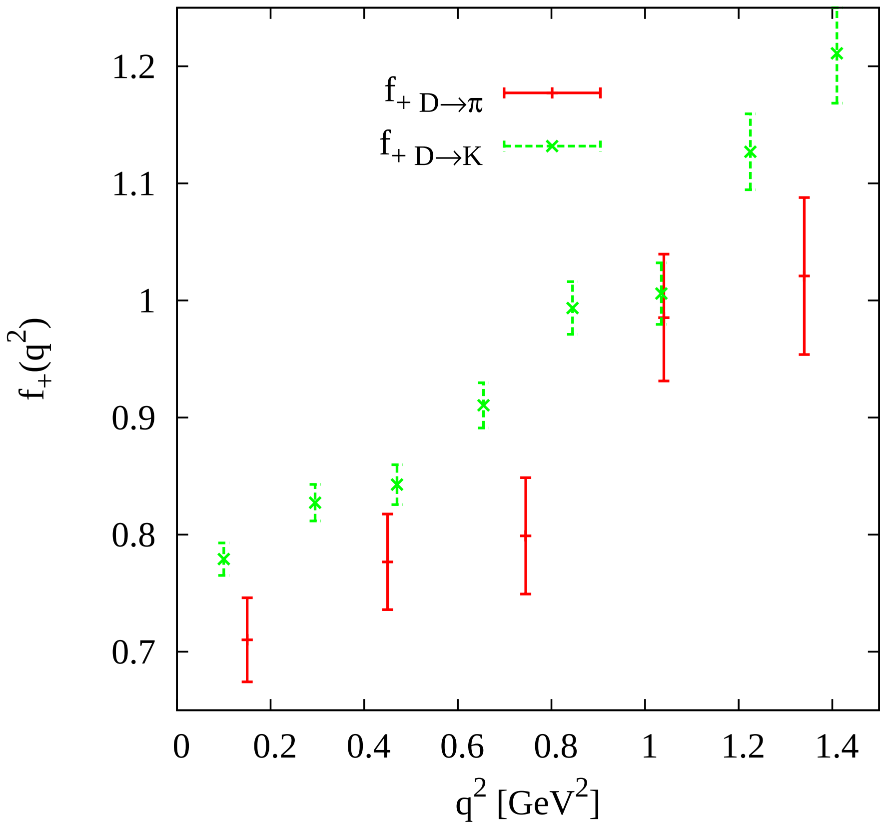

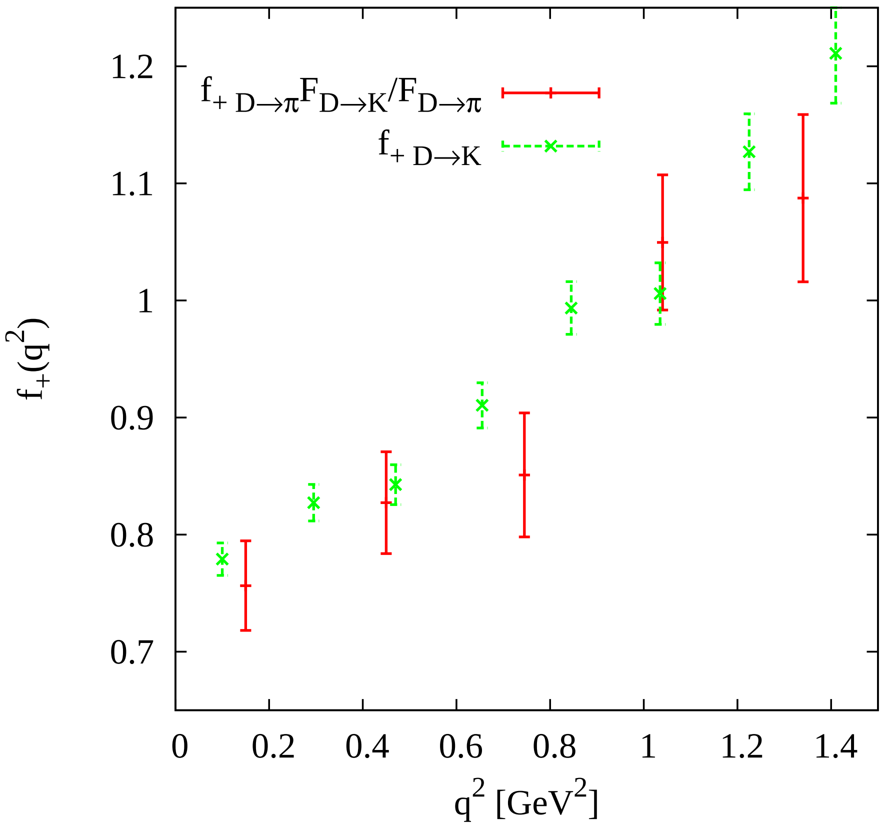

$$f_{+D\to\pi} = f_{+D\to K} F_{D\to\pi} / F_{D\to K}$$

Applications to charmonium

- We look at decays $\chi_{c0}, \chi_{c2} \to \pi\pi, KK, \eta\eta$
- $J/\psi, \psi(nS), \chi_{c1}$ decays to the same final state break isospin or U-spin or V-spin, they thus proceed via electromagnetism or quark mass differences: more difficult.
- So construct a Lagrangian with a chiral singlet scalar and tensor field.
- $\mathcal{L}_{\chi_c} = E_1 F_0^2 \, \chi_0 \, \langle u^\mu u_\mu \rangle + E_2 F_0^2 \, \chi_2^{\mu\nu} \, \langle u_\mu u_\nu \rangle \, .$

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- No chiral logarithm corrections
- Expanding the energy-momentum tensor result Donoghue-Leutwyler at large q^2 agrees.
- These decays should have small $SU(3)_V$ breaking

Charmonium

- Phase space correction: $|\vec{p}_1| = \sqrt{m_\chi^2 - 4m_P^2}/2$.
- χ_{c0}:
  - $A \propto p_1 \cdot p_2 = (m_\chi^2 - 2m_P^2)/2$.
  - $\Longrightarrow$ $\boxed{G_0 = \sqrt{BR/|\vec{p}_1|}/(p_1.p_2)}$.
- χ_{c2}:
  - $A \propto T_\chi^{\mu\nu} p_{1\mu} p_{2\nu}$. (polarization tensor)
  - $|A|^2 \propto \frac{1}{5}\sum_{pol} T_\chi^{\mu\nu} p_{1\mu} p_{2\nu} T_\chi^{*\alpha\beta} p_{1\alpha} p_{2\beta} =$ $\frac{1}{30}\left(m_\chi^2 - 4m_P^2\right)^2 \propto |\vec{p}_1|^4$.
  - $\Longrightarrow$ $\boxed{G_2 = \sqrt{BR/|\vec{p}_1|}/|\vec{p}_1|^2}$.
- $\times 2$ for $K_S^0 K_S^0$ to $K^0 \overline{K^0}$, $\times 2/3$ for $\pi\pi$ to $\pi^+\pi^-$.

Charmonium

	χ_{c0}		χ_{c2}	
Mass	3414.75 ± 0.31 MeV		3556.20 ± 0.09 MeV	
Width	10.4 ± 0.6 MeV		1.97 ± 0.11 MeV	
Final state	10^3 BR	$10^{10}\, G_0[\mathrm{MeV}^{-5/2}]$	10^3 BR	$10^{10} G_2[\mathrm{MeV}^{-5/2}]$
$\pi\pi$	8.5 ± 0.4	3.15 ± 0.07	2.42 ± 0.13	3.04 ± 0.08
K^+K^-	6.06 ± 0.35	3.45 ± 0.10	1.09 ± 0.08	2.74 ± 0.10
$K_S^0 K_S^0$	3.15 ± 0.18	3.52 ± 0.10	0.58 ± 0.05	2.83 ± 0.12
$\eta\eta$	3.03 ± 0.21	2.48 ± 0.09	0.59 ± 0.05	2.06 ± 0.09
$\eta'\eta'$	2.02 ± 0.22	2.43 ± 0.13	< 0.11	< 1.2

Experimental results for $\chi_{c0}, \chi_{c2} \to PP$ and the factors corrected for the known m^2 effects.

- $\pi\pi$ and KK are good to 10% (Note: 20% for F_K/F_π)
- $\eta\eta$ OK

Caveat utilitor: let the user beware

- This is not a simple straightforward process
- Especially the proof that it all reduces to a single type of lowest order term can be tricky.
- Some examples where it does not work easily:
  - VV two-point function has two types of lowest order terms: $\langle LR\rangle$ and $\langle LL\rangle + \langle RR\rangle$ (no derivative structure indicated)
  - Scalar form factors in three flavour ChPT, again two types of lowest order terms $\langle\chi_+\rangle$ and $\langle\chi_+\rangle\langle u_\mu u^\mu\rangle$

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  - Scalar form factors in three flavour ChPT, again two types of lowest order terms $\langle \chi_+ \rangle$ and $\langle \chi_+ \rangle \langle u_\mu u^\mu \rangle$
    - In $SU(2)$ these two types are the same hence our check still worked for the scalar form-factor
    - For the vector formfactor the second type vanishes or gives for the $SU(2)$ case no contribution because of G-parity.

