THE MUON ANOMALOUS MAGNETIC MOMENT

Johan Bijnens

Lund University

bijnens@thep.lu.se
http://thep.lu.se/~bijnens

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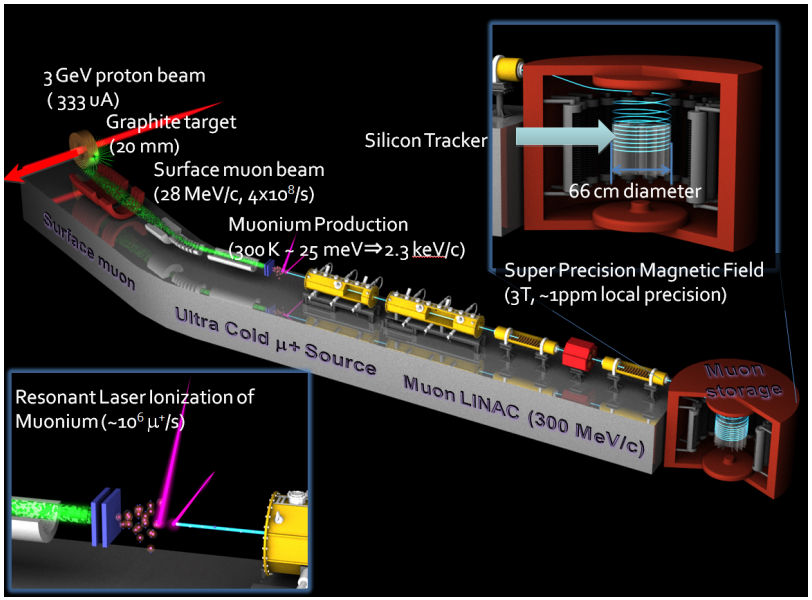
Introduction

- Magnetic moment: $\vec{\mu} = g\frac{q}{2m}\vec{S}$
- For angular momentum: $g = 1$
- Dirac equation: $g = 2$
- Structure and/or QFT give different values
- Anomaly: $a = \frac{g-2}{2}$
- QED (Schwinger): $a = 1 + \frac{\alpha}{2\pi} + \ldots$

Jacob Bourjaily/Wikipedia

Why do we do this?

The muon $a_\mu = \dfrac{g_\mu - 2}{2}$ will be measured more precisely



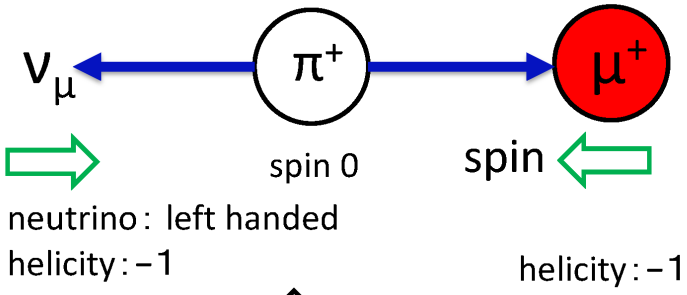
J-PARC

Fermilab

Principle of the measurement

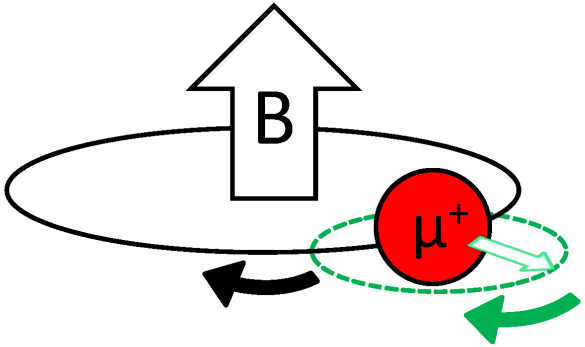
Three steps of g-2 measurement

1. Prepare a polarized muon beam.

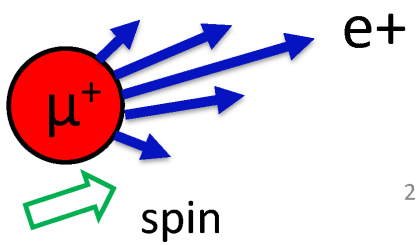


2. Store in a magnetic field (muon's spin precesses)

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$



3. Measure decay positron



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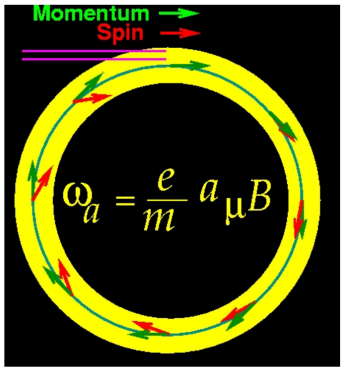
Principle of the measurement

muon g-2 and EDM measurements



In uniform magnetic field, muon spin rotates ahead of momentum due to g-2 ≠ 0

general form of spin precession vector:

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

BNL E821 approach
γ=30 (P=3 GeV/c)

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

FNAL E989

J-PARC approach
E = 0 at any γ

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B}\right)\right]$$

J-PARC E34

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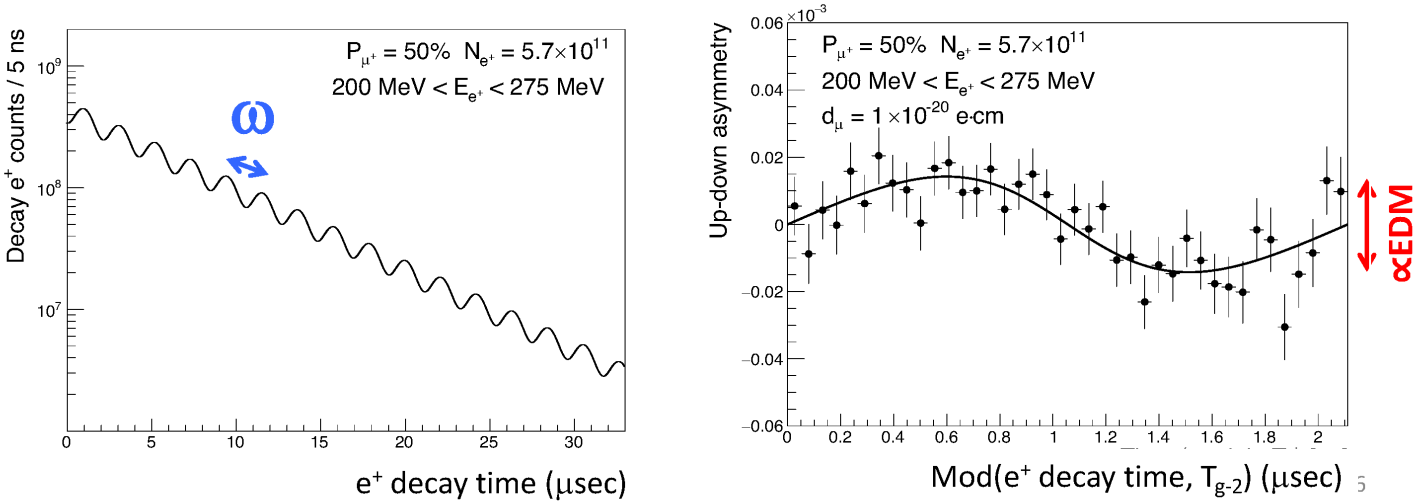
Principle of the measurement

Expected results

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B}\right)\right]$$

Expected time spectrum of e^+ in $\mu \rightarrow e^+\nu\nu$ decay



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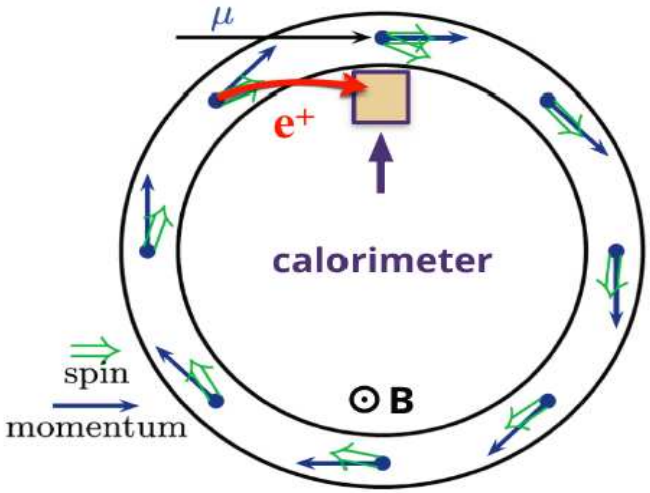
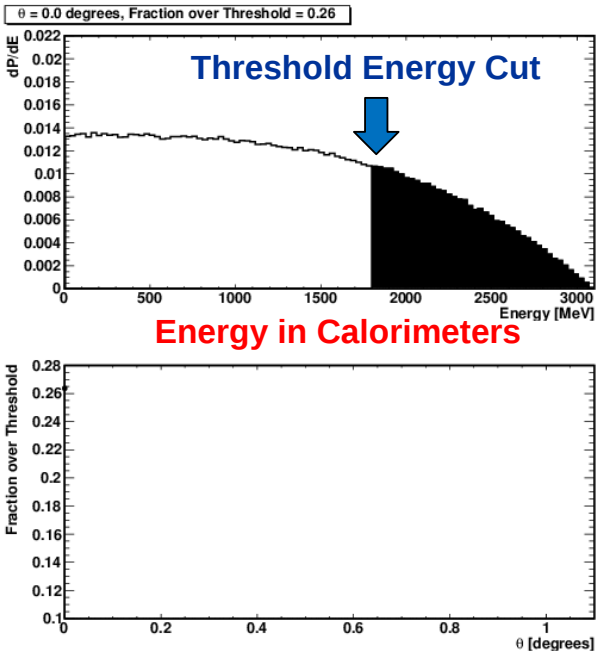
Principle of the measurement

