



Eta and Eta' Physics

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Various ChPT: `http://www.thep.lu.se/~bijnens/chpt.html`

Other talks

- Production of η and η' (or in medium):
 - Plenary: Krusche
 - Parallel: He, Vankova, Nanova, Shklyar, Glazier, Jain, Papenbrock, Dugger, Klaja, Przerwa, Pettersson, Takizawa
 - Posters: Klaja, Moskal, Czyzykiewicz
- Decays of η and/or η' :
 - Plenary: Wolke
 - Parallel: Borasoy, Prakhov, Roy
 - Posters: Stepaniak, Duniec, Jany, Redmer, Janusz, Yurev
- And I probably missed some

Overview

- No Production
- No weak decays: Typically: $\eta : \text{BR} \lesssim 10^{-11}$
 $\eta' : \text{BR} \lesssim 10^{-12}$
- Both η and η' decays are suppressed
 \implies good laboratories to study nondominant strong interaction effects
- Give an idea of why we look at η and η'
- Overview of the known theory, puzzles etc

Contents

- Useful proceedings/conferences ETA01 (Uppsala), ETA05 (Cracow), ETA06(Julich), ETA07 (Peniscola)
- Why are pseudoscalars special
- Chiral Perturbation Theory (ChPT, CHPT, χ PT)
- $\eta \rightarrow 3\pi$: Main part of talk
- $\eta' \rightarrow \eta\pi\pi, \pi\pi\pi$
- Reminder: many reactions can probe the anomaly

Eta Physics Handbook: ETA01

Physica Scripta, Vol. T99, 2002

Contents

Preface	5
The η and η' Mesons from Lattice QCD. <i>C. Michael</i>	7
Mixing of Pseudoscalar Mesons. <i>Th. Feldmann and P. Kroll</i>	13
On Searches for CP, T, CPT and C Violation in Flavour-Changing and Flavour-Conserving Interactions. <i>C. Jarlskog and E. Shabalin</i>	23
Eta Decays at and Beyond p^4 in Chiral Perturbation Theory. <i>Johan Bijnens and Jürg Gasser</i>	34
η Decays Involving Photons. <i>L. Ametller</i>	45
Allowed Eta-Decay Modes and Chiral Symmetry. <i>Barry R. Holstein</i>	55
Chiral Dynamics with Strange Quarks: Mysteries and Opportunities. <i>Ulf-G. Meißner</i>	68
$\eta'(\eta) \rightarrow \gamma\gamma$: A Tale of Two Anomalies. <i>G. M. Shore</i>	84
Gluonic Effects in η and η' Physics. <i>Steven D. Bass</i>	96
Decays of η and η' Mesons Caused by the Weak Interaction. <i>E. Shabalin</i>	104
Testing Discrete Symmetries with the Decay $\eta \rightarrow \pi^+\pi^-\gamma$. <i>C. Q. Geng and J. N. Ng</i>	109
Eta Decays and Physics Beyond the Standard Model. <i>Peter Herczeg</i>	113
The Neutral Decay Modes of the Eta-Meson. <i>B. M. K. Nefkens and J. W. Price</i>	114
η, η' Studies with the KLOE Detector at DAΦNE. <i>A. Aloisio, F. Ambrosino, A. Antonelli, M. Antonelli, C. Bacci, G. Bencivenni, S. Bertolucci, C. Bini, C. Bloise, V. Bocci, F. Bossi, P. Branchini, S. A. Bulychjov, R. Caloi, P. Campana, G. Capon, G. Carboni, M. Casarsa, V. Casavola, G. Cataldi, F. Ceradini, F. Cervelli, F. Cevenini, G. Chiefari, P. Ciambone, S. Conetti, E. De Lucia, G. De Robertis, P. De Simone, G. De Zorzi, S. Dell'Agnello, A. Denig, A. Di Domenico, C. Di Donato, S. Di Falco, A. Doria, M. Dreucci, O. Erriquez, A. Farilla, G. Felici, A. Ferrari, M. L. Ferrer, G. Finocchiaro, C. Forti, A. Franceschi, P. Franzini, C. Gatti, P. Gauzzi, S. Giovannella, E. Gorini, F. Grancagnolo, E. Graziani, S. W. Han, M. Incagli, L. Ingrosso, W. Kluge, C. Kuo, V. Kulikov, F. Lacava, G. Lanfranchi, J. Lee-Franzini, D. Leone, F. Lu, M. Martemianov, M. Matsyuk, W. Mei, L. Merola,</i>	

ETA05: Acta Phys.Slov. 56(2006) No 3

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Pseudoscalars are special

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $U(3)_V$

But
$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if $m_q = 0$ then $U(3)_L \times U(3)_R$.