Conclusions HPChPT

Why is this useful:

- Lattice works actually around the strange quark mass
- need only extrapolate in m_u and m_d.
- Applicable in momentum regimes where usual ChPT might not work
- Three flavour case useful for B, D, χ_c decays

Leading Logarithms

- Take a quantity with a single scale: $F(M)$
- The dependence on the scale in field theory is typically logarithmic
- $L = \log(\mu/M)$
- $F = F_0 + F_1^1 L + F_0^1 + F_2^2 L^2 + F_1^2 L + F_0^2 + F_3^3 L^3 + \cdots$
- Leading Logarithms: The terms $F_m^m L^m$

The F_m^m can be more easily calculated than the full result

- $\mu\,(dF/d\mu) \equiv 0$
- Ultraviolet divergences in Quantum Field Theory are always local

Renormalizable theories

- Loop expansion $\equiv \alpha$ expansion
- $F = \alpha + f_1^1 \alpha^2 L + f_0^1 \alpha^2 + f_2^2 \alpha^3 L^2 + f_1^2 \alpha^3 L + f_0^2 \alpha^3 + f_3^3 \alpha^4 L^3 + \cdots$
- f_i^j are pure numbers
- $\mu \dfrac{d\alpha}{d\mu} = \beta_0 \alpha^2 + \beta_1 \alpha^3 + \cdots$

Renormalizable theories

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- f_i^j are pure numbers
- $\mu \frac{d\alpha}{d\mu} = \beta_0 \alpha^2 + \beta_1 \alpha^3 + \cdots$
- $\mu \frac{dF}{d\mu} = 0 \Longrightarrow \boxed{\beta_0 = -f_1^1 = f_2^2 = -f_3^3 = \cdots}$

Renormalization Group

- Can be extended to other operators as well
- Underlying argument always $\mu \frac{dF}{d\mu} = 0$.
- Gell-Mann–Low, Callan–Symanzik, Weinberg–'t Hooft
- In great detail: J.C. Collins, *Renormalization*
- Relies on the α the same in all orders
- LL one-loop β_0
- NLL two-loop β_1, f_0^1

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- Relies on the α the same in all orders
- LL one-loop β_0
- NLL two-loop β_1, f_0^1
- In effective field theories: different Lagrangian at each order
- The recursive argument does not work

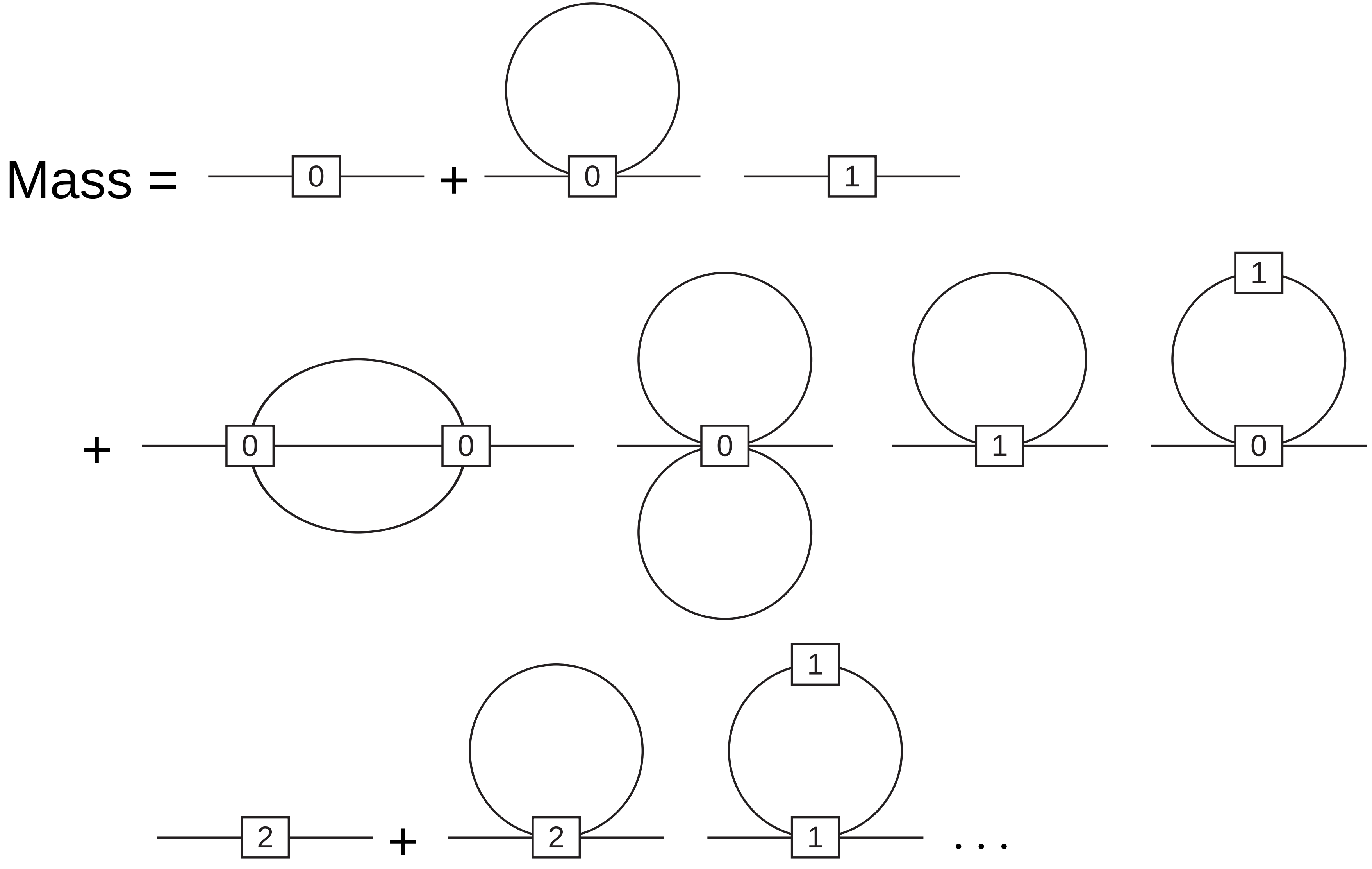
Weinberg's argument

- Weinberg, Physica A96 (1979) 327
- Two-loop leading logarithms can be calculated using only one-loop
- Weinberg consistency conditions
- $\pi\pi$ at 2-loop: Colangelo, hep-ph/9502285
- General at 2 loop: JB, Colangelo, Ecker, hep-ph/9808421
- Proof at all orders using β-functions
  Büchler, Colangelo, hep-ph/0309049
- Proof with diagrams: JB, Carloni, arXiv:0909.5086

Weinberg's argument

- μ: dimensional regularization scale
- $d = 4 - w$
- loop-expansion $\equiv$ $\hbar$-expansion
- $\mathcal{L}^{\text{bare}} = \sum_{n\geq 0} \hbar^n \mu^{-nw} \mathcal{L}^{(n)} =$
  $\sum_{n\geq 0} \hbar^n \mu^{-nw} \sum_i \left(\sum_{k=0,n} \frac{c_{ki}^{(n)}}{w^k}\right) \mathcal{O}_i^{(n)}$
- L_l^n l-loop contribution at order $\hbar^n$
- Expand in divergences from the loops (not from the counterterms) $L_l^n = \sum_{k=0,l} \frac{1}{w^k} L_{kl}^n$
- $\mathcal{L}^{(n)} =$ $\boxed{\text{n}}$

Weinberg's argument



Weinberg's argument

- $\hbar^0$: L_0^0
- $\hbar^1$: $\dfrac{1}{w}\left(\mu^{-w}L_{00}^1(\{c\}_1^1) + L_{11}^1\right) + \mu^{-w}L_{00}^1(\{c\}_0^1) + L_{10}^1$

Weinberg's argument

- $\hbar^0$: L^0_0
- $\hbar^1$: $\frac{1}{w}\left(\mu^{-w}L^1_{00}(\{c\}^1_1)+L^1_{11}\right)+\mu^{-w}L^1_{00}(\{c\}^1_0)+L^1_{10}$
  - Expand $\mu^{-w}=1-w\log\mu+\frac{1}{2}w^2\log^2\mu+\cdots$
  - $1/w$ must cancel: $L^1_{00}(\{c\}^1_1)+L^1_{11}=0$
    this determines the c^1_{1i}
  - Explicit $\log\mu$: $-\log\mu\, L^1_{00}(\{c\}^1_0)\equiv\log\mu\, L^1_{11}$

All orders

- $\hbar^n$:
$$\frac{1}{w^n}\Big(\mu^{-nw}L^n_{00}(\{c\}^n_n) + \mu^{-(n-1)w}L^n_{11}(\{c\}^{n-1}_{n-1}) + \cdots$$
$$+\mu^{-w}L^n_{n-1\ n-1}(\{c\}^1_1) + L^n_{nn}\Big) + \frac{1}{w^{n-1}}\cdots$$

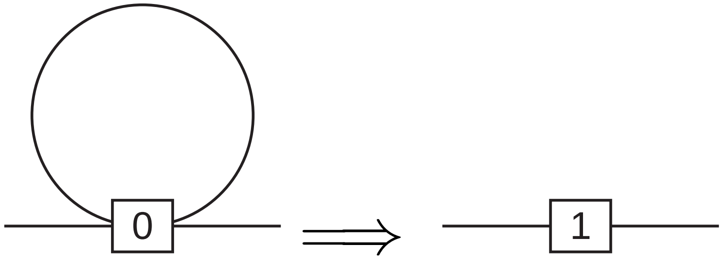
All orders

- $\hbar^n$:
$$\frac{1}{w^n}\Big(\mu^{-nw}L^n_{00}(\{c\}^n_n)+\mu^{-(n-1)w}L^n_{11}(\{c\}^{n-1}_{n-1})+\cdots$$
$$+\mu^{-w}L^n_{n-1\;n-1}(\{c\}^1_1)+L^n_{nn}\Big)+\frac{1}{w^{n-1}}\cdots$$
- $1/w^n, \log\mu/w^{n-1},\ldots,\log^{n-1}\mu/w$ cancel:
$$\sum_{i=0}^{n} i^j L^n_{n-i\;n-i}(\{c\}^i_i)=0 \qquad j=0,..,n-1.$$

