KAON DECAYS AND CHIRAL PERTURBATION THEORY

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Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- Kaon ?
- A bit of history of Kaons
- Motivation
- Effective Field Theory
- Chiral Perturbation Theory
- $K_{\ell 3}$
- Some rare Kaon decays
- $K \to 3\pi$

Kaon: a first guess

Know All On Nothing

Kill Any Old Nerd

Keep All Original Neanderthals

Try Google

http://www.tech-ex.com/article_images1/324984/DSC02855.JPG

Try Google once more

http://www.mountaindesigns.com.au/
product_images/full/04305_S04.jpg

http://tn3-2.deviantart.com/300W/fs7.deviantart.com/
i/2005/179/d/6/Anti__Kaon_by_Segzhan.jpg

Wikipedia

This article describes the subatomic particle called the kaon. For the ontology infrastructure of the same name, see KAON.

In particle physics, a kaon (also called K-meson and denoted K) is any one of a group of four mesons distinguished by the fact that they carry a quantum number called strangeness. In the quark model they are understood to contain a single strange quark (or antiquark).

One of the first Kaons

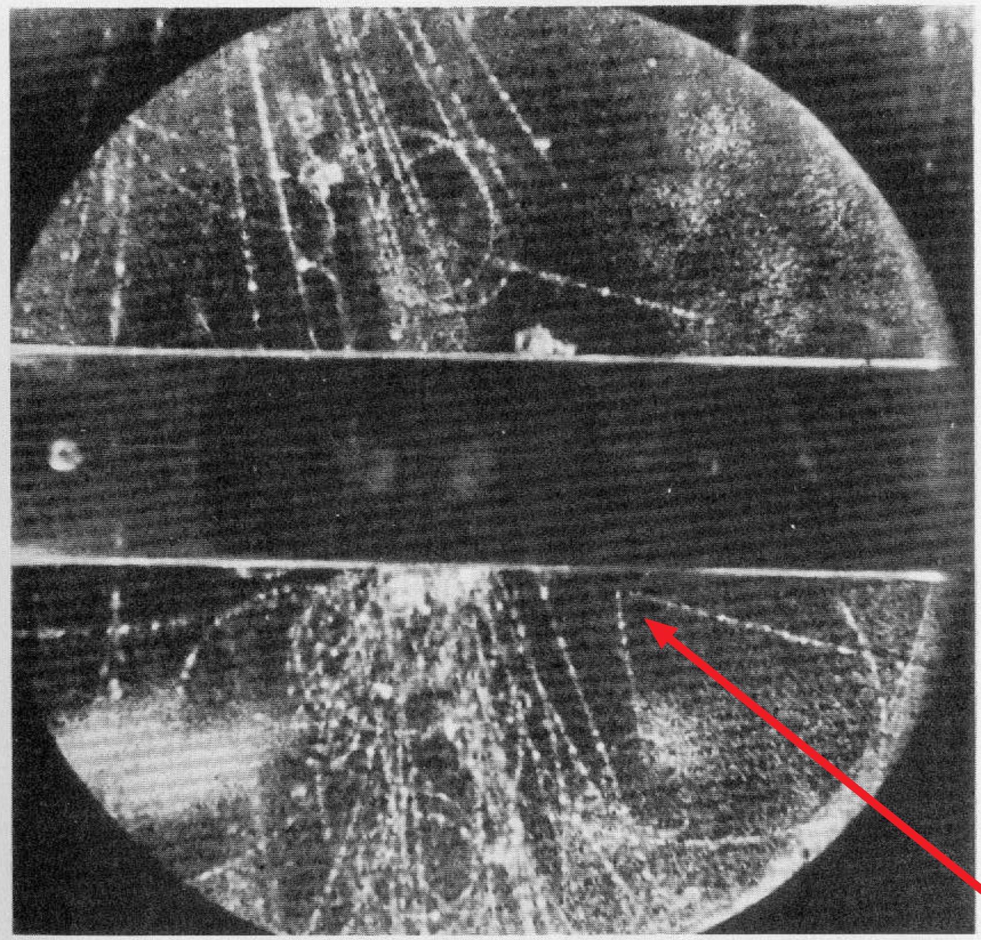

Figure 13-2 First picture showing a neutral particle decaying into 2 pions. Such a neutral particle is now called a K^0 particle. Originally the authors called them V particles. [From G. D. Rochester and C. C. Butler, *Nature*, **160**, 855 (1947).]

a V particle

One of the first Kaons

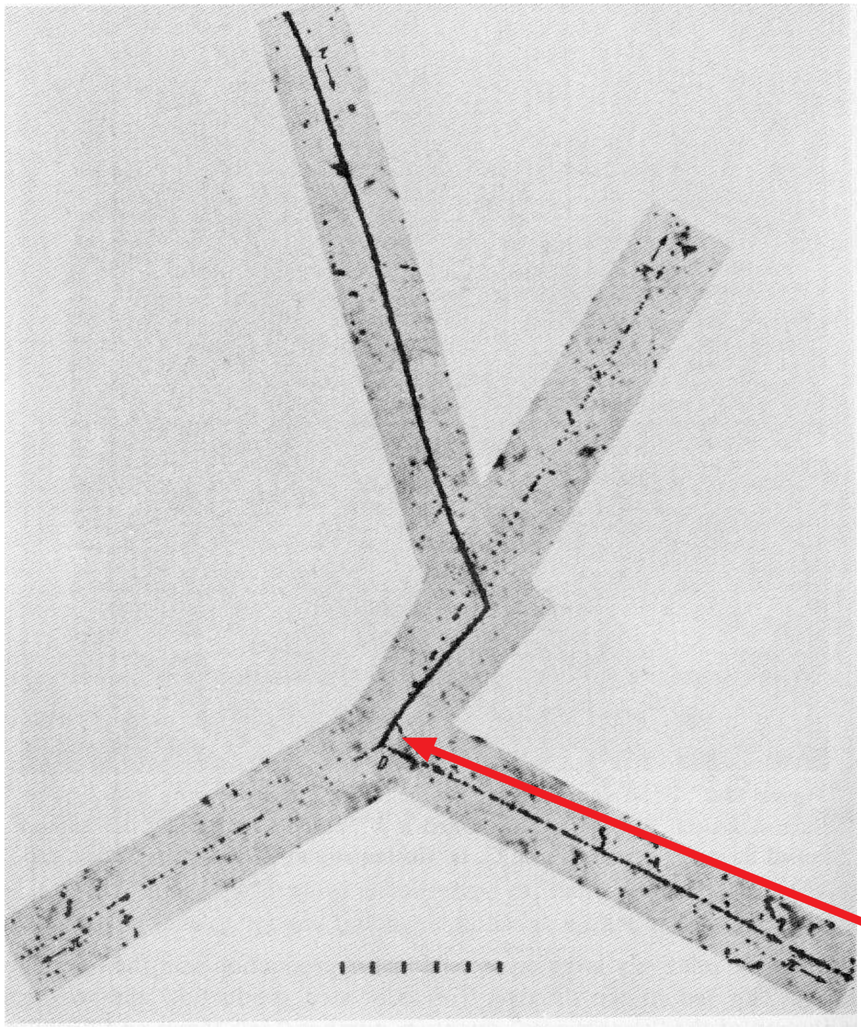

Figure 13-3 Decay of a τ meson (in modern notation $K^{\pm}$) into three pions. [Hodgson, 1951, from (PFP 59).]

a $\tau^+ \to \pi^+\pi^+\pi^-$ decay

Early Kaon results

- Produced with strong interaction rates, decay weakly: Introduction of strangeness: Gell-Mann–Pais
- θ, τ, κ, ...: All the same mass
- The θ-τ puzzle
  - $\tau \to 3\pi \Longrightarrow$ negative parity,
  - $\theta \to 2\pi \Longrightarrow$ positive parity,
  - Two particles or parity broken
- K^0-$\overline{K}^0$: Two states with *very* different lifetimes
  - K_L and K_S are the CP even and odd states
  - CP-violation
- $\Delta I = 1/2$ rule: $\Gamma(K_S \to \pi^0\pi^0) \ggg \Gamma(K^+ \to \pi^+\pi^0)$

More recent Kaon results

- Direct CP-violation ε'/ε.
- Determination of V_{us}
- $\pi\pi$ scattering lengths
- $\ldots$

More recent Kaon results

I will talk about ChPT for:

- $K_{\ell 3}$
- A few rare kaon decays
- $K \to 3\pi$

There are many more ChPT calculations relevant for Kaons

- Masses and decay constants
- Rare decays
- Radiative decays
- $K_{\ell 4}$
- Constraints in calculating the nonleptonic matrix elements: $\Delta I = 1/2$ and ε'/ε
- $\ldots$

The Standard Model

The Standard Model Lagrangian has four parts:

$$\underbrace{\mathcal{L}_H(\phi)}_{\text{Higgs}} + \underbrace{\mathcal{L}_G(W,Z,G)}_{\text{Gauge}}$$

$$\underbrace{\sum_{\psi=\text{fermions}} \bar\psi i \not{D}\,\psi}_{\text{gauge-fermion}} + \underbrace{\sum_{\psi,\psi'=\text{fermions}} g_{\psi\psi'}\bar\psi\phi\psi'}_{\text{Yukawa}}$$

The Standard Model

What is tested ?

gauge-fermion	Very well tested
Higgs	Limits only, real tests coming up
Gauge	Well tested, QCD nonperturbative a small E
Yukawa	Flavour Physics

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- C Charge Conjugation
- P Parity
- T Time Reversal

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Discrete symmetries:

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QCD and QED conserve C,P,T separately,
Weak breaks C and P, only Yukawa breaks CP
Field theory implies CPT

Weak interaction: quarks to mesons

ENERGY SCALE	FIELDS	Effective Theory
M_W	W, Z, γ, g; τ, μ, e, ν_ℓ; t, b, c, s, u, d	Standard Model
	$\Downarrow$ using OPE	
$\lesssim m_c$	γ, g; μ, e, ν_ℓ; s, d, u	QCD,QED,$\mathcal{H}_{\text{eff}}^{\vert\Delta S\vert=1,2}$
	$\Downarrow$???	
M_K	γ; μ, e, ν_ℓ; π, K, η	ChPT

Effective Field Theory

Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples: $\left\{\begin{array}{l}\text{Solid state physics: conductors: neglect the empty bands above the partially filled one} \\ \text{Atomic physics: Blue sky: neglect atomic structure}\end{array}\right.$

Power Counting

- gap in the spectrum $\Longrightarrow$ separation of scales
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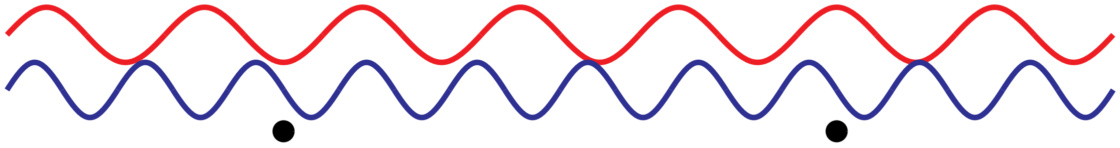
- Taylor series expansion does not work (convergence radius is zero)
- Continuum of excitation states need to be taken into account

Why is the sky blue ?

System: Photons of visible light and neutral atoms

Length scales: a few 1000 Å versus 1 Å

Atomic excitations suppressed by $\approx 10^{-3}$



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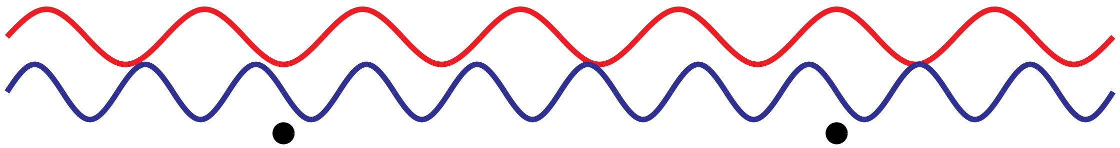

$$\mathcal{L}_A = \Phi_v^\dagger \partial_t \Phi_v + \dots \qquad \mathcal{L}_{\gamma A} = G F_{\mu\nu}^2 \Phi_v^\dagger \Phi_v + \dots$$

Units with $\hbar = c = 1$: G energy dimension -3:

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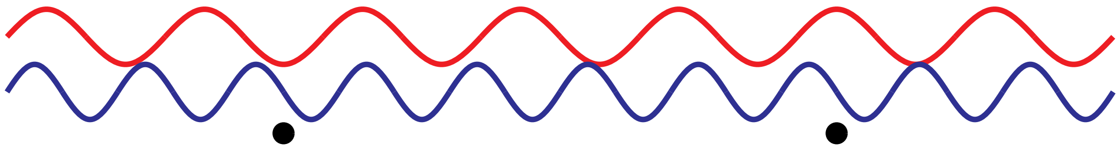

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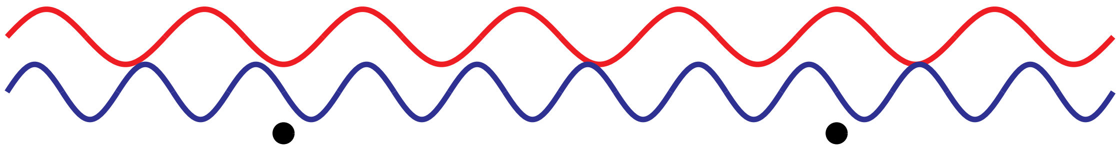

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blue light scatters a lot more than red $\left\{\begin{array}{l} \Longrightarrow \text{red sunsets} \\ \Longrightarrow \text{blue sky} \end{array}\right.$

Higher orders suppressed by 1 Å$/\lambda_\gamma$.

Why Field Theory ?

