RECENT ISSUES IN ETAS AND KAONS

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Overview

- Motivation: Standard Model
- Method: Chiral Perturbation Theory
- Two examples:
  - $\eta \to 3\pi$: $m_u - m_d$
  - $K_{\ell 3}$: V_{us}
- Conclusions

The Standard Model

The Standard Model Lagrangian has four parts:

$$\underbrace{\mathcal{L}_H(\phi)}_{\text{Higgs}} + \underbrace{\mathcal{L}_G(W,Z,G)}_{\text{Gauge}}$$

$$\underbrace{\sum_{\psi=\text{fermions}} \bar\psi i \not{D}\, \psi}_{\text{gauge-fermion}} + \underbrace{\sum_{\psi,\psi'=\text{fermions}} g_{\psi\psi'} \bar\psi \phi \psi'}_{\text{Yukawa}}$$

The Standard Model

What is tested ?

gauge-fermion	Very well tested
Higgs	Limits only, real tests coming up
Gauge	Well tested,
	QCD at low-energy nonperturbative
Yukawa	Flavour Physics

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Discrete symmetries:

- C Charge Conjugation
- P Parity
- T Time Reversal

QCD and QED conserve C,P,T separately,
Weak breaks C and P, only Yukawa breaks CP
Field theory implies CPT

Standard Model

We want to study Low-energy QCD as well as possible and determine couplings in the Yukawa sector as precisely as possible

Effective Field Theory

Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples:
- Solid state physics: conductors: neglect the empty bands above the partially filled one
- Atomic physics: Blue sky: neglect atomic structure

Power Counting

- gap in the spectrum $\Longrightarrow$ separation of scales
- with the lower degrees of freedom, build the most general effective Lagrangian

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$\Longrightarrow$ Need some ordering principle: power counting

- Taylor series expansion does not work (convergence radius is zero)
- Continuum of excitation states need to be taken into account

Why Field Theory ?

- Only known way to combine QM and special relativity
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Drawbacks

- Many parameters (but finite number at any order)
  any model has few parameters but model-space is large
- expansion: it might not converge or only badly

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Advantages

- Calculations are (relatively) simple
- It is general: model-independent
- Theory $\Longrightarrow$ errors can be estimated
- Systematic: ALL effects at a given order can be included
- Even if no convergence: classification of models often useful

Chiral Perturbation Theory

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$

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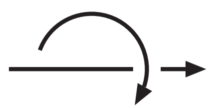
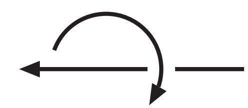


$v < c,\ m_q \neq 0 \Longrightarrow$

$v = c,\ m_q = 0 \not\Longrightarrow$



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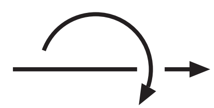

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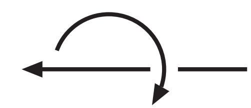

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

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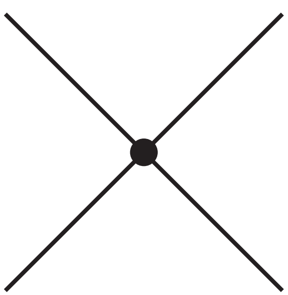
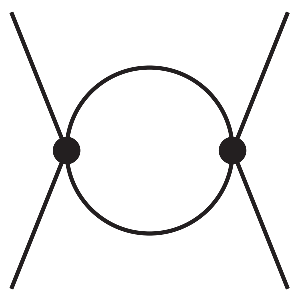
Power counting in momenta:



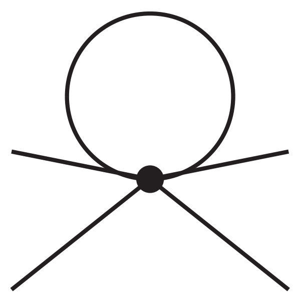
p^2



$(p^2)^2 \, (1/p^2)^2 \, p^4 = p^4$

$1/p^2$

$\int d^4p$ $\quad p^4$



$(p^2) \, (1/p^2) \, p^4 = p^4$

Chiral Perturbation Theory

Perturbation Theory with mesons, expansion in momenta

The physics of pions, kaons and eta

Actually useful:

- tree level: sixties
- one-loop: eighties
- two-loop now (and especially in Lund ☺)
- Similar techniques: technicolour, little Higgs,. . .

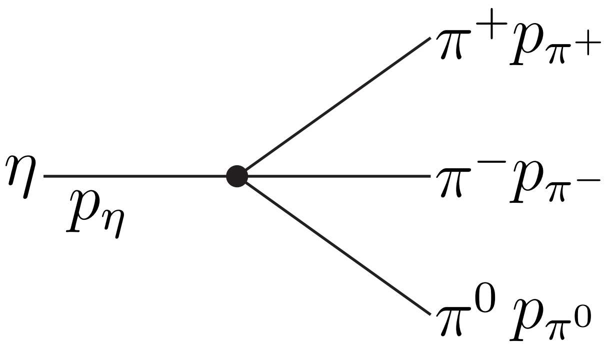
Remember: Eta Physics Handbook

Physica Scripta, Vol. T99, 2002

Contents

$\eta \to 3\pi$ beyond p^4: Basic

Review: JB, Gasser, Phys.Scripta T99(2002)34 [hep-ph/0202242]



$$
\begin{aligned}
s &= (p_{\pi^+}+p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2 \\
t &= (p_{\pi^-}+p_{\pi^0})^2 = (p_\eta - p_{\pi^+})^2 \\
u &= (p_{\pi^+}+p_{\pi^0})^2 = (p_\eta - p_{\pi^-})^2
\end{aligned}
$$

$$s+t+u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0\,.$$

$$\langle \pi^0\pi^+\pi^-\text{out}|\eta\rangle = i\,(2\pi)^4\;\delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right)\;A(s,t,u)\,.$$

$$\langle \pi^0\pi^0\pi^0\text{out}|\eta\rangle = i\,(2\pi)^4\;\delta^4\left(p_\eta - p_1 - p_2 - p_3\right)\;\overline{A}(s_1,s_2,s_3)$$

$$\overline{A}(s_1,s_2,s_3) = A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2)\,,$$

$\eta \to 3\pi$ beyond p^4: Lowest order

Pions are in $I = 1$ state $\Longrightarrow A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \to \pi^+\pi^-\pi^0\gamma$ needs to be included directly

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ChPT: Cronin 67: $A(s,t,u) = \dfrac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \dfrac{3(s - s_0)}{m_\eta^2 - m_\pi^2}\right\}$

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or with $Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$, $\qquad \hat{m} = \frac{1}{2}(m_u + m_d)$

$$A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\frac{M(s,t,u)}{3\sqrt{3}F_\pi^2},$$

with at lowest order $M(s,t,u) = \dfrac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$.

$\eta \to 3\pi$ beyond p^4: p^2 and p^4

$\Gamma\left(\eta \to 3\pi\right) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

$\boxed{Q \approx 24}$ gives lowest order $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 66$ eV .

Other Source from $m^2_{K^+} - m^2_{K^0} \sim Q^{-2} \Longrightarrow \boxed{Q = 20.0 \pm 1.5}$

Lowest order prediction $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 140$ eV .

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At order p^4: $$\frac{\int dLIPS|A_2 + A_4|^2}{\int dLIPS|A_2|^2} = 2.4\,,$$

($LIPS$=Lorentz invariant phase-space)

Major source: large S-wave final state rescattering

Experiment: $\Gamma(\eta \to 3\pi) = 295 \pm 17$ eV.

$\eta \to 3\pi$ beyond p^4: Dispersive

Try to resum the S-wave rescattering:

Anisovich-Leutwyler (AL), Kambor,Wiesendanger,Wyler (KWW)

Different method but similar approximations

Here: simplified version of AL

Up to p^8: No absorptive parts from $\ell \geq 2$

$\Longrightarrow M(s,t,u) =$

$$M_0(s) + (s-u)M_1(t) + (s-t)M_1(t) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

M_I: “roughly” contributions with isospin 0,1,2

$\eta \to 3\pi$ beyond p^4: Dispersive

3 body dispersive: difficult: keep only 2 body cuts

start from $\pi\eta \to \pi\pi$ ($m_\eta^2 < 3m_\pi^2$) standard dispersive analysis

analytically continue to physical m_η^2.

$$M_I(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \, \frac{\mathrm{Im} M_I(s')}{s' - s - i\varepsilon}$$

$$\mathrm{Im} M_I(s') \longrightarrow \mathrm{disc} M_I(s) = \frac{1}{2i}\left(M_I(s+i\varepsilon) - M_I(s-i\varepsilon)\right)$$

$$\begin{aligned}
M_0(s) &= a_0 + b_0 s + c_0 s^2 + \frac{s^3}{\pi}\int \frac{ds'}{s'^3}\,\frac{\mathrm{disc} M_0(s')}{s' - s - i\varepsilon}\,, \\
M_1(s) &= a_1 + b_1 s + \frac{s^2}{\pi}\int \frac{ds'}{s'^2}\,\frac{\mathrm{disc} M_1(s')}{s' - s - i\varepsilon}\,, \\
M_2(s) &= a_2 + b_2 s + c_2 s^2 + \frac{s^3}{\pi}\int \frac{ds'}{s'^3}\,\frac{\mathrm{disc} M_2(s')}{s' - s - i\varepsilon}\,.
\end{aligned}$$

$\eta \rightarrow 3\pi$ beyond p^4

AL: Lowest order is $M(s,t,u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$

zero at $s_4/3\,m_\pi^2$: remains in the neighbourhood:

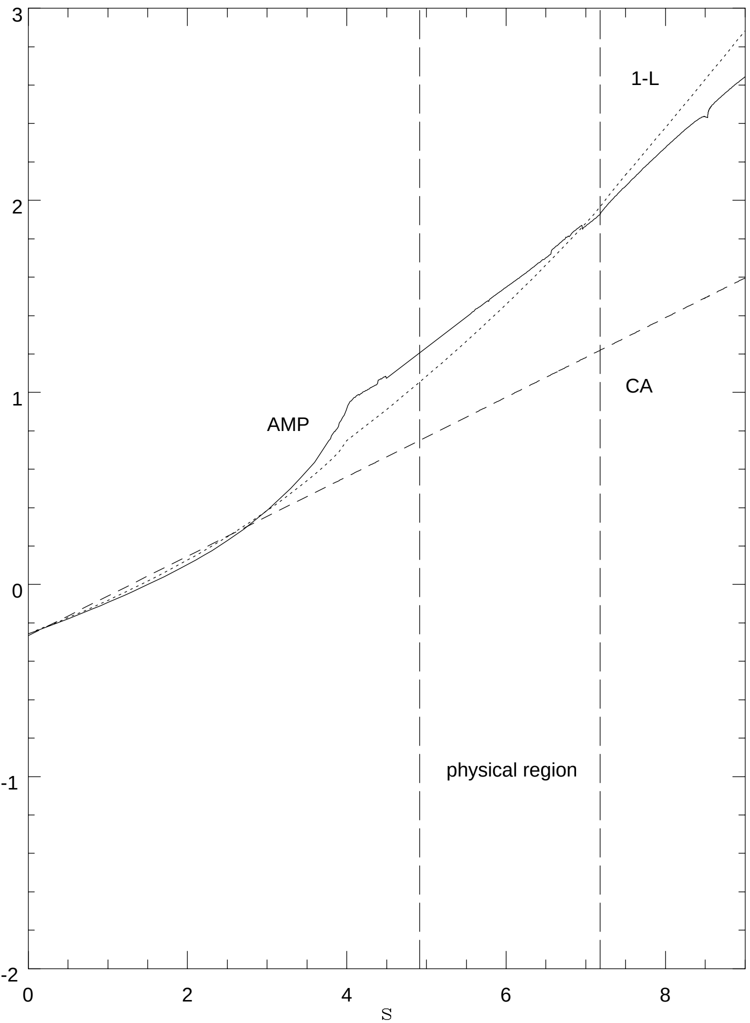
match position of s_A and slope of Adler zero.

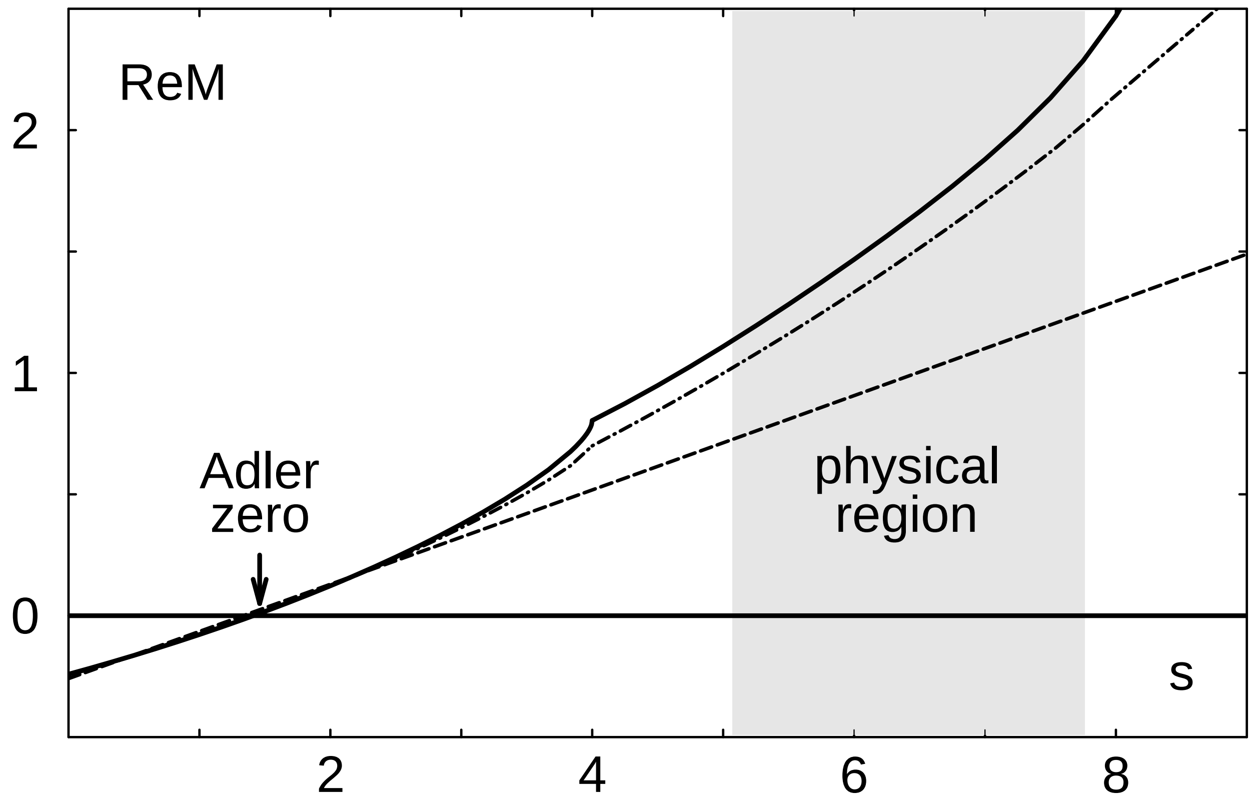
KWW: fix amplitude at some place(s) in s,t,u plane to be equal to p^4

Both find moderate increases over p^4: $\sim 15\%$ in amplitude

Dalitzplot distributions provide a check

$\eta \to 3\pi$ beyond p^4





Two Loop Calculation: why

- In $K_{\ell 4}$ dispersive gave about half of p^6 in amplitude
- Same order in ChPT as masses for consistency check on m_u/m_d
- Check size of 3 pion dispersive part
- Technology exists:
  - Two-loops: Amorós,JB,Dhonte,Talavera,...
  - Dealing with the mixing π^0-η: Amorós,JB,Dhonte,Talavera 01

Status

$A(s,t,u) = \sin\epsilon \; M(s,t,u)$ $\qquad \epsilon \approx \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s - \hat{m}}$

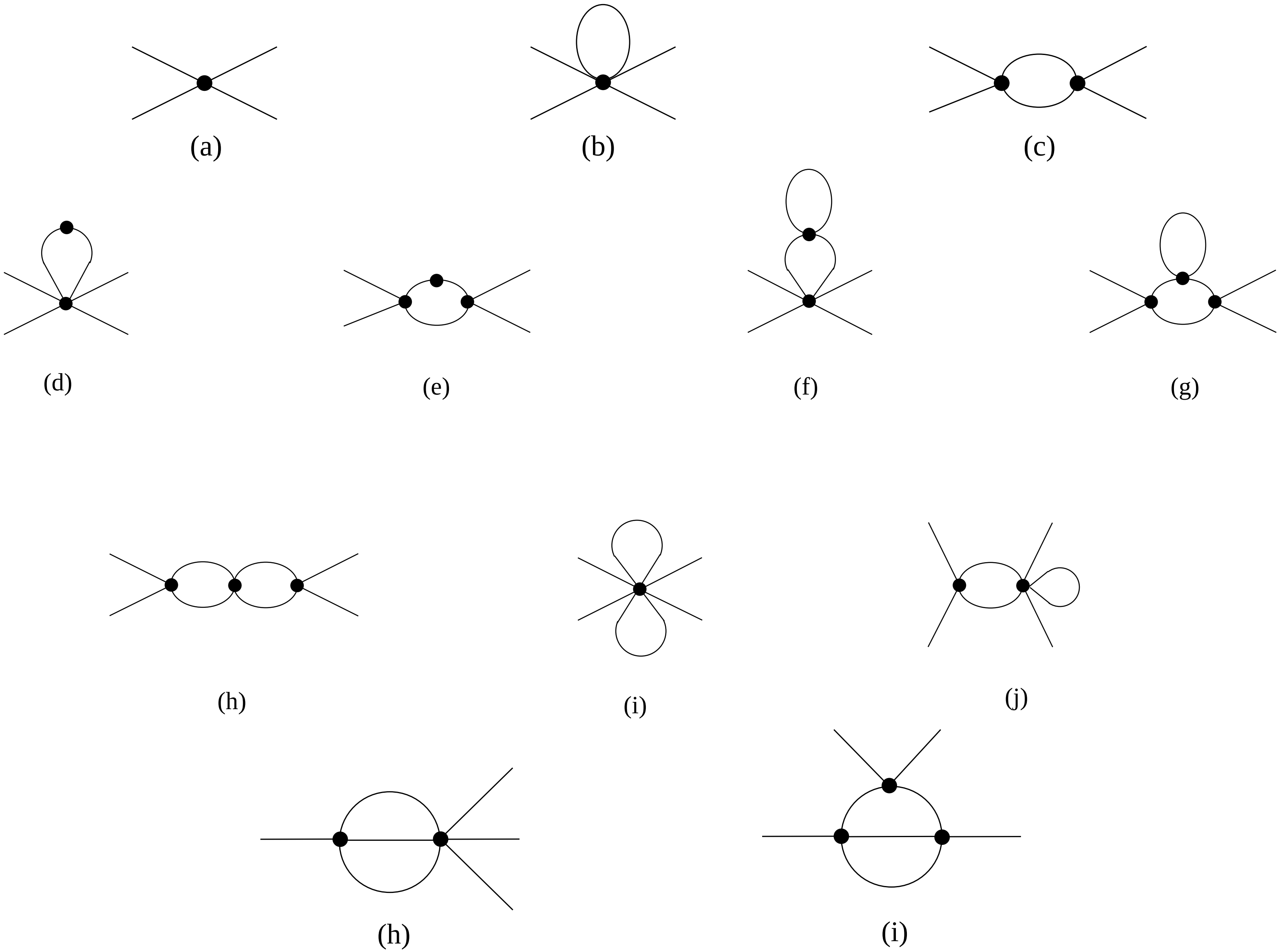
$M(s,t,u) =$

$M_0(s) + (s-u)M_1(t) + (s-t)M_1(t) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$

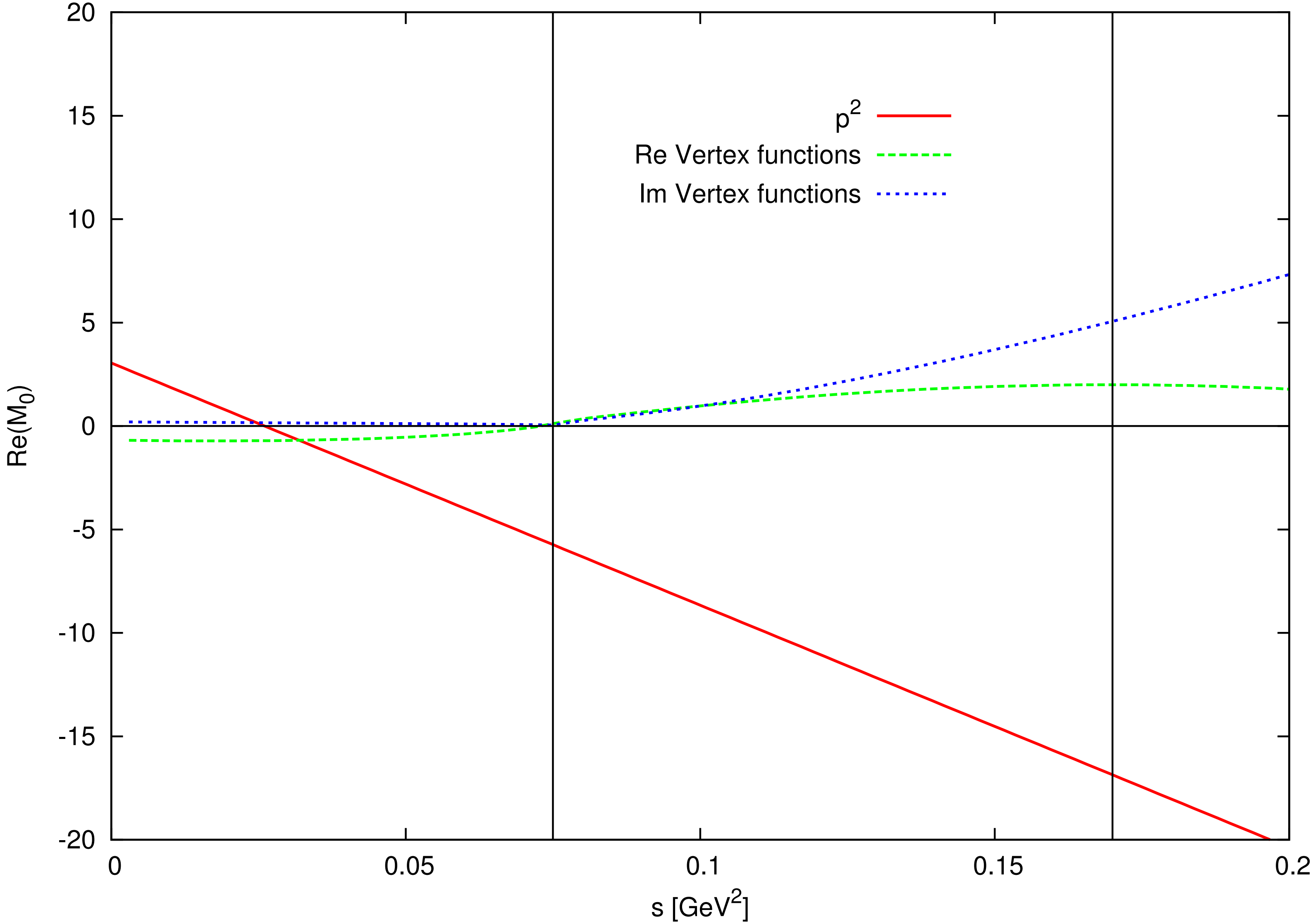
$M^{(2)} = \frac{1}{F_\pi^2}\left(\frac{4}{3}m_\pi^2 - s\right)$

	JB	Ghorbani	Talavera
p^4	All agree and with GL		
p^6 ∞	All cancel		
p^6 Vertex	Checked and agree		in progress
p^6 Rest	Comparison in progress		
p^6 Numerics	Very preliminary		Very preliminary

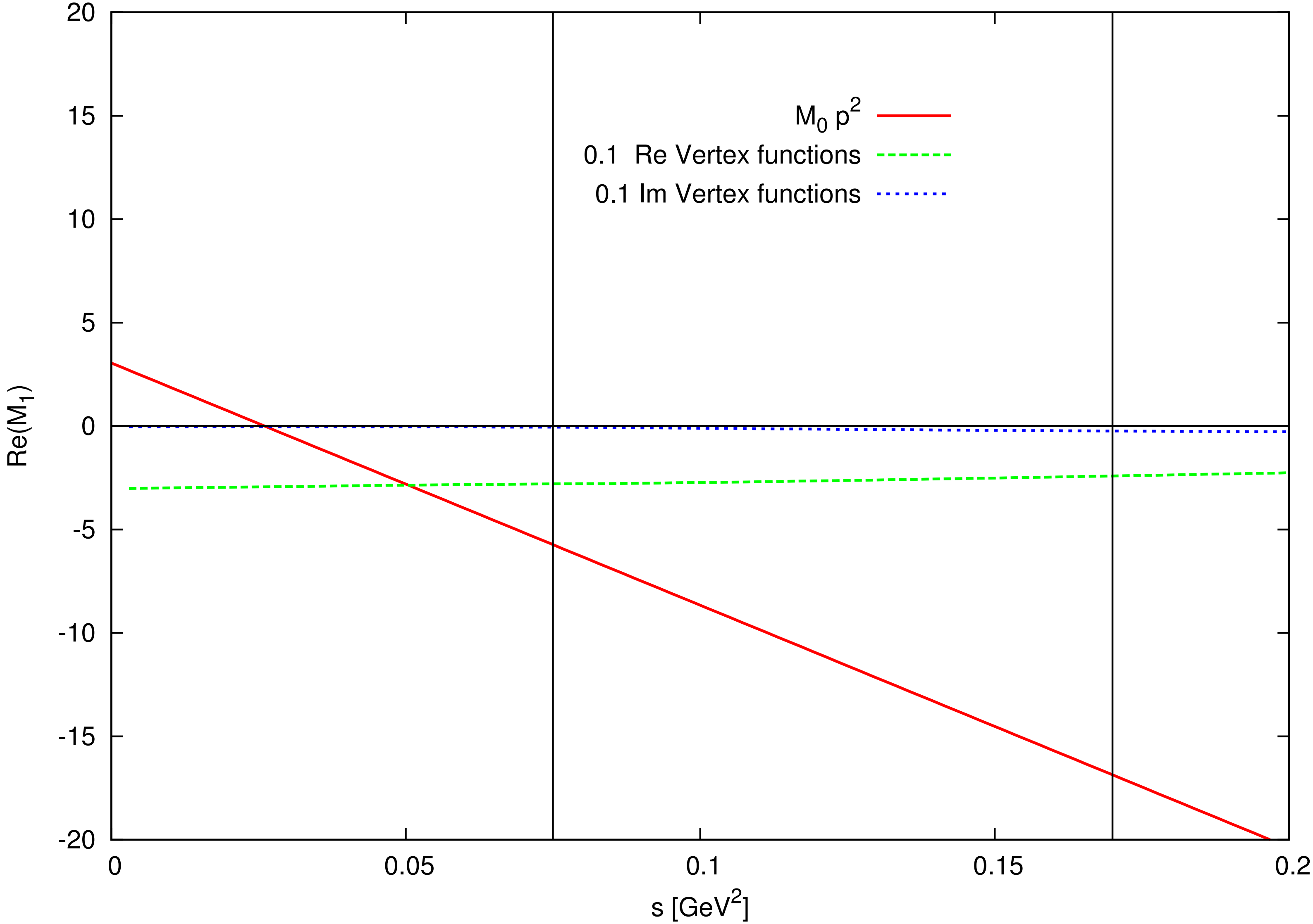
Diagrams



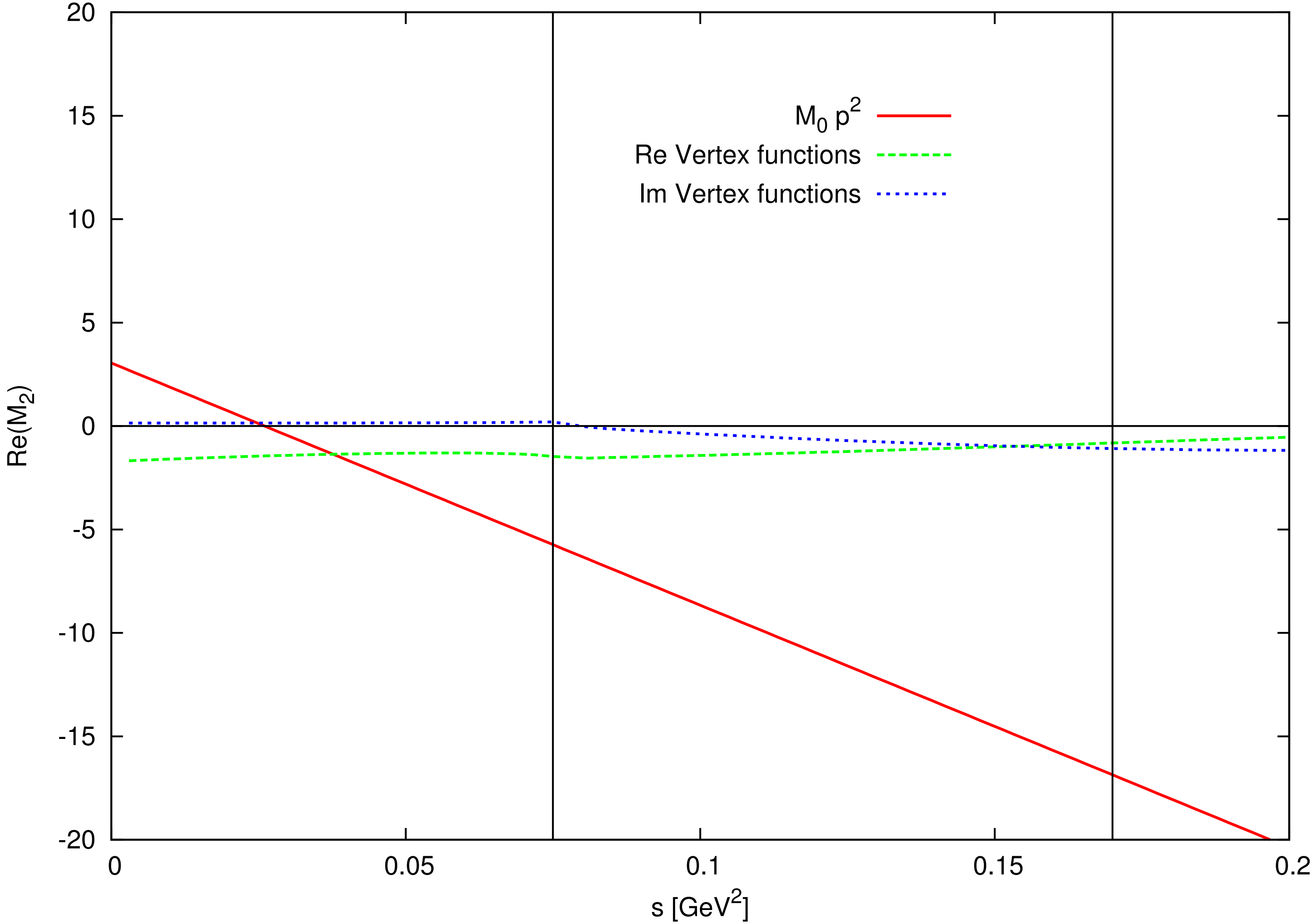
M_0: Vertex function contribution



M_1: Vertex function contribution



M_2: Vertex function contribution



$K_{\ell 3}$ Definitions

$$K^+_{\ell 3}: \quad K^+(p) \rightarrow \pi^0(p')\ell^+(p_\ell)\nu_\ell(p_\nu)$$

$$K^0_{\ell 3}: \quad K^0(p) \rightarrow \pi^-(p')\ell^+(p_\ell)\nu_\ell(p_\nu)$$

$$K^+_{\ell 3}: \quad T = \frac{G_F}{\sqrt{2}} V^\star_{us} \ell^\mu F_\mu{}^+(p', p)$$

$$\ell^\mu = \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_\ell)$$

$$F_\mu{}^+(p', p) = < \pi^0(p') \mid V_\mu^{4-i5}(0) \mid K^+(p) >$$

$$= \frac{1}{\sqrt{2}}[(p'+p)_\mu f_+^{K^+\pi^0}(t) + (p-p')_\mu f_-^{K^+\pi^0}(t)]$$

Isospin: $f_+^{K^0\pi^-}(t) = f_+^{K^+\pi^0}(t) = f_+(t)$

$f_-^{K^0\pi^-}(t) = f_-^{K^+\pi^0}(t) = f_-(t)$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization: $$f_{+,0}(t) = f_+(0)\left(1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right)$$

$|V_{us}|$:

- Know theoretically $f_+(0) = 1 + \cdots$
  - Short distance correction to G_F from G_μ Marciano-Sirlin
  - Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
  - Isospin Breaking Leuwyler-Roos
- Know experimentally $f_+(0)$

V_{us}

PDG2002:

$|V_{ud}| = 0.9734 \pm 0.0008$ $\qquad$ $|V_{us}| = 0.2196 \pm 0.0026$

$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) =$
0.9957 ± 0.0019

PDG2006:

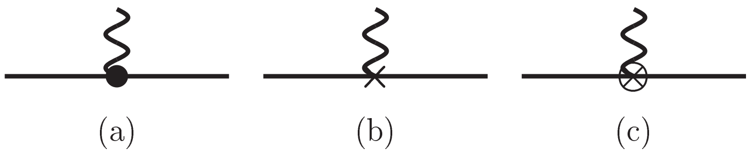
$|V_{ud}| = 0.97377 \pm 0.00027$ $\qquad$ $|V_{us}| = 0.2257 \pm 0.0021$

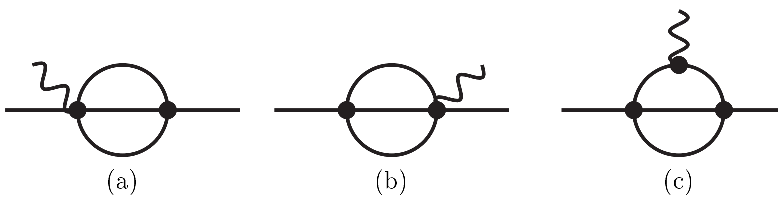
$|V_{ud}|^2 + |V_{us}|^2 = (0.94823 \pm 0.00054) + (0.05094 \pm 0.00095) =$
0.99917 ± 0.00110

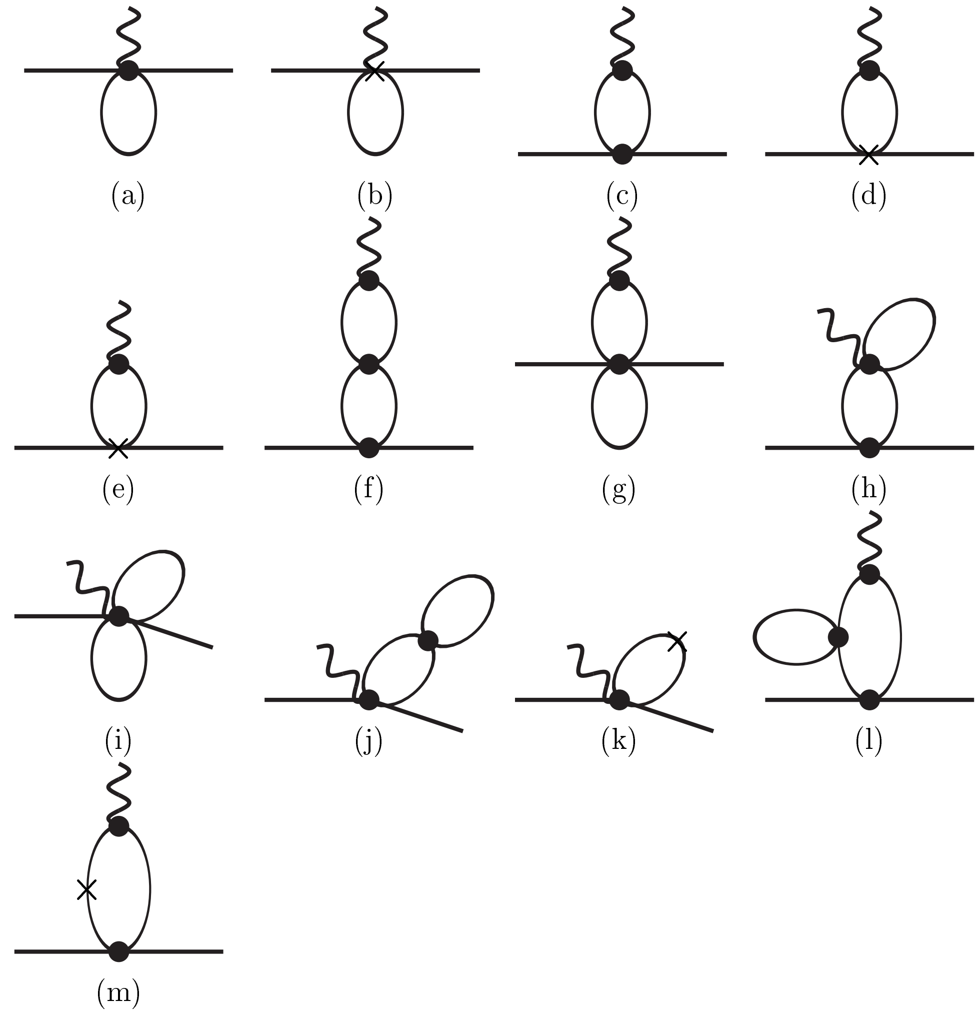
Problems:

- Ignores $\Delta(0) = 0.0113$ from pure two-loop
- Conflicts between experiments

$K_{\ell 3}$ Diagrams







● : p^2 vertex
$\times$: p^4 vertex
$\otimes$: p^6 vertex

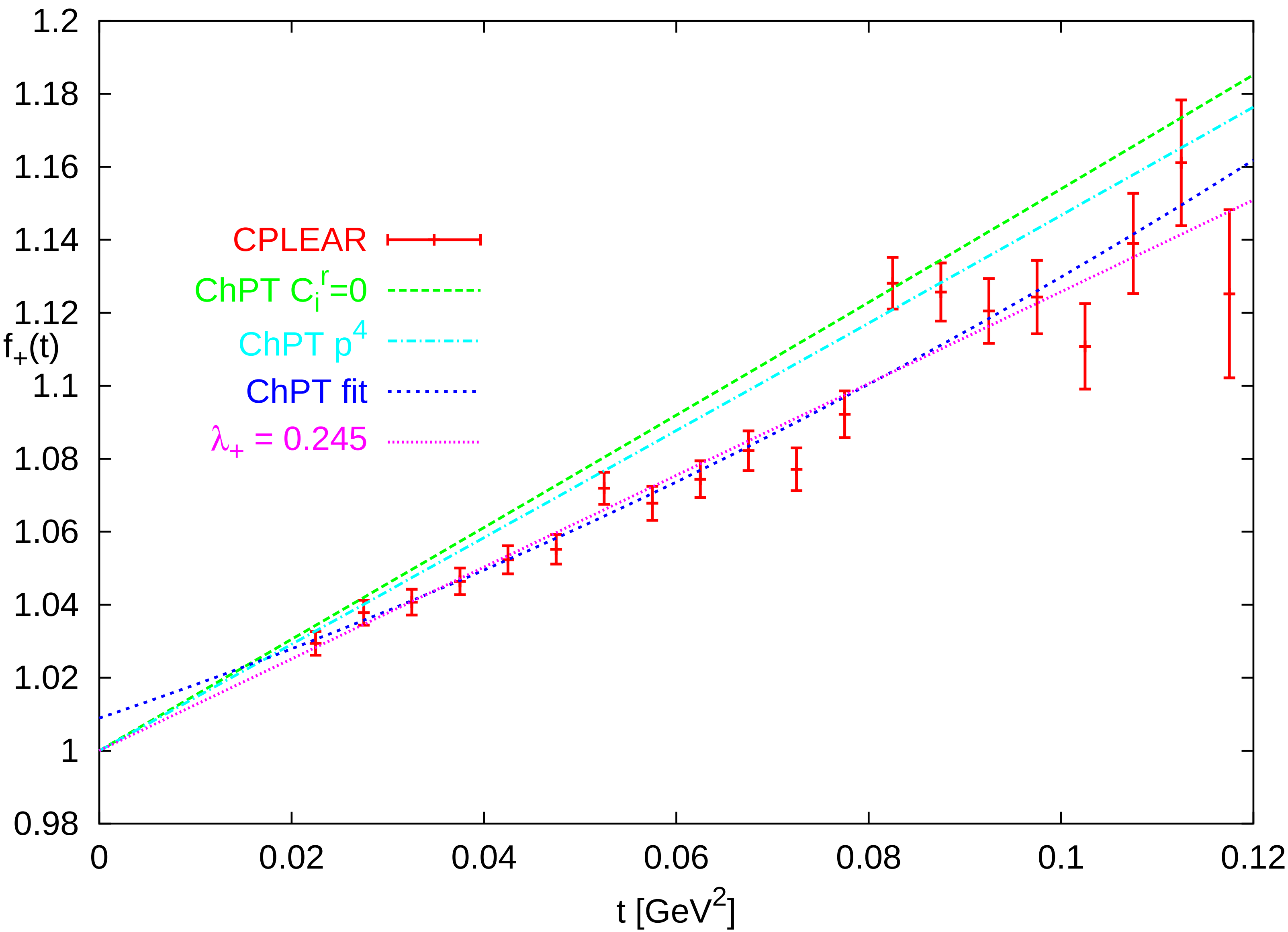
$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4}(-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

ChPT fit to $f_+(t)$



$f_0(t)$

Main Result:

$$f_0(t) = 1 - \frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + 8\frac{t}{F_\pi^4}(2C_{12}^r + C_{34}^r)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}(F_K/F_\pi - 1) - \frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6

$\Longrightarrow$

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.$$

Experiment

Form Factor comparison

KTeV [PRD 70(2004)]

K^0_{e3} quadratic fit: $\lambda''_+ \neq 0$ @ 4σ level

$K^0_{\mu 3}$ quadraqtic fit: $\lambda_0 = (13.72 \pm 1.31)\ 10^{-3}$

Slopes consistent for K^0_{e3} and $K^0_{\mu 3}$

ISTRA+ [PLB 581(2004), PLB589(2004)]

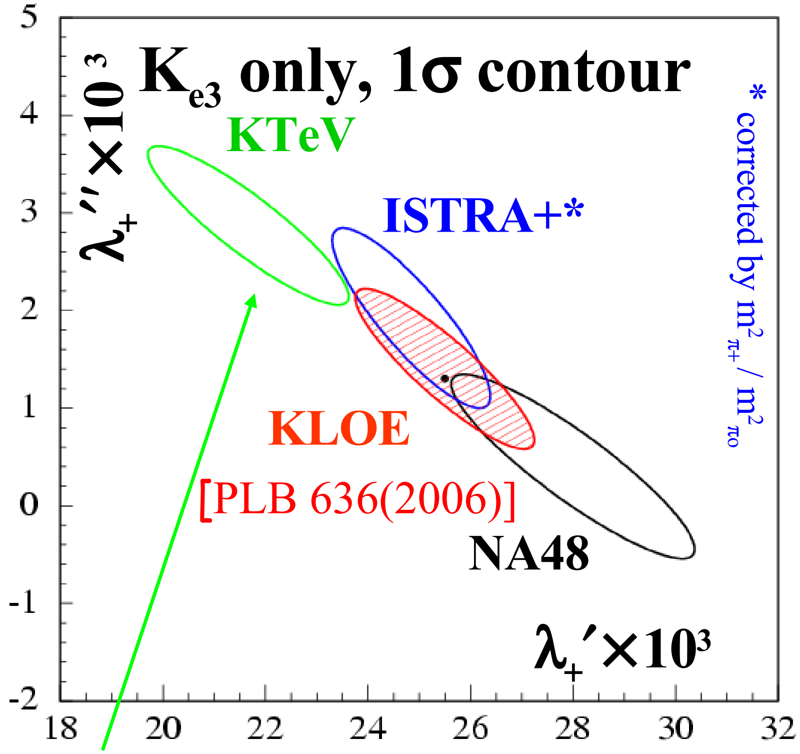
K^-_{e3} quadratic fit: $\lambda''_+ \neq 0$ @ 2σ level

$K^-_{\mu 3}$ quadratic fit: $\lambda_0 = (17.11 \pm 2.31)\ 10^{-3}$

NA48 [PLB 604(2004), HEP2005 289]

K^0_{e3}: No evidence for quadratic term

$K^0_{\mu 3}$ linear fit: $\lambda_0 = (12.0 \pm 1.7)\ 10^{-3}$



$\lambda''_+ \to -1\%$ phase space integral

$\to +0.5\%$ for V_{us}

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Experiment

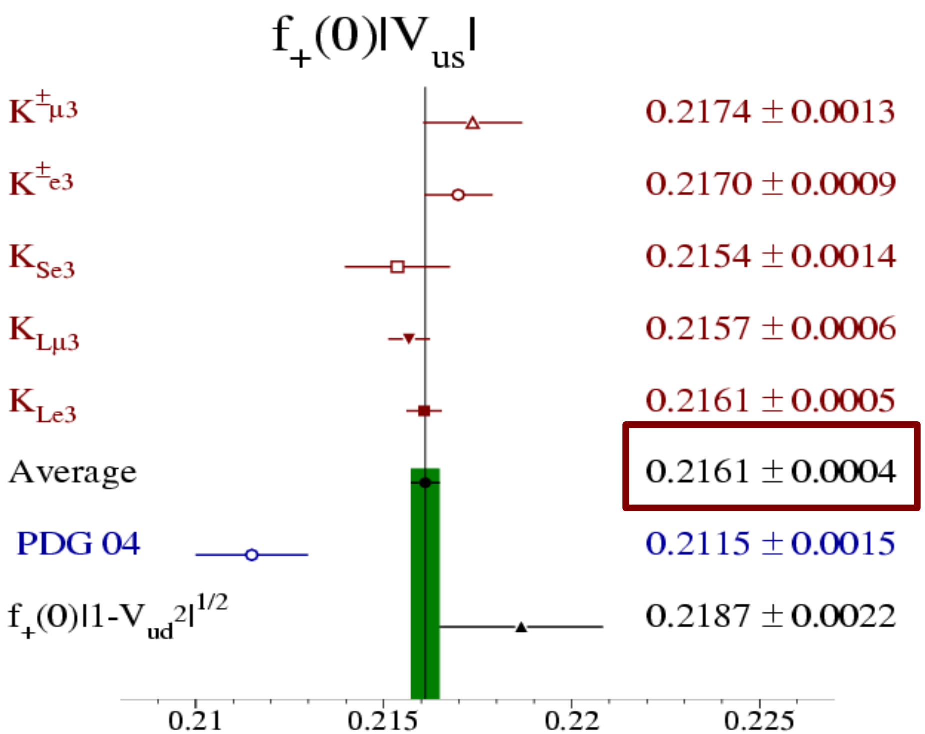
V_{us} determination from Kl3

	$K_L\,e3$	$K_L\,\mu3$	$K_S\,e3$	$K^{\pm}e3$	$K^{\pm}\mu3$
BR	0.4046(8)	0.2697(7)	$7.046(91)\times10^{-4}$	0.05043(31)	0.03383(27)
τ	51.10(19) ns		89.58(6) ps	12.386(22) ns	

$\lambda'_+ = 0.02496(79)$

$\lambda''_+ = 0.0016(3)$

$\lambda_0 = 0.01587(95)$



- $f_+(0) = 0.961(8)$
  Leutwyler and Roos Z. [Phys. **C25**, 91, 1984]
- $V_{ud} = 0.97377(27)$
  Marciano and Sirlin [Phys.Rev.Lett.96 032002,2006]

$$V_{us} = 0.2249 \pm 0.0019$$

From $R_{e\mu}$:

$$\frac{g_e}{g_\mu} = 1.0014 \pm 0.0037$$

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Experiment

$V_{us} - V_{ud}$ plane

Combining the experimental value of $\Gamma(K\to\mu\nu(\gamma))/\Gamma(\pi\to\mu\nu(\gamma))$

with the ratio f_K/f_π obtained from lattice calculations we can extract $|V_{us}|/|V_{ud}|$

(Marciano hep-ph/0406324) $\Gamma(K\to\mu\nu(\gamma))/\Gamma(\pi\to\mu\nu(\gamma)) \propto |V_{us}|^2/|V_{ud}|^2 f_K^2/f_\pi^2$

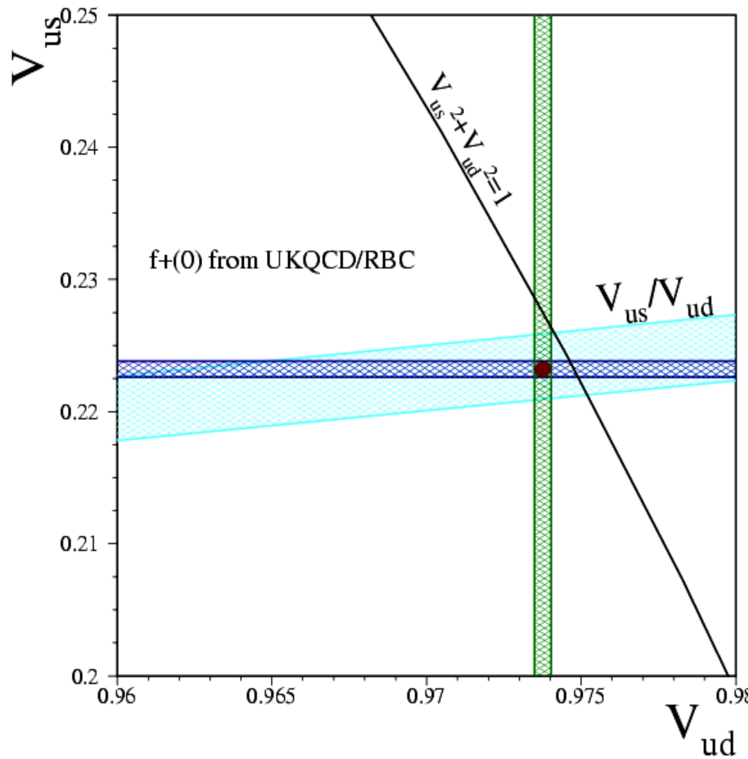
Using f_K/f_π =1.198(3)($^{+16}_{-5}$) from MILC

and **KLOE BR($K^+ \to \mu^+\nu$)**

we get **V_{us}/V_{ud} = 0.2294 ± 0.0026**

$\delta=|Vus|^2+|Vus|^2-1=-0.0020\pm0.0006$

NEED TO BE CONFIRMED



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Conclusions

- Chiral Perturbation Theory doing fine
- $\eta \to 3\pi$: All three have expressions: do we agree?
- $K_{\ell 3}$: useful even with very many parameters
- Other recent work: connection with lattice QCD: Partially quenched ChPT
  - PQChPT at two loops: basic calculations done
  - Photons now also included (but NLO)
- For formulas: `http://www.thep.lu.se/~bijnens/chpt.html`
- Many more calculations have been done