All orders

- $\hbar^n$:
$$\frac{1}{w^n}\left(\mu^{-nw}L^n_{00}(\{c\}^n_n)+\mu^{-(n-1)w}L^n_{11}(\{c\}^{n-1}_{n-1})+\cdots\right.$$
$$\left.+\mu^{-w}L^n_{n-1\;n-1}(\{c\}^1_1)+L^n_{nn}\right)+\frac{1}{w^{n-1}}\cdots$$
- $1/w^n, \log\mu/w^{n-1},\ldots,\log^{n-1}\mu/w$ cancel:
$$\sum_{i=0}^{n} i^j L^n_{n-i\;n-i}(\{c\}^i_i)=0 \qquad j=0,..,n-1.$$
- Solution: $L^n_{n-i\;n-i}(\{c\}^i_i)=(-1)^i\begin{pmatrix} n \\ i\end{pmatrix}L^n_{nn}$
- explicit leading $\log\mu$ dependence and divergence
$$\log^n\mu\frac{(-1)^{n-1}}{n}L^n_{11}(\{c\}^{n-1}_{n-1}) \qquad L^n_{00}(\{c\}^n_n)=-\frac{1}{n}L^n_{11}(\{c\}^{n-1}_{n-1})$$

Mass to $\hbar^2$

- $\hbar^1$: 

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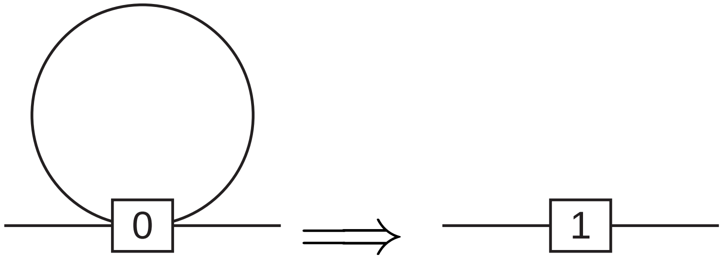
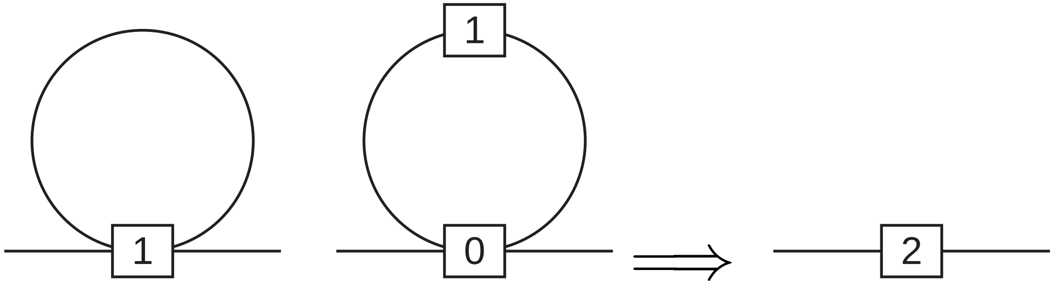


- $\hbar^2$:



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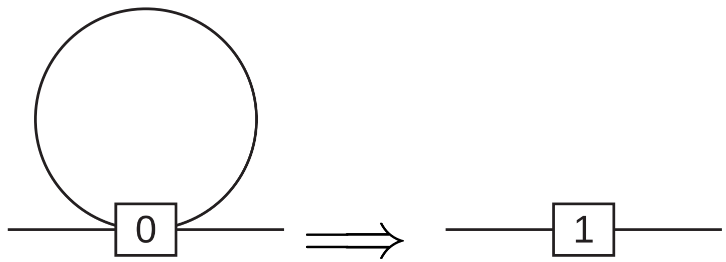
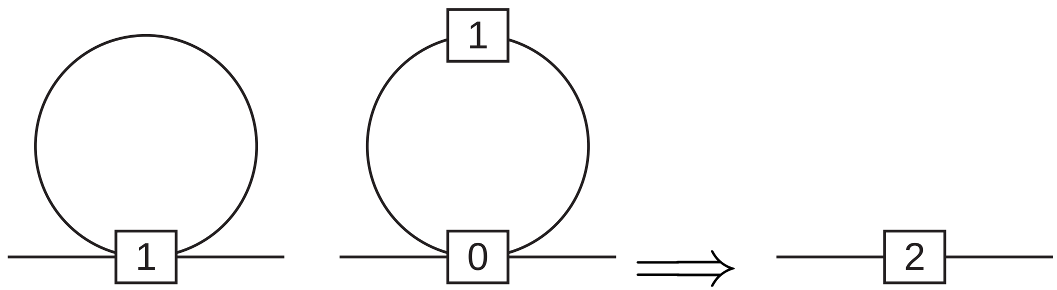
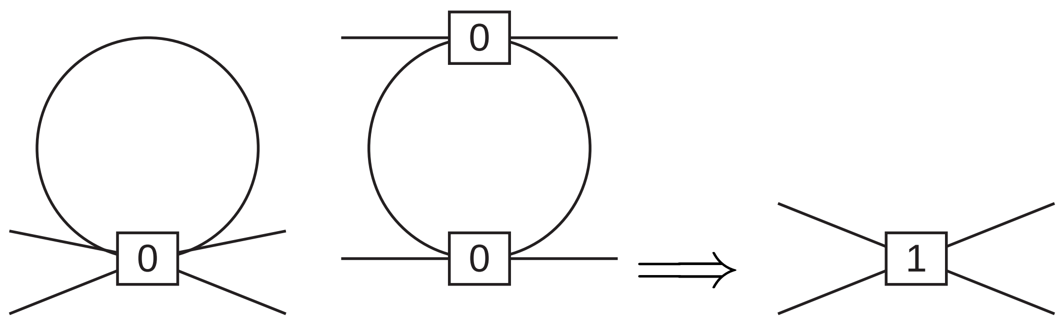
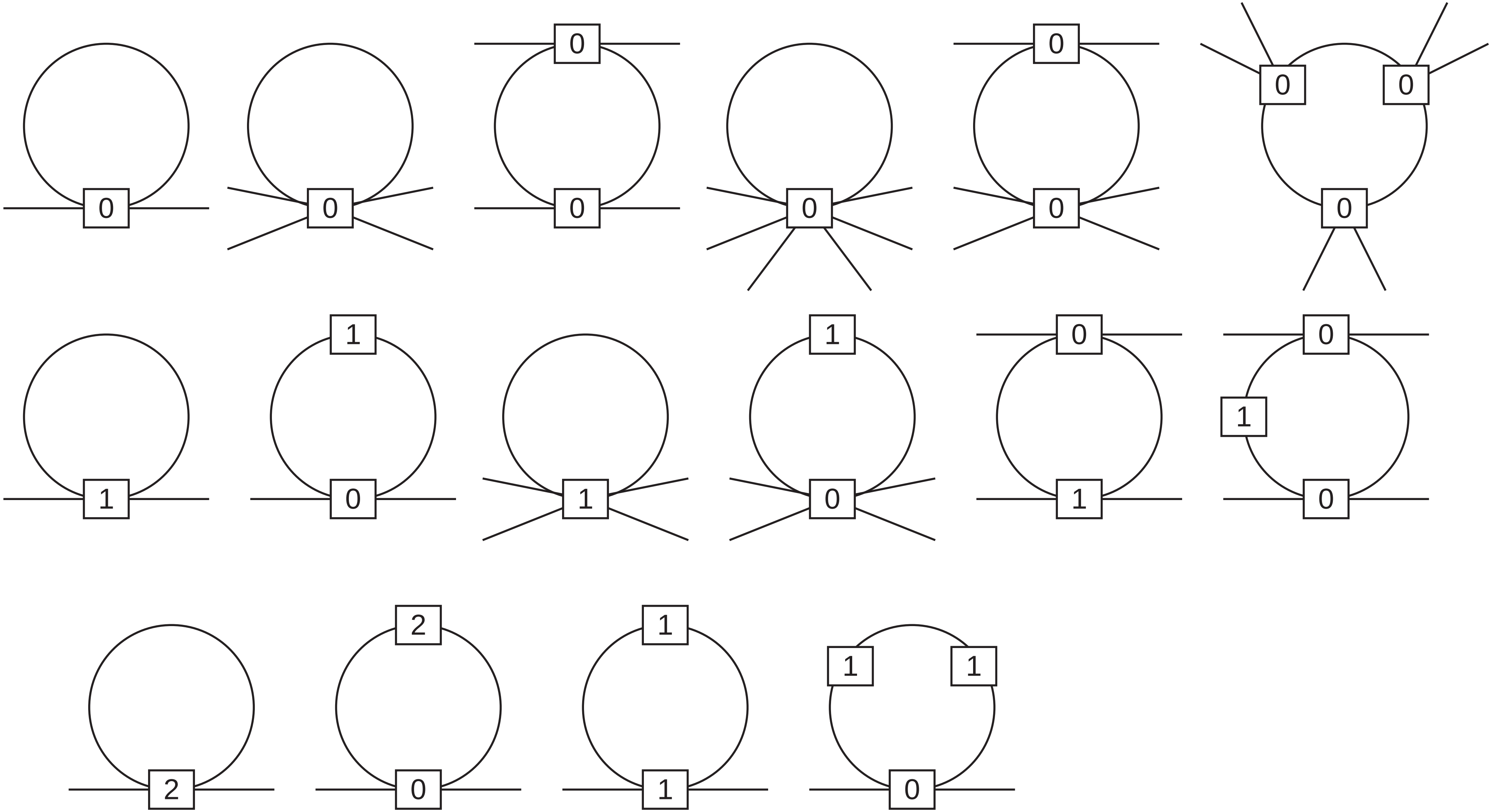
- $\hbar^1$: 