The precession frequency is the difference between the spin and cyclotron frequencies, encoded as an oscillation in the positron energy spectrum as seen by the calorimeters

$$\omega_a = \omega_s - \omega_c = \left(\frac{g-2}{2}\right)\frac{eB}{m}$$





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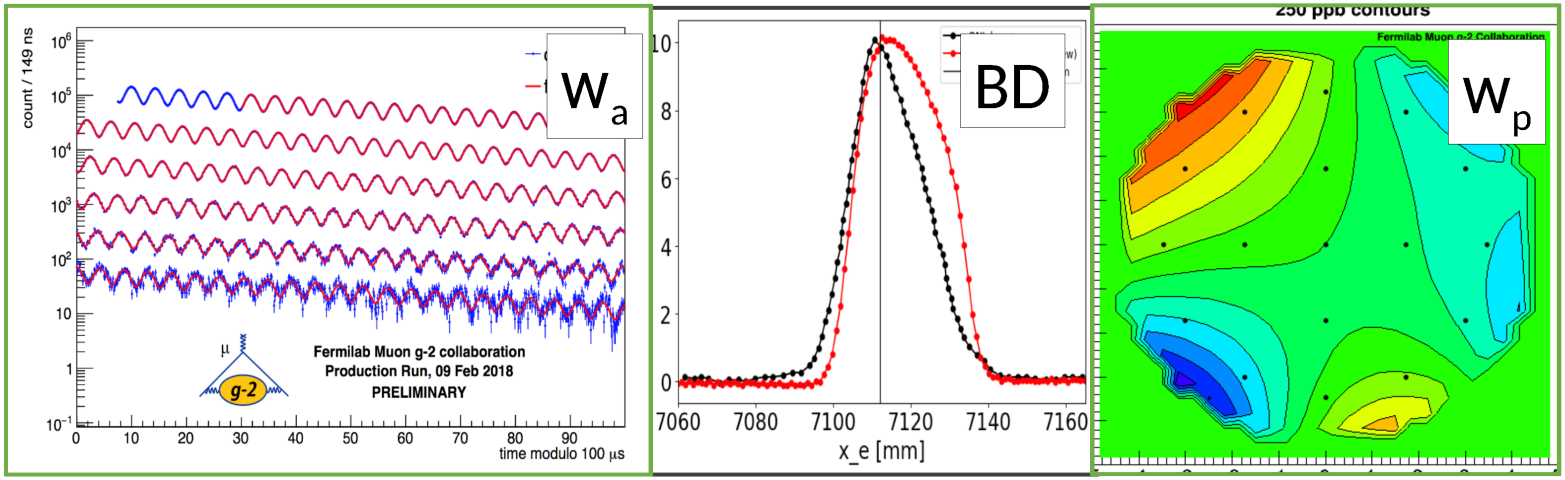
Conclusions

- E989 is in the thick of simultaneous Analysis / Hardware Tweaking / Running.
  - We are busy!!!
- People are working hard to get out a much-anticipated result, still in 2019.
  - BUT !!! We cannot afford to rush it so we are making lots and lots of checks.
- As we go "beyond BNL" we are learning a lot.
- Thanks



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Introduction

- Experiments done for electron and muon: very precise
- $a_e^{\rm exp} = 11596521.8073(0.0028)\ 10^{-10}$ Harvard
- $a_\mu^{\rm exp} = 11659209.1(5.4)(3.3)\ 10^{-10}$ BNL
- Main difference: electron loops enhanced by $\log(m_e/m_\mu)$
- $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$
- $a_\mu^{QED} = \frac{\alpha}{2\pi} + 0.765857425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050996(32)\left(\frac{\alpha}{\pi}\right)^3 + 130.8796(63)\left(\frac{\alpha}{\pi}\right)^4 + 753.3(1.0)\left(\frac{\alpha}{\pi}\right)^5$
- up to 4 loops essentially analytically, 5-loops numerically
- $a_\mu^{QED} = 11658471.895(0.008)\ 10^{-10}$
- $a_\mu^{EW} = 15.36(0.10)\ 10^{-10}$
- $a_\mu^{\rm exp} - a_\mu^{QED} - a_\mu^{EW} = 7218(63)\ 10^{-11}$

Why do we do this?

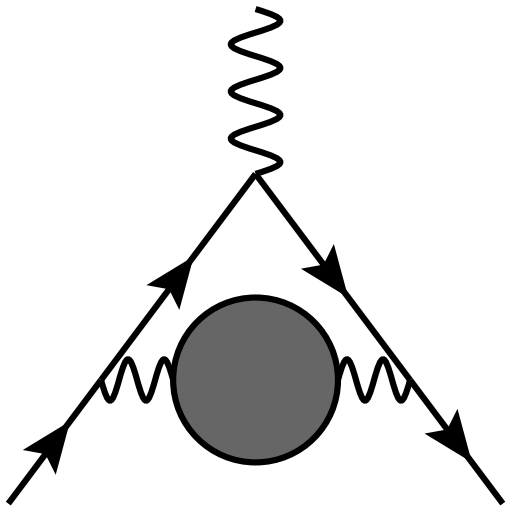
- Experiment dominated by BNL, FNAL error down by four
- Theory taken from PDG2018
- $a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{Had}}$
- $a_\mu^{\mathrm{Had}} = a_\mu^{\mathrm{LO\text{-}HVP}} + a_\mu^{\mathrm{HO\text{-}HVP}} + a_\mu^{\mathrm{HLbL}}$
- Impressive agreement with g_μ to 2×10^{-9}

Part	value	errors	units
a_μ^{EXP}:	116 592 091.x	(54)(33)	$\times 10^{-11}$
a_μ^{SM}:	116 591 823.x	(1)(34)(26)	$\times 10^{-11}$
Δa_μ:	268.x	(63)(43)	$\times 10^{-11}$
a_μ^{QED}:	116 584 719.0	(0.1)	$\times 10^{-11}$
a_μ^{EW} :	153.6	(1.0)	$\times 10^{-11}$
$a_\mu^{\mathrm{LO\text{-}HVP}}$:	6 931.x	(33)(7)	$\times 10^{-11}$
$a_\mu^{\mathrm{HO\text{-}HVP}}$:	-86.3	(0.9)	$\times 10^{-11}$
a_μ^{HLbL} :	105.x	(26)	$\times 10^{-11}$

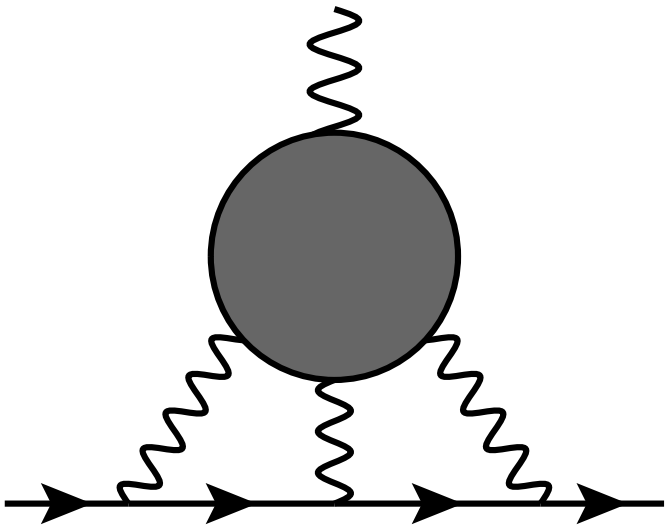
Numerical summary

- Long standing discrepancy with SM
- Theory number has had slowly shrinking error (especially LO HVP)
- $a_\mu^{\rm exp} - a_\mu^{SM} = (27.0 - 33.6) \pm (7.3 - 7.8)\ 10^{-10}$
- $3.6 - 4.6\sigma$ discrepancy
- Many BSM papers explaining this (experiment has ≥ 1950 citations)

Hadronic contributions



LO-HVP



HLbL

- The blobs are hadronic contributions
- I will present some results for LO-HVP and HLbL short-distance as well as give an overview
- There are higher order contributions of both types: known accurately enough

My recent work related to the muon $g-2$

[1] J. Bijnens and J. Relefors, "Pion light-by-light contributions to the muon $g-2$," JHEP **1609** (2016) 113 [arXiv:1608.01454 [hep-ph]]

[2] J. Bijnens, N. Hermansson-Truedsson and A. Rodriguez-Sanchez, "Short-distance constraints for the HLbL contribution to the muon anomalous magnetic moment," Phys. Lett. B **798C** (2019) 134994 [arxiv:1908.03331].

[3] J. Bijnens and J. Relefors, "Connected, Disconnected and Strange Quark Contributions to HVP," JHEP **1611** (2016) 086 [1609.01573 [hep-lat]].