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Advantages

- Calculations are (relatively) simple
- It is general: model-independent
- Theory $\Longrightarrow$ errors can be estimated
- Systematic: ALL effects at a given order can be included
- Even if no convergence: classification of models often useful

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

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Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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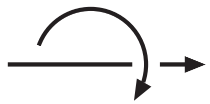
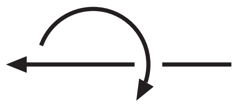
Can also see that via  $v < c,\ m_q \neq 0 \Longrightarrow$ / $v = c,\ m_q = 0 \not\Longrightarrow$ 

Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
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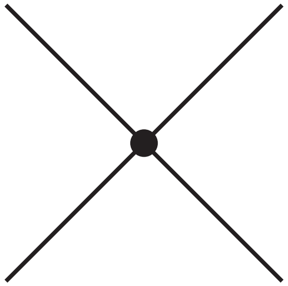
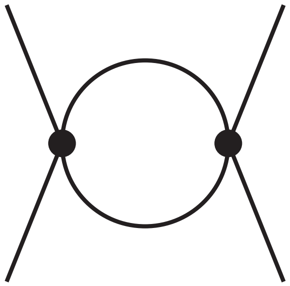
Power counting in momenta:



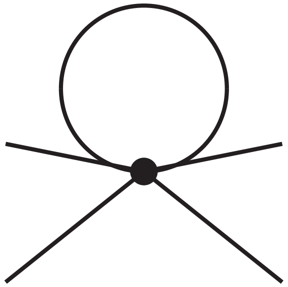
p^2



$$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$$

$1/p^2$

$\int d^4p$ $\qquad p^4$



$$(p^2)\,(1/p^2)\,p^4 = p^4$$

Chiral Perturbation Theory

Large subject:

- Steven Weinberg, Physica A96:327,1979: 1662 citations
- Juerg Gasser and Heiri Leutwyler, Nucl.Phys.B250:465,1985: 2126 citations
- Juerg Gasser and Heiri Leutwyler, Annals Phys.158:142,1984: 2072 citations
- Sum: 3509
- Checked on 9/1/2007 in SPIRES

For lectures, review articles: see
http://www.thep.lu.se/~bijnens/chpt.html

Two Loop: Lagrangians

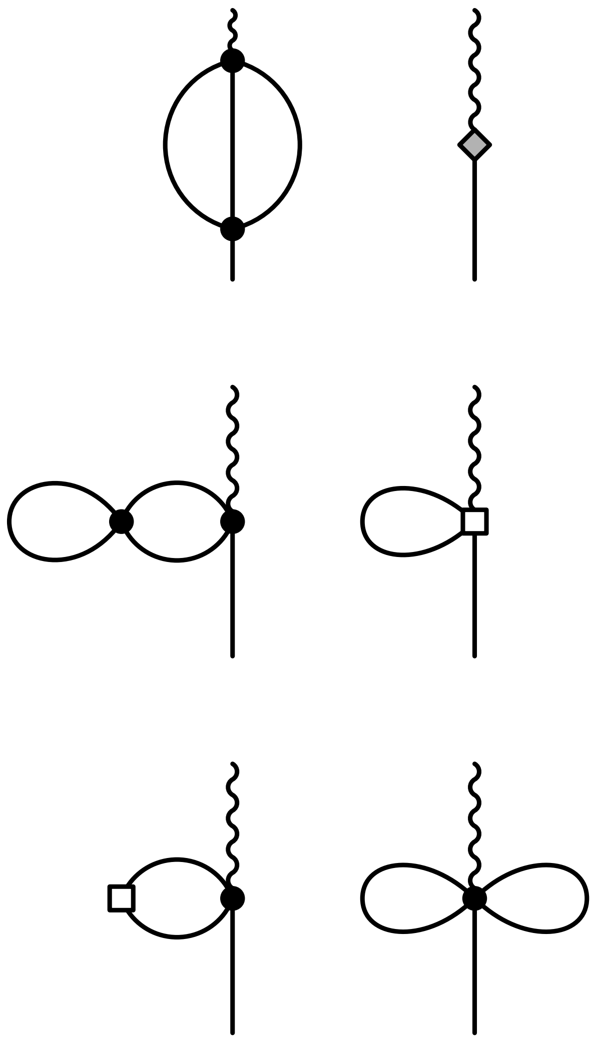
Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966
p^4: Gasser, Leutwyler 84,85
p^6: JB, Colangelo, Ecker 99,00

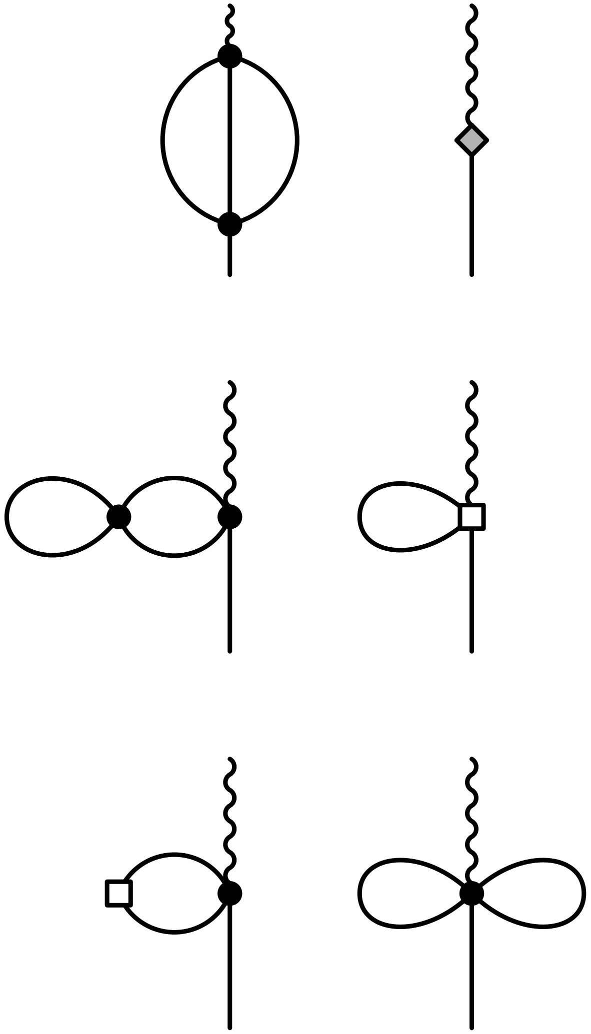
Note
- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour is a special case of 3+3 PQ:
  $\hat{L}_i^r, K_i^r \to L_i^r, C_i^r$

Long Expressions



$\Longrightarrow$

Long Expressions



$\Longrightarrow$

$$
\begin{aligned}
\delta^{(6)22}_{\text{loops}} = \;& \pi_{16}\, L^r_0\;\left[4/9\,\chi_\eta\chi_4 - 1/2\,\chi_1\chi_3 + \chi_{13}^2 - 13/3\,\bar\chi_1\chi_{13} - 35/18\,\bar\chi_2\right] \;-\; 2\,\pi_{16}\,L^r_1\,\chi_{13}^2 \\
-\;& \pi_{16}\,L^r_2\;\left[11/3\,\chi_\eta\chi_4 + \chi_{13}^2 + 13/3\,\bar\chi_2\right] + \pi_{16}\,L^r_3\;\left[4/9\,\chi_\eta\chi_4 - 7/12\,\chi_1\chi_3 + 11/6\,\chi_{13}^2 - 17/6\,\bar\chi_1\chi_{13} - 43/36\,\bar\chi_2\right] \\
+\;& \pi_{16}^2\;\left[-15/64\,\chi_\eta\chi_4 - 59/384\,\chi_1\chi_3 + 65/384\,\chi_{13}^2 - 1/2\,\bar\chi_1\chi_{13} - 43/128\,\bar\chi_2\right] \;-\; 48\,L^r_4 L^r_5\,\bar\chi_1\chi_{13} \;-\; 72\,L^{r2}_4\,\bar\chi_1^2 \\
-\;& 8\,L^{r2}_5\,\chi_{13}^2 \;+\; \bar A(\chi_p)\,\pi_{16}\;\left[-1/24\,\chi_p + 1/48\,\bar\chi_1 - 1/8\,\bar\chi_1\,R^p_{q\eta} + 1/16\,\bar\chi_1\,R^c_p - 1/48\,R^p_{q\eta}\,\chi_p - 1/16\,R^p_{q\eta}\,\chi_q\right. \\
+\;& \left.1/48\,R^\eta_{pp}\,\chi_\eta + 1/16\,R^c_p\,\chi_{13}\right] \;+\; \bar A(\chi_p)\,L^r_0\;\left[8/3\,R^p_{q\eta}\,\chi_p + 2/3\,R^c_p\,\chi_p + 2/3\,R^d_p\right] \;+\; \bar A(\chi_p)\,L^r_3\;\left[2/3\,R^p_{q\eta}\,\chi_p\right. \\
+\;& \left.5/3\,R^c_p\,\chi_p + 5/3\,R^d_p\right] \;+\; \bar A(\chi_p)\,L^r_4\;\left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta0} - 2\,\bar\chi_1\,R^p_{q\eta} + 3\,\bar\chi_1\,R^c_p\right] \;+\; \bar A(\chi_p)\,L^r_5\;\left[-2/3\,\bar\chi^{pp}_{\eta\eta1} - R^p_{q\eta}\,\chi_p\right. \\
+\;& \left.1/3\,R^p_{q\eta}\,\chi_q + 1/2\,R^c_p\,\chi_p - 1/6\,R^c_p\,\chi_q\right] \;+\; \bar A(\chi_p)^2\;\left[1/16 + 1/72\,(R^p_{q\eta})^2 - 1/72\,R^p_{q\eta}R^c_p + 1/288\,(R^c_p)^2\right] \\
+\;& \bar A(\chi_p)\bar A(\chi_{ps})\;\left[-1/36\,R^p_{q\eta} - 5/72\,R^p_{s\eta} + 7/144\,R^c_p\right] \;-\; \bar A(\chi_p)\bar A(\chi_{qs})\;\left[1/36\,R^p_{q\eta} + 1/24\,R^p_{s\eta} + 1/48\,R^c_p\right] \\
+\;& \bar A(\chi_p)\bar A(\chi_\eta)\;\left[-1/72\,R^p_{q\eta}R^v_{\eta13} + 1/144\,R^c_pR^v_{\eta13}\right] \;+\; 1/8\,\bar A(\chi_p)\bar A(\chi_{13}) \;+\; 1/12\,\bar A(\chi_p)\bar A(\chi_{46})\,R^\eta_{pp} \\
+\;& \bar A(\chi_p)\bar B(\chi_p,\chi_p;0)\;\left[1/4\,\chi_p - 1/18\,R^p_{q\eta}R^c_p\,\chi_p - 1/72\,R^p_{q\eta}R^d_p + 1/18\,(R^c_p)^2\,\chi_p + 1/144\,R^c_pR^d_p\right] \\
+\;& \bar A(\chi_p)\bar B(\chi_p,\chi_\eta;0)\;\left[1/18\,R^\eta_{pp}R^c_p\,\chi_p - 1/18\,R^\eta_{13}R^c_p\,\chi_p\right] \;+\; \bar A(\chi_p)\bar B(\chi_q,\chi_q;0)\;\left[-1/72\,R^p_{q\eta}R^d_q + 1/144\,R^c_pR^d_q\right] \\
-\;& 1/12\,\bar A(\chi_p)\bar B(\chi_{ps},\chi_{ps};0)\,R^p_{s\eta}\,\chi_{ps} \;-\; 1/18\,\bar A(\chi_p)\bar B(\chi_1,\chi_3;0)\,R^q_{p\eta}R^c_p\,\chi_p \\
+\;& 1/18\,\bar A(\chi_p)\bar C(\chi_p,\chi_p,\chi_p;0)\,R^c_pR^d_p\,\chi_p \;+\; \bar A(\chi_p;\varepsilon)\,\pi_{16}\;\left[1/8\,\bar\chi_1R^p_{q\eta} - 1/16\bar\chi_1\,R^c_p - 1/16\,R^c_p\,\chi_p - 1/16\,R^d_p\right] \\
+\;& \bar A(\chi_{ps})\,\pi_{16}\;\left[1/16\,\chi_{ps} - 3/16\,\chi_{qs} - 3/16\,\bar\chi_1\right] \;-\; 2\,\bar A(\chi_{ps})\,L^r_0\,\chi_{ps} \;-\; 5\,\bar A(\chi_{ps})\,L^r_3\,\chi_{ps} \;-\; 3\,\bar A(\chi_{ps})\,L^r_4\,\bar\chi_1 \\
+\;& \bar A(\chi_{ps})\,L^r_5\,\chi_{13} \;+\; \bar A(\chi_{ps})\bar A(\chi_\eta)\;\left[7/144\,R^\eta_{pp} - 5/72\,R^\eta_{ps} - 1/48\,R^\eta_{qq} + 5/72\,R^\eta_{qs} - 1/36\,R^\eta_{13}\right] \\
+\;& \bar A(\chi_{ps})\bar B(\chi_p,\chi_p;0)\;\left[1/24\,R^p_{s\eta}\,\chi_p - 5/24\,R^p_{s\eta}\,\chi_{ps}\right] \;+\; \bar A(\chi_{ps})\bar B(\chi_p,\chi_\eta;0)\;\left[-1/18\,R^\eta_{ps}R^z_{qp\eta}\,\chi_p\right. \\
-\;& \left.1/9\,R^\eta_{ps}R^z_{qp\eta}\,\chi_{ps}\right] \;-\; 1/48\,\bar A(\chi_{ps})\bar B(\chi_q,\chi_q;0)\,R^d_q \;+\; 1/18\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0)\,R^q_{s\eta}\,\chi_s \\
+\;& 1/9\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0,k)\,R^q_{s\eta} \;+\; 3/16\,\bar A(\chi_{ps};\varepsilon)\,\pi_{16}\;\left[\chi_s + \bar\chi_1\right] \;-\; 1/8\,\bar A(\chi_{p4})^2 \;-\; 1/8\,\bar A(\chi_{p4})\bar A(\chi_{p6}) \\
+\;& 1/8\,\bar A(\chi_{p4})\bar A(\chi_{q6}) \;-\; 1/32\,\bar A(\chi_{p6})^2 \;+\; \bar A(\chi_\eta)\,\pi_{16}\;\left[1/16\,\bar\chi_1\,R^v_{\eta13} - 1/48\,R^v_{\eta13}\,\chi_\eta + 1/16\,R^v_{\eta13}\,\chi_{13}\right] \\
+\;& \bar A(\chi_\eta)\,L^r_0\;\left[4R^\eta_{13}\,\chi_\eta + 2/3\,R^v_{\eta13}\,\chi_\eta\right] \;-\; 8\,\bar A(\chi_\eta)\,L^r_1\,\chi_\eta \;-\; 2\,\bar A(\chi_\eta)\,L^r_2\,\chi_\eta \;+\; \bar A(\chi_\eta)\,L^r_3\;\left[4R^\eta_{13}\,\chi_\eta + 5/3\,R^v_{\eta13}\chi_\eta\right] \\
+\;& \bar A(\chi_\eta)\,L^r_4\;\left[4\,\chi_\eta + \bar\chi_1\,R^v_{\eta13}\right] \;-\; \bar A(\chi_\eta)\,L^r_5\;\left[1/6\,R^\eta_{pp}\,\chi_q + R^\eta_{13}\,\chi_{13} + 1/6\,R^v_{\eta13}\,\chi_\eta\right] \;+\; 1/288\,\bar A(\chi_\eta)^2\,(R^v_{\eta13})^2 \\
+\;& 1/12\,\bar A(\chi_\eta)\bar A(\chi_{46})\,R^v_{\eta13} \;+\; \bar A(\chi_\eta)\bar B(\chi_p,\chi_p;0)\;\left[-1/36\,\bar\chi^{pp\eta}_{\eta\eta1} - 1/18\,R^p_{q\eta}R^\eta_{pp}\,\chi_p + 1/18\,R^\eta_{pp}R^c_p\,\chi_p\right. \\
+\;& \left.1/144\,R^d_pR^v_{\eta13}\right] \;+\; \bar A(\chi_\eta)\bar B(\chi_p,\chi_\eta;0)\;\left[-1/18\,\bar\chi^{\eta p\eta}_{p\eta1} + 1/18\,\bar\chi^{\eta p\eta}_{q\eta1} + 1/18\,(R^\eta_{pp})^2R^z_{qp\eta}\,\chi_p\right] \\
-\;& 1/12\,\bar A(\chi_\eta)\bar B(\chi_{ps},\chi_{ps};0)\,R^\eta_{ps}\,\chi_{ps} \;-\; \bar A(\chi_\eta)\bar B(\chi_\eta,\chi_\eta;0)\;\left[1/216\,R^v_{\eta13}\,\chi_4 + 1/27\,R^v_{\eta13}\,\chi_6\right] \\
-\;& 1/18\,\bar A(\chi_\eta)\bar B(\chi_1,\chi_3;0)\,R^1_{\eta\eta}R^3_{\eta\eta}\,\chi_\eta \;+\; 1/18\,\bar A(\chi_\eta)\bar C(\chi_p,\chi_p,\chi_p;0)\,R^\eta_{pp}R^d_p\,\chi_p \;+\; \bar A(\chi_\eta;\varepsilon)\,\pi_{16}\;\left[1/8\,\chi_\eta\right. \\
-\;& \left.1/16\,\bar\chi_1\,R^v_{\eta13} - 1/8\,R^\eta_{13}\,\chi_\eta - 1/16\,R^v_{\eta13}\,\chi_\eta\right] \;+\; \bar A(\chi_1)\bar A(\chi_3)\;\left[-1/72\,R^p_{q\eta}R^c_q + 1/36\,R^1_{3\eta}R^3_{1\eta} + 1/144\,R^c_1R^c_3\right] \\
-\;& 4\,\bar A(\chi_{13})\,L^r_1\,\chi_{13} \;-\; 10\,\bar A(\chi_{13})\,L^r_2\,\chi_{13} \;+\; 1/8\,\bar A(\chi_{13})^2 \;-\; 1/2\,\bar A(\chi_{13})\bar B(\chi_1,\chi_3;0,k) \\
+\;& 1/4\,\bar A(\chi_{13};\varepsilon)\,\pi_{16}\,\chi_{13} \;+\; 1/4\,\bar A(\chi_{14})\bar A(\chi_{34}) \;+\; 1/16\,\bar A(\chi_{16})\bar A(\chi_{36}) \;-\; 24\,\bar A(\chi_4)\,L^r_1\,\chi_4 \;-\; 6\,\bar A(\chi_4)\,L^r_2\,\chi_4 \\
+\;& 12\,\bar A(\chi_4)\,L^r_4\,\chi_4 \;+\; 1/12\,\bar A(\chi_4)\bar B(\chi_p,\chi_p;0)\,(R^p_{4\eta})^2\,\chi_4 \;+\; 1/6\,\bar A(\chi_4)\bar B(\chi_p,\chi_\eta;0)\;\left[R^p_{4\eta}R^\eta_{p4}\,\chi_4 - R^p_{4\eta}R^\eta_{q4}\,\chi_4\right] \\
-\;& 1/24\,\bar A(\chi_4)\bar B(\chi_\eta,\chi_\eta;0)\,R^v_{\eta13}\,\chi_4 \;-\; 1/6\,\bar A(\chi_4)\bar B(\chi_1,\chi_3;0)\,R^1_{4\eta}R^3_{4\eta}\,\chi_4 \;+\; 3/8\,\bar A(\chi_4;\varepsilon)\,\pi_{16}\,\chi_4 \\
-\;& 32\,\bar A(\chi_{46})\,L^r_1\,\chi_{46} \;-\; 8\,\bar A(\chi_{46})\,L^r_2\,\chi_{46} \;+\; 16\,\bar A(\chi_{46})\,L^r_4\,\chi_{46} \;+\; \bar A(\chi_{46})\bar B(\chi_p,\chi_p;0)\;\left[1/9\,\chi_{46} + 1/12\,R^\eta_{pp}\,\chi_p\right. \\
+\;& \left.1/36\,R^\eta_{pp}\,\chi_4 + 1/9\,R^\eta_{p4}\,\chi_6\right] \;+\; \bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0)\;\left[-1/18\,R^\eta_{pp}\,\chi_4 - 1/9\,R^\eta_{p4}\,\chi_6 + 1/9\,R^\eta_{q4}\,\chi_6 + 1/18\,R^\eta_{13}\,\chi_4\right] \\
-\;& 1/6\,\bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0,k)\;\left[R^\eta_{pp} - R^\eta_{13}\right] \;+\; 1/9\,\bar A(\chi_{46})\bar B(\chi_\eta,\chi_\eta;0)\,R^v_{\eta13}\,\chi_{46} \;-\; \bar A(\chi_{46})\bar B(\chi_1,\chi_3;0)\;\left[2/9\,\chi_{46}\right. \\
+\;& \left.1/9\,R^\eta_{p4}\,\chi_6 + 1/18\,R^\eta_{13}\,\chi_4\right] \;-\; 1/6\,\bar A(\chi_{46})\bar B(\chi_1,\chi_3;0,k)\,R^\eta_{13} \;+\; 1/2\,\bar A(\chi_{46};\varepsilon)\,\pi_{16}\,\chi_{46} \\
+\;& \bar B(\chi_p,\chi_p;0)\,\pi_{16}\;\left[1/16\,\bar\chi_1\,R^d_p + 1/96\,R^d_p\,\chi_p + 1/32\,R^d_p\,\chi_q\right] \;+\; 2/3\,\bar B(\chi_p,\chi_p;0)\,L^r_0\,R^d_p\,\chi_p \\
+\;& 5/3\,\bar B(\chi_p,\chi_p;0)\,L^r_3\,R^d_p\,\chi_p \;+\; \bar B(\chi_p,\chi_p;0)\,L^r_4\;\left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta0}\chi_p - 4\,\bar\chi_1R^p_{q\eta}\,\chi_p + 4\,\bar\chi_1R^c_p\,\chi_p + 3\,\bar\chi_1R^d_p\right] \\
+\;& \bar B(\chi_p,\chi_p;0)\,L^r_5\;\left[-2/3\bar\chi^{pp}_{\eta\eta1}\chi_p - 4/3\,R^p_{q\eta}\,\chi_p^2 + 4/3\,R^c_p\,\chi_p^2 + 1/2\,R^d_p\,\chi_p - 1/6\,R^d_p\,\chi_q\right] \\
+\;& \bar B(\chi_p,\chi_p;0)\,L^r_6\;\left[4\,\bar\chi_1\bar\chi^{pp}_{\eta\eta1} + 8\,\bar\chi_1R^p_{q\eta}\,\chi_p - 8\,\bar\chi_1R^c_p\,\chi_p\right] \;+\; 4\,\bar B(\chi_p,\chi_p;0)\,L^r_7\,(R^d_p)^2 \\
+\;& \bar B(\chi_p,\chi_p;0)\,L^r_8\;\left[4/3\,\bar\chi^{pp}_{\eta\eta2} + 8/3\,R^p_{q\eta}\,\chi_p^2 - 8/3\,R^c_p\,\chi_p^2\right] \;+\; \bar B(\chi_p,\chi_p;0)^2\;\left[-1/18\,R^p_{q\eta}R^d_p\,\chi_p + 1/18\,R^c_pR^d_p\,\chi_p\right. \\
+\;& \left.1/288\,(R^d_p)^2\right] \;+\; 1/18\,\bar B(\chi_p,\chi_p;0)\bar B(\chi_p,\chi_\eta;0)\;\left[R^\eta_{pp}R^d_p\,\chi_p - R^\eta_{13}R^d_p\,\chi_p\right]
\end{aligned}
$$

plus several more pages

Usual ChPT two-loop: A list

Review paper on Two-Loops: JB, LU TP 06-16 hep-ph/0604043

Two-Loop Two-Flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \to \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \to \pi^+\pi^-$, F_π, m_π: 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi$, F_π, m_π: 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t)$, $F_{S\pi}$: 1998
- JB-Talavera: $\pi \to \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \to \pi^0\pi^0$, $\gamma\gamma \to \pi^+\pi^-$: 2005-2006

Two-Loops Three flavours

- $\Pi_{VV\pi}$, $\Pi_{VV\eta}$, Π_{VVK} — Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
- $\Pi_{VV\rho\omega}$ — Maltman
- $\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η — Kambor, Golowich; Amorós, JB, Talavera
- Π_{SS} — Moussallam $\boxed{L_4^r, L_6^r}$

Usual ChPT two-loop: A list

- $\Pi_{VVK}, \Pi_{AAK}, F_K, m_K$ — Amorós, JB, Talavera
- $K_{\ell 4}$, $\langle \overline{q}q \rangle$ — Amorós, JB, Talavera $\boxed{L_1^r, L_2^r, L_3^r}$
- $F_M, m_M, \langle \overline{q}q \rangle$ $(m_u \neq m_d)$ — Amorós, JB, Talavera $\boxed{L^r_{5,7,8}, m_u/m_d}$
- $F_{V\pi}, F_{VK^+}, F_{VK^0}$ — Post, Schilcher; JB, Talavera $\boxed{L_9^r}$
- $K_{\ell 3}$ — Post, Schilcher; JB, Talavera $\boxed{V_{us}}$
- $F_{S\pi}, F_{SK}$ (includes σ-terms) — JB, Dhonte $\boxed{L_4^r, L_6^r}$
- $K, \pi \to \ell\nu\gamma$ — Geng, Ho, Wu $\boxed{L_{10}^r}$
- $\pi\pi$ — JB,Dhonte,Talavera
- πK — JB,Dhonte,Talavera

$K_{\ell 3}$

- H. Leutwyler and M. Roos, Z.Phys.C25:91,1984.
- J. Gasser and H. Leutwyler, Nucl.Phys.B250:517-538,1985.
- J. Bijnens and P. Talavera, hep-ph/0303103, Nucl. Phys. B669 (2003) 341-362
- V. Cirigliano et al., hep-ph/0110153, Eur.Phys.J.C23:121-133,2002.

$K_{\ell 3}$ Definitions

$$K^+_{\ell 3}: \quad K^+(p) \rightarrow \pi^0(p')\ell^+(p_\ell)\nu_\ell(p_\nu)$$

$$K^0_{\ell 3}: \quad K^0(p) \rightarrow \pi^-(p')\ell^+(p_\ell)\nu_\ell(p_\nu)$$

$$
\begin{aligned}
K^+_{\ell 3}: \quad T &= \frac{G_F}{\sqrt{2}} V^\star_{us} \ell^\mu F_\mu{}^+(p',p) \\
\ell^\mu &= \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_\ell) \\
F_\mu{}^+(p',p) &= <\pi^0(p') \mid V_\mu^{4-i5}(0) \mid K^+(p)> \\
&= \frac{1}{\sqrt{2}}\left[(p'+p)_\mu f_+^{K^+\pi^0}(t) + (p-p')_\mu f_-^{K^+\pi^0}(t)\right]
\end{aligned}
$$

Isospin: $f_+^{K^0\pi^-}(t) = f_+^{K^+\pi^0}(t) = f_+(t)$

$f_-^{K^0\pi^-}(t) = f_-^{K^+\pi^0}(t) = f_-(t)$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization: $$f_{+,0}(t) = f_+(0)\left(1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right)$$

$|V_{us}|$:

- Know theoretically $f_+(0) = 1 + \cdots$
  - Short distance correction to G_F from G_μ Marciano-Sirlin
  - Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
  - Isospin Breaking Leutwyler-Roos
  - Radiative corrections: Cirigliano et al
- Know experimentally $f_+(0)$

V_{us}

PDG2002:

$|V_{ud}| = 0.9734 \pm 0.0008$ $\qquad$ $|V_{us}| = 0.2196 \pm 0.0026$

$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) =$
0.9957 ± 0.0019

PDG2006:

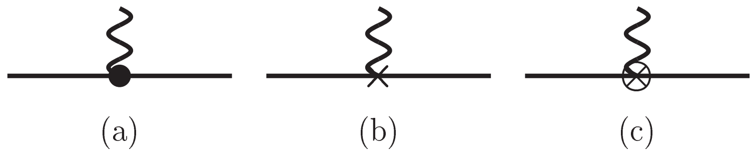
$|V_{ud}| = 0.97377 \pm 0.00027$ $\qquad$ $|V_{us}| = 0.2257 \pm 0.0021$

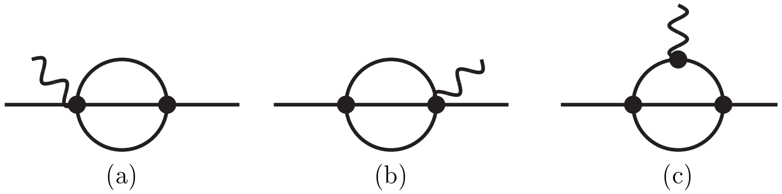
$|V_{ud}|^2 + |V_{us}|^2 = (0.94823 \pm 0.00054) + (0.05094 \pm 0.00095) =$
0.99917 ± 0.00110

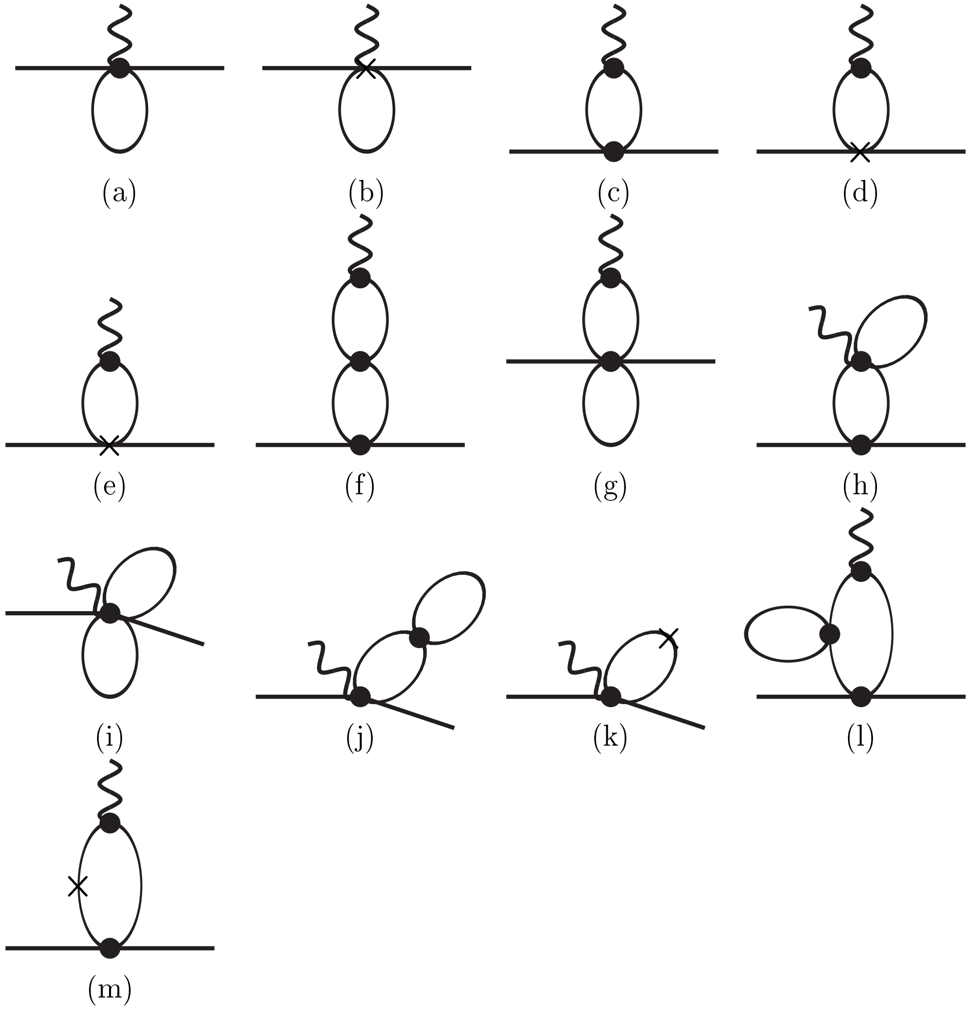
Problems:

- Ignores $\Delta(0) = 0.0113$ from pure two-loop
- Conflicts between experiments

$K_{\ell 3}$ Diagrams







● : p^2 vertex

× : p^4 vertex

⊗ : p^6 vertex

$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4}(-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

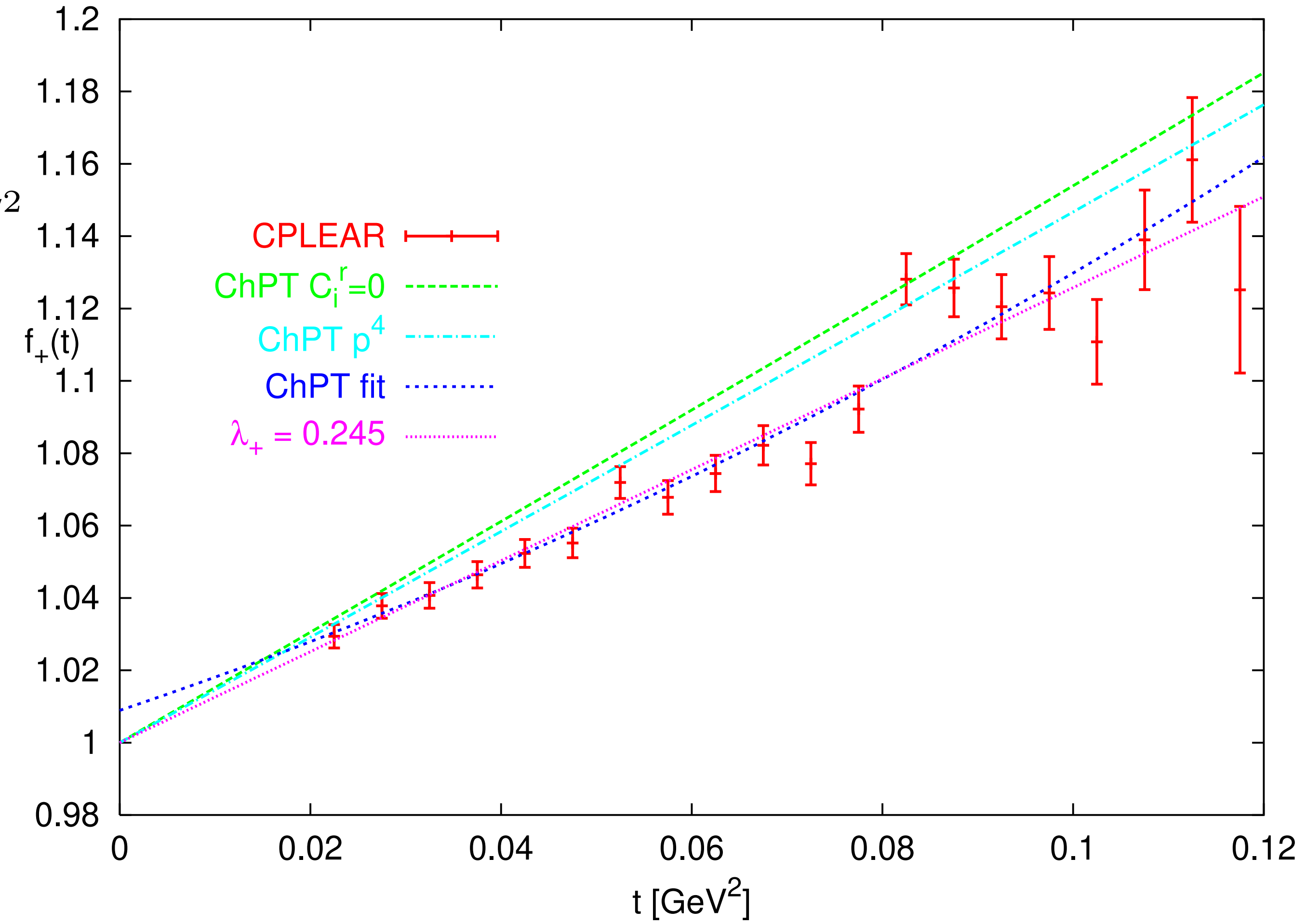
ChPT fit to $f_+(t)$

$\Rightarrow R^{K\pi}_{+1} =$
$-(4.7 \pm 0.5)\ 10^{-5}$ GeV^2

$(c_+ = 3.2$ GeV$^{-4})$

$\Rightarrow a_+ = 1.009 \pm 0.004$

$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$



$f_0(t)$

Main Result:

$$
\begin{aligned}
f_0(t) \;=\; & 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 \\
& + 8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
& - \frac{8}{F_\pi^4} t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6

$\Longrightarrow$

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.$$

Experiment

Form Factor comparison

KTeV [PRD 70(2004)]

K^0_{e3} quadratic fit: $\lambda''_+ \neq 0$ @ 4σ level

$K^0_{\mu 3}$ quadraqtic fit: $\lambda_0 = (13.72 \pm 1.31)\ 10^{-3}$

Slopes consistent for K^0_{e3} and $K^0_{\mu 3}$

ISTRA+ [PLB 581(2004), PLB589(2004)]

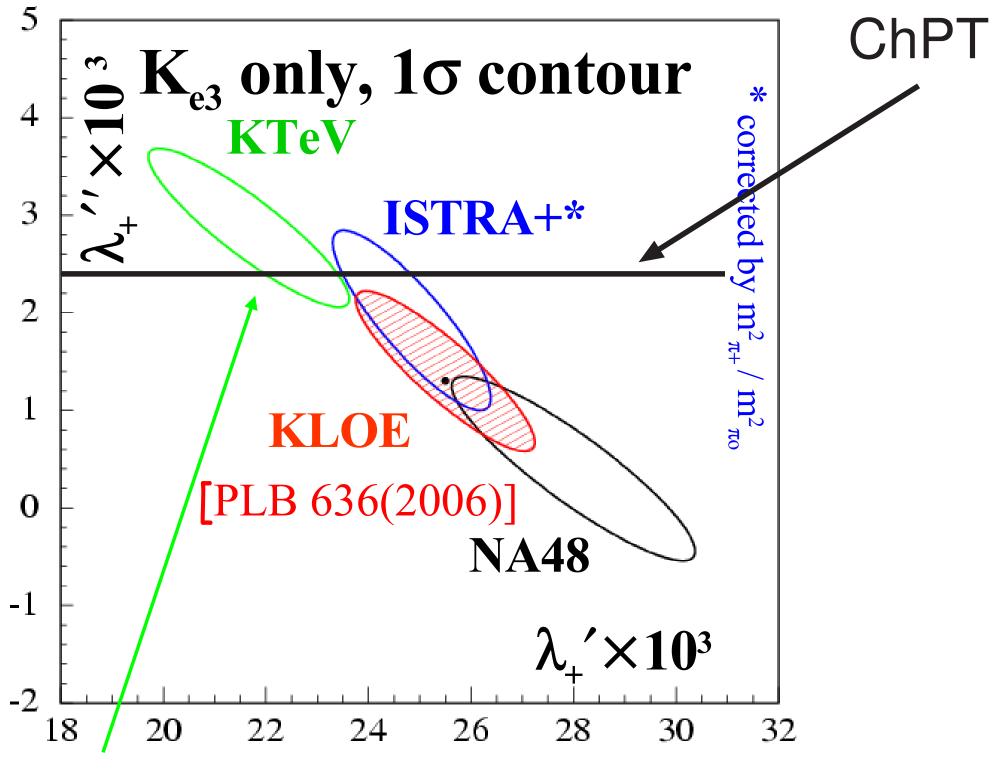
K^-_{e3} quadratic fit: $\lambda''_+ \neq 0$ @ 2σ level

$K^-_{\mu 3}$ quadratic fit: $\lambda_0 = (17.11 \pm 2.31)\ 10^{-3}$

NA48 [PLB 604(2004), HEP2005 289]

K^0_{e3}: No evidence for quadratic term

$K^0_{\mu 3}$ linear fit: $\lambda_0 = (12.0\ \pm 1.7)\ 10^{-3}$



$\lambda''_+ \rightarrow -1\%$ phase space integral

$\rightarrow +0.5\%$ for V_{us}

A WG on Precise SM tests in K decays M.Antonelli LNF-INFN – FLAVIANET meeting Barcelona

Experiment

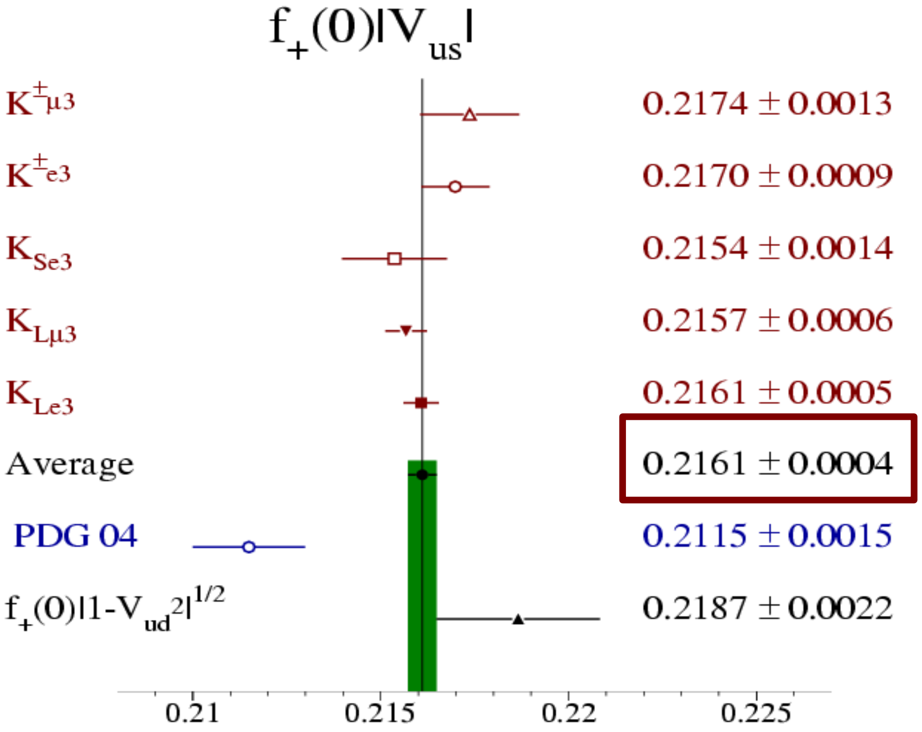
V_{us} determination from Kl3

	$K_L e3$	$K_L \mu3$	$K_S e3$	$K^{\pm} e3$	$K^{\pm} \mu3$
BR	0.4046(8)	0.2697(7)	$7.046(91)\times10^{-4}$	0.05043(31)	0.03383(27)
τ	51.10(19) ns		89.58(6) ps	12.386(22) ns	

$\lambda'_+ = 0.02496(79)$

$\lambda''_+ = 0.0016(3)$

$\lambda_0 = 0.01587(95)$



- $f_+(0)=0.961(8)$

  Leutwyler and Roos Z. [Phys. **C25**, 91, 1984]
- $V_{ud}=0.97377(27)$

  Marciano and Sirlin [Phys.Rev.Lett.96 032002,2006]

$$\boxed{V_{us} = 0.2249 \pm 0.0019}$$

From $R_{e\mu}$:

$$\boxed{\frac{g_e}{g_\mu} = 1.0014 \pm 0.0037}$$

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Experiment

$V_{us} - V_{ud}$ plane

Combining the experimental value of $\Gamma(K\to\mu\nu(\gamma))/\Gamma(\pi\to\mu\nu(\gamma))$

with the ratio f_K/f_π obtained from lattice calculations we can extract $|V_{us}|/|V_{ud}|$

(Marciano hep-ph/0406324) $\Gamma(K\to\mu\nu(\gamma))/\Gamma(\pi\to\mu\nu(\gamma)) \propto |V_{us}|^2/|V_{ud}|^2 f_K^2/f_\pi^2$

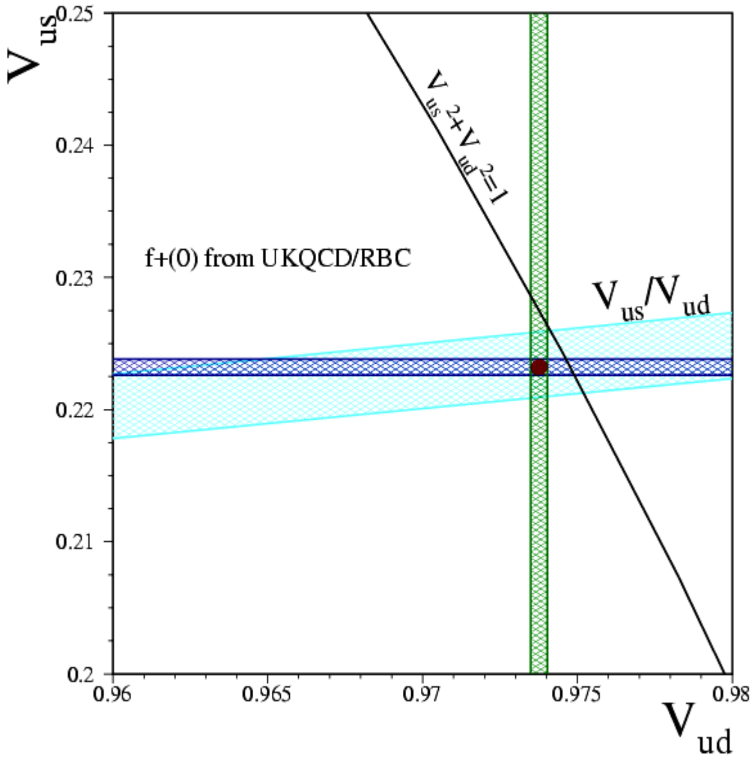
Using $f_K/f_\pi = 1.198(3)(^{+16}_{-5})$ from MILC

and **KLOE BR($K^+ \to \mu^+\nu$)**

we get $V_{us}/V_{ud} = 0.2294 \pm 0.0026$

$\delta = |Vus|^2 + |Vus|^2 - 1 = -0.0020 \pm 0.0006$

NEED TO BE CONFIRMED



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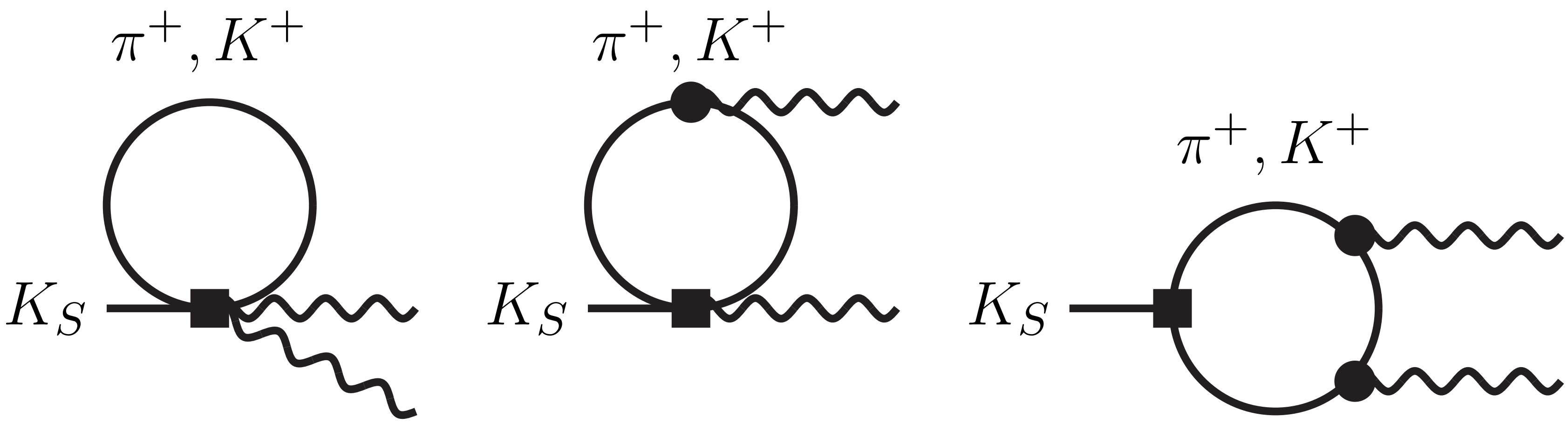
$K_S \to \gamma\gamma$

Well predicted by CHPT at order p^4 from Goity, D'Ambrosio, Espriu



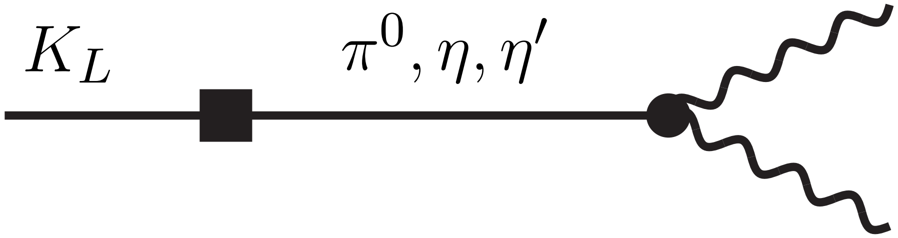
Prediction was: BR = $2.1 \cdot 10^{-6}$

NA48: $2.78(6)(4) \cdot 10^{-6}$

Some other rare decays

- $K_L \to \gamma\gamma$
  Needs work: main contribution is full of cancellations:

difficult



- $K_L \to \pi^0\gamma\gamma$
  $K_L \to \pi^0\gamma\gamma$ OK predicted by CHPT
  Main succes: events must be at high $m_{\gamma\gamma}$
  Rate still a problem
- $K_S \to \pi^0\gamma\gamma$
  Similar problems as in $K_L \to \gamma\gamma$
- Ecker, Pich, de Rafael, D'Ambrosio,. . .

$K \to 3\pi$

- $K \to 3\pi$ Decays in Chiral Perturbation Theory, J. Bijnens, P. Dhonte and F. Persson, hep-ph/0205341, Nucl. Phys. B648 (2003) 317-344.
- Isospin Breaking in $K \to 3\pi$ Decays I: Strong Isospin Breaking, J. Bijnens and F. Borg, hep-ph/0405025, Nuclear Physics B697 (2004) 319-342.
- Isospin Breaking in $K \to 3\pi$ Decays II: Radiative Corrections, J. Bijnens and F. Borg, hep-ph/0410333, Eur. Phys. J. C39 (2005) 347-357.
- Isospin Breaking in $K \to 3\pi$ Decays III: Bremsstrahlung and Fit to Experiment, J. Bijnens and F. Borg, hep-ph/0501163,Eur. Phys. J. C40 (2005) 383-394

Note: Fredrik Borg = Fredrik Persson

$K \to 3\pi$: Overview

- ChPT in the nonleptonic mesonic sector
- Lagrangians
- $K \to 3\pi$ kinematics and isospin
- Overview of calculations and results
- Data and Fits

ChPT in the nonleptonic sector

- some earlier work: especially on decays with photons
- Kambor, Missimer, Wyler (KMW) : Constructed $\mathcal{L}$ and ∞ 1990
- G. Esposito-Farese: Checked $\mathcal{L}$ and ∞ 1991
- KMW : Calculated $K \to 2\pi$ and $K \to 3\pi$ 1991
- Donoghue + Holstein + KMW : clarified the relations between observables 1992
- BUT: explicit formulas lost (US Mail)
- Kambor, Ecker, Wyler : simplified octet $\mathcal{L}$ 1993
- $K \to 2\pi$ redone: Bijnens, Pallante, Prades 1998

ChPT in the nonleptonic sector

- First paper: redo $K \to 3\pi$
- Ecker Isidori Muller Neufeld Pich: Electromagnetic octet $\mathcal{L}$ Lagrangian plus ∞ 2000
- applications to $K \to 2\pi$: Several papers
- Isospin breaking in $K \to 3\pi$: remaining papers $\mathcal{O}\left(p^4, p^2(m_u - m_d), e^2p^2\right)$.

Lagrangians: p^2 and e^2

$$\mathcal{L}_2 = \mathcal{L}_{S2} + \mathcal{L}_{W2} + \mathcal{L}_{E2} \qquad\qquad \mathcal{L}_{S2} = \frac{F_0^2}{4}\left\langle u_\mu u^\mu + \chi_+ \right\rangle$$

$$u_\mu = i u^\dagger D_\mu U \, u^\dagger = u_\mu^\dagger \,, \quad u^2 = U \,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \,,$$

U contains the Goldstone boson fields

$$U = \exp\left(\frac{i\sqrt{2}}{F_0} M\right) \,, \qquad M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & \frac{-2}{\sqrt{6}}\eta_8 \end{pmatrix} .$$

Here $\quad \chi = 2B_0 \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \quad$ and $\quad D_\mu U = \partial_\mu U - ie\,[Q, U]\,.$

Lagrangians: p^2 and e^2

$$\mathcal{L}_{W2} = C F_0^4 \left[G_8 \langle \Delta_{32} u_\mu u^\mu \rangle + G_8' \langle \Delta_{32} \chi_+ \rangle + G_{27} t^{ij,kl} \, \langle \Delta_{ij} u_\mu \rangle \langle \Delta_{kl} u^\mu \rangle \right]$$

$$\Delta_{ij} \equiv u \lambda_{ij} u^\dagger \,, \qquad (\lambda_{ij})_{ab} \equiv \delta_{ia} \, \delta_{jb} \,, \qquad t^{21,13} = t^{13,21} = 1/3, \ldots$$

Lagrangians: p^2 and e^2

$$\mathcal{L}_{W2} = C F_0^4 \left[G_8 \langle \Delta_{32} u_\mu u^\mu \rangle + G_8' \langle \Delta_{32} \chi_+ \rangle + G_{27} t^{ij,kl} \langle \Delta_{ij} u_\mu \rangle \langle \Delta_{kl} u^\mu \rangle \right]$$

$$\Delta_{ij} \equiv u \lambda_{ij} u^\dagger \,, \quad (\lambda_{ij})_{ab} \equiv \delta_{ia} \delta_{jb} \,, \quad t^{21,13} = t^{13,21} = 1/3, \ldots$$

$C = -\frac{3}{5} \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^*$: chiral and large N_c limits $G_8 = G_{27} = 1$

Lagrangians: p^2 and e^2

$$\mathcal{L}_{W2} = C F_0^4 \left[G_8 \langle \Delta_{32} u_\mu u^\mu \rangle + G_8' \langle \Delta_{32} \chi_+ \rangle + G_{27} t^{ij,kl} \, \langle \Delta_{ij} u_\mu \rangle \langle \Delta_{kl} u^\mu \rangle \right]$$

$$\Delta_{ij} \equiv u \lambda_{ij} u^\dagger \,, \qquad (\lambda_{ij})_{ab} \equiv \delta_{ia} \, \delta_{jb} \,, \qquad t^{21,13} = t^{13,21} = 1/3, \ldots$$

$C = -\frac{3}{5} \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^*$: chiral and large N_c limits $G_8 = G_{27} = 1$

Electromagnetic:

$$\mathcal{L}_{E2} = e^2 F_0^4 Z \langle \mathcal{Q}_L \mathcal{Q}_R \rangle + \left(e^2 C F_0^6 G_E \langle \Delta_{32} \mathcal{Q}_R \rangle \right)$$

$$\mathcal{Q}_L = u Q u^\dagger \,, \qquad \mathcal{Q}_R = u^\dagger Q u \quad \text{with} \quad Q = \text{diag}\,(2/3, -1/3, -1/3)$$

Lagrangians: p^2 and e^2

$$\mathcal{L}_{W2} = C F_0^4 \left[G_8 \langle \Delta_{32} u_\mu u^\mu \rangle + G_8' \langle \Delta_{32} \chi_+ \rangle + G_{27} t^{ij,kl} \, \langle \Delta_{ij} u_\mu \rangle \langle \Delta_{kl} u^\mu \rangle \right]$$

$$\Delta_{ij} \equiv u \lambda_{ij} u^\dagger \,, \qquad (\lambda_{ij})_{ab} \equiv \delta_{ia} \, \delta_{jb} \,, \qquad t^{21,13} = t^{13,21} = 1/3, \ldots$$

$C = -\frac{3}{5} \frac{G_F}{\sqrt{2}} V_{ud} \, V_{us}^*$: chiral and large N_c limits $G_8 = G_{27} = 1$

Electromagnetic:

$$\mathcal{L}_{E2} = e^2 F_0^4 Z \langle \mathcal{Q}_L \mathcal{Q}_R \rangle + \left(e^2 C F_0^6 G_E \langle \Delta_{32} \mathcal{Q}_R \rangle \right)$$

$$\mathcal{Q}_L = u Q u^\dagger \,, \qquad \mathcal{Q}_R = u^\dagger Q u \quad \text{with} \quad Q = \text{diag}\,(2/3, -1/3, -1/3)$$

Note: F_0 not well known, fit $C F_0^4 G_8, \ldots$

use *numerically* $F_0 = F_\pi$ to quote $G_8, \ldots$

Lagrangians: p^4 and e^2p^2

$$\mathcal{L}_4 = \mathcal{L}_{S4} + \mathcal{L}_{W4} + \mathcal{L}_{S2E2} + \mathcal{L}_{W2E2}(G8)\,.$$

$\mathcal{L}_{S4}$ values of L_i^r use Amoros, Bijnens, Talavera p^4 fit

$\mathcal{L}_{W4}$ thirteen N_i^r (octet) and twelve D_i^r might contribute (two N_i^r and two D_i^r extra for photon-reducible)

$\mathcal{L}_{S2E2}$ eleven K_i^r can contribute

$\mathcal{L}_{W2E2}$ fourteen Z_i^r can contribute, 27 part not classified

Lagrangians: p^4 and e^2p^2

Done here :

Isospin conserved: 11 combinations of N_i^r, D_i^r relevant: $\tilde{K}_i$

7 coefficients order m_K^4, 4 order $M_K^2 m_\pi^2$

2 (virtually) indistinguishable from G_8 and G_{27}.

Isospin Broken: 30 combinations of N_i^r, D_i^r, Z_i^r relevant

Results from isospin breaking with $K_i^r = Z_i^r = 0$

$K \to 3\pi$ Kinematics and Isospin

$$
\begin{aligned}
K_L(k) &\rightarrow \pi^0(p_1)\pi^0(p_2)\pi^0(p_3)\,, \quad [A^L_{000}]\,,\\
K_L(k) &\rightarrow \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)\,, \quad [A^L_{+-0}]\,,\\
K_S(k) &\rightarrow \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)\,, \quad [A^S_{+-0}]\,,\\
K^+(k) &\rightarrow \pi^0(p_1)\pi^0(p_2)\pi^+(p_3)\,, \quad [A_{00+}]\,,\\
K^+(k) &\rightarrow \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\,, \quad [A_{++-}]\,,
\end{aligned}
$$

plus charge conjugate ones
even under $p_1 \leftrightarrow p_2$ except A^S_{+-0} odd

$K \to 3\pi$ Kinematics and Isospin

Kinematics: $s_1 = (k-p_1)^2\ ,\ s_2 = (k-p_2)^2\ ,\ s_3 = (k-p_3)^2\ .$

Dalitz plot variables:

$$y = (s_3 - s_0)/m^2_{\pi^+}\ , \ x = (s_2 - s_1)/m^2_{\pi^+}\ , \ s_0 = (s_1 + s_2 + s_3)\,/3\,.$$

Bremsstrahlung problem: s_i from E_i or $m^2_{\pi\pi}$ different

Decay amplitudes squared expanded as

$$\left|\frac{A(s_1,s_2,s_3)}{A(s_0,s_0,s_0)}\right|^2 = 1 + gy + hy^2 + kx^2$$

$K_L \to \pi^0\pi^0\pi^0$: $g = 0$ and $k = h/3$.

Kinematics and Isospin

Valid also at p^6

$$
\begin{aligned}
A^L_{000} &= M_0(s_1)+M_0(s_2)+M_0(s_3)\,,\\
A^L_{+-0} &= M_1(s_3)+M_2(s_1)+M_2(s_2)+M_3(s_1)(s_2-s_3)+M_3(s_2)(s_1-s_3)\,,\\
A^S_{+-0} &= M_4(s_1)-M_4(s_2)+M_5(s_1)(s_2-s_3)-M_5(s_2)(s_1-s_3)+M_6(s_3)(s_1-s_2)\,,\\
A_{00+} &= M_7(s_3)+M_8(s_1)+M_8(s_2)+M_9(s_1)(s_2-s_3)+M_9(s_2)(s_1-s_3)\,,\\
A_{++-} &= M_{10}(s_3)+M_{11}(s_1)+M_{11}(s_2)+M_{12}(s_1)(s_2-s_3)+M_{12}(s_2)(s_1-s_3)\,.
\end{aligned}
$$

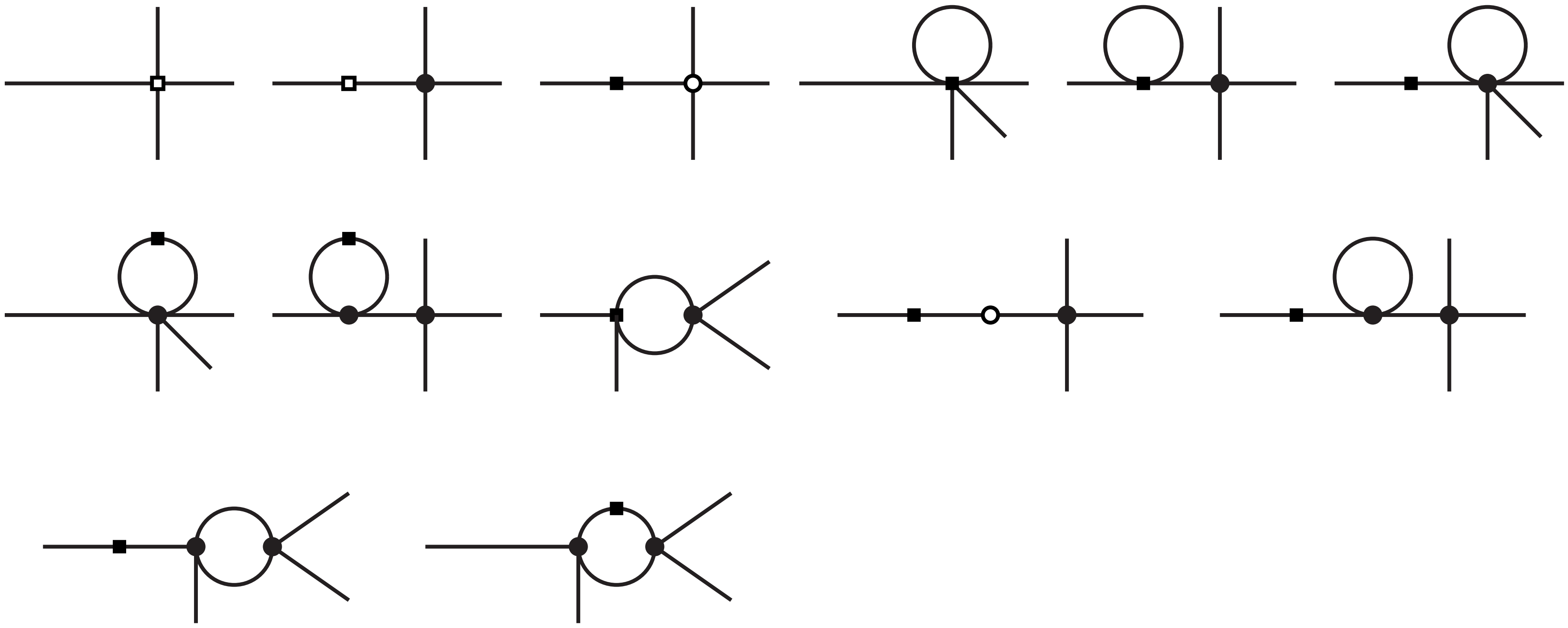
polynomial ambiguity from $s_1 + s_2 + s_3 = \sum_i m_i^2$

Isospin:

$$
\begin{aligned}
M_0(s) &= M_1(s) + 2M_2(s)\,,\\
2M_7(s) + 4M_8(s) &= M_{10}(s) + 2M_{11}(s)\,,\\
M_4(s) &= \frac{1}{3}\left(M_7(s) - M_8(s) + M_{10}(s) - M_{11}(s)\right)\\
M_5(s) - M_6(s) &= M_9(s) + M_{12}(s)\,.
\end{aligned}
$$

Calculations: isospin limit

The diagrams of order p^4



Expressions for the $M_i(s)$: see first paper

Confirmed by Gamiz, Prades, Scimemi, hep-ph/0305164
Lashin, hep-ph/0308200 (how not clear)

Calculations: Strong Isospin Breaking

Includes:

- $m_u - m_d$ and the effects of $\mathcal{L}_{E2}$, $\mathcal{L}_{E2S2}$ and $\mathcal{L}_{W2E2}(G_8)$
- π^0-η mixing (η' via LECs)
- Same diagrams as before
- Formulas immediately a lot longer:
  see

`http://www.thep.lu.se/~bijnens/chpt.html`

Calculations: Strong Isospin Breaking

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- Same diagrams as before
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  see
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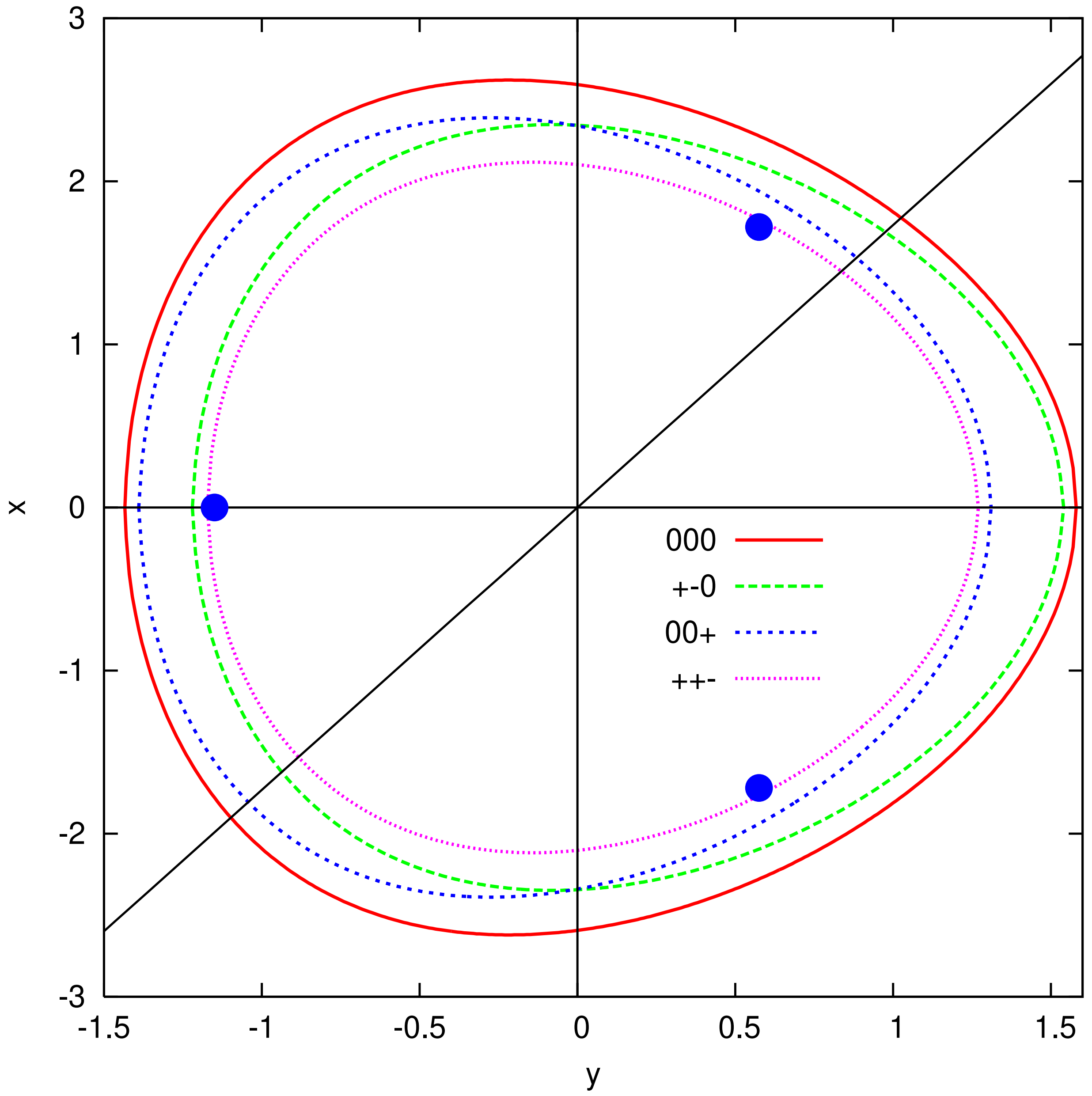
Effects:

- F_{π^+}, F_{K^+} overall factor
- Mass difference π^+-π^0
- Others typically a few %

Phasespace



Curves:

Phasespace boundaries

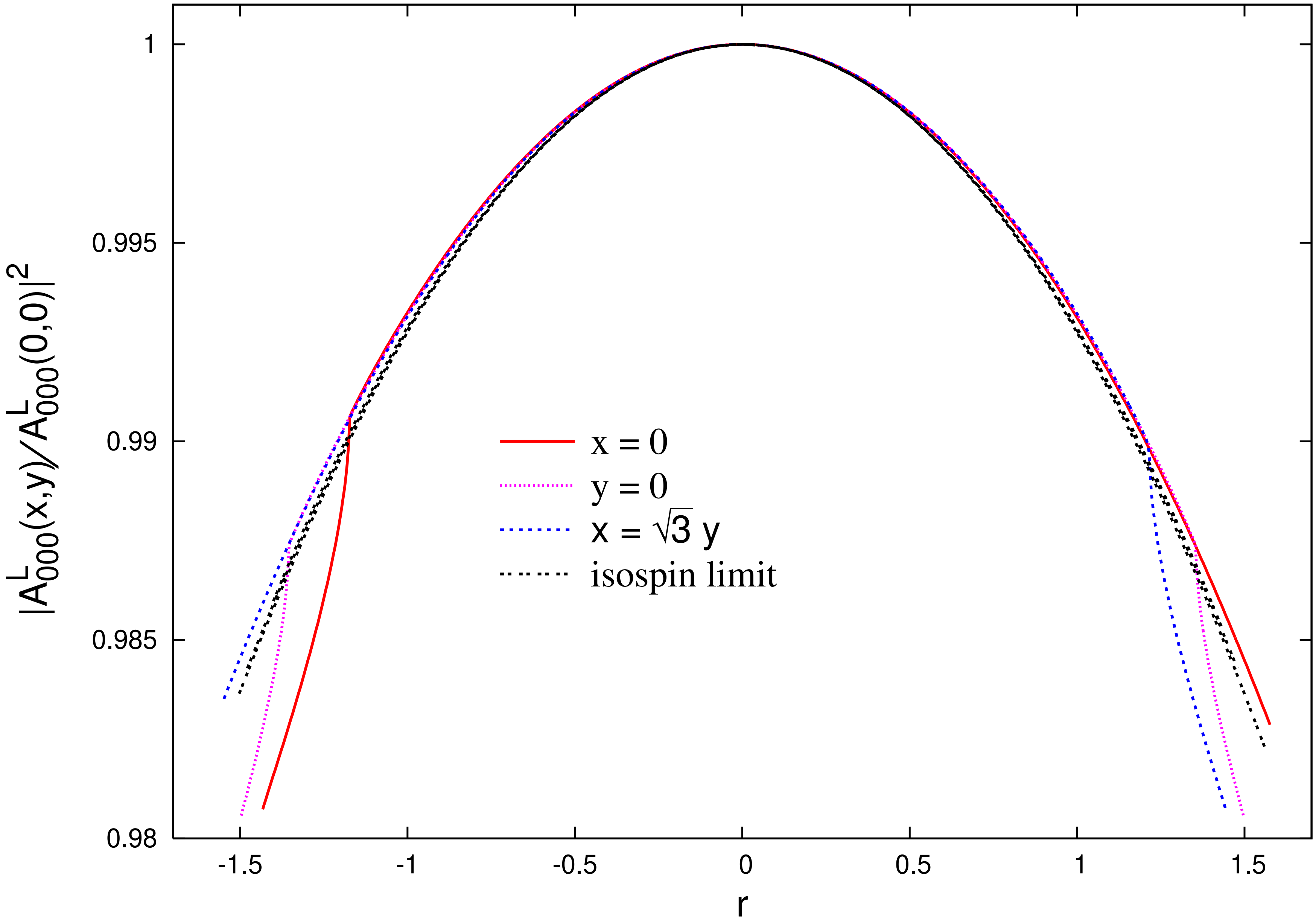
Dots:

$\pi\pi$ pair at rest

Lines:

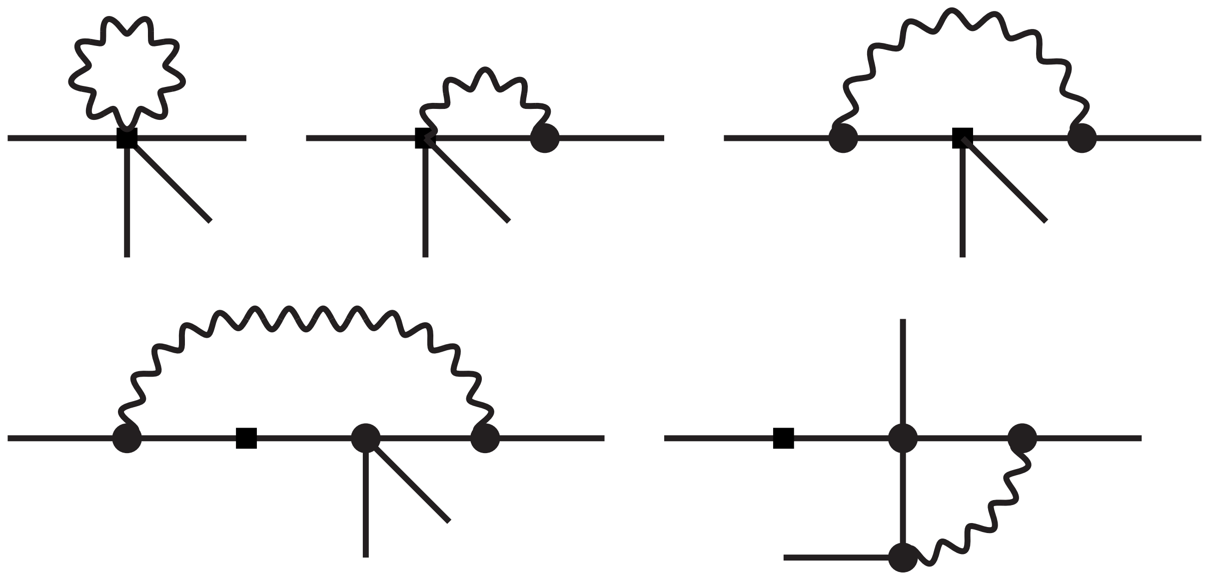
Show $|A|^2$ along these

$K_L \to \pi^0\pi^0\pi^0$



Calculations: Radiative Corrections

- Photon Loops



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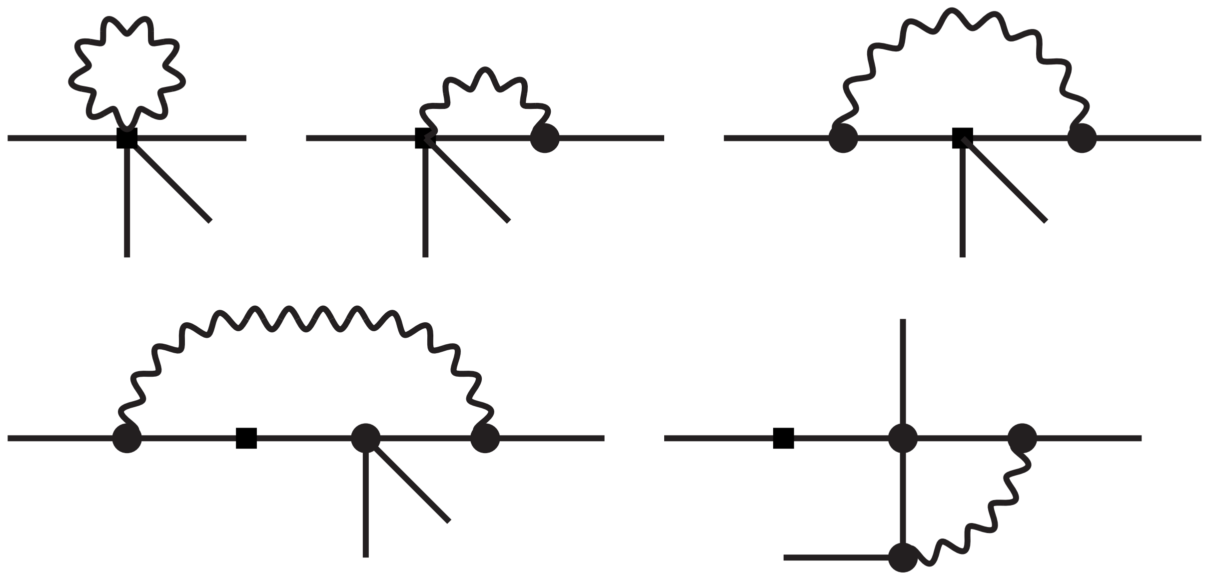
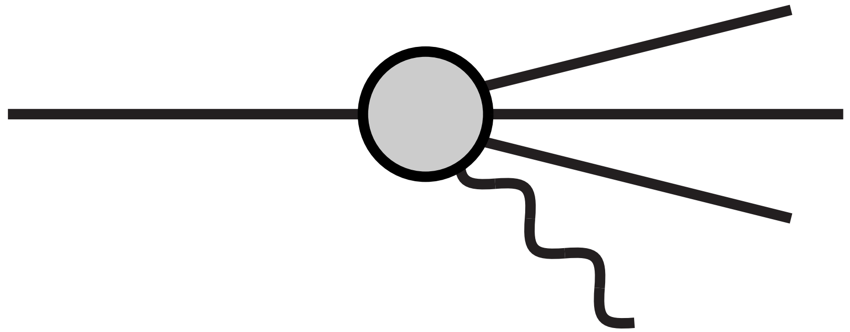
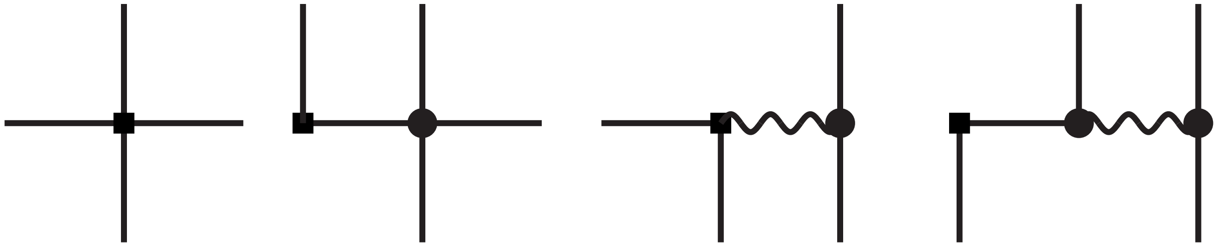


- Soft Bremsstrahlung



Calculations: Radiative Corrections

- Photon reducible diagrams

- Tree Level: 

Calculations: Radiative Corrections

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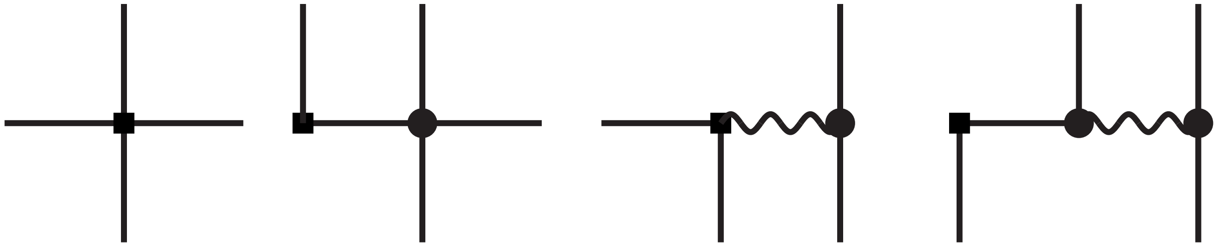

Extra vanish exactly

Calculations: Radiative Corrections

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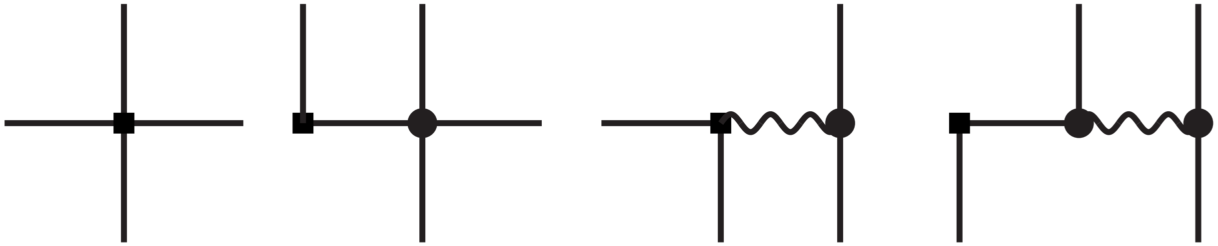
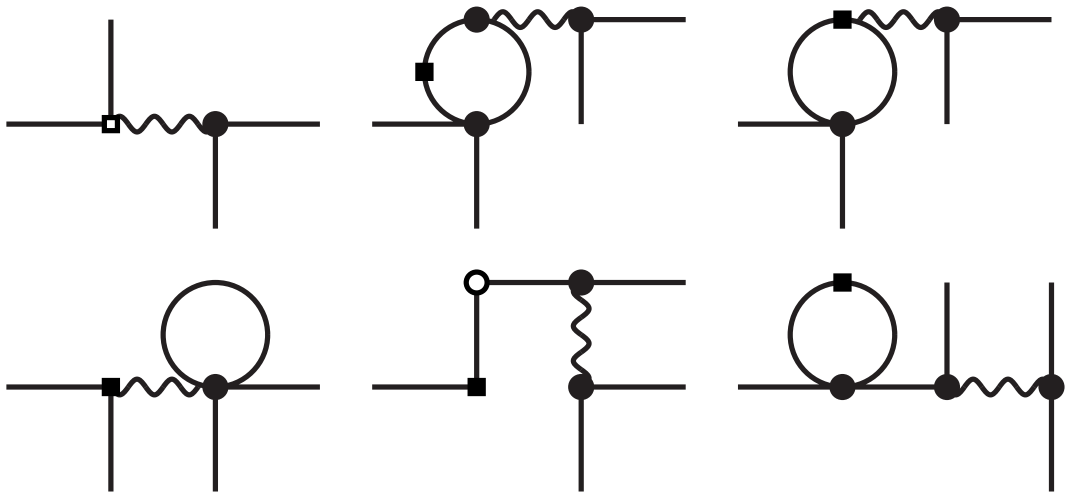
- Tree Level:



Extra vanish exactly

- One-loop Level:

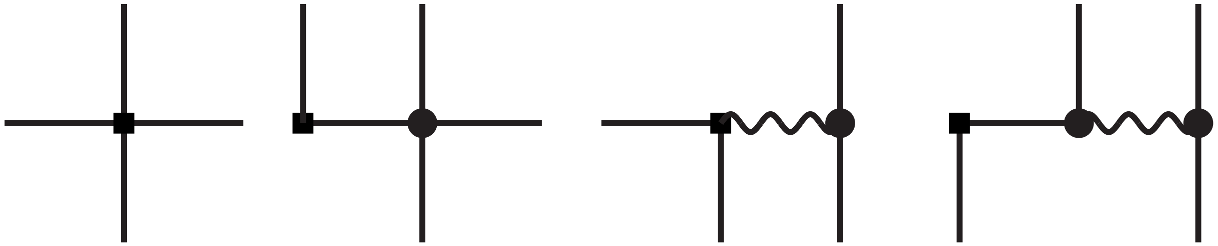


Extra N_i^r, D_i^r from $K \to \pi \ell^+ \ell^-$

Calculations: Radiative Corrections

- Photon reducible diagrams

- Tree Level:



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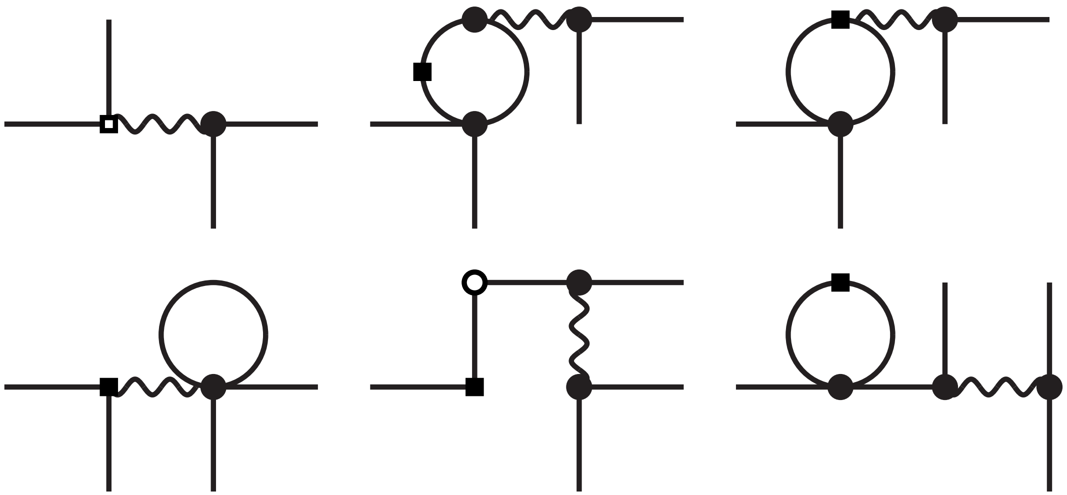
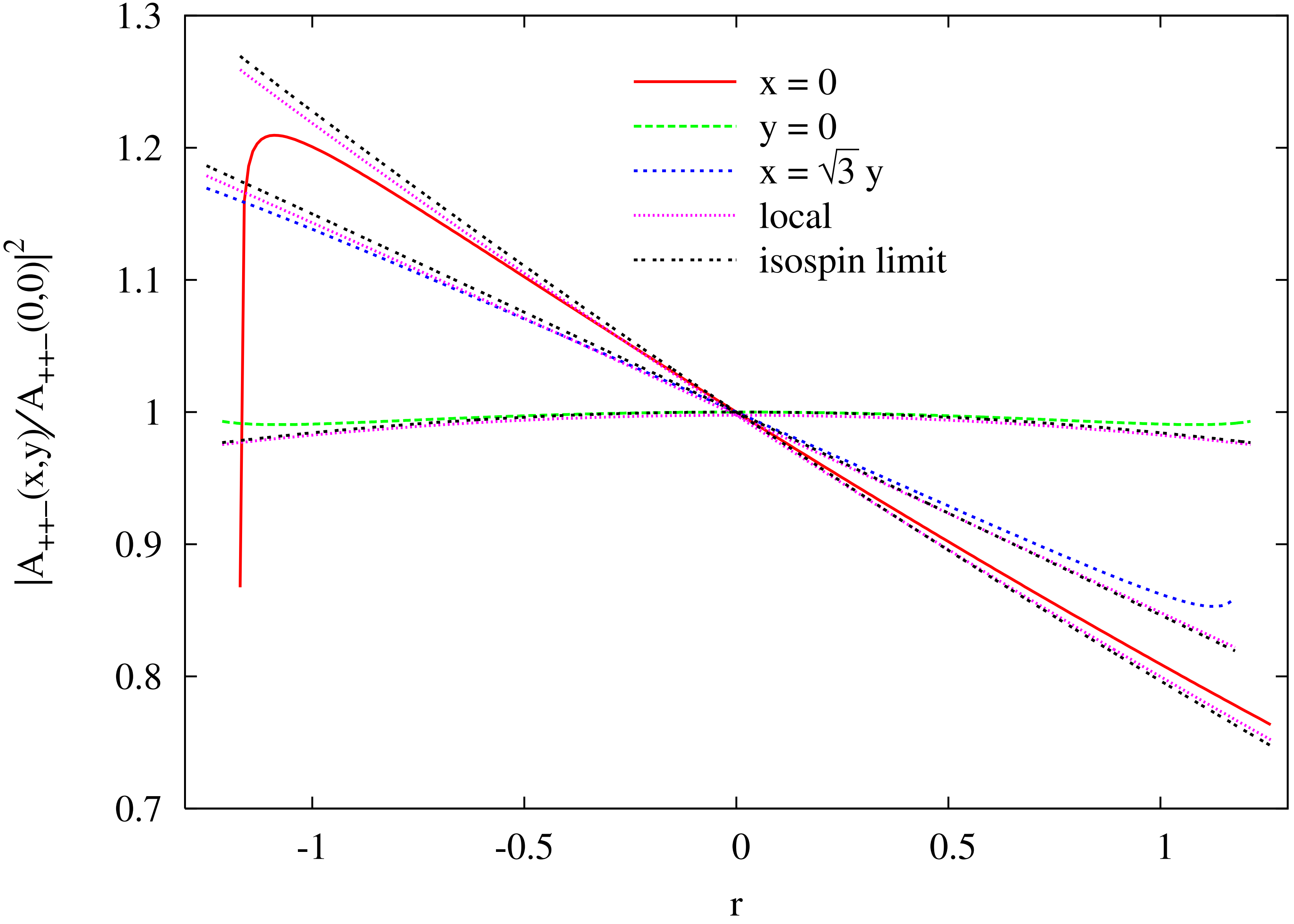


Extra N_i^r, D_i^r from $K \to \pi \ell^+ \ell^-$

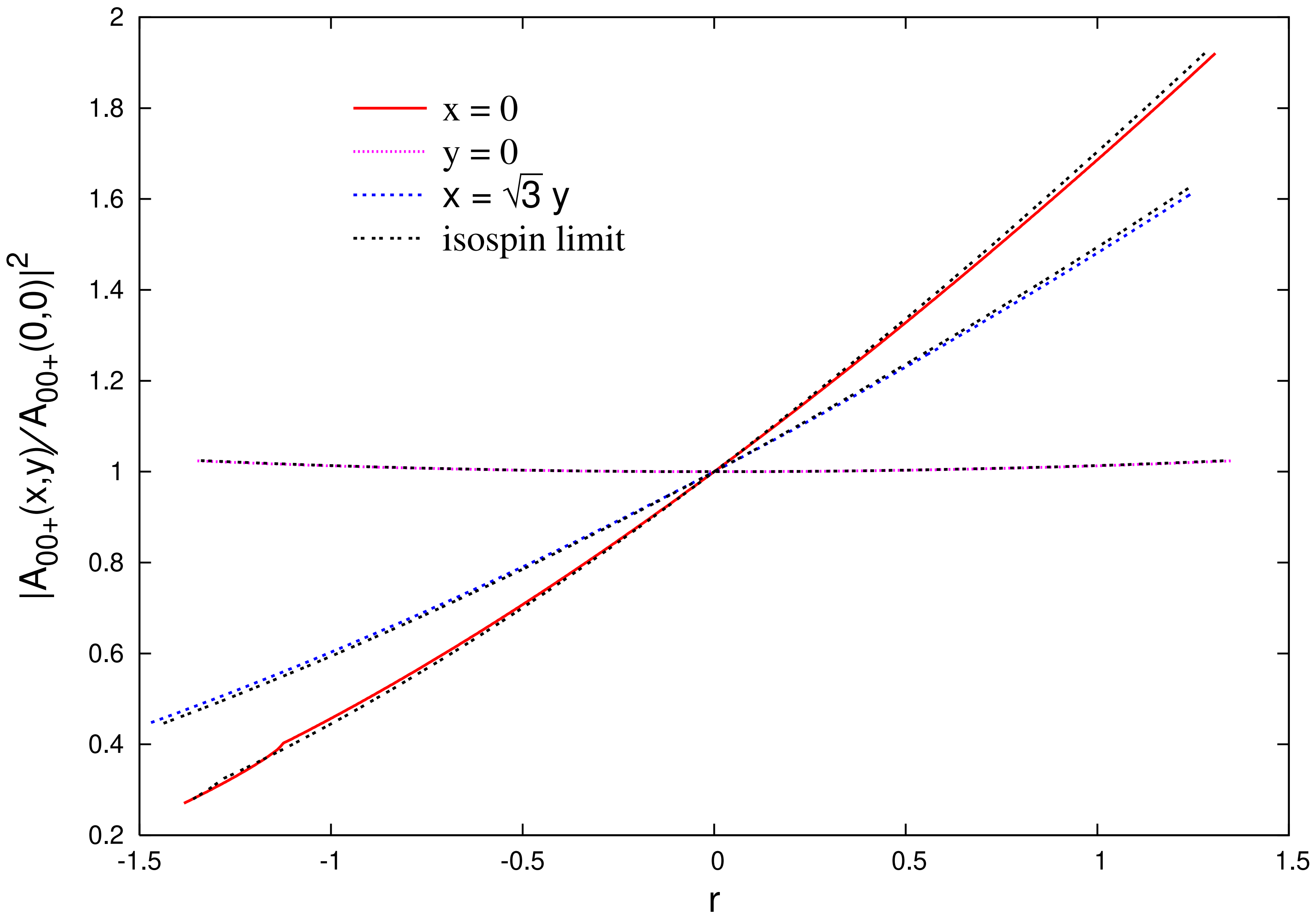
Numerically Negligible

$K^+ \to \pi^+\pi^+\pi^-$



x = 0
y = 0
x = √3 y
local
isospin limit
1.3
1.2
1.1
1
0.9
0.8
0.7
-1
-0.5
0
0.5
1
r
|A++−(x,y)/A++−(0,0)|²

$K^+ \to \pi^0\pi^0\pi^+$



Note: Disagree numerically with Nehme, hep-ph/0406209

Calculations: Hard Bremsstrahlung

- Include hard photon Bremsstrahlung to lowest order
  Full amplitude from Low's theorem
- Do everything also for $K \to 2\pi$ to have same treatment
  Note: some new pieces, e.g., $e^2 p^2 G_{27}$

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  Full amplitude from Low's theorem
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Checks:
- $\circledast$ Everything done twice indepently
- $\circledast$ UV infinities cancel analytically: no $1/(d-4)$
- $\circledast$ IR infinities cancel analytically: no m_γ
- $\circledast$ soft-hard Bremsstrahlung match: no E_γ
  (except for F_π, F_K parts)

Treatment of Bremsstrahlung

Old experiments: what to do?

Typically: Decay rate and distribution measured in part of phasespace and then extrapolated to whole decay region

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Old experiments: what to do?

Typically: Decay rate and distribution measured in part of phasespace and then extrapolated to whole decay region

We adopted:

- Subtract hard Bremsstrahlung from Decay Rate
- From remainder get $|A(s_0, s_0, s_0)|^2$ using experimental g, h, k
- Fit $|A(s_0, s_0, s_0)|^2$, g, h, k with soft Bremsstrahlung and loops, . . .

Advantages:

- Mimicks experimental determination (well, sort of)
- Coulomb singularity avoided

Results

$K \to \pi\pi$ only: essentially same as Ecker et al.

Tree level:	$G_8 = 10.36$	$G_{27} = 0.550$
Full:	$G_8 = 5.39$	$G_{27} = 0.359$

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Tree level: $G_8 = 10.36$ $G_{27} = 0.550$

Full: $G_8 = 5.39$ $G_{27} = 0.359$

Full fit:

μ	0.77 GeV	1.0 GeV	0.6 GeV	0.77 GeV
G_8	5.39(1)	4.60(1)	6.43(1)	5.39(1)
G_{27}	0.359(2)	0.301(1)	0.438(2)	0.359(2)
$\delta_2 - \delta_0$	$-57.9(1.5)^o$	$-57.3(1.4)^o$	$-58.9(1.4)^o$	$-57.9(1.4)^o$
$10^3 \tilde{K}_1/G_8$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$
$10^3 \tilde{K}_2/G_8$	48.5(2.4)	56.5(2.4)	41.2(1.9)	46.6(1.6)
$10^3 \tilde{K}_3/G_8$	2.6(1.2)	−1.7(1.1)	6.7(1.0)	3.5(0.8)
$10^3 \tilde{K}_4/G_{27}$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$
$10^3 \tilde{K}_5/G_{27}$	−41.2(16.9)	−52.0(17.7)	−31.1(12.0)	−27.0(8.3)
$10^3 \tilde{K}_6/G_{27}$	−102(105)	−114(105)	$-93(76)$	$\equiv 0$
$10^3 \tilde{K}_7/G_{27}$	78.6(33)	78.0(33.5)	79.6(22.7)	50.0(13.0)
χ^2/DOF	29.3/10	27.2/10	33.0/10	30.5/11

vary $\tilde{K}_1$, $\tilde{K}_4$: see first paper

Octet and μ-variation

μ	octet 0.77 GeV	μ variation 1.0 GeV	μ variation 0.6 GeV
G_8	4.84(1)	–	–
G_{27}	0.430(1)	–	–
$\delta_2 - \delta_0$	$-57.9(0.2)^o$	–	–
$10^3\tilde{K}_1/G_8$	2.0(1)	-5.88	5.61
$10^3\tilde{K}_2/G_8$	63.0(1.5)	-2.69	2.57
$10^3\tilde{K}_3/G_8$	$-6.0(7)$	0.159	-0.152
$10^3\tilde{K}_4/G_{27}$	$\equiv 0$	-9.93	9.48
$10^3\tilde{K}_5/G_{27}$	$\equiv 0$	0	0
$10^3\tilde{K}_6/G_{27}$	$\equiv 0$	27.0	-25.8
$10^3\tilde{K}_7/G_{27}$	$\equiv 0$	-21.5	20.5
$10^3\tilde{K}_8/G_8$	20.4(1)	-0.546	0.521
$10^3\tilde{K}_9/G_8$	9.1(1)	-2.92	2.79
$10^3\tilde{K}_{10}/G_8$	$\equiv 0$	11.6	-11.1
$10^3\tilde{K}_{11}/G_8$	$\equiv 0$	-1.66	1.58
χ^2/DOF	33.3/10	–	–

Octet: $G_{27} = 0$
keep also $\tilde{K}_i$
with $m_K^2 m_\pi^2$ factors

μ variation
$\tilde{K}_i(\mu) - \tilde{K}_i(0.77\ GeV)$

Fit about same as before, some experiments fit better, others smaller error

Models

See Ecker, Kambor, Wyler 1993 for explanations

- Vector octet dominance $\Longrightarrow \tilde{K}_3 = -\frac{1}{2}\tilde{K}_2$: Not at all

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$$
\begin{aligned}
N_1^r &= 2k_f(32/3\,L_1^r + 4\,L_3^r + 2/3\,L_9^r)\,,\\
N_2^r &= 2k_f(16/3L_1^r + 4L_3^r + 10/3\,L_9^r)\,,\\
N_3^r &= 2k_f(8L_2^r - 2L_9^r)\,,\\
N_4^r &= 2k_f(-16/3\,L_1^r - 8/3\,L_3^r - 4/3\,L_9^r)\,,\\
N_5^r &= 2k_f(-L_5^r)\,,\\
N_6^r &= 2k_f(2/3\,L_5^r)\,,\\
N_7^r &= 2k_f(L_5^r)\,,\\
N_8^r &= 2k_f(4\,L_4^r + 2\,L_5^r)\,,\\
N_9^r &= N_{10}^r = N_{11}^r = N_{12}^r = N_{13}^r = 0
\end{aligned}
$$

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N_4^r &= 2k_f(-16/3\,L_1^r - 8/3\,L_3^r - 4/3\,L_9^r)\,,\\
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N_6^r &= 2k_f(2/3\,L_5^r)\,,\\
N_7^r &= 2k_f(L_5^r)\,,\\
N_8^r &= 2k_f(4\,L_4^r + 2\,L_5^r)\,,\\
N_9^r &= N_{10}^r = N_{11}^r = N_{12}^r = N_{13}^r = 0
\end{aligned}
$$

μ	0.77 GeV	0.9 GeV	0.842 GeV
G_8	4.18(1)	4.42	4.22(1)
G_{27}	0.360(2)	0.326(10)	0.339(10)
k_F	2.61(1)	4.94(2)	3.60(5)
χ^2/DOF	109/14	182/14	60.4/13

WDM or $k_f = 1/2$ No

Factorization: surprisingly OK

But k_f not naive one

Data and Fits

Decay	Width [GeV]	ChPT [GeV]	Fact. [GeV]
$K^+ \to \pi^+\pi^0$	$(1.1231 \pm 0.0078) \cdot 10^{-17}$	$1.123 \cdot 10^{-17}$	$1.127 \cdot 10^{-17}$
$K_S \to \pi^0\pi^0$	$(2.2828 \pm 0.0104) \cdot 10^{-15}$	$2.282 \cdot 10^{-15}$	$2.283 \cdot 10^{-15}$
$K_S \to \pi^+\pi^-$	$(5.0691 \pm 0.0108) \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$
$K_L \to \pi^0\pi^0\pi^0$	$(2.6748 \pm 0.0358) \cdot 10^{-18}$	$2.618 \cdot 10^{-18}$	$2.698 \cdot 10^{-18}$
$K_L \to \pi^+\pi^-\pi^0$	$(1.5998 \pm 0.0271) \cdot 10^{-18}$	$1.658 \cdot 10^{-18}$	$1.711 \cdot 10^{-18}$
$K^+ \to \pi^0\pi^0\pi^+$	$(9.195 \pm 0.0213) \cdot 10^{-19}$	$8.934 \cdot 10^{-19}$	$8.816 \cdot 10^{-19}$
$K^+ \to \pi^+\pi^+\pi^-$	$(2.9737 \pm 0.0174) \cdot 10^{-18}$	$2.971 \cdot 10^{-18}$	$2.933 \cdot 10^{-18}$

Decay	Quantity	Experiment	ChPT	Fact.
$K_L \to \pi^0\pi^0\pi^0$	h	-0.0050 ± 0.0014	-0.0062	-0.0025
$K_L \to \pi^+\pi^-\pi^0$	g	0.678 ± 0.008	0.678	0.654
	h	0.076 ± 0.006	0.088	0.083
	k	0.0099 ± 0.0015	0.0057	0.0068
$K_S \to \pi^+\pi^-\pi^0$	γ_S	$(3.3 \pm 0.5) \cdot 10^{-8}$	$3.0 \cdot 10^{-8}$	$2.9 \cdot 10^{-8}$
$K^\pm \to \pi^0\pi^0\pi^\pm$	g	0.638 ± 0.020	0.636	0.648
	h	0.051 ± 0.013	0.077	0.080
	k	0.004 ± 0.007	0.0047	0.0069
$K^+ \to \pi^+\pi^+\pi^-$	g	-0.2154 ± 0.0035	−0.215	−0.226
	h	0.012 ± 0.008	0.012	0.019
	k	-0.0101 ± 0.0034	−0.0034	−0.0033

Conclusions

- ChPT is a useful tool for Kaon decays
- $K_{\ell 3}$
  - Predicts λ''_+
  - Sizable pure two-loop corrections to $f_+(0)$
  - Can combine measurements + dispersive for $f_0(t)$ to get fully at $f_+(0)$
- A very short summary of some rare decays
- $K \to 3\pi$
  - Calculated $K \to 3\pi$ to first nontrivial order in isospin breaking fully
  - New fit done and tested a few models
  - Fit with/without isospin breaking seems similar
  - CP violation: partly done by Gamiz et al.