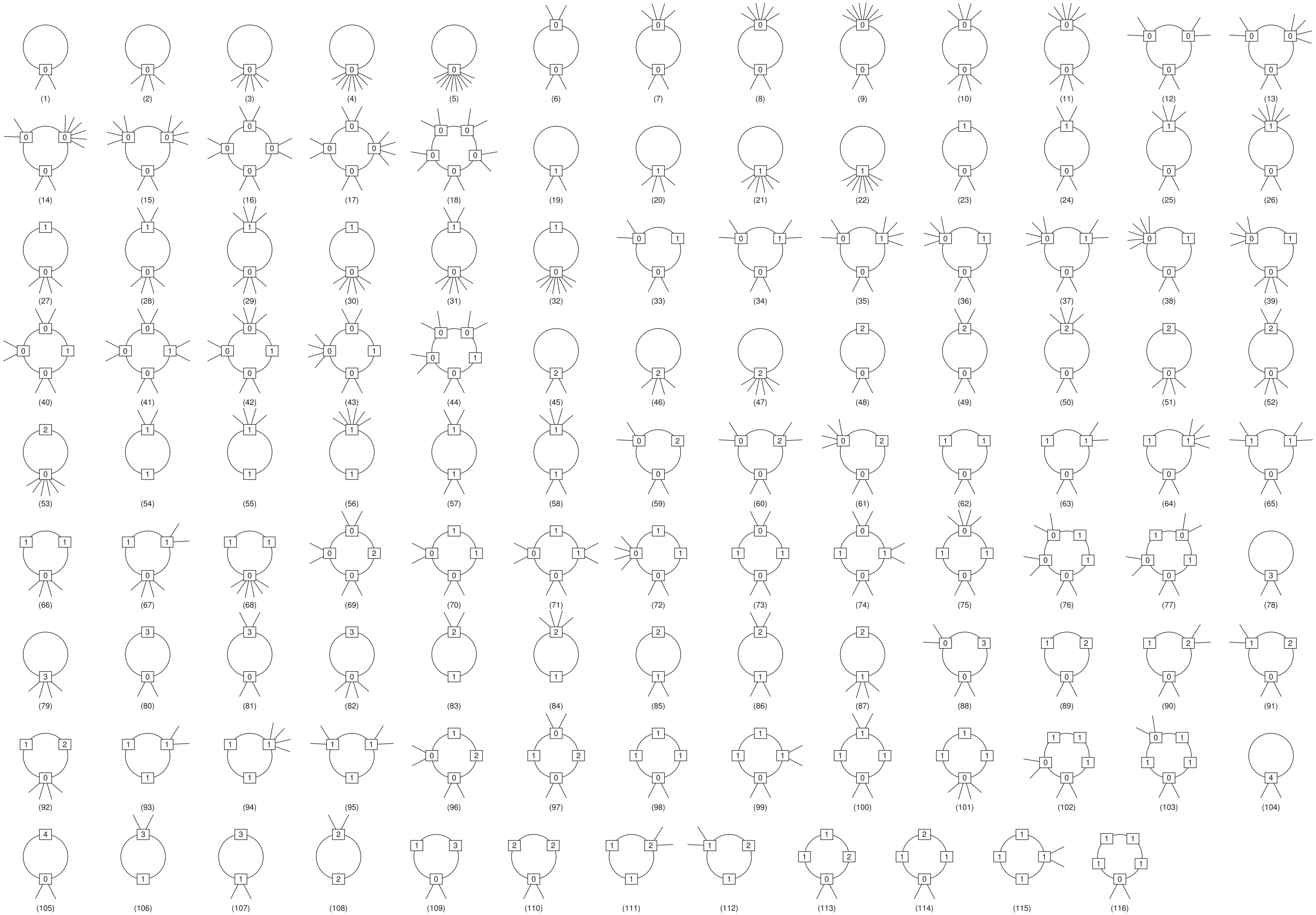
- $\hbar^2$: 

- but also needs $\hbar^1$: 

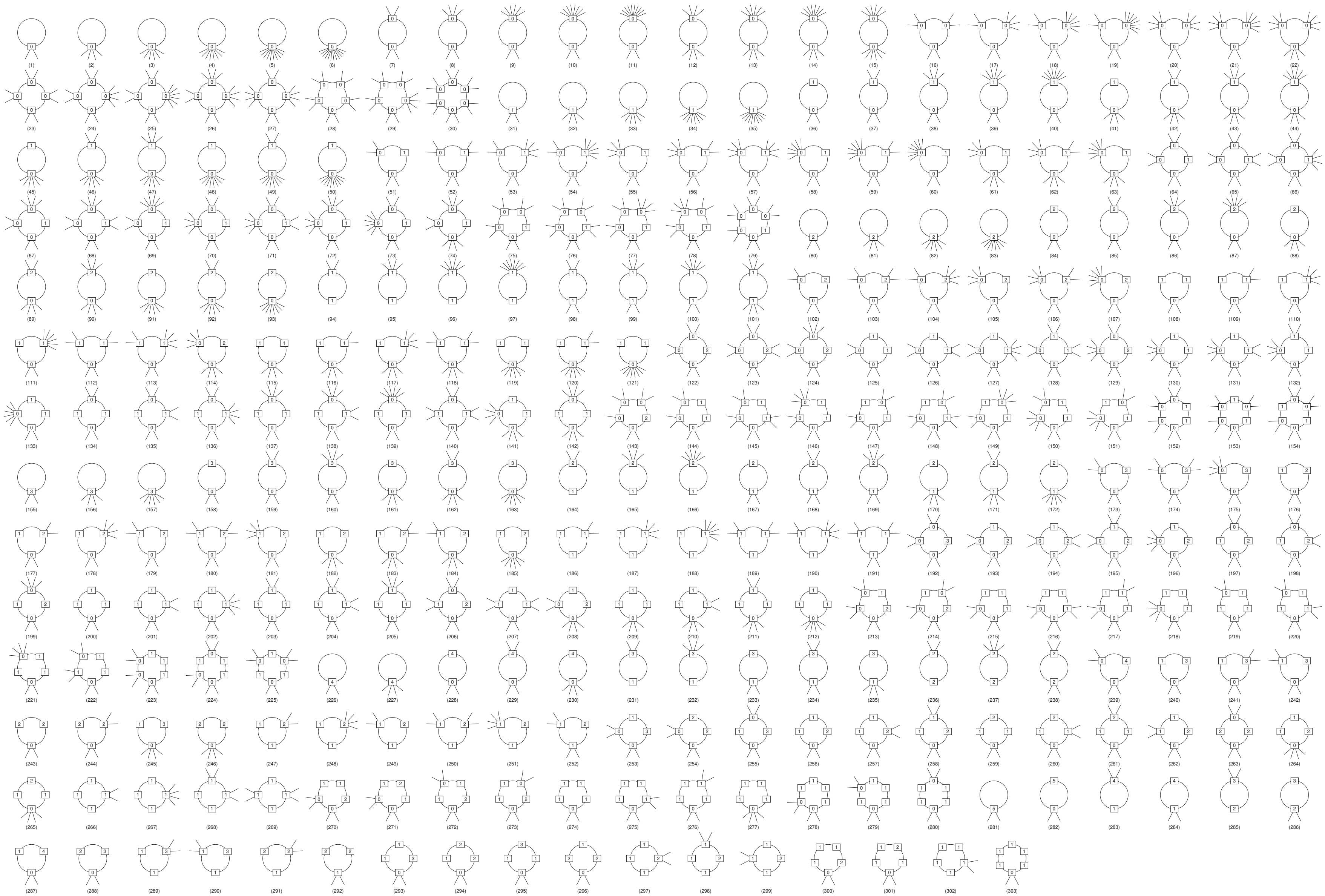
Mass to order $\hbar^3$



Mass to order $\hbar^5$



Mass to order $\hbar^6$



Mass+decay to $\hbar^5$

- $\hbar^1$: 18 + 27
- $\hbar^2$: 26 + 45
- $\hbar^3$: 33 + 51
- $\hbar^4$: 26 + 33
- $\hbar^5$: 13 + 13

- Calculate the divergence
- rewrite it in terms of a local Lagrangian
- Luckily: symmetry kept: we know result will be symmetrical, hence do not need to explicitly rewrite the Lagrangians in a nice form
- We keep all terms to have all 1PI (one particle irreducible) diagrams finite

Massive $O(N)$ sigma model

- $O(N+1)/O(N)$ nonlinear sigma model
- $\mathcal{L}_{n\sigma} = \frac{F^2}{2}\partial_\mu\Phi^T\partial^\mu\Phi + F^2\chi^T\Phi\,.$
- Φ is a real $N+1$ vector; $\Phi \to O\Phi$; $\Phi^T\Phi = 1$.
- Vacuum expectation value $\langle\Phi^T\rangle = (1\ 0\ldots 0)$
- Explicit symmetry breaking: $\chi^T = \left(M^2\ 0\ldots 0\right)$
- Both spontaneous and explicit symmetry breaking
- N-vector ϕ
- N (pseudo-)Nambu-Goldstone Bosons
- $N=3$ is two-flavour Chiral Perturbation Theory