Can also see that via



$$\begin{aligned} v < c, m_q \neq 0 &\implies \\ v = c, m_q = 0 &\not\implies \end{aligned}$$



Pseudoscalars are special

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $U(3)_V$

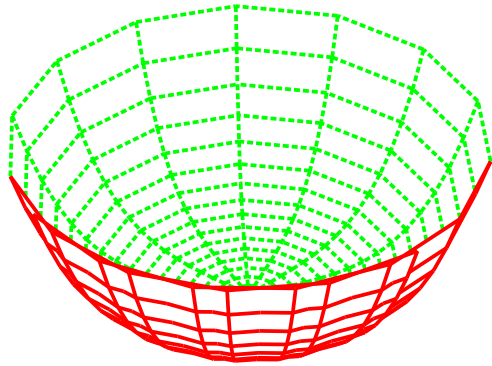
But
$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if $m_q = 0$ then $U(3)_L \times U(3)_R$.

- Hadrons do not come in parity doublets: symmetry must be broken
- A few very light hadrons: $\pi^0 \pi^+ \pi^-$ and also K, η
- Both can be understood from spontaneous Chiral Symmetry Breaking

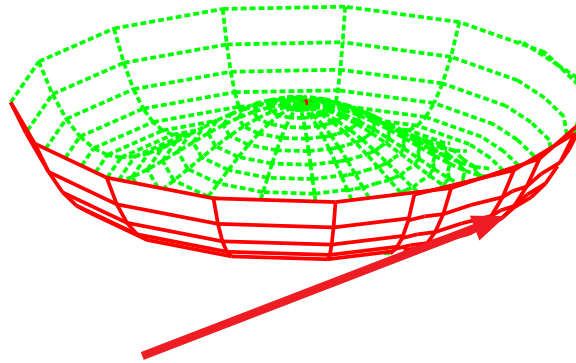
Goldstone Modes

UNBROKEN: $V(\phi)$



Only massive modes
around lowest energy
state (=vacuum)

BROKEN: $V(\phi)$



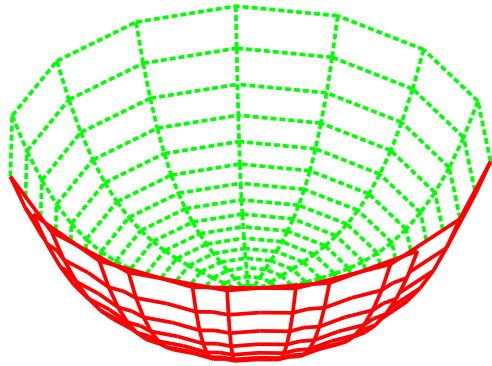
Need to pick a vacuum
 $\langle \phi \rangle \neq 0$: Breaks symmetry
No parity doublets
Massless mode along ridge

For QCD: $\langle \phi \rangle \neq 0 \longrightarrow \langle \bar{q}q \rangle \neq 0$ $U(3)_L \times U(3)_R \rightarrow U(3)_V$

Explains why pions light

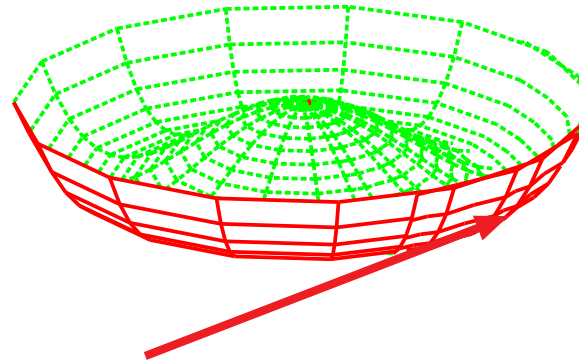
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Need to pick a vacuum
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Massless mode along ridge

For QCD: $\langle \phi \rangle \neq 0 \longrightarrow \langle \bar{q}q \rangle \neq 0 \quad U(3)_L \times U(3)_R \longrightarrow U(3)_V$

Explains why pions light but need NINE light particles

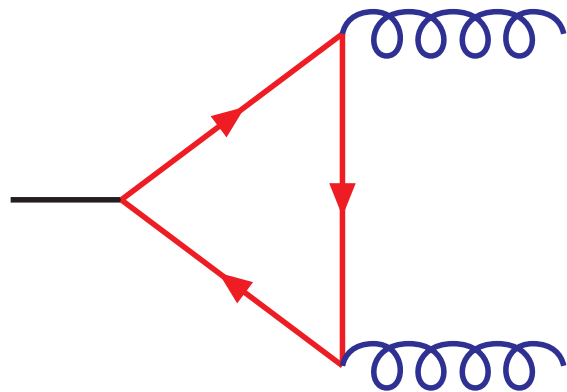
So WHY is the η' NOT light?