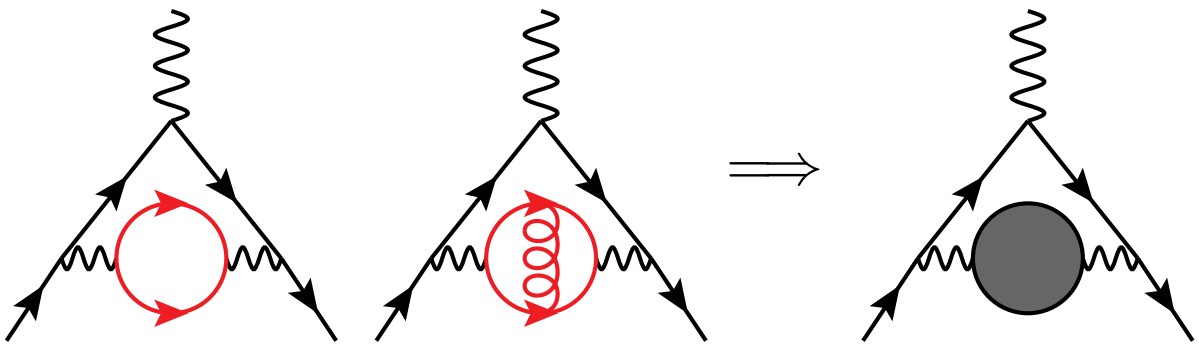
[4] J. Bijnens and J. Relefors, "Vector two-point functions in finite volume using partially quenched chiral perturbation theory at two loops," JHEP **1712** (2017) 114 [arXiv:1710.04479 [hep-lat]].

[5] J. Bijnens, J. Harrison, N. Hermansson-Truedsson, T. Janowski, A. Jüttner and A. Portelli, "Electromagnetic finite-size effects to the hadronic vacuum polarization," Phys. Rev. D100 (2019) 014508 [arXiv:1903.10591 [hep-lat]].

[1,2,3]: HLbL, [4,5]: HVP

I will concentrate on [2,5]

Muon $g-2$: LO hadronic from experiment



- $a_\mu^{\mathrm{LO\ HVP}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R^{(0)}(s)$
- $R^{(0)}(s)$ bare cross-section ratio $\dfrac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$
- Bare, many different evaluations,...
- e^+e^- versus τ-decays
- Main problem: combining data from many sources and data sometimes not compatible with errors

Numerical summary: HVP

- LO-HVP latest evaluations (talks at workshops KEK 2018/Seattle 2019)
  - Paris (Davier et al.) $a_\mu^{LO\ HVP} = 693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{QCD}\ 10^{-10}$
  - Liverpool (Keshavarzi et al) $a_\mu^{LO\ HVP} = 693.8(2.4)\ 10^{-10}$
  - Jegerlehner $a_\mu^{LO\ HVP} = 689.46(3.25)\ 10^{-10}$
  - HLS (Benayoun et al) $a_\mu^{LO\ HVP} = 686.1(3.8)\ 10^{-10}$
  - Slight differences in data, how put together plus what theory
  - Discrepancy Babar/KLOE in $\pi\pi$-channel
  - Can be with or without τ-data
  - Numbers vary slightly whenever new experiments included
- $a_\mu^{NLO\ HVP} = -9.82(0.04)\ 10^{-10}$
- $a_\mu^{NNLO\ HVP} = 1.24(0.01)\ 10^{-10}$
- Errors: LO-HVP and HLbL, rest negligible

HVP: Summary of results

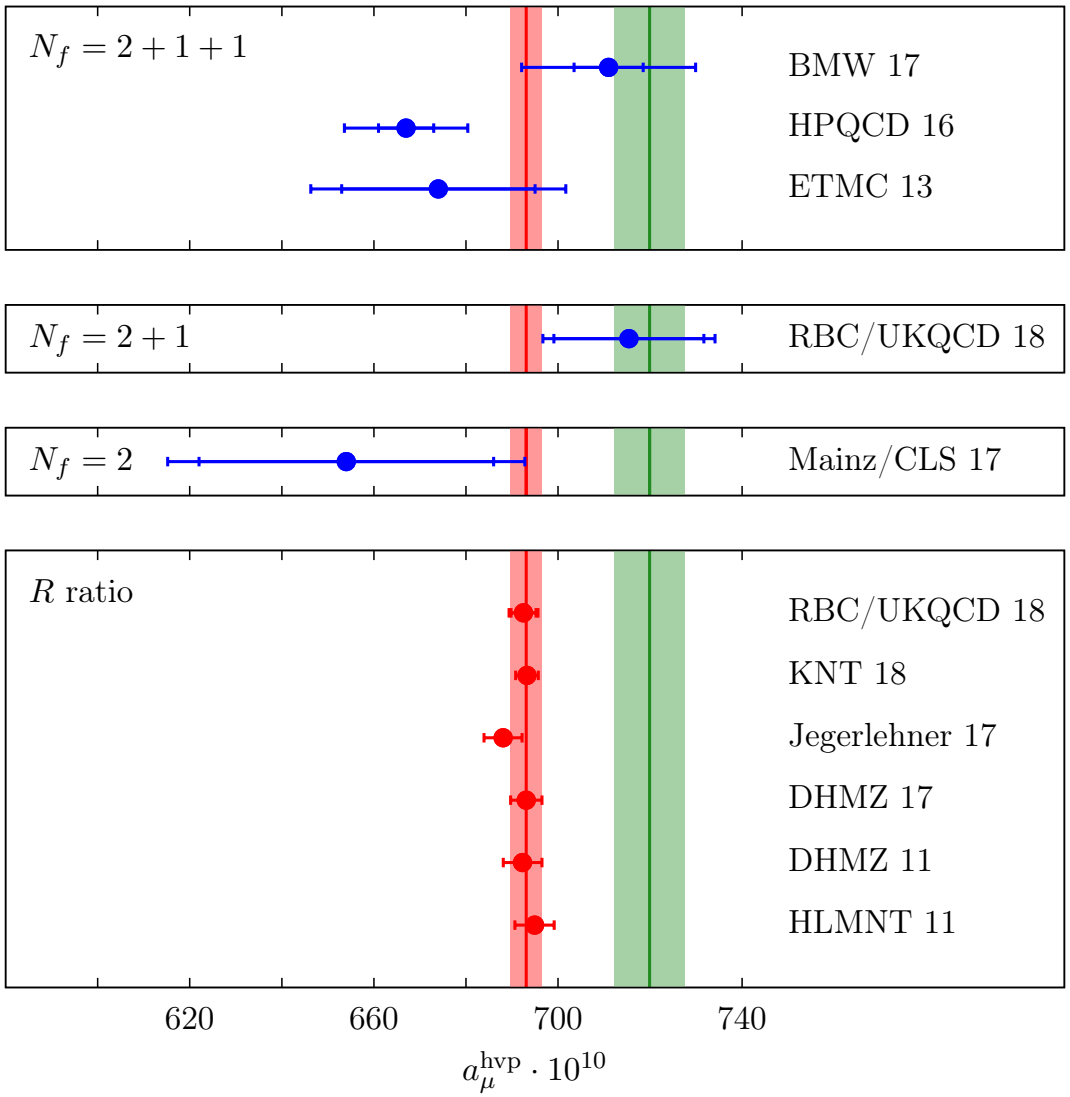


Figure from 1807.09370

Some discrepancies, lattice accuracies improve

Status lattice results

- Dispersive has reached below 0.5%
- Lattice accuracies a few %
- Improvement continuous
- Finite volume corrections known to two-loop order in ChPT [4]
- Next: isospin breaking
  - $m_u - m_d$: "easy"
  - Electromagnetism: possibly large finite volume corrections since $1/L^n$ rather than $\exp(-m_\pi L)$
- J. Bijnens, J. Harrison, N. Hermansson-Truedsson, T. Janowski, A. Jüttner and A. Portelli, "Electromagnetic finite-size effects to the hadronic vacuum polarization," arXiv:1903.10591 [hep-lat].

  Presented at $g-2$ meetings in KEK february 2018, Mainz June 2018

  They are expected to be small

Conventions

- Main object: $\boxed{\Pi^{\mu\nu}_{ab}(q) = i \int d^4x e^{iq\cdot x} \langle 0| T(j^\mu_a(x) j^{\nu\dagger}_b(0)|0\rangle}$
- $j^\mu_{EM} = (2/3)j^\mu_U - (1/3)j^\mu_D - (1/3)j^\mu_S \qquad j^\mu_Q = \overline{q}\gamma^\mu q$
- Continuum/infinite volume:
  $\Pi^{\mu\nu}_{EM}(q) = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right) \Pi_{EM}(q^2)$
- Known positive weight functions v, w and $Q^2 = -q^2$:
  - $a_\mu = \int_{\text{threshold}}^{\infty} dq^2 w(q^2) \frac{1}{\pi} \text{Im}\Pi_{EM}(q^2)$
  - $a_\mu = \int_0^\infty dQ^2 v(Q^2) \left(-\Pi(Q^2) + \Pi(0)\right)$
- Dispersion relation:
  $$\Pi(q^2) = \Pi(0) + \frac{q^2}{\pi} \int_{\text{threshold}}^{\infty} ds \frac{1}{s(s-q^2)} \frac{1}{\pi} \text{Im}\Pi(s)$$

First: Scalar QED

- Pion loops: finite volume effects suppressed by $e^{-m_\pi L}$ (if off-shell)
- Photon loops have suppression only by powers of $1/L$
- Dynamical photons: large finite volume effects possible
- Scalar QED in usual $\overline{MS}$ ($\mu^2_{ChPT} = \mu^2_{\overline{MS}} e$, $e = 2.71\ldots$)
- $\mathcal{L} = (\partial_\mu \Phi^* + ieA_\mu \Phi^*)(\partial_\mu \Phi - ieA_\mu \Phi) - m_0^2 \Phi^* \Phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ ($-\lambda (\phi^* \phi)^2$ not needed)