Massive $O(N)$ sigma model: Φ vs ϕ

- $$\Phi_1 = \begin{pmatrix} \sqrt{1 - \frac{\phi^T\phi}{F^2}} \\ \frac{\phi^1}{F} \\ \vdots \\ \frac{\phi^N}{F} \end{pmatrix} = \begin{pmatrix} \sqrt{1 - \frac{\phi^T\phi}{F^2}} \\ \frac{\phi}{F} \end{pmatrix}$$ Gasser, Leutwyler

- $$\Phi_2 = \frac{1}{\sqrt{1 + \frac{\phi^T\phi}{F^2}}} \begin{pmatrix} 1 \\ \frac{\phi}{F} \end{pmatrix}$$ similar to Weinberg

  $$\Phi_3 = \begin{pmatrix} 1 - \frac{1}{2}\frac{\phi^T\phi}{F^2} \\ \sqrt{1 - \frac{1}{4}\frac{\phi^T\phi}{F^2}}\,\frac{\phi}{F} \end{pmatrix}$$ only mass term

- $$\Phi_4 = \begin{pmatrix} \cos\sqrt{\frac{\phi^T\phi}{F^2}} \\ \sin\sqrt{\frac{\phi^T\phi}{F^2}}\,\frac{\phi}{\sqrt{\phi^T\phi}} \end{pmatrix}$$ CCWZ

Massive $O(N)$ sigma model: Checks

Need (many) checks:

- use the four different parametrizations
- compare with known results:

  $$M^2_{phys} = M^2\left(1 - \frac{1}{2}L_M + \frac{17}{8}L_M^2 + \cdots\right),$$

  $$L_M = \frac{M^2}{16\pi^2 F^2}\log\frac{\mu^2}{\mathcal{M}^2}$$

  Usual choice $\mathcal{M} = M$.
- large N (but known results only for massless case) Coleman, Jackiw, Politzer 1974
- large N massive later found partly in appendix of Kivel, Polyakov, Vladimirov on distribution functions.

Results

$$M^2_{\text{phys}} = M^2(1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \ldots)$$

$$L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{M^2}$$

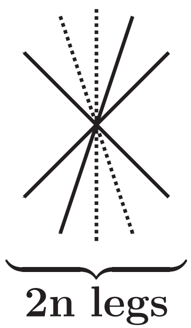
i	a_i, $N = 3$	a_i for general N
1	$-\frac{1}{2}$	$1 - \frac{N}{2}$
2	$\frac{17}{8}$	$\frac{7}{4} - \frac{7N}{4} + \frac{5\ N^2}{8}$
3	$-\frac{103}{24}$	$\frac{37}{12} - \frac{113N}{24} + \frac{15\ N^2}{4} - N^3$
4	$\frac{24367}{1152}$	$\frac{839}{144} - \frac{1601\ N}{144} + \frac{695\ N^2}{48} - \frac{135\ N^3}{16} + \frac{231\ N^4}{128}$
5	$-\frac{8821}{144}$	$\frac{33661}{2400} - \frac{1151407\ N}{43200} + \frac{197587\ N^2}{4320} - \frac{12709\ N^3}{300} + \frac{6271\ N^4}{320} - \frac{7\ N^5}{2}$

F_{phys}, $\langle \bar{q}_i q_i \rangle$ as well done

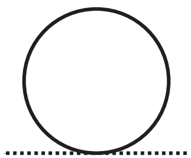
Anyone recognize any funny functions?

Large N

Power counting: pick $\mathcal{L}$ extensive in $N \Rightarrow F^2 \sim N$, $M^2 \sim 1$

- 

$\Leftrightarrow F^{2-2n} \sim \frac{1}{N^{n-1}}$


$\Leftrightarrow N$

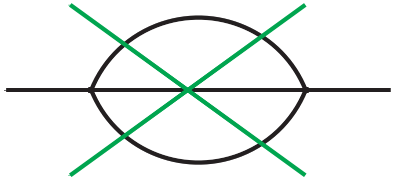
- 1PI diagrams:

$$\left.\begin{array}{l} N_L = N_I - \sum_n N_{2n} + 1 \\ 2N_I + N_E = \sum_n 2nN_{2n} \end{array}\right\} \Rightarrow N_L = \sum_n (n-1)N_{2n} - \frac{1}{2}N_E + 1$$

- diagram suppression factor: $\dfrac{N^{N_L}}{N^{N_E/2-1}}$

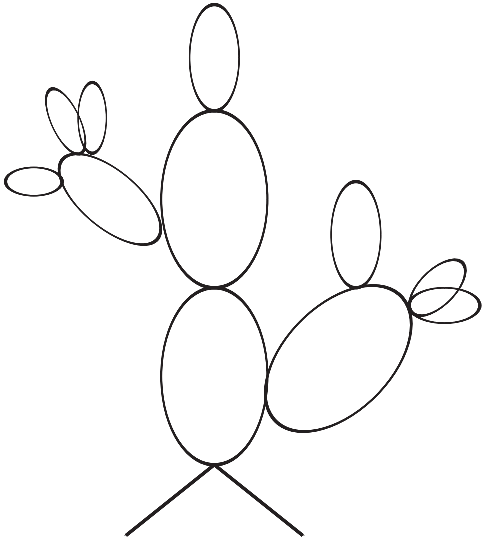
Large N

- diagrams with shared lines are suppressed



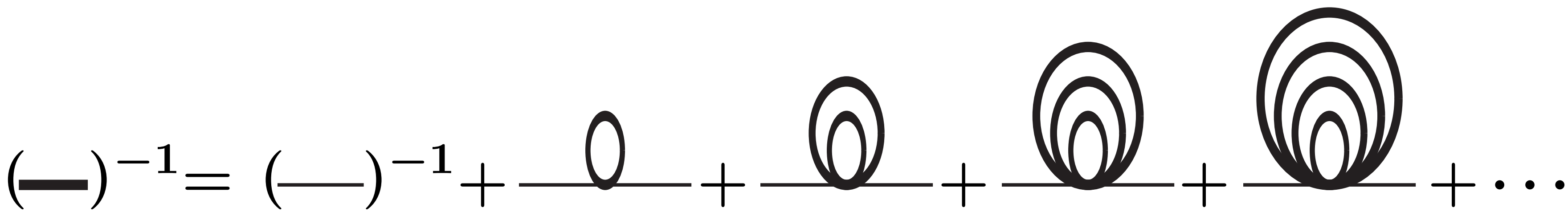
each new loop needs also a new flavour loop

- in the large N limit only "*cactus*" diagrams survive:



large N: propagator

Generate recursively via a Gap equation



$\Rightarrow$ resum the series and look for the pole

$$M^2 = M^2_{\text{phys}}\sqrt{1+\frac{N}{F^2}\overline{A}(M^2_{\text{phys}})}$$

$$\overline{A}(m^2) = \frac{m^2}{16\pi^2}\log\frac{\mu^2}{m^2}.$$

Solve recursively, agrees with other result

Note: can be done for all parametrizations

large N

$$F_{\text{phys}} = F\sqrt{1 + \frac{N}{F^2}\overline{A}(M^2_{\text{phys}})}$$

$$\langle \bar{q}q\rangle_{\text{phys}} = \langle \bar{q}q\rangle_0 \sqrt{1 + \frac{N}{F^2}\overline{A}(M^2_{\text{phys}})}$$

Comments:

- These are the full* leading N results, not just leading log
- But depends on the choice of N-dependence of higher order coefficients
- Assumes higher LECs zero ($< N^{n+1}$ for $\hbar^n$)
- Large N as in $O(N)$ not large N_c

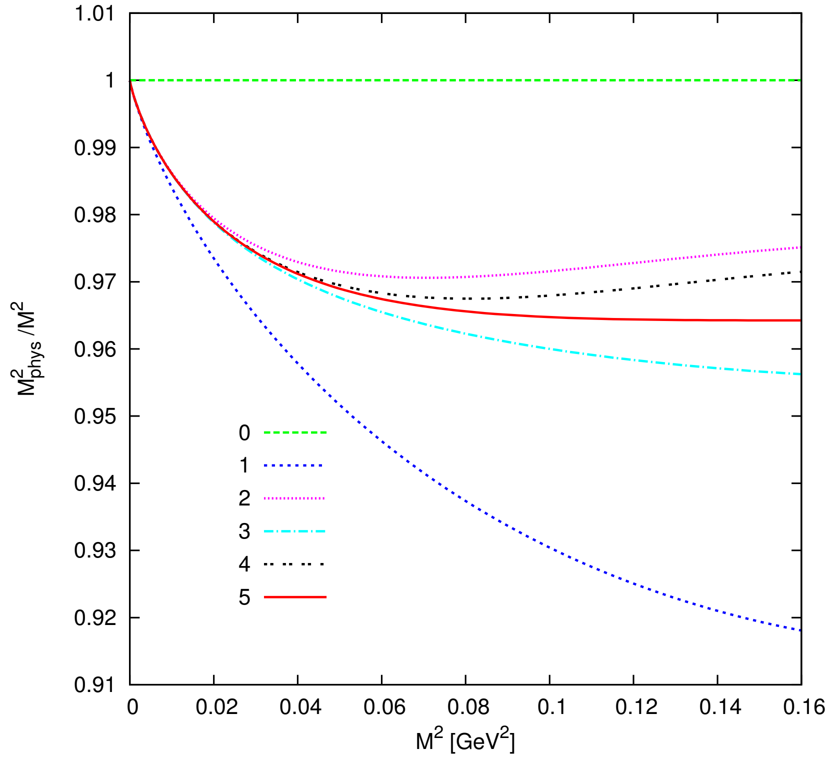
Large N: Checking expansions

$$M^2 = M^2_{\text{phys}} \sqrt{1 + \frac{N}{F^2}\overline{A}(M^2_{\text{phys}})}$$

much smaller expansion coefficients than the table, try

$$M^2 = M^2_{\text{phys}}(1 + d_1 L_{M_{\text{phys}}} + d_2 L^2_{M_{\text{phys}}} + d_3 L^3_{M_{\text{phys}}} + \ldots)$$

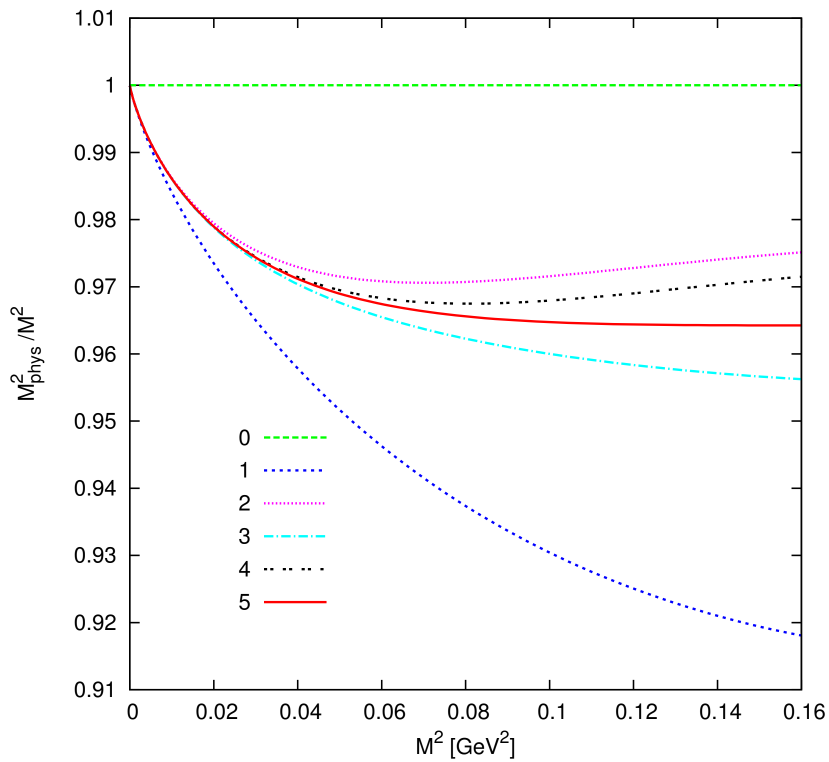
Numerical results

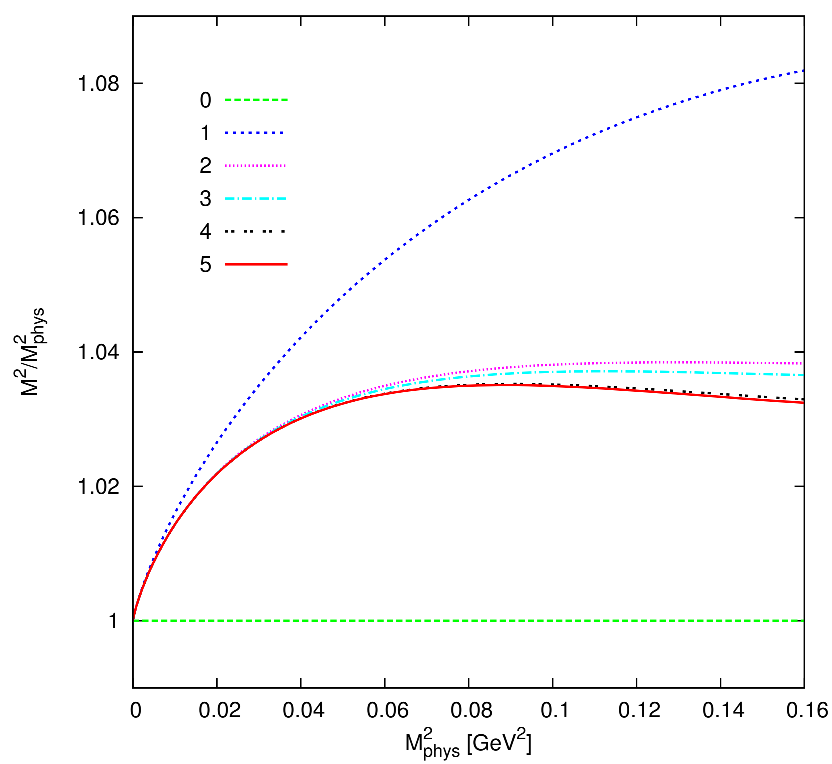


Left: $\frac{M^2_{\rm phys}}{M^2} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots$

F = 90 MeV, $\mu = 0.77$ GeV

Numerical results





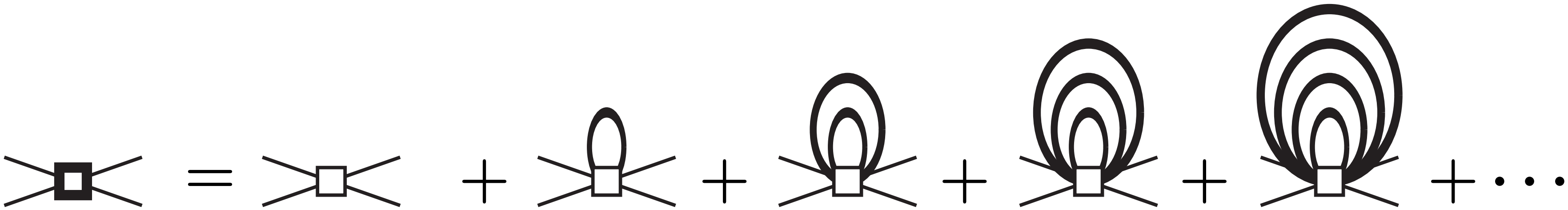
Left: $\frac{M^2_{\text{phys}}}{M^2} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots$

Right: $\frac{M^2}{M^2_{\text{phys}}} = 1 + d_1 L_{M_{\text{phys}}} + d_2 L^2_{M_{\text{phys}}} + d_3 L^3_{M_{\text{phys}}} + \cdots$

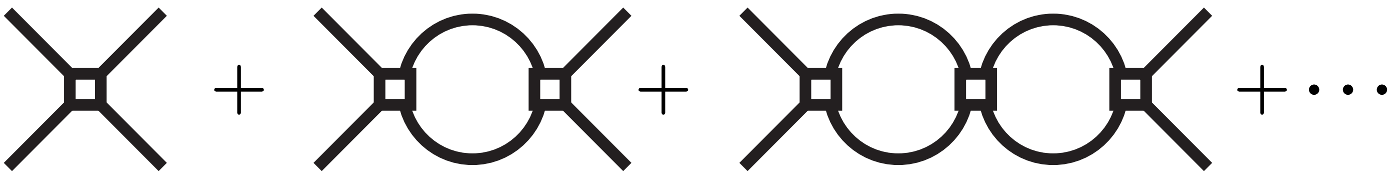
F = 90 MeV, $\mu = 0.77$ GeV

Large N: $\pi\pi$-scattering

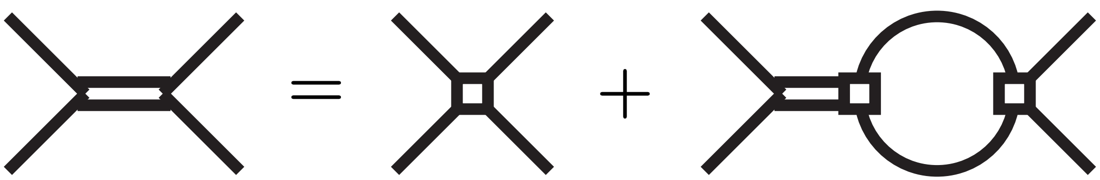
- Cactus diagrams for $A(s,t,u)$
- Branch with no momentum: resummed by —
- Branch starting at vertex: resum by



- The full result is then



- Can be summarized by a recursive equation



Large N: $\pi\pi$ scattering

$$y = \frac{N}{F^2}\overline{A}(M^2_{\rm phys})$$

$$A(s,t,u) = \frac{\frac{s}{F^2(1+y)} - \frac{M^2}{F^2(1+y)^{3/2}}}{1 - \frac{N}{2}\left(\frac{s}{F^2(1+y)} - \frac{M^2}{F^2(1+y)^{3/2}}\right)\overline{B}(M^2_{\rm phys}, M^2_{\rm phys}, s)}$$

or

$$A(s,t,u) = \frac{\frac{s-M^2_{\rm phys}}{F^2_{\rm phys}}}{1 - \frac{N}{2}\frac{s-M^2_{\rm phys}}{F^2_{\rm phys}}\overline{B}(M^2_{\rm phys}, M^2_{\rm phys}, s)}$$

- $M^2 \to 0$ agrees with the known results
- Agrees with our 4-loop results

Other results

- Bissegger, Fuhrer, hep-ph/0612096 Dispersive methods, massless Π_S to five loops
- Kivel, Polyakov, Vladimirov, 0809.3236, 0904.3008, 1004.2197, 1012.4205
  - In the massless case tadpoles vanish
  - $\Longrightarrow$ number of external legs needed does not grow
  - All 4-meson vertices via Legendre polynomials
  - can do divergence of all one-loop diagrams analytically
  - algebraic (but quadratic) recursion relations
  - massless $\pi\pi$, F_V and F_S to arbitrarily high order
  - large N agrees with Coleman, Wess, Zumino
  - large N is not a good approximation

Other results

- JB,Carloni, arXiv:1008.3499
  - massive case: $\pi\pi$, F_V and F_S to 4-loop order
  - large N for these cases also for massive $O(N)$.
  - done using bubble resummations or recursion eqation which can be solved analytically
- JB, Kampf, Lanz, in preparation
  - Mass, F_π to six loops
  - Anomaly: $\gamma^* 3\pi$ (five) and $\pi^0\gamma^*\gamma^*$ (six loops)
  - large N not relevant in this case

Conclusions Leading Logs

- Several quantities in massive $O(N)$ LL known to high loop order
- Large N in massive $O(N)$ model solved
- Had hoped: recognize the series also for general N
- Limited essentially by CPU time and size of intermediate files
- Some first studies on convergence etc.
- $\pi\pi$, F_V and F_S to four-loop order
- The technique can be generalized to other models/theories
  - $SU(N) \times SU(N)/SU(N)$
  - One nucleon sector

Conclusions

Three new surroundings for ChPT:

- Hard Pion ChPT: a new application domain for EFT and first results
  - Many processes but limited domain
  - power counting proof lacking so far, SCET?
- Leading Logarithms and large N: some progress in getting results at high loop orders, but hoped for patterns not seen (except large N calculated)
  - Anybody recognize some funny functions?
  - Method applicable to many more cases
- Two-loop results for the equal mass case for different symmetry patterns. $SU(N) \times SU(N)/SU(N)$, $SU(2N)/SO(2N)$, $SU(2N)/Sp(2N)$
  - If you insist, there are more slides coming for this

QCDlike and/or technicolor theories

A typical gaugegroup and N_F fermions:

- QCD or complex: $q^T = (q_1\ q_2 \ldots\ q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R$
    $q_L \to g_L q_L$ and $g_R \to g_R q_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \overline{q}_j q_i \rangle \propto \delta_{ij}$
  - Conserved $H = SU(N_F)$ $g_L = g_R$ $\Sigma_{ij} \to \Sigma_{ij}$
  - q in complex prepresentation of gauge group

QCDlike and/or technicolor theories

A typical gaugegroup and N_F fermions:

- Real (e.g. adjoint):
  - $\tilde{q}_{Ri} \equiv C\bar{q}_{Li}^T$ is in the same gauge group representation as q_{Ri}
  - $\hat{q}^T = (q_{R1} \ \ldots \ q_{RN_F} \ \tilde{q}_{R1} \ \ldots \ \tilde{q}_{RN_F})$
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - Vacuum condensate $\langle \overline{q}_j q_i \rangle$ is really $\langle (\hat{q}_j)^T C \hat{q}_i \rangle \propto J_{Sij}$

    $$J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$
  - Conserved symmetry part has $g J_S g^T = J_S$
  - $H = SO(2N_F)$
  - some Goldstone bosons have baryonnumber

QCDlike and/or technicolor theories

A typical gaugegroup and N_F fermions:

- Pseudoreal (e.g. two-colours):
  - $\tilde{q}_{R\alpha i} \equiv \epsilon_{\alpha\beta} C \bar{q}^T_{L\beta i}$ is in the same gauge group representation as $q_{R\alpha i}$
  - $\hat{q}^T = (q_{R1} \; \dots \; q_{RN_F} \; \tilde{q}_{R1} \; \dots \; \tilde{q}_{RN_F})$
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - Vacuum condensate $\langle \overline{q}_j q_i \rangle$ is really

    $$\epsilon_{\alpha\beta}\langle(\hat{q}_{\alpha j})^T C \hat{q}_{\beta i}\rangle \propto J_{Aij} \; J_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$
  - Conserved symmetry part has $g J_A g^T = J_A$
  - $H = Sp(2N_F)$
  - some Goldstone bosons have baryonnumber

Lagrangians

In arXiv:0910.5424 we showed that there is a very similar way of phrasing the two theories using $u = \exp\left(\frac{i}{\sqrt{2}F}\phi^a X^a\right)$

But the matrices X^a are:

- Complex or $SU(N) \times SU(N)/SU(N)$: all $SU(N)$ generators
- Real or $SU(2N)/SO(2N)$: $SU(2N)$ generator with $X^a J_S = J_S X^{aT}$
- Pseudoreal or $SU(2N)/Sp(2N)$: $SU(2N)$ generator with $X^a J_A = J_A X^{aT}$
- Note that the latter are not the usual ways of parametrizing $SO(2N)$ and $Sp(2N)$ matrices

Divergences etc

Calculating for equal mass case goes through using:

$$
\begin{aligned}
\mathrm{QCD}: \quad & \langle X^a A X^a B\rangle = \langle A\rangle\langle B\rangle - \frac{1}{N_F}\langle AB\rangle\ ,\\
& \langle X^a A\rangle\langle X^a B\rangle = \langle AB\rangle - \frac{1}{N_F}\langle A\rangle\langle B\rangle\ .\\
\mathrm{Adjoint}: \quad & \langle X^a A X^a B\rangle = \frac{1}{2}\langle A\rangle\langle B\rangle + \frac{1}{2}\left\langle A J_S B^T J_S\right\rangle - \frac{1}{2N_F}\langle AB\rangle\ ,\\
& \langle X^a A\rangle\langle X^a B\rangle = \frac{1}{2}\langle AB\rangle + \frac{1}{2}\left\langle A J_S B^T J_S\right\rangle - \frac{1}{2N_F}\langle A\rangle\langle B\rangle\ .\\
2-\mathrm{colour}: \quad & \langle X^a A X^a B\rangle = \frac{1}{2}\langle A\rangle\langle B\rangle + \frac{1}{2}\left\langle A J_A B^T J_A\right\rangle - \frac{1}{2N_F}\langle AB\rangle\ ,\\
& \langle X^a A\rangle\langle X^a B\rangle = \frac{1}{2}\langle AB\rangle - \frac{1}{2}\left\langle A J_A B^T J_A\right\rangle - \frac{1}{2N_F}\langle A\rangle\langle B\rangle
\end{aligned}
$$

So can do the calculations for all cases

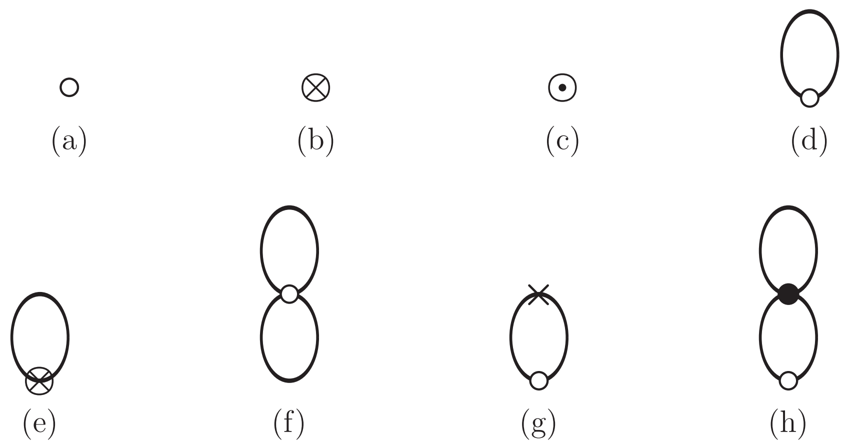
Vacuum expectation value

All cases: $\langle \overline{q}q\rangle_{\mathrm{LO}} \equiv \sum_{i=1,N_F}\langle \overline{q}_{Ri}q_{Li} + \overline{q}_{Li}q_{Ri}\rangle_{\mathrm{LO}} = -N_F B_0 F^2$

$M^2 = 2B_0\hat{m}$ and $\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}\,.$
$\langle \overline{q}q\rangle = \langle \overline{q}q\rangle_{\mathrm{LO}} + \langle \overline{q}q\rangle_{\mathrm{NLO}} + \langle \overline{q}q\rangle_{\mathrm{NNLO}}\,.$

$$\langle \overline{q}q\rangle_{\mathrm{NLO}} = \langle \overline{q}q\rangle_{\mathrm{LO}}\left(a_V\frac{\overline{A}(M^2)}{F^2} + b_V\frac{M^2}{F^2}\right),$$
$$\langle \overline{q}q\rangle_{\mathrm{NNLO}} = \langle \overline{q}q\rangle_{\mathrm{LO}}\left(c_V\frac{\overline{A}(M^2)^2}{F^4} + \frac{M^2\overline{A}(M^2)}{F^4}\left(d_V + \frac{e_V}{16\pi^2}\right) + \frac{M^4}{F^4}\left(f_V + \frac{g_V}{16\pi^2}\right)\right).$$

Diagrams:



Vacuum expectation value

	QCD	
a_V	$n - \frac{1}{n}$	
b_V	$16nL_6^r + 8L_8^r + 4H_2^r$	
c_V	$\frac{3}{2}\left(-1 + \frac{1}{n^2}\right)$	
d_V	$-24\left(n^2 - 1\right)\left(L_A + \frac{1}{n}L_B\right)$	$L_A = L_4^r - 2L_6^r$
e_V	$1 - \frac{1}{n^2}$	$L_B = L_5^r - 2L_8^r$
f_V	$48\left(K_{25}^r + nK_{26}^r + n^2K_{27}^r\right)$	
g_V	$8\left(n^2 - 1\right)\left(L_A + \frac{1}{n}L_B\right)$	
	Adjoint	**2-colour**
a_V	$n + \frac{1}{2} - \frac{1}{2n}$	$n - \frac{1}{2} - \frac{1}{2n}$
b_V	$32nL_6^r + 8L_8^r + 4H_2^r$	$32nL_6^r + 8L_8^r + 4H_2^r$
c_V	$\frac{3}{8}\left(-1 + \frac{1}{n^2} - \frac{2}{n} + 2n\right)$	$\frac{3}{8}\left(-1 + \frac{1}{n^2} + \frac{2}{n} - 2n\right)$
d_V	$-12\left(2n^2 + n - 1\right)\left(2L_A + \frac{1}{n}L_B\right)$	$-12\left(2n^2 - n - 1\right)\left(2L_A + \frac{1}{n}L_B\right)$
e_V	$\frac{1}{4}\left(1 - \frac{1}{n^2} + \frac{2}{n} - 2n\right)$	$\frac{1}{4}\left(1 - \frac{1}{n^2} - \frac{2}{n} + 2n\right)$
f_V	r_{VA}^r	r_{VT}^r
g_V	$4\left(2n^2 + n - 1\right)\left(2L_A + \frac{1}{n}L_B\right)$	$4\left(2n^2 - n - 1\right)\left(2L_A + \frac{1}{n}L_B\right)$

$\phi\phi \to \phi\phi$

- $\pi\pi$ scattering
  - Amplitude in terms of $A(s,t,u)$

    $M_{\pi\pi}(s,t,u) = \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,s,t)\,.$
  - Three intermediate states $I = 0, 1, 2$
- Our three cases
  - Two amplitudes needed $B(s,t,u)$ and $C(s,t,u)$

$$
\begin{aligned}
M(s,t,u) \quad = \quad & \left[\langle X^aX^bX^cX^d\rangle + \langle X^aX^dX^cX^b\rangle\right]B(s,t,u)\\
& + \left[\langle X^aX^cX^dX^b\rangle + \langle X^aX^bX^dX^c\rangle\right]B(t,u,s)\\
& + \left[\langle X^aX^dX^bX^c\rangle + \langle X^aX^cX^bX^d\rangle\right]B(u,s,t)\\
& + \delta^{ab}\delta^{cd}C(s,t,u) + \delta^{ac}\delta^{bd}C(t,u,s) + \delta^{ad}\delta^{bc}C(u,s,t)\,.
\end{aligned}
$$

$$B(s,t,u) = B(u,t,s) \qquad C(s,t,u) = C(s,u,t)\,.$$

  - 7, 6 and 6 possible intermediate states

$\phi\phi \to \phi\phi$

- calculate all the diagrams
- Do all integrals, renormalize,. . .
- Construct states for all the presentations and their projection operators
- Get the amplitudes for all intermediate states
- Get all scattering lengths
- All formulas similar length to $\pi\pi$ cases but there are so many of them
- arXiv:1102.0172:
  - Very long appendix part
  - References for the Young diagrams, tensor algebra we did ourselves but probably exists (e.g. Cvitanovic group theory book)

$\phi\phi \to \phi\phi$

Some curious large $N_F = n$ relations

Leading in n:

$$
\begin{aligned}
a_0^I|_{\text{complex}} &= a_0^I|_{\text{real}} = a_0^I|_{\text{pseudoreal}} =_{LO} \frac{x_2}{\pi}\frac{n}{8}\,,\\
a_0^S|_{\text{complex}} &= a_0^S|_{\text{real}} = a_0^A|_{\text{pseudoreal}} =_{LO} \frac{x_2}{\pi}\frac{n}{16}\,,\\
a_1^A|_{\text{complex}} &= a_1^A|_{\text{real}} = a_1^S|_{\text{pseudoreal}} =_{LO} \frac{x_2}{\pi}\frac{n}{48}\,,
\end{aligned}
$$

Subleading:

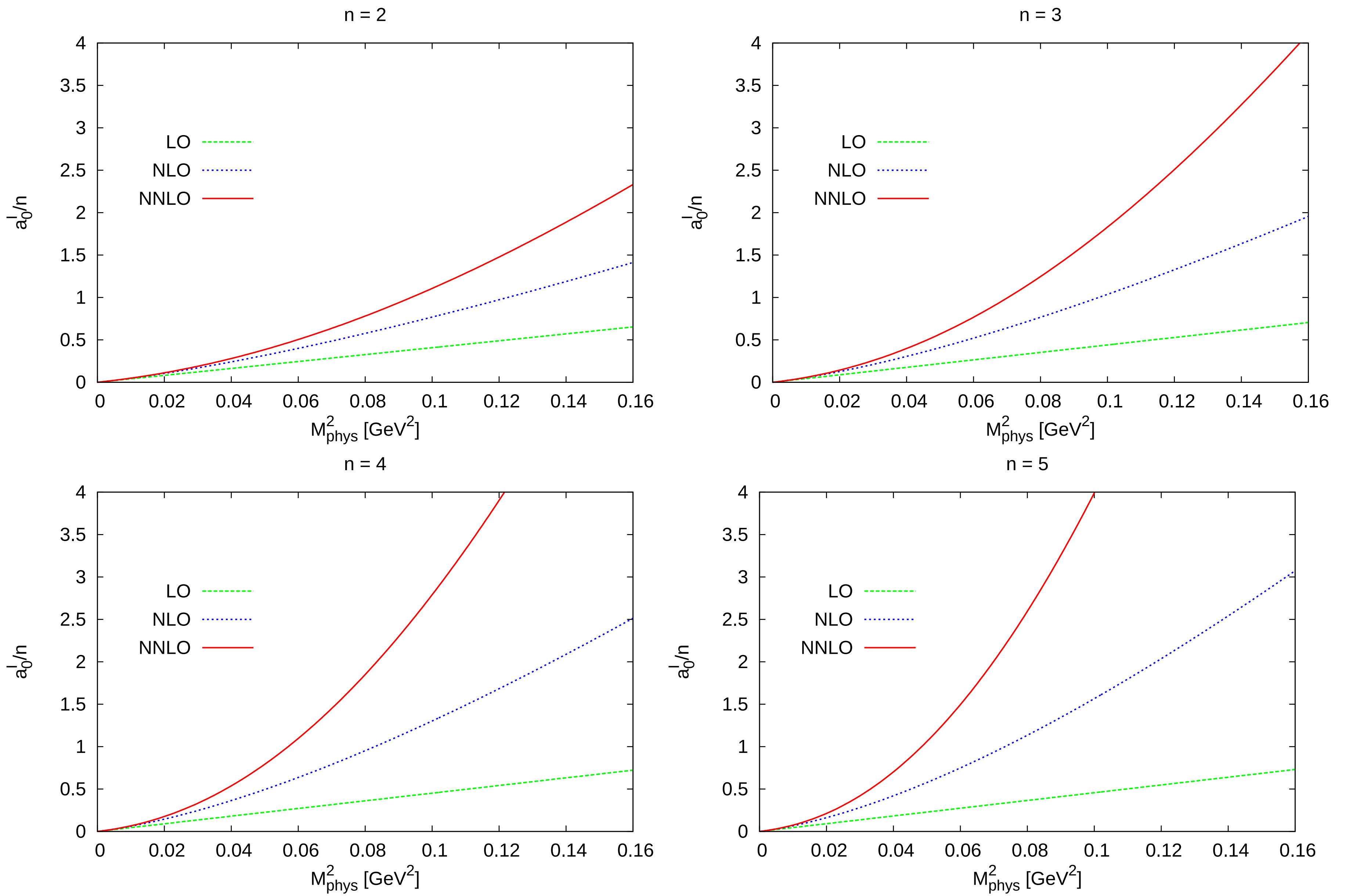
$$
\begin{aligned}
a_0^{SS}|_{\text{complex}} &= a_0^{FS}|_{\text{real}} = 2a_0^{MS}|_{\text{pseudoreal}} =_{LO} \frac{x_2}{\pi}\frac{-1}{16}\,,\\
a_0^{AA}|_{\text{complex}} &= 2a_0^{MS}|_{\text{real}} = a_0^{FA}|_{\text{pseudoreal}} =_{LO} \frac{x_2}{\pi}\frac{1}{16}\,.
\end{aligned}
$$

Subsubleading:

$$
a_1^{SA}|_{\text{complex}} = a_1^{AS}|_{\text{complex}} = 2a_1^{MA}|_{\text{real}} = 2a_1^{MA}|_{\text{pseudoreal}} =_{LO} 0\,.
$$

At NNLO here violated by an $L_4^r L_6^r$ term

$\phi\phi \to \phi\phi$: a_0^I/n



Other results: fully to NNLO

- M^2_{phys}
- F_{phys}
- Meson-meson scattering
- Equal mass case: allows to get fully analytical result just as for 2-flavour ChPT
- Note: large N_F here not cactus but planar diagrams (in flavour lines)

QCDlike: conclusions

- Different symmetry patterns can appear for different gaugegroups and fermion representations
- Nonperturbative: lattice needs extrapolation formulae
- Masses, decay constant and VEV: done to NNLO
- Meson-meson scattering: done to NNLO and some large N_F relations at NNLO.
- Two-pointfunctions and formfactors (S-parameter) for precision observables

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