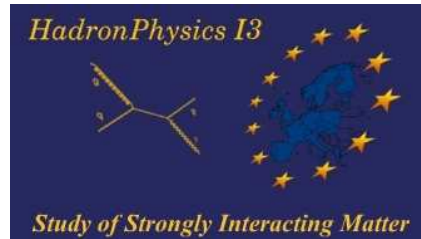




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RECENT ISSUES IN ETAS AND KAONS

Johan Bijnens

Lund University

bijnens@thep.lu.se

<http://www.thep.lu.se/~bijnens>

Overview

- Motivation: Standard Model
- Method: Chiral Perturbation Theory
- Two examples:
 - $\eta \rightarrow 3\pi: m_u - m_d$
 - $K_{\ell 3}: V_{us}$
- Conclusions

The Standard Model

The Standard Model Lagrangian has four parts:

$$\underbrace{\mathcal{L}_H(\phi)}_{\text{Higgs}} + \underbrace{\mathcal{L}_G(W, Z, G)}_{\text{Gauge}}$$

$$\underbrace{\sum_{\psi=\text{fermions}} \bar{\psi} i \not{D} \psi}_{\text{gauge-fermion}} + \underbrace{\sum_{\psi, \psi'=\text{fermions}} g_{\psi\psi'} \bar{\psi} \phi \psi'}_{\text{Yukawa}}$$

The Standard Model

What is tested ?

gauge-fermion Very well tested

Higgs Limits only, real tests coming up

Gauge Well tested,
QCD at low-energy nonperturbative

Yukawa Flavour Physics

The Standard Model

What is tested ?

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Yukawa Flavour Physics

Discrete symmetries:

- C Charge Conjugation
- P Parity
- T Time Reversal

QCD and QED conserve C,P,T separately,

Weak breaks C and P, only Yukawa breaks CP

Field theory implies CPT

Standard Model

We want to study **Low-energy QCD** as well as possible and determine couplings in the **Yukawa** sector as precisely as possible

Effective Field Theory

Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples:

Solid state physics: conductors: neglect the empty bands above the partially filled one

Atomic physics: Blue sky: neglect atomic structure

Power Counting

- ▣ gap in the spectrum \implies separation of scales
- ▣ with the lower degrees of freedom, build the most general effective Lagrangian

Power Counting

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- ▶▶▶ with the lower degrees of freedom, build the most general effective Lagrangian

- ▶▶▶ ∞ # parameters
- ▶▶▶ Where did my predictivity go ?

Power Counting

- ⇒ gap in the spectrum \implies separation of scales
 - ⇒ with the lower degrees of freedom, build the most general effective Lagrangian
 - ⇒ $\infty \#$ parameters
 - ⇒ Where did my predictivity go ?
- ⇒ Need some ordering principle: power counting

Power Counting

- ▮ gap in the spectrum \implies separation of scales
- ▮ with the lower degrees of freedom, build the most general effective Lagrangian

- ▮ $\infty \#$ parameters

- ▮ Where did my predictivity go ?

\implies Need some ordering principle: power counting

- ▮ Taylor series expansion does not work (convergence radius is zero)
- ▮ Continuum of excitation states need to be taken into account

Why Field Theory ?

- ⇒ Only known way to combine QM and special relativity
- ⇒ Off-shell effects: there as new free parameters

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- ➡ Only known way to combine QM and special relativity
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Drawbacks

- Many parameters (but finite number at any order)
any model has few parameters but model-space is large
- expansion: it might not converge or only badly

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Advantages

- Calculations are (relatively) simple
- It is general: model-independent
- Theory \implies errors can be estimated
- Systematic: ALL effects at a given order can be included
- Even if no convergence: classification of models often useful

Chiral Perturbation Theory

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But
$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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$$v = c, m_q = 0 \not\implies$$





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Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Perturbation Theory

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken \implies 8 massless degrees of freedom
and interaction vanishes at zero momentum

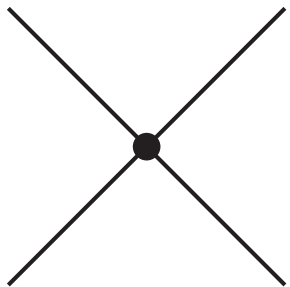
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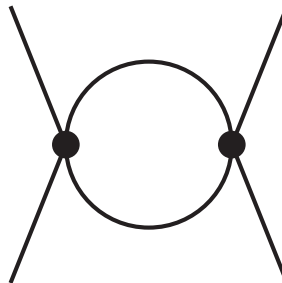
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

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Power counting in momenta:



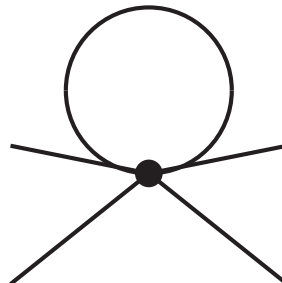
$$p^2$$



$$(p^2)^2 (1/p^2)^2 p^4 = p^4$$



$$1/p^2$$



$$(p^2) (1/p^2) p^4 = p^4$$

$$\int d^4 p$$

$$p^4$$

Chiral Perturbation Theory

Perturbation Theory with mesons, expansion in momenta

The physics of pions, kaons and eta

Actually useful:

- tree level: sixties
- one-loop: eighties
- two-loop now (and especially in Lund 😊)
- Similar techniques: technicolour, little Higgs, . . .

Remember: Eta Physics Handbook

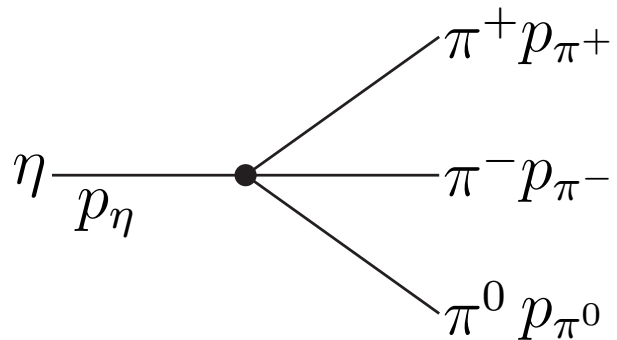
Physica Scripta, Vol. T99, 2002

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$\eta \rightarrow 3\pi$ beyond p^4 : Basic

Review: JB, Gasser, Phys.Scripta T99(2002)34 [hep-ph/0202242]


$$\begin{aligned} s &= (p_{\pi^+} + p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2 \\ t &= (p_{\pi^-} + p_{\pi^0})^2 = (p_\eta - p_{\pi^+})^2 \\ u &= (p_{\pi^+} + p_{\pi^0})^2 = (p_\eta - p_{\pi^-})^2 \end{aligned}$$

$$s + t + u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0.$$

$$\langle \pi^0 \pi^+ \pi^- \text{out} | \eta \rangle = i (2\pi)^4 \delta^4 (p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}) A(s, t, u).$$

$$\langle \pi^0 \pi^0 \pi^0 \text{out} | \eta \rangle = i (2\pi)^4 \delta^4 (p_\eta - p_1 - p_2 - p_3) \bar{A}(s_1, s_2, s_3)$$

$$\bar{A}(s_1, s_2, s_3) = A(s_1, s_2, s_3) + A(s_2, s_3, s_1) + A(s_3, s_1, s_2),$$

$\eta \rightarrow 3\pi$ beyond p^4 : Lowest order

Pions are in $I = 1$ state $\implies A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \rightarrow \pi^+ \pi^- \pi^0 \gamma$ needs to be included directly

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ChPT:Cronin 67:
$$A(s, t, u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2} \left\{ 1 + \frac{3(s - s_0)}{m_\eta^2 - m_\pi^2} \right\}$$

$\eta \rightarrow 3\pi$ beyond p^4 : Lowest order

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$$\text{ChPT:Cronin 67: } A(s, t, u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2} \left\{ 1 + \frac{3(s - s_0)}{m_\eta^2 - m_\pi^2} \right\}$$

$$\text{or with } Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}, \quad \hat{m} = \frac{1}{2}(m_u + m_d)$$

$$A(s, t, u) = \frac{1}{Q^2} \frac{m_K^2}{m_\pi^2} (m_\pi^2 - m_K^2) \frac{M(s, t, u)}{3\sqrt{3}F_\pi^2},$$

$$\text{with at lowest order } M(s, t, u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}.$$

$\eta \rightarrow 3\pi$ beyond p^4 : p^2 and p^4

$\Gamma(\eta \rightarrow 3\pi) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

$Q \approx 24$ gives lowest order $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 66 \text{ eV}$.

Other Source from $m_{K^+}^2 - m_{K^0}^2 \sim Q^{-2} \implies Q = 20.0 \pm 1.5$

Lowest order prediction $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 140 \text{ eV}$.

$\eta \rightarrow 3\pi$ beyond p^4 : p^2 and p^4

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Lowest order prediction $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 140$ eV.

$$\text{At order } p^4: \frac{\int dLIPS |A_2 + A_4|^2}{\int dLIPS |A_2|^2} = 2.4,$$

(LIPS=Lorentz invariant phase-space)

Major source: large S -wave final state rescattering

Experiment: $\Gamma(\eta \rightarrow 3\pi) = 295 \pm 17$ eV.

$\eta \rightarrow 3\pi$ beyond p^4 : Dispersive

Try to resum the S -wave rescattering:

Anisovich-Leutwyler (AL), Kambor, Wiesendanger, Wyler (KWW)

Different method but similar approximations

Here: simplified version of AL

Up to p^8 : No absorptive parts from $\ell \geq 2$

$\implies M(s, t, u) =$

$$M_0(s) + (s - u)M_1(t) + (s - t)M_1(t) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

M_I : “roughly” contributions with isospin 0, 1, 2

$\eta \rightarrow 3\pi$ beyond p^4 : Dispersive

3 body dispersive: difficult: keep only 2 body cuts

start from $\pi\eta \rightarrow \pi\pi$ ($m_\eta^2 < 3m_\pi^2$) standard dispersive analysis

analytically continue to physical m_η^2 .

$$M_I(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}M_I(s')}{s' - s - i\varepsilon}$$

$$\text{Im}M_I(s') \longrightarrow \text{disc}M_I(s) = \frac{1}{2i} (M_I(s + i\varepsilon) - M_I(s - i\varepsilon))$$

$$M_0(s) = a_0 + b_0s + c_0s^2 + \frac{s^3}{\pi} \int \frac{ds'}{s'^3} \frac{\text{disc}M_0(s')}{s' - s - i\varepsilon},$$

$$M_1(s) = a_1 + b_1s + \frac{s^2}{\pi} \int \frac{ds'}{s'^2} \frac{\text{disc}M_1(s')}{s' - s - i\varepsilon},$$

$$M_2(s) = a_2 + b_2s + c_2s^2 + \frac{s^3}{\pi} \int \frac{ds'}{s'^3} \frac{\text{disc}M_2(s')}{s' - s - i\varepsilon}.$$

$\eta \rightarrow 3\pi$ beyond p^4

AL: Lowest order is $M(s, t, u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$

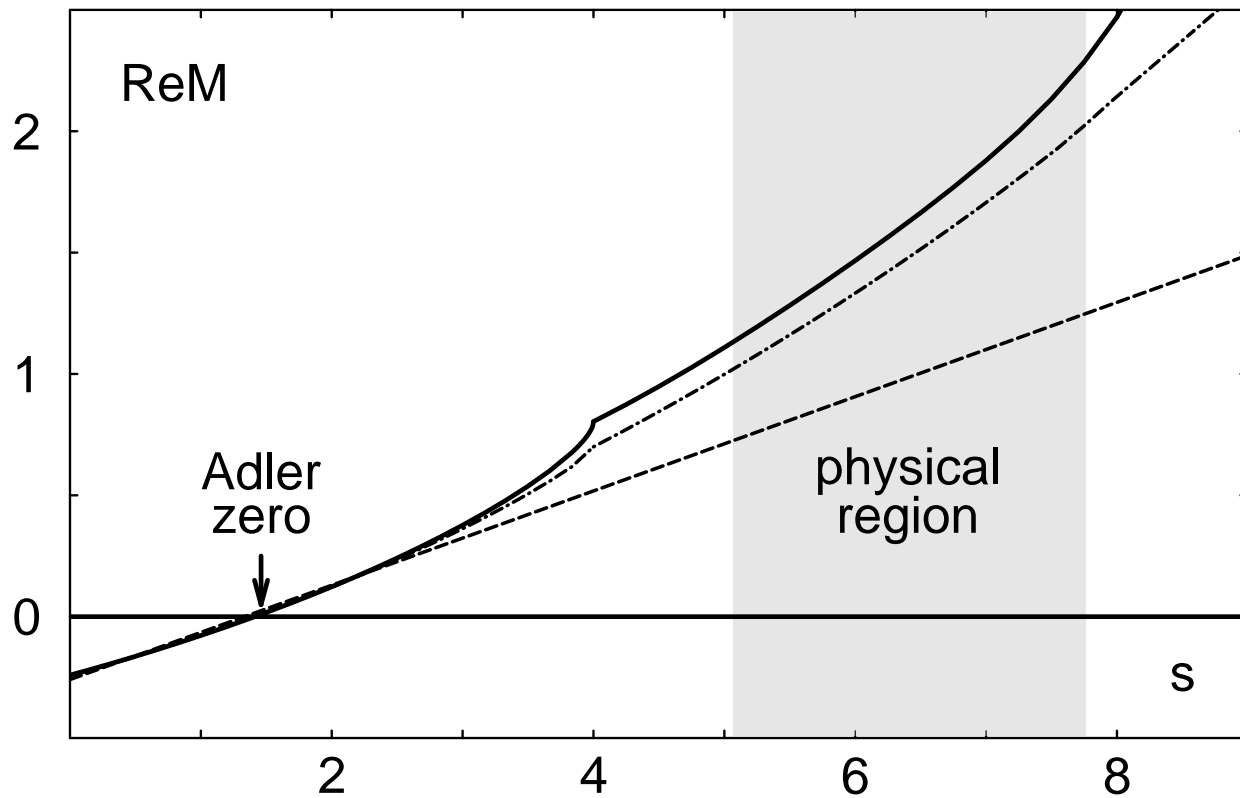
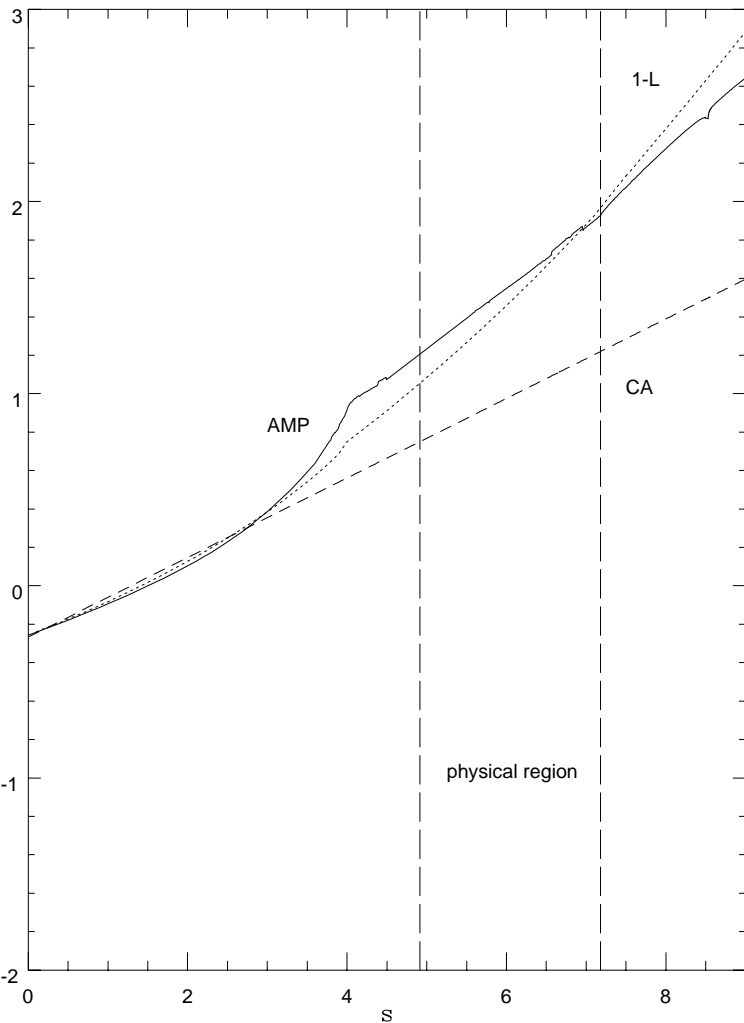
zero at $s_4/3 m_\pi^2$: remains in the neighbourhood:
match position of s_A and slope of Adler zero.

KWW: fix amplitude at some place(s) in s, t, u plane to be equal to p^4

Both find moderate increases over p^4 : $\sim 15\%$ in amplitude

Dalitzplot distributions provide a check

$\eta \rightarrow 3\pi$ beyond p^4



Two Loop Calculation: why

- In $K_{\ell 4}$ dispersive gave about half of p^6 in amplitude
- Same order in ChPT as masses for consistency check on m_u/m_d
- Check size of 3 pion dispersive part
- Technology exists:
 - Two-loops: Amorós, JB, Dhonte, Talavera, . . .
 - Dealing with the mixing π^0 - η :
Amorós, JB, Dhonte, Talavera 01

Status

$$A(s, t, u) = \sin \epsilon M(s, t, u) \quad \epsilon \approx \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \hat{m}}$$

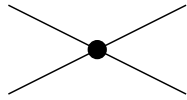
$$M(s, t, u) =$$

$$M_0(s) + (s - u)M_1(t) + (s - t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

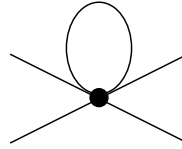
$$M^{(2)} = \frac{1}{F_\pi^2} \left(\frac{4}{3}m_\pi^2 - s \right)$$

	JB	Ghorbani	Talavera
p^4	All agree and with GL		
$p^6 \infty$	All cancel		
p^6 Vertex	Checked and agree	in progress	
p^6 Rest	Comparison in progress		
p^6 Numerics	Very preliminary		Very preliminary

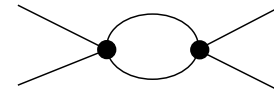
Diagrams



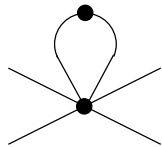
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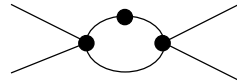
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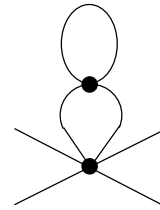
(c)



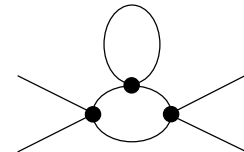
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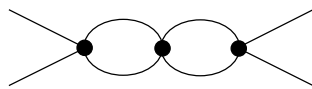
(e)



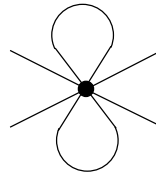
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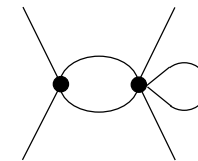
(g)



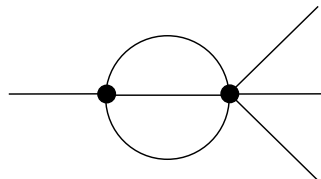
(h)



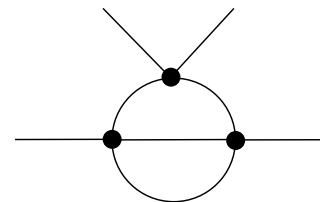
(i)



(j)

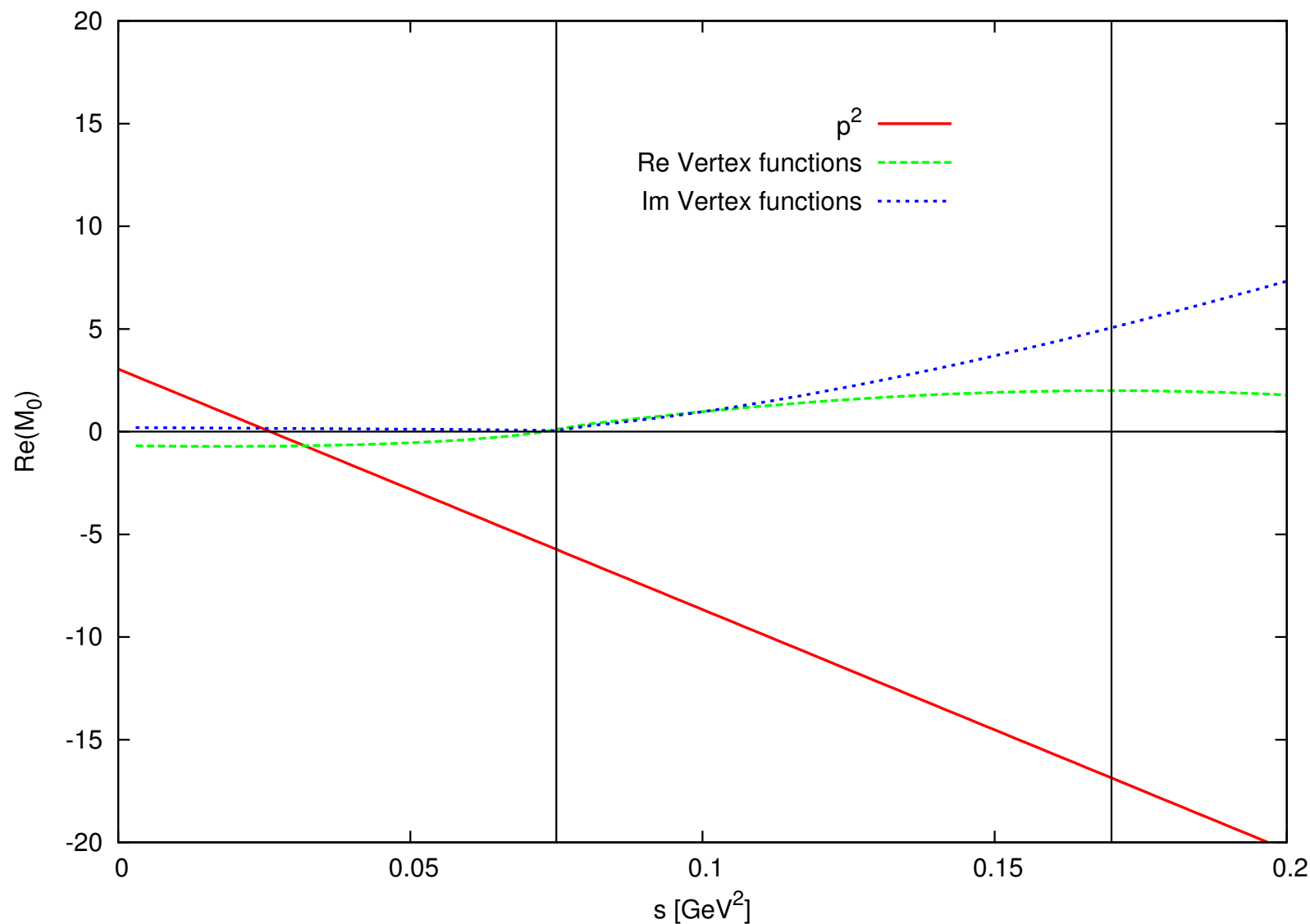


(h)

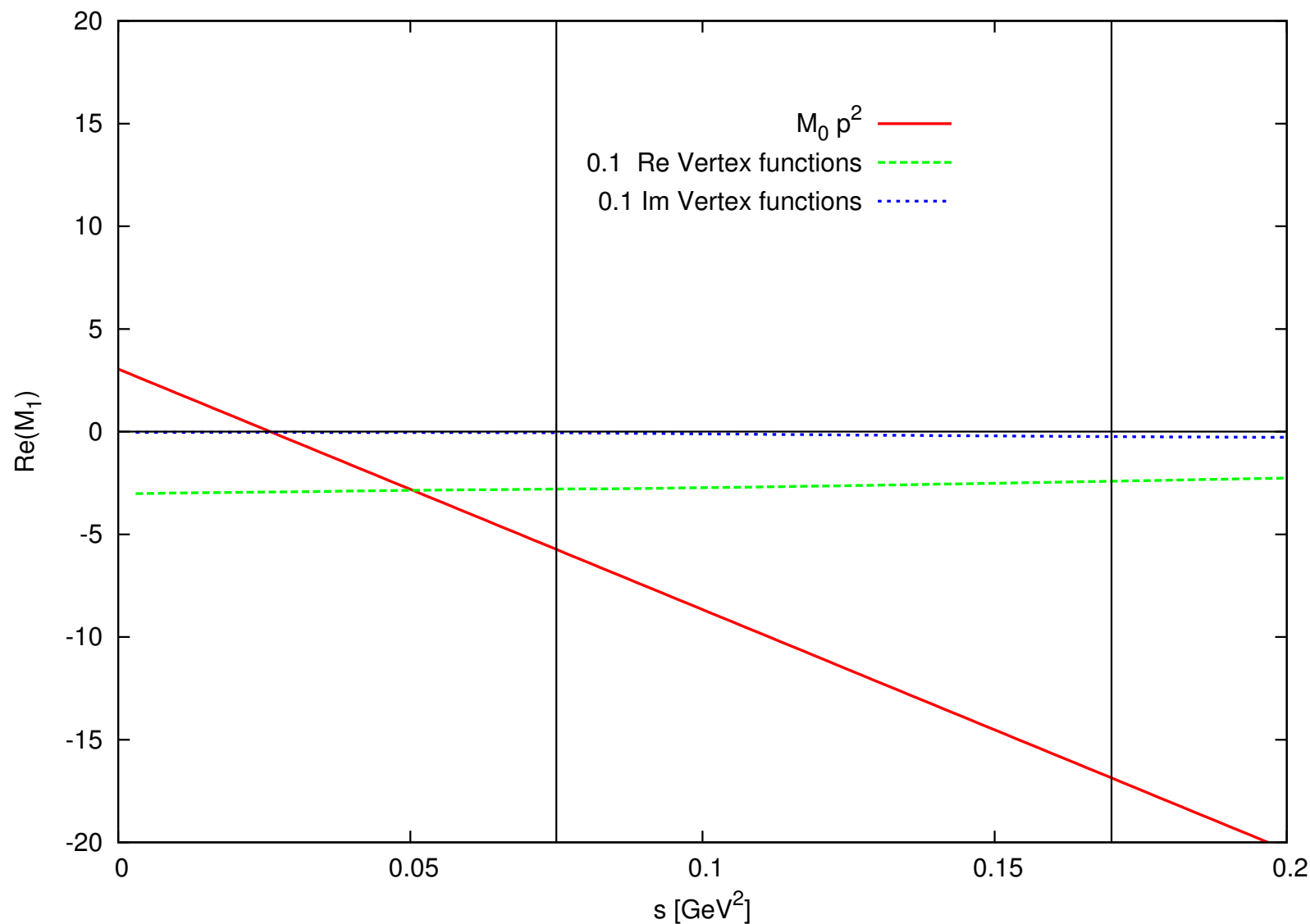


(i)

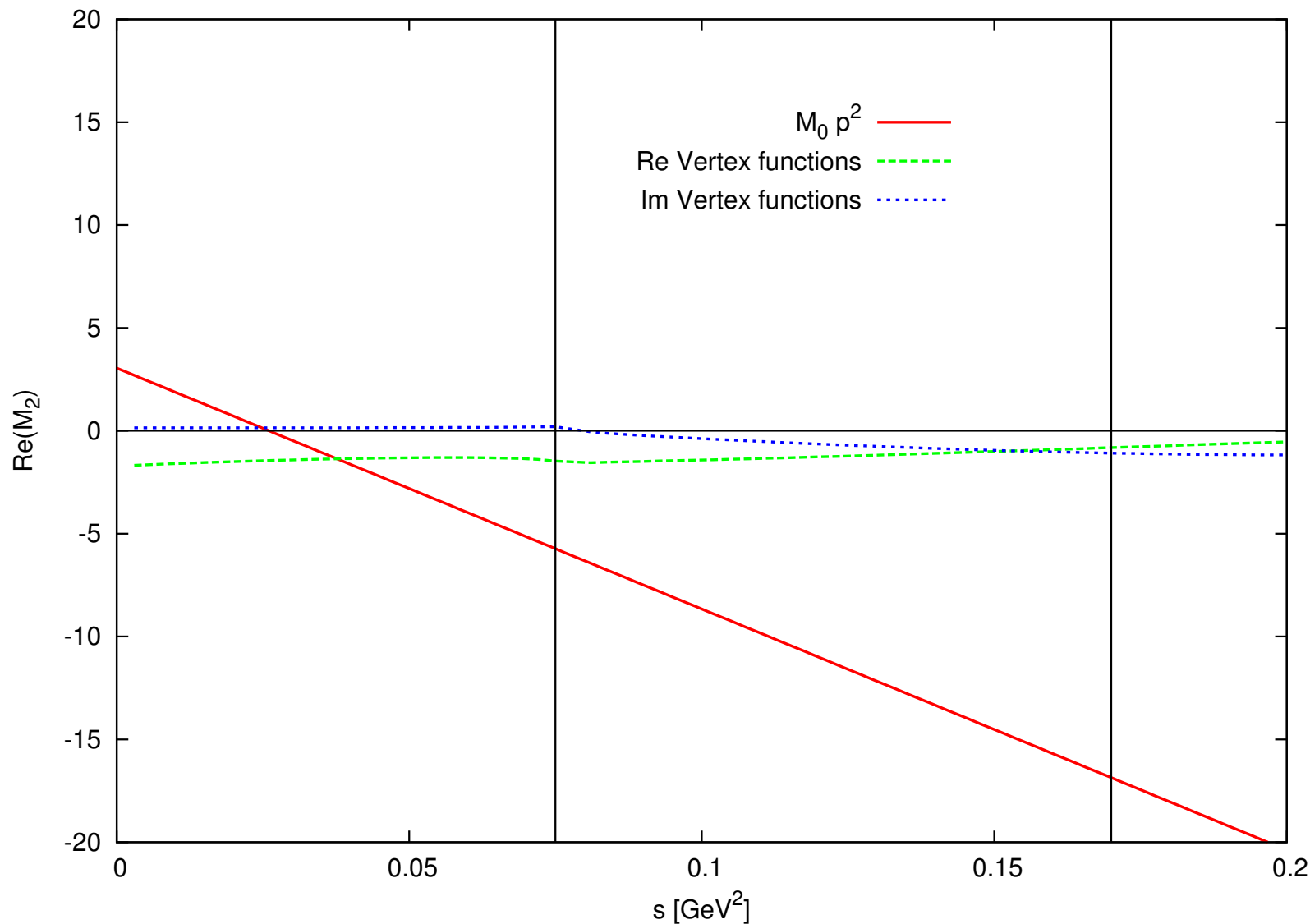
M_0 : Vertex function contribution



M_1 : Vertex function contribution



M_2 : Vertex function contribution



$K_{\ell 3}$ Definitions

$$K_{\ell 3}^+ : \quad K^+(p) \rightarrow \pi^0(p') \ell^+(p_\ell) \nu_\ell(p_\nu)$$

$$K_{\ell 3}^0 : \quad K^0(p) \rightarrow \pi^-(p') \ell^+(p_\ell) \nu_\ell(p_\nu)$$

$$K_{\ell 3}^+ : \quad T = \frac{G_F}{\sqrt{2}} V_{us}^* \ell^\mu F_\mu^+(p', p)$$

$$\ell^\mu = \bar{u}(p_\nu) \gamma^\mu (1 - \gamma_5) v(p_\ell)$$

$$F_\mu^+(p', p) = \langle \pi^0(p') | V_\mu^{4-i5}(0) | K^+(p) \rangle$$

$$= \frac{1}{\sqrt{2}} [(p' + p)_\mu f_+^{K^+ \pi^0}(t) + (p - p')_\mu f_-^{K^+ \pi^0}(t)]$$

Isospin: $f_+^{K^0 \pi^-}(t) = f_+^{K^+ \pi^0}(t) = f_+(t)$

$$f_-^{K^0 \pi^-}(t) = f_-^{K^+ \pi^0}(t) = f_-(t)$$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor:
$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization:
$$f_{+,0}(t) = f_+(0) \left(1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right)$$

- $|V_{us}|$:
- Know theoretically $f_+(0) = 1 + \dots$
 - Short distance correction to G_F from G_μ
Marciano-Sirlin
 - Ademollo-Gatto-Behrends-Sirlin theorem:
 $(m_s - \hat{m})^2$
 - Isospin Breaking Leuwylers-Roos
 - Know experimentally $f_+(0)$

V_{us}

PDG2002:

$$|V_{ud}| = 0.9734 \pm 0.0008 \quad |V_{us}| = 0.2196 \pm 0.0026$$
$$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) =$$
$$0.9957 \pm 0.0019$$

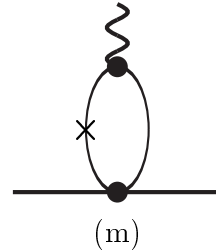
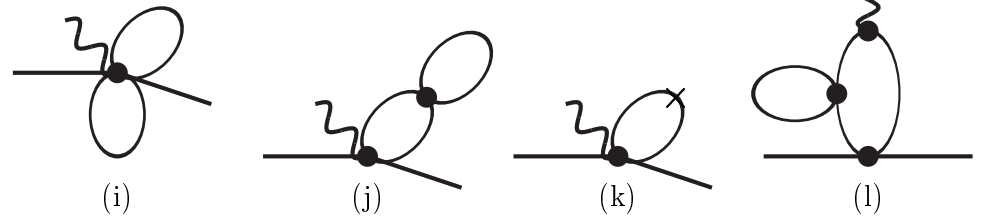
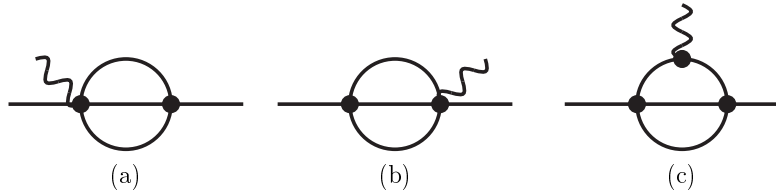
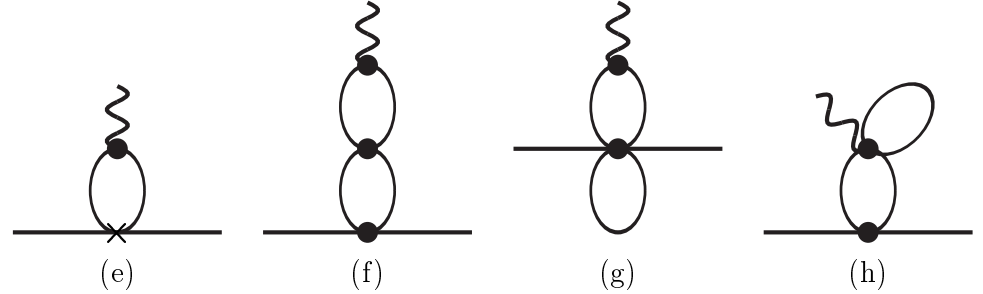
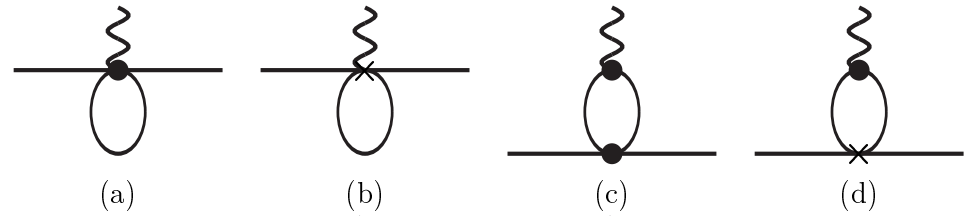
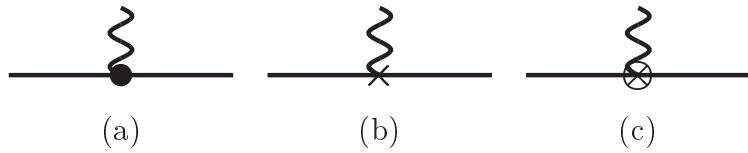
PDG2006:

$$|V_{ud}| = 0.97377 \pm 0.00027 \quad |V_{us}| = 0.2257 \pm 0.0021$$
$$|V_{ud}|^2 + |V_{us}|^2 = (0.94823 \pm 0.00054) + (0.05094 \pm 0.00095) =$$
$$0.99917 \pm 0.00110$$

Problems:

- Ignores $\Delta(0) = 0.0113$ from pure two-loop
- Conflicts between experiments

$K_{\ell 3}$ Diagrams



● : p^2 vertex
 × : p^4 vertex
 ⊗ : p^6 vertex

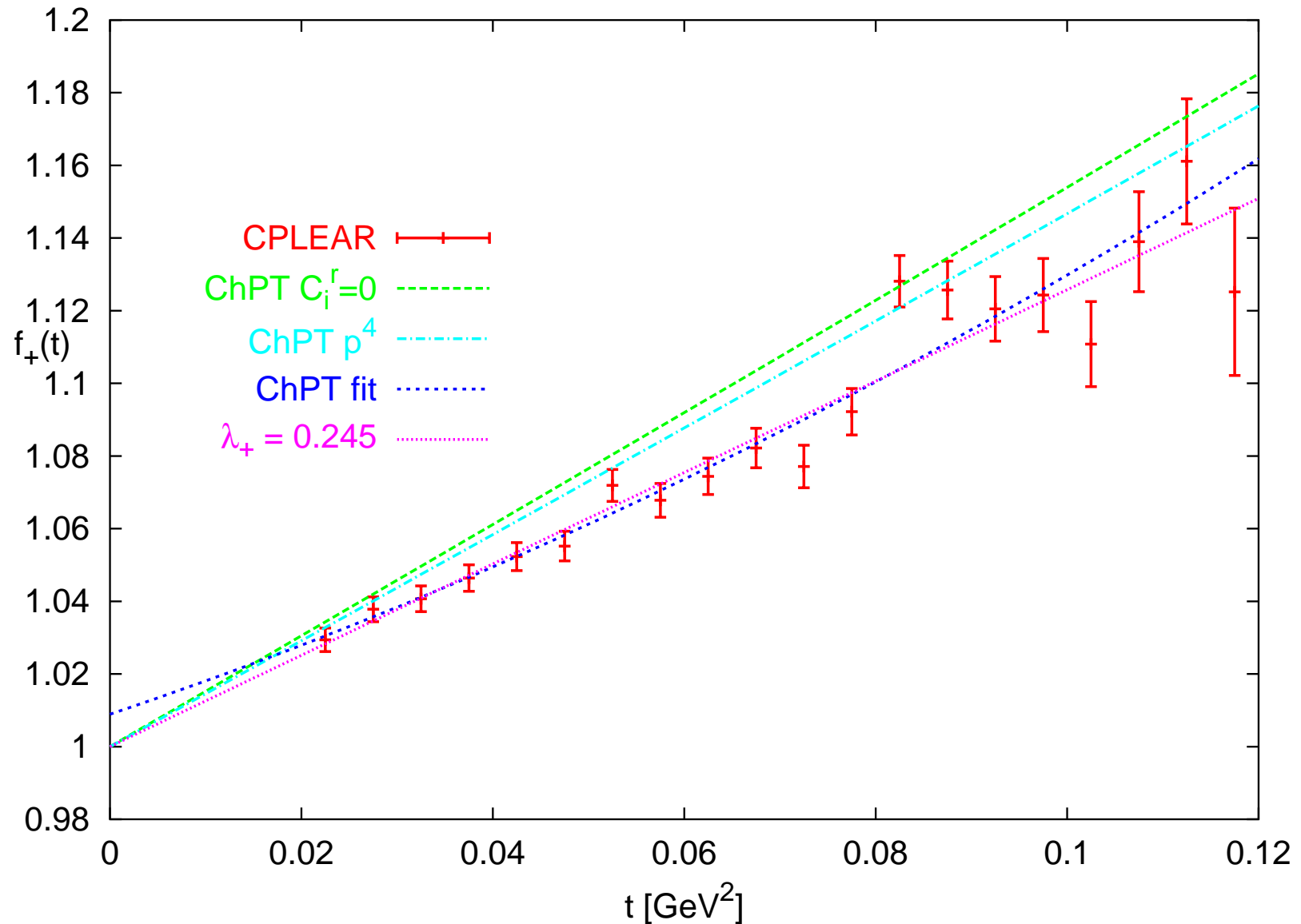
$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} \\ + \frac{t^2}{F_\pi^4} (-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

ChPT fit to $f_+(t)$



$f_0(t)$

Main Result:

$$f_0(t) = 1 - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 \\ + 8 \frac{t}{F_\pi^4} (2C_{12}^r + C_{34}^r) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) \\ - \frac{8}{F_\pi^4} t^2 C_{12}^r + \bar{\Delta}(t) + \Delta(0).$$

$\bar{\Delta}(t)$ and $\Delta(0)$ contain **NO** C_i^r and only depend on the L_i^r at order p^6

\implies

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r].$$

Experiment

Form Factor comparison

KTeV [PRD 70(2004)]

K_{e3}^0 quadratic fit: $\lambda''_+ \neq 0$ @ 4σ level

$K_{\mu 3}^0$ quadratic fit: $\lambda_0 = (13.72 \pm 1.31) 10^{-3}$

Slopes consistent for K_{e3}^0 and $K_{\mu 3}^0$

ISTRA+ [PLB 581(2004), PLB589(2004)]

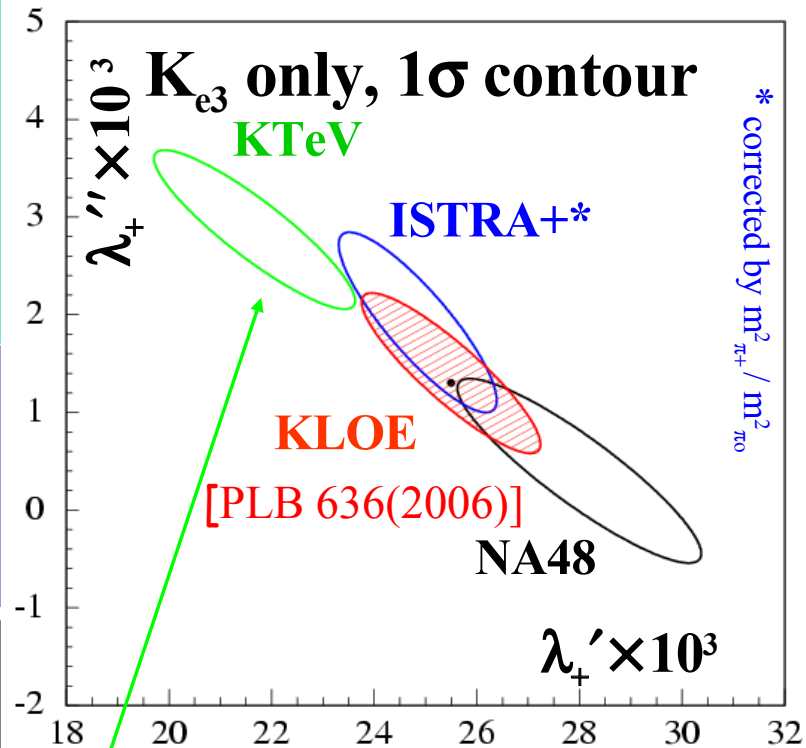
K_{e3}^- quadratic fit: $\lambda''_+ \neq 0$ @ 2σ level

$K_{\mu 3}^-$ quadratic fit: $\lambda_0 = (17.11 \pm 2.31) 10^{-3}$

NA48 [PLB 604(2004), HEP2005 289]

K_{e3}^0 : No evidence for quadratic term

$K_{\mu 3}^0$ linear fit: $\lambda_0 = (12.0 \pm 1.7) 10^{-3}$



$\lambda''_+ \rightarrow -1\%$ phase space integral

$\rightarrow +0.5\%$ for V_{us}

Experiment

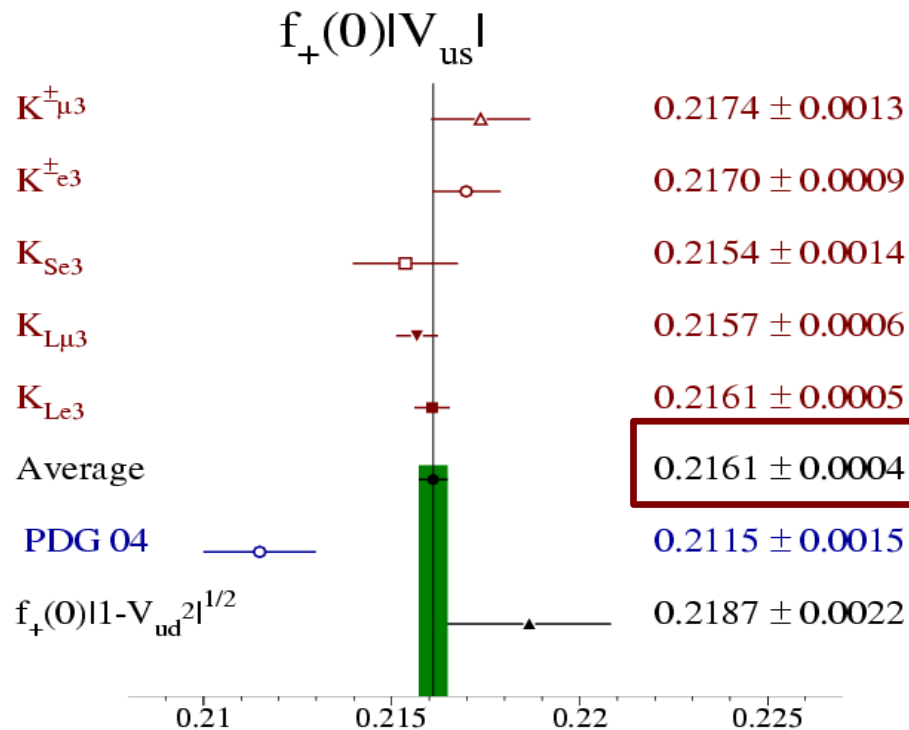
V_{us} determination from $Kl3$

	$K_L e3$	$K_L \mu3$	$K_S e3$	$K^\pm e3$	$K^\pm \mu3$
BR	0.4046(8)	0.2697(7)	$7.046(91) \times 10^{-4}$	0.05043(31)	0.03383(27)
τ	51.10(19) ns		89.58(6) ps	12.386(22) ns	

$$\lambda'_+ = 0.02496(79)$$

$$\lambda''_+ = 0.0016(3)$$

$$\lambda_0 = 0.01587(95)$$



- $f_+(0) = 0.961(8)$

Leutwyler and Roos Z.
[Phys. C25, 91, 1984]

- $V_{ud} = 0.97377(27)$

Marciano and Sirlin
[Phys.Rev.Lett.96 032002,2006]

$$V_{us} = 0.2249 \pm 0.0019$$

From $R_{e\mu}$:

$$\frac{g_e}{g_\mu} = 1.0014 \pm 0.0037$$

Experiment

$V_{us} - V_{ud}$ plane

Combining the experimental value of $\Gamma(\text{K} \rightarrow \mu\nu(\gamma))/\Gamma(\pi \rightarrow \mu\nu(\gamma))$

with the ratio f_{K}/f_{π} obtained from lattice calculations we can extract $|V_{us}|/|V_{ud}|$

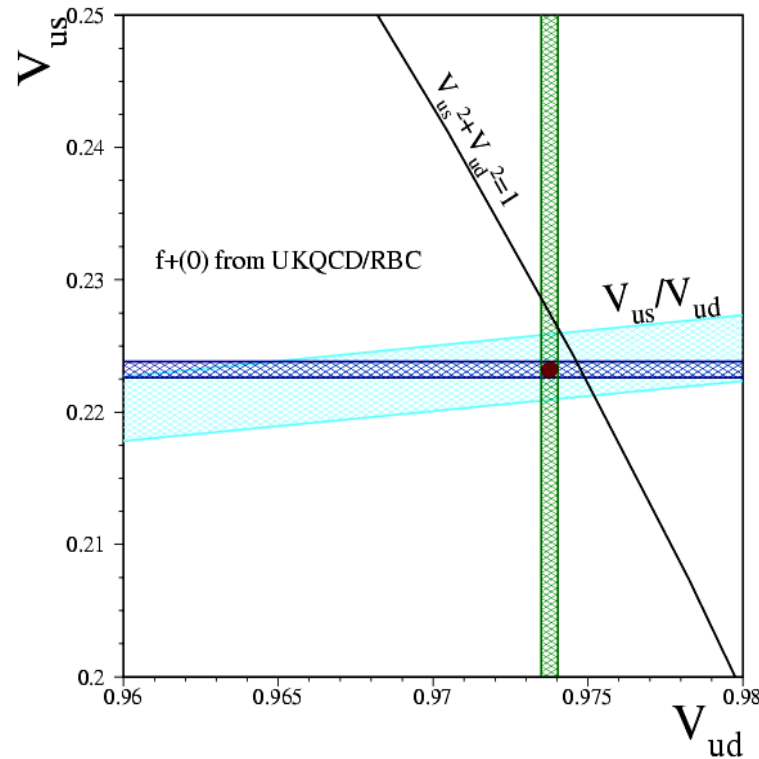
(Marciano hep-ph/0406324) $\Gamma(\text{K} \rightarrow \mu\nu(\gamma))/\Gamma(\pi \rightarrow \mu\nu(\gamma)) \propto |V_{us}|^2/|V_{ud}|^2 f_{\text{K}}^2/f_{\pi}^2$

Using $f_{\text{K}}/f_{\pi} = 1.198(3)^{(+16_{-5})}$ from MILC
and KLOE $\text{BR}(\text{K}^+ \rightarrow \mu^+\nu)$

we get $V_{us}/V_{ud} = 0.2294 \pm 0.0026$

$\delta = |V_{us}|^2 + |V_{ud}|^2 - 1 = -0.0020 \pm 0.0006$

NEED TO BE CONFIRMED



Conclusions

- Chiral Perturbation Theory doing fine
- $\eta \rightarrow 3\pi$: All three have expressions: do we agree?
- $K_{\ell 3}$: useful even with very many parameters
- Other recent work: connection with lattice QCD:
Partially quenched ChPT
 - PQChPT at two loops: basic calculations done
 - Photons now also included (but NLO)
- For formulas:
<http://www.thep.lu.se/~bijmens/chpt.html>
- Many more calculations have been done