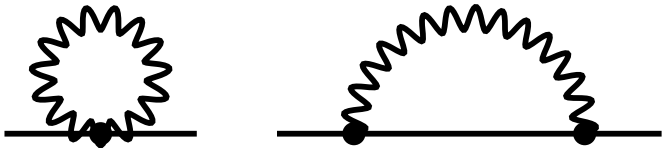
Integrals at finite volume

- Finite volume for photons started in earnest with
  N. Hayakawa, S. Uno, Prog. Theor. Phys. **120**(2008)413 [0804.2044]
- Get the explicit $1/L$ behaviour from one-loop integrals:
  Z. Davoudi, M. Savage, Phys. Rev. D90(2014)054503 [1402,6471]
  S. Borsyani et al, Science 347(2015)1452 [1406.4088]
- $S = \frac{1}{i^2} \int \frac{d^d l}{(2\pi)^d} \frac{d^d k}{(2\pi)^d} \frac{1}{k^2(l^2-m^2)((k+l-p)^2-m^2))}$
- do l^0, k^0 integrals via contour integration
- $\vec{k} = \frac{2\pi}{L}\vec{n}$ and expand in $1/L$
- Write the $\vec{k}$ part as

$$\frac{1}{L^{d-1}} \sum_{\vec{n}\neq\vec{0}} = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} + \left[\frac{1}{L^{d-1}} \sum_{\vec{n}\neq\vec{0}} - \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \right]$$

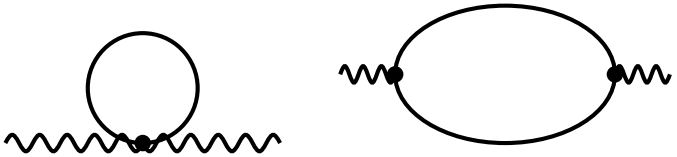
- First term: infinite volume contribution
- Call the quantity in brackets $\left(1/L^{d-1}\right) \Delta'_{\vec{n}}$
- Define $c_m = \Delta_{\vec{n}} \frac{1}{|\vec{n}|^m}$

Example: mass

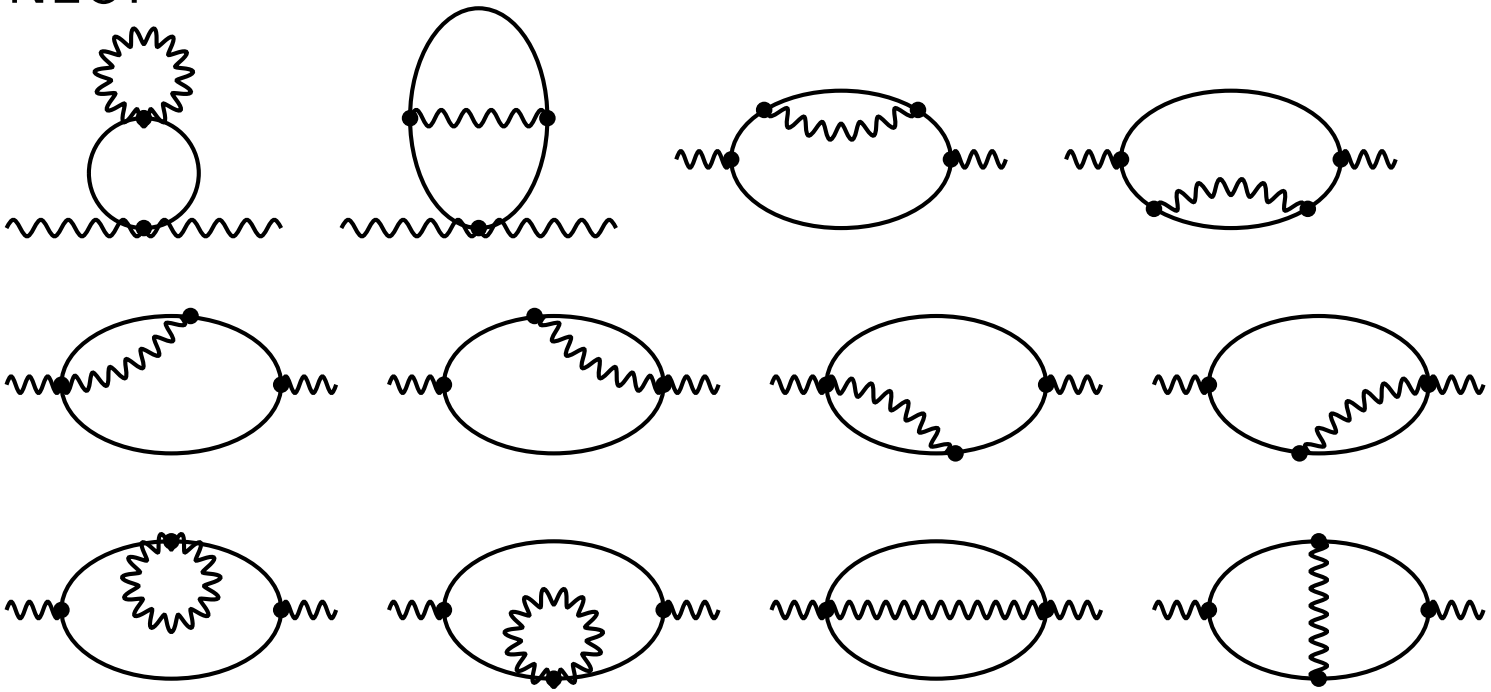
- 
- $$\frac{\Delta_V m^2}{m^2} = e^2\left(\frac{4c_2}{16\pi^2 mL} + \frac{2c_1}{16\pi^2 m^2 L^2} + \mathcal{O}\left(\frac{1}{L^4}, e^{-mL}\right)\right)$$
- Agrees with earlier results
- c_2 shows up since BOTH propagators can be 'on-shell'
- For $\Pi_{\mu\nu}$ below threshold only photon line goes 'on-shell' $\Longrightarrow$ corrections start only at $1/L^2$
- Only true if infinite volume mass is used in the expressions at LO
- The finite volume correction is 30% of the total electromagnetic for "standard" pion masses

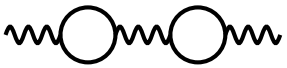
Diagrams

- Lowest order:



- NLO:



contributions from counterterms and 

Twopoint function

- Do the k^0, l^0 integral
- expand for large L as explained earlier
- Rest can be expressed in terms of
  $$Z_{ij}(m^2, p^2) = \int \frac{d^{d-1}l}{(2\pi)^{d-1}} \frac{1}{(\vec{l}^2 + m^2)^{i/2}(4\vec{l}^2 + 4m^2 - p^2)^j}$$
- these correspond to one-loop integrals with masses
- $\Omega_{ij} \equiv Z_{ij}(m^2, p^2) m^{i+2j-d+1}$
- Calculate in center of mass frame: $p = (p^0, \vec{0})$
- QED$_L$: no disconnected contribution
- $t_{\mu\nu}$ spatial part of $g_{\mu\nu}$
- $\widetilde{\Pi}((p^0)^2) \equiv \frac{-1}{3p^2} t_{\mu\nu} \left(\Pi^{\mu\nu}(p) - \Pi^{\mu\nu}(p=0)\right)$
- Infinite volume: $\widetilde{\Pi}(p^2) = \Pi(p^2)$

Results

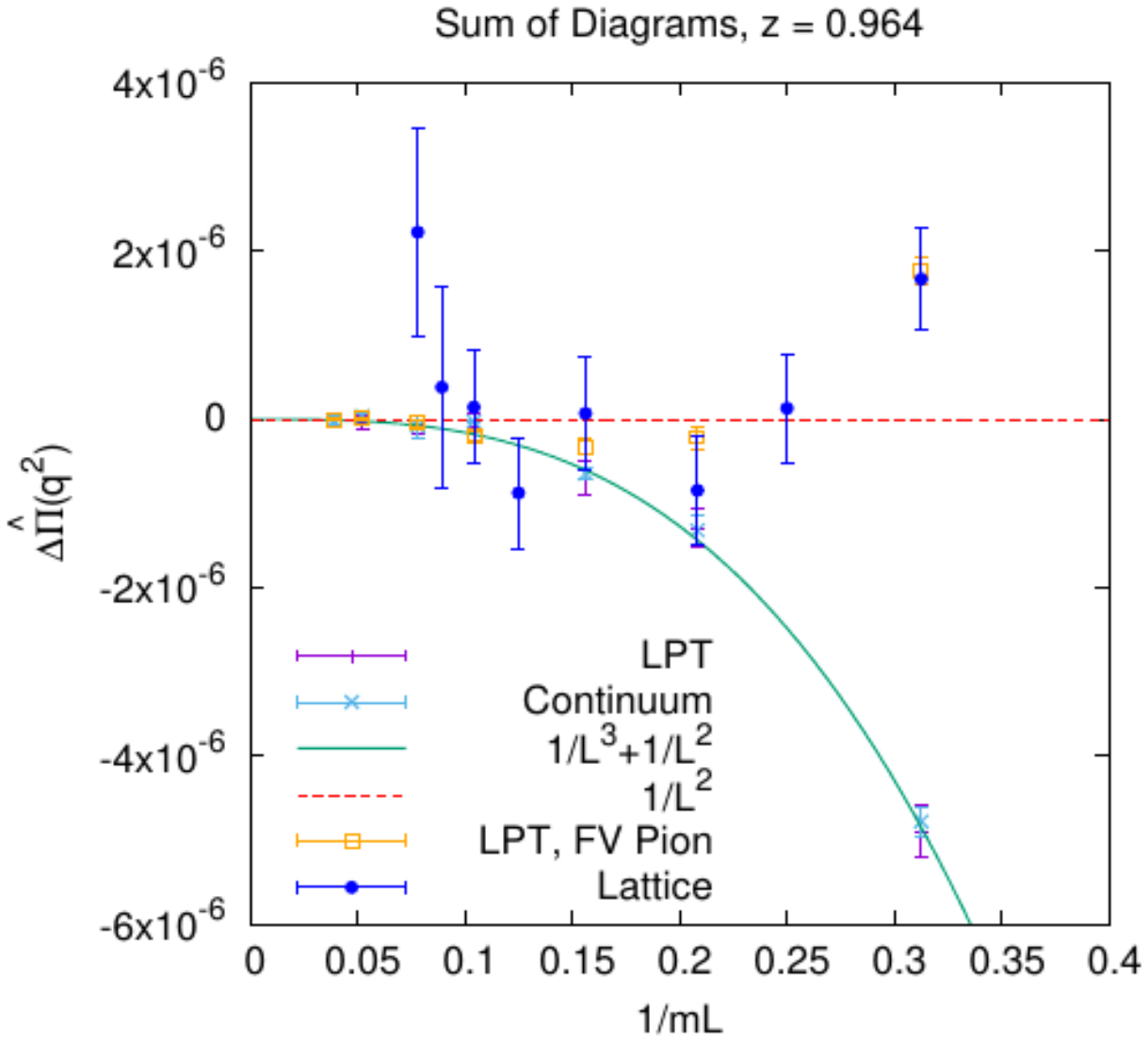
$$\widetilde{\Pi}(p^2) = \frac{c_1}{\pi m^2 L^2}\left(\frac{16}{3}\Omega_{-1,3} - \frac{1}{3}\Omega_{1,2} - \frac{32}{3}\Omega_{1,3} - \frac{2}{3}\Omega_{3,2} + \frac{16}{3}\Omega_{3,3} - \frac{1}{8}\Omega_{5,1} + \Omega_{5,2}\right)$$
$$+ \frac{c_0}{m^3 L^3}\left(-\frac{128}{3}\Omega_{-2,4} + \frac{256}{3}\Omega_{0,4} - \frac{5}{3}\Omega_{2,2} + \frac{8}{3}\Omega_{2,3} - \frac{128}{3}\Omega_{2,4}\right.$$
$$\left. - \frac{3}{8}\Omega_{4,1} + \frac{7}{6}\Omega_{4,2} - \frac{8}{3}\Omega_{4,3}\right) + \mathcal{O}\left(\frac{1}{L^4}, e^{-mL}\right)$$

Simplify using relations of the Ω_{ij} to

$$\widetilde{\Pi}(p^2) = + \frac{c_0}{m^3 L^3}\left(-\frac{16}{3}\Omega_{0,3} - \frac{5}{3}\Omega_{2,2} + \frac{40}{9}\Omega_{2,3} - \frac{3}{8}\Omega_{4,1} + \frac{7}{6}\Omega_{4,2} + \frac{8}{9}\Omega_{4,3}\right)$$
$$+ \mathcal{O}\left(\frac{1}{L^4}, e^{-mL}\right)$$

the $1/L^2$ cancels: expected: far away the photon sees no charge since it is a neutral current: only dipole effect

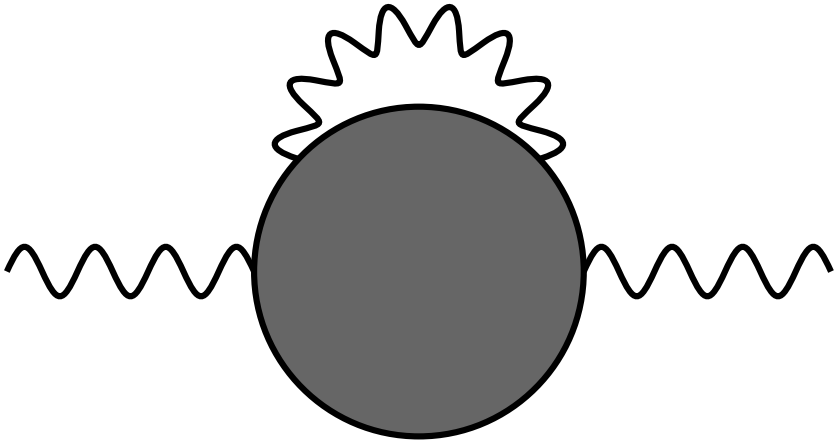
Scalar QED on a lattice



Correction for $mL \sim 5$ less than 1%

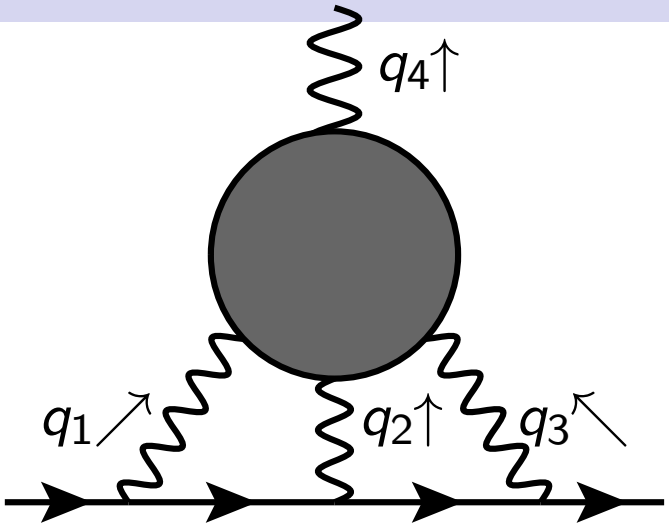
Generality

the $1/L^2$ cancels: expected: far away the photon sees no charge since it is a neutral current: only dipole effect



- The blob (hadrons) is a four-point function of electromagnetic currents and has no singularities in the regime needed
- The conclusion about $1/L^3$ is general

HLbL: the main object to calculate



- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks
- Two main evaluations around 1995

Numerical summary: HLbL

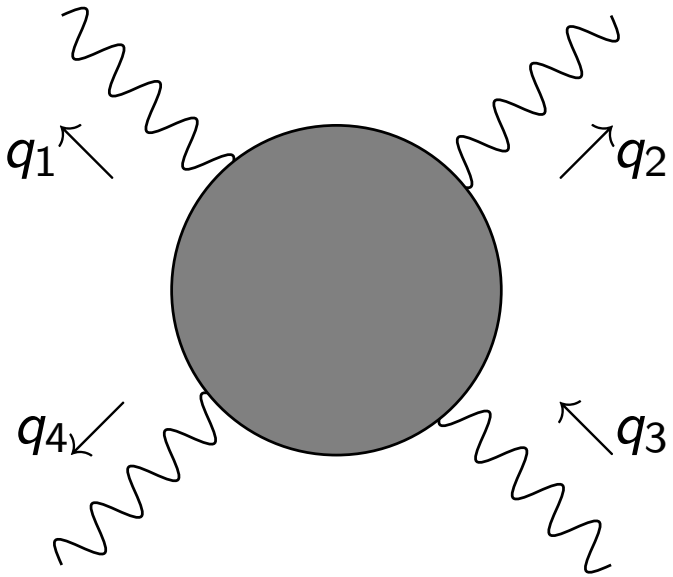
- The largest constribution is π^0 (and η, η') exchange/pole
  - Beware: pole/exchange not quite the same
  - Most evaluations are in reasonable agreement
- The pion loop can be sizable but a large difference between the two evaluations
  - For the pure pion loop part, even larger numbers have been proposed by Engel, Ramsey-Musolf
  - Scalar exchange versus $\pi\pi$-rescattering
- There are other contributions but the sum is smaller than the leading pseudo-scalar exchange
- BPP: $(8.3 \pm 3.2)\ 10^{-10}$ HKS: $(8.96 \pm 1.54)\ 10^{-10}$

Numerical summary: HLbL

- π^0-exchange (in units of 10^{-10})
  - BPP: 5.9(0.9)
  - pointlike VMD 5.6
  - Dispersive (1808.04823): $6.26^{+0.30}_{-0.25}$
- η, η' add about 1.5 each
- Pion loop (in units of 10^{-10})
  - BPP: $-1.9(1.3)$ large error to include other model
  - dispersive (1702.07347): $-2.4(1)$
- a_1: 0.7
- Scalars -0.7
- Quark loop: here is the main double counting problem 0–2
- Short distance constraints might enhance a bit
- Everything else
- $a_\mu^{HLbL} = 10.5(2.6)\ 10^{-10}$ (error up to 4 10^{-10} defendable)

General properties

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) =$$



Actually we really need $\left.\dfrac{\delta \Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)}{\delta q_{4\rho}}\right|_{q_4=0}$

General properties

$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)$:

- In general 138 Lorentz structures (136 in 4 dimensions)
- Using $q_{1\mu}\Pi^{\mu\nu\lambda\sigma} = q_{2\nu}\Pi^{\mu\nu\lambda\sigma} = q_{3\lambda}\Pi^{\mu\nu\lambda\sigma} = q_{4\sigma}\Pi^{\mu\nu\lambda\sigma} = 0$
  43 (41) gauge invariant structures
- 41 helicity amplitudes
- Bose symmetry relates some of them
- Compare HVP: one function, one variable
- General calculation from experiment via dispersion relations: recent progress

  Colangelo, Hoferichter, Kubis, Procura, Stoffer,...
- Well defined separation between different contributions
- Theory initiative: paper under preparation
- One remaining problem: intermediate- and short-distances

General properties

- Formalism of Colangelo et al., JHEP 1704 (2017) 161 [1702.07347]
- $\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \sum_{i=1,54} \hat{T}^{\mu\nu\lambda\sigma} \hat{\Pi}_i$
- $Q_i^2 = -q_i^2$
- $Q_3^2 = Q_1^2 + Q_2^2 + 2\tau Q_1 Q_3$
- $a_\mu = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 dQ_2 Q_1^3 Q_2^3 \int_{-1}^{1} d\tau \sqrt{1-\tau^2} \sum_{i=1,12} \hat{T}_i \overline{\Pi}_i$
- The 12 $\overline{\Pi}_i$ are related to 6 $\hat{\Pi}_i$ with $q_4 \to 0$.

$$\overline{\Pi}_1 = \hat{\Pi}_1 \,,\ \overline{\Pi}_2 = C_{23}\left[\hat{\Pi}_1\right] \,,\ \overline{\Pi}_3 = \hat{\Pi}_4 \,,\ \overline{\Pi}_4 = C_{23}\left[\hat{\Pi}_4\right] \,,$$

$$\overline{\Pi}_5 = \hat{\Pi}_7 \,,\ \overline{\Pi}_6 = C_{12}\left[C_{13}\left[\hat{\Pi}_7\right]\right] \,,\ \overline{\Pi}_7 = C_{23}\left[\hat{\Pi}_7\right] \,,$$

$$\overline{\Pi}_8 = C_{13}\left[\hat{\Pi}_{17}\right] \,,\ \overline{\Pi}_9 = \hat{\Pi}_{17} \,,\ \overline{\Pi}_{10} = \hat{\Pi}_{39} \,,$$

$$\overline{\Pi}_{11} = -C_{23}\left[\hat{\Pi}_{54}\right] \,,\ \overline{\Pi}_{12} = \hat{\Pi}_{54} \,,$$

General properties

- Colangelo et al.
- Dispersion relations can be written for the $\hat{\Pi}_i$
- Can cleanly separate different contributions: $\pi^0, \eta, \eta', \pi\pi, \ldots$
- Well defined separations (other ways of doing it can be fine as well)
- Caveat: separate contributions must satisfy certain sum rules so the dispersion relations work
- White-paper in progress (actually both HVP and HLbL)

π^0-pole

- Depends on the double off-shell form-factor $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)$
- Old days: VMD
- Now: normalization from experiment, form-factors mixture experiment/theory/lattice
- Short-distance constraints important
- Fully dispersive Hoferichter et al., 1808.04823
  $a_\mu^{\pi^0\text{-pole}} = 63.0^{+2.7}_{-2.1}\ 10^{-11}$
- Padé/Canterbury approximants
  Masjuan, Sanchez-Puertas 1701.05829
  $a_\mu^{\pi^0\text{-pole}} = 63.6(2.7)\ 10^{-11}$
- Compatible with all other estimates (except MV)

η, η'-pole

- Depends on the double off-shell form-factor $F_{\eta^{(\prime)}\gamma^*\gamma^*}(Q_1^2, Q_2^2)$
- Old days: VMD
- Now: normalization from experiment, form-factors mixture experiment/theory/lattice
- Short-distance constraints important
- Padé/Canterbury approximants

  Masjuan, Sanchez-Puertas 1701.05829

  $a_\mu^{\eta\text{-pole}} = 16.3(1.4)\ 10^{-11}$

  $a_\mu^{\eta'\text{-pole}} = 14.5(1.9)\ 10^{-11}$
- Compatible with all other estimates (except MV)

π-loop

- BPP: $-1.9\ 10^{-10}$ (ENJL/Full-VMD)
- HKS: $-0.5\ 10^{-10}$ (HLS model)
- Since then:
  - HLS does not satisfy shortdistance JB, Prades 2007
  - Polarizability expected to give small enhancement $-20(5)\ 10^{-11}$ JB, Relefors, 2016
  - rescattering as parametrized by scalars expected to give enhancement beyond this $-6.8(2.0)\ 10^{-11}$ JB, Relefors, 2016
- Dispersive: Pion-box and rescattering has all contributions
- Well defined separation
- Pion box: needs $F_V(Q^2)$: the pion form-factor

  Colangelo, Hoferichter, Procura, Stoffer, 1702.07347

  $a_\mu^{\pi\text{-box}} = -15.8(2)\ 10^{-11}$
- Pion S-wave rescattering

  $a_\mu^{\pi\text{-box}} = -8(1)\ 10^{-11}$
- Other rescatterings: work in progress

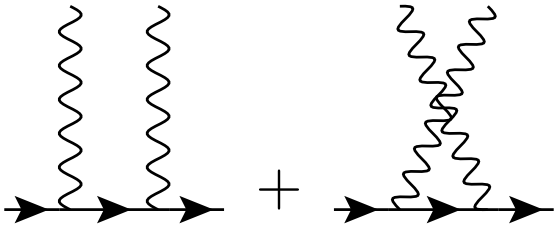
Other hadrons

- There are very many states above 1 GeV
- Not that much is known about their photon couplings
- Even less (essentially nothing) about form-factors in the couplings to photons
- Many estimates

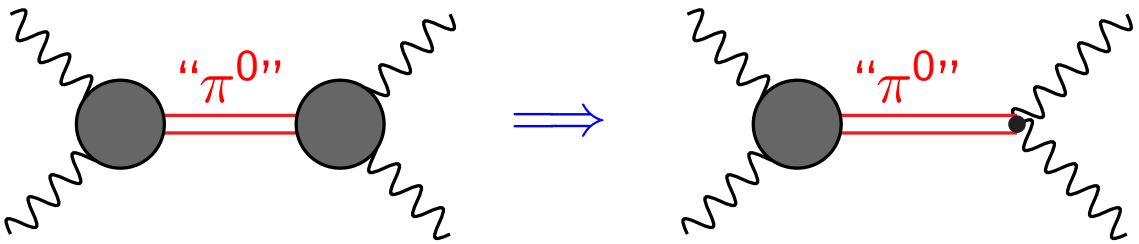
  Pauk-Vanderhaeghen,Jegerlehner,BPP,Melnikov-Vainshtein,...
- Estimates:
  - Scalars/tensors: $a_\mu^{\text{scalars+tensors}} = -2(3)\ 10^{-11}$
  - Axials: $a_\mu^{\text{axials}} = +8(?)\ 10^{-11}$
- Melnikov-Vainshtein, 2003 have a much large estimate (~ 22)

MV short-distance

- K. Melnikov, A. Vainshtein, Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment revisited, Phys. Rev. **D70** (2004) 113006. [hep-ph/0312226]
- take $Q_1^2 \approx Q_2^2 \gg Q_3^2$: Leading term in OPE of two vector currents is proportional to axial current
- $\Pi^{\rho\nu\alpha\beta} \propto \frac{P_\rho}{Q_1^2} \langle 0 | T \left(J_{A\nu} J_{V\alpha} J_{V\beta} \right) | 0 \rangle$
- J_A comes from

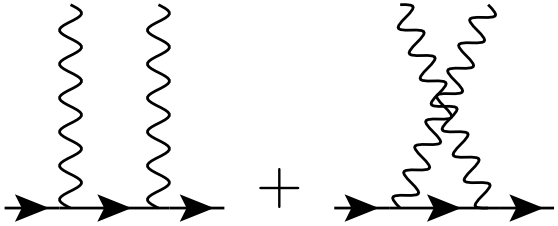


- AVV triangle anomaly: extra info
- Implemented via setting one blob = 1



- $a_\mu^{\pi^0} = 7.7 \times 10^{-10}$

Short-distance: MV

- The pointlike vertex implements shortdistance part, not only π^0-exchange
- 

  Are these part of the quark-loop? JB, Prades, 2007

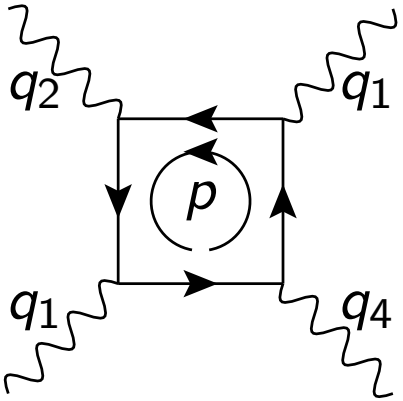
  See also in Dorokhov,Broniowski, Phys.Rev. D78(2008)07301
- BPP quarkloop + π^0-exchange $\approx$ MV π^0-exchange
- Important short-distance constraint but numerical effect probably overestimated

Quarkloop

- Use (constituent) quark loop
- Used for full estimates since the beginning (1970s)
- Used for short-distance estimates with mass as a cut-off

  JB, Pallante, Prades, 1996

- We recalculated:



- In agreement with quarkloop formulae from

  Hoferichter, Stoffer, private communication

- In agreement with known numerics

Charm loop

- Kühn et al., hep-ph/0301151, Phys.Rev. D68 (2003) 033018
- $a_\mu = \left(\frac{\alpha}{\pi}\right)^3 N_c e_q^4 \left[\frac{m_\mu^2}{M^2}\left(\frac{3}{3}\zeta_3 - \frac{19}{16}\right) + \cdots\right]$
- Up to m_μ^{10}/M^{10} in paper
- $m_c = 1.27$ GeV
- $a_\mu^{\mathrm{HLbLc}} = (3.165 - 0.0786 - 0.00033 + \cdots) \times 10^{-11}$
  $= 3.1(1) \times 10^{-11}$
- $m_b = 4.18$ GeV
- $a_\mu^{\mathrm{HLbLb}} = 1.8 \times 10^{-13}$

Quarkloop: u, d, s

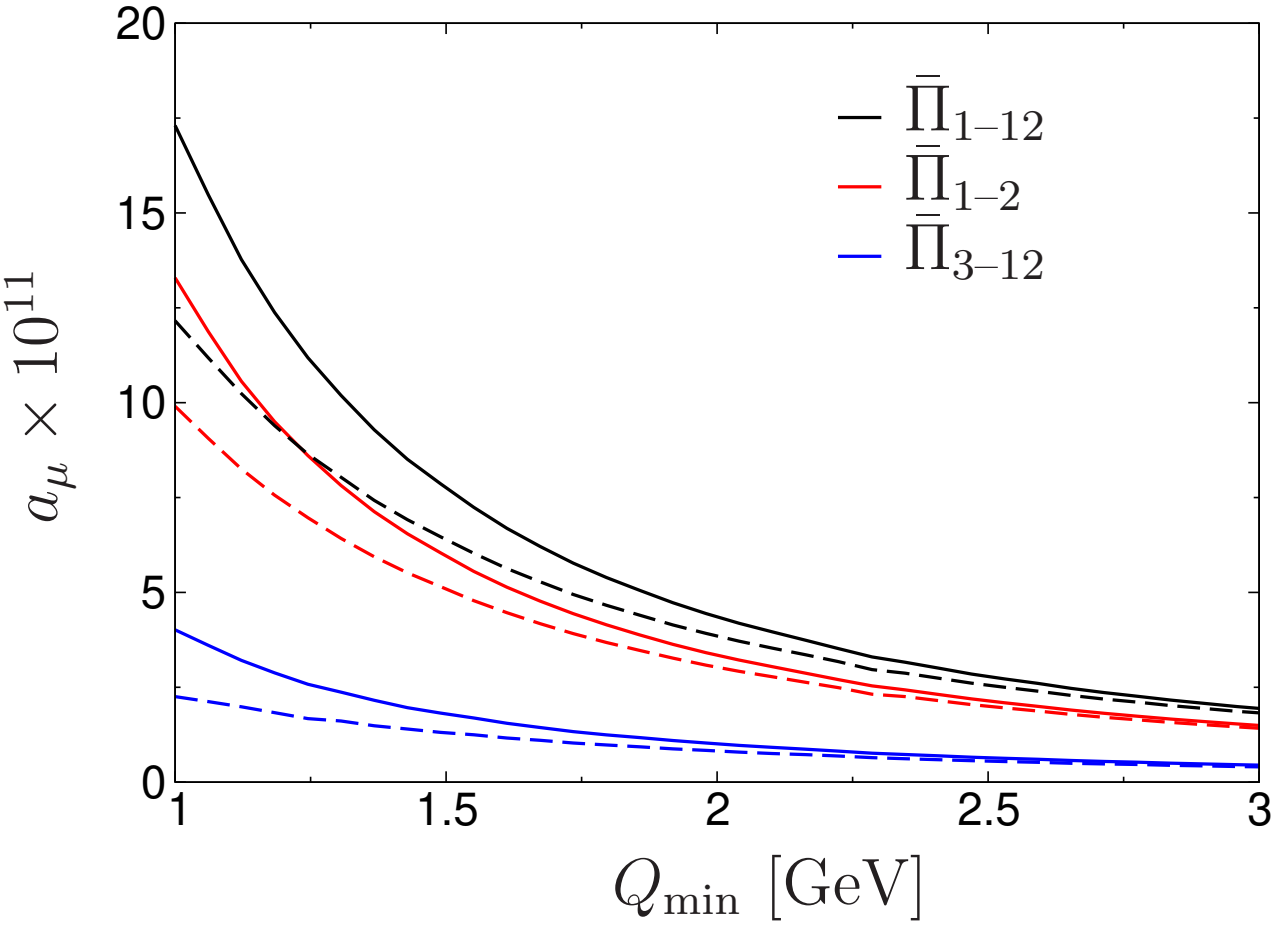


figure: Hoferichter

$Q_1, Q_2, Q_3 > Q_{\rm min}$

$M_Q = 0$: full

$M_Q = 0.3$ GeV dashed

$a_\mu^{\rm HLbLQ} = 54 \times 10^{-11}$

- M_Q provides an infrared cut-off, $M_Q \to 0$ divergent
- About 12×10^{-11} from above 1 GeV for $M_Q = 0.3$ GeV
- About 17×10^{-11} from above 1 GeV for $M_Q = 0$

Quarkloop

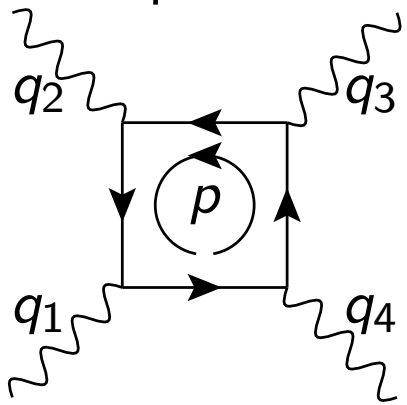
- Is it a first term in a systematic OPE?
- OPE has been used as constraints on specific contributions
  - $\pi^0\gamma^*\gamma^*$ asymptotic behaviour
  - Constraints on many other hadronic formfactors
  - $q_1^2 \approx q_2^2 \gg q_3^2$ Melnikhov, Vainshtein 2003
  - These are discussed in the next two talks

Short-distance: first attempt

$$\Pi^{\mu\nu\lambda\sigma} = -i \int d^4x d^4y d^4z e^{-i(q_1\cdot x + q_2\cdot y + q_3\cdot z)} \left\langle T\left(j^\mu(x) j^\nu(y) j^\lambda(z) j^\sigma(0)\right)\right\rangle$$

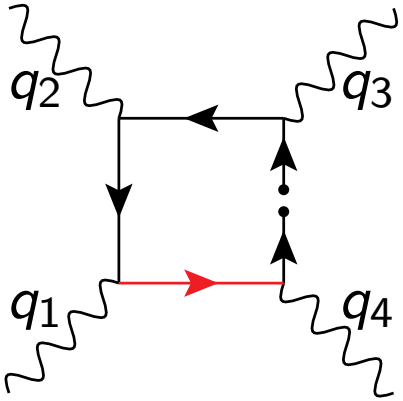
- Usual OPE: x, y, z all small
- First term in the expansion is the quark-loop
  no problem with $\partial/\partial q_4^\rho$ and $q_4 \to 0$

  p in loop $\Rightarrow$ no singular propagators:



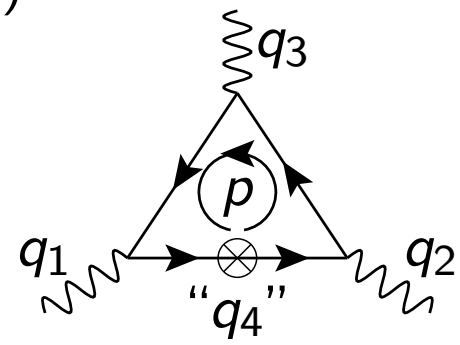
- Next term problems: no loop momentum;

  $q_4 \to 0$ propagator diverges:



Short-distance: correctly

- Similar problem in QCD sum rules for electromagnetic radii and magnetic moments
- Ioffe, Smilga, 1984
- For the q_4-leg use a constant background field and do the OPE in the presence of that constant background field
- Use radial gauge: $A_4^\lambda(w) = \frac{1}{2} w_\mu F^{\mu\lambda}$
  whole calculation is immediately with $q_4 = 0$.
- First term is exactly the usual quark loop
  (even including quark masses)

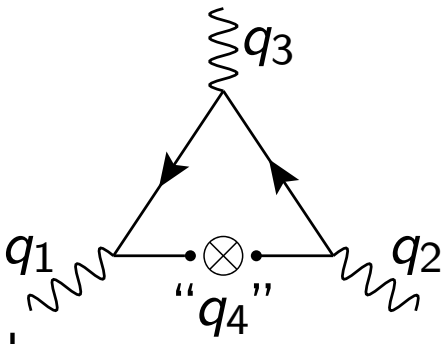


Short-distance: next term(s)

- Do the usual QCD sum rule expansion in terms of vacuum condensates
- There are new condensates, induced by the constant magnetic field: $\langle \bar{q}\sigma_{\alpha\beta}q\rangle \equiv e_q F_{\alpha\beta} X_q$
- Lattice QCD Bali et al., arXiv:1206.4205
  $X_u = 40.7 \pm 1.3$ MeV,
  $X_d = 39.4 \pm 1.4$ MeV,
  $X_s = 53.0 \pm 7.2$ MeV
- Could have started at order $1/Q$, only starts at $1/Q^2$ via $m_q X_q$ corrections to the leading quark-loop result
- X_q and m_q are very small, only a very small correction
- Next order: very many condensates contribute, work in progress

Short-distance

- Result derived from:



- $N_c = 3$ and one quark

$$\hat{\Pi}_1 = m_q X_q e_q^4 \frac{-4(Q_1^2 + Q_2^2 - Q_3^2)}{Q_1^2 Q_2^2 Q_3^4} \qquad \hat{\Pi}_7 = 0$$

$$\hat{\Pi}_4 = m_q X_q e_q^4 \frac{8}{Q_1^2 Q_2^2 Q_3^2} \qquad \hat{\Pi}_{17} = m_q X_q e_q^4 \frac{8}{Q_1^2 Q_2^2 Q_3^4}$$

$$\hat{\Pi}_{54} = m_q X_q e_q^4 \frac{-4(Q_1^2 - Q_2^2)}{Q_1^4 Q_2^4 Q_3^2} \qquad \hat{\Pi}_{39} = 0$$

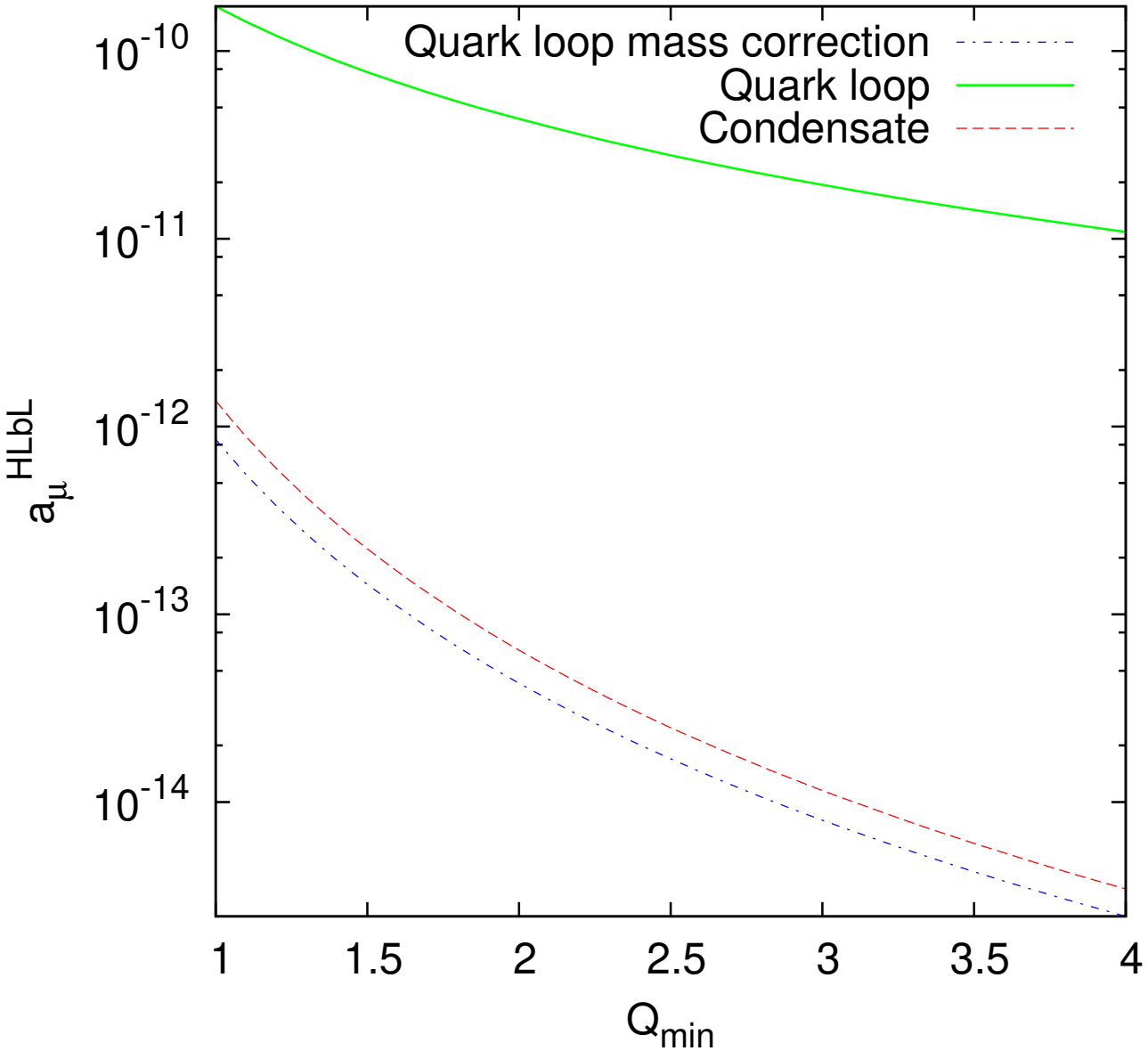
Short-distance: numerical results

- preliminary
- $Q_1, Q_2, Q_3 \geq Q_{\text{min}}$
- $m_u = m_d = m_s = 0$ for quark-loop
- $m_u = m_d = 5$ MeV and $m_s = 100$ MeV for $m_q X_q$

Q_{min}	quarkloop	$m_u X_u + m_d X_d$	$m_s X_s$
1 GeV	17.3×10^{-11}	5.40×10^{-13}	8.29×10^{-13}
2 GeV	4.35×10^{-11}	3.40×10^{-14}	5.22×10^{-14}

- Above 1 GeV still 15% of total value of HLbL
- Quarkloop goes as $1/Q_{\text{min}}^2$
  $m_q X_q$ goes as $1/Q_{\text{min}}^4$
  (dimensional analysis)
- Naive suppression is $m_q X_q / Q_{\text{min}}^2 \sim 2 \times 10^{-3}$
- Observed is roughly that

Numerical results



Short-distance: $1/Q^2_{\min}$

- Can we understand scaling with $Q_{\min}$?
- $a_\mu = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 dQ_2 Q_1^3 Q_2^3 \int_{-1}^{1} d\tau \sqrt{1-\tau^2} \sum_{i=1,12} \hat{T}_i \overline{\Pi}_i$
- Do $Q_i \to \lambda Q_i$
- overall factor goes as λ^8
- Quark loop has no scale thus $\hat{\Pi}_i$ scale with their dimension
  $\hat{\Pi}_1, \hat{\Pi}_4 \sim \lambda^{-4}, \qquad \hat{\Pi}_7, \hat{\Pi}_{17}, \hat{\Pi}_{39}, \hat{\Pi}_{54} \sim \lambda^{-6}$
- $\Longrightarrow \overline{\Pi}_{1,\ldots,4} \sim \lambda^{-4} \qquad \overline{\Pi}_{5,\ldots,12} \sim \lambda^{-6}$
- Expand the T_i for $Q_i \gg m_\mu$: $T_1 \sim m_\mu^4$, $T_{i\neq 1} \sim m_\mu^2$
  $T_1 \sim \lambda^{-8}$, $T_{2,3,4} \sim \lambda^{-6}$, $T_{5,\ldots,12} \sim \lambda^{-4}$
- Put all together: quark-loop scales as $a_\mu^{\mathrm{SD\ ql}} \sim \lambda^{-2}$
- $m_q X_q$ adds an overall factor $\Longrightarrow a_\mu^{\mathrm{SD}X_q} \sim \lambda^{-4}$

Short distance HLbL Conclusions

- We have shown that the quarkloop really is the first term of a proper OPE expansion for the HLbL
- We have calculated the next term which is suppressed by quark masses and a small X_q: negligible
- The next term contains both the usual vacuum and a large number of induced condensates but will not be suppressed by small quark masses
- Why do this: matching of the sum over hadronic contributions to the expected short distance domain
- Finding the onset of the asymptotic domain
- Still to be worked out: overlap with what is in the other parts

Putting together

- LO-HVP:
  - Paris (Davier et al.)
    $a_\mu^{LO\ HVP} = 6939 \pm 10 \pm 34 \pm 16 \pm 1_\psi \pm 7_{QCD}\ 10^{-11}$
  - Liverpool (Keshavarzi et al) $a_\mu^{LO\ HVP} = 6938(24)\ 10^{-11}$
- HO-HVP: $a_\mu^{HO\ HVP} = -85.8(4)\ 10^{-11}$
- HLbL: put all numbers together
  - π^0, η, η': $93.8(4.0)\ 10^{-11}$
  - π-loop: $-24(1)\ 10^{-11}$
  - Kaon-loop: $-0.5\ 10^{-11}$
  - HO-HLbL Colangelo et al., 2014 $3(2)\ 10^{-11}$
  - Charm loop: $a_\mu^{\mathrm{HLbLc}} = 3.1(1)\ ^{-11}$
  - Other hadronic: $6(?)\ 10^{-11}$
  - Short-distance: $10(10)\ 10^{-11}$
  - Sum: $a_\mu^{HLbL} = 91(17?)\ 10^{-11}$

Putting together

- So what is the status?
- $a_\mu^{\rm exp} - a_\mu^{QED} - a_\mu^{EW} = 7218(63)\ 10^{-11}$
- LO-HVP: $a_\mu^{LO\ HVP} = 6939(40?)\ 10^{-11}$
  Note: Babar/KLOE discrepancy
- HO-HVP: $a_\mu^{HO\ HVP} = -85.8(4)\ 10^{-11}$
- HLbL: $a_\mu^{HLbL} = 91(17?)\ 10^{-11}$
  Note: error from overlap and guessing putting together
- Sum: $6952(44?)\ 10^{-11}$
- Difference: $\boxed{\Delta a_\mu = 274(77?)\ 10^{-11}}$

Putting together

- Difference: $\boxed{\Delta a_\mu = 274(77?)\ 10^{-11}}$
- HVP: Babar/KLOE: Belle II, Novosibirsk
- Lattice HVP: not competitive now but will get there
- HLbL: improvements have happened
- Lattice HLbL: numbers compatible
- Theory: errors are slowly going down
  no large changes in numbers expected, in fact numbers quite stable since 2000
- Experiment: FNAL is running;
  first result expected late this year or early next year
- Thank you for listening