Anomaly

$$U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$$

$$SU(3)_L \times SU(3)_R \longrightarrow SU(3)_V \implies \pi, K, \eta \text{ light FINE}$$

$U(1)_A$: Is NOT a good quantum symmetry



The diagram shows a fermion loop (red lines with arrows) connected to two gluon vertices (blue wavy lines). The loop is a triangle with a vertical side. The gluon vertices are attached to the top and bottom vertices of the triangle. The diagram is followed by an implication arrow pointing to the equation $\partial_\mu A^{0\mu} = 2\sqrt{N_f}\omega$.

$$\implies \partial_\mu A^{0\mu} = 2\sqrt{N_f}\omega$$

$$\omega = \frac{1}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr} G_{\mu\nu} G_{\alpha\beta}$$

ω is gluons: strongly interacting: η' heavy

But

So quantum effects break $U(1)_A$

BUT ω is a total derivative \implies How does it have an effect?

But

So quantum effects break $U(1)_A$

BUT ω is a total derivative \implies How does it have an effect?

't Hooft:

- winding number $\nu = \int d^4x \omega$
- instantons lead to an effect

Creates a new problem: $\mathcal{L}_{QCD} \longrightarrow \mathcal{L}_{QCD} - \theta\omega$

(Strong CP problem) BUT it solved the η' problem

η' has possibly large and very interesting nonperturbative effects and interaction with gluons as no other hadron

$m_s \neq \hat{m}$: This also affects η physics

Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

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Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory*,
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

Chiral Perturbation Theory

- Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown (without η')
- Power counting: Dimensional counting in momenta/masses
- Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

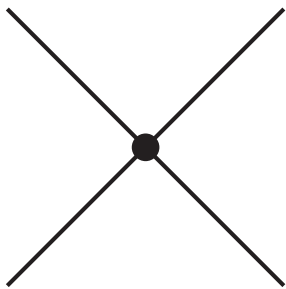
Chiral Perturbation Theory

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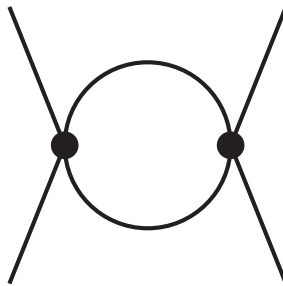
Power counting: Dimensional counting in momenta/masses

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Power counting in momenta: **Meson loops**



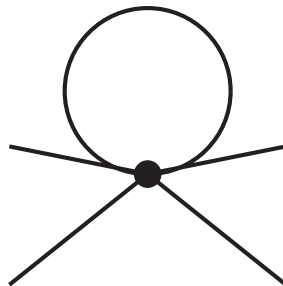
$$p^2$$



$$(p^2)^2 (1/p^2)^2 p^4 = p^4$$



$$1/p^2$$



$$(p^2) (1/p^2) p^4 = p^4$$

$$\int d^4p$$

$$p^4$$

Lagrangians

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}.$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4} \{ \langle D_\mu U^\dagger D^\mu U \rangle + \langle \chi^\dagger U + \chi U^\dagger \rangle \},$

$$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu,$$

left and right external currents: $r(l)_\mu = v_\mu + (-)a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$

quark masses via scalar density: $s = \mathcal{M} + \dots$

$$\langle A \rangle = Tr_F (A)$$

Lagrangians

$$\begin{aligned}\mathcal{L}_4 = & L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\ & + L_3 \langle D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U \rangle + L_4 \langle D^\mu U^\dagger D_\mu U \rangle \langle \chi^\dagger U + \chi U^\dagger \rangle \\ & + L_5 \langle D^\mu U^\dagger D_\mu U (\chi^\dagger U + U^\dagger \chi) \rangle + L_6 \langle \chi^\dagger U + \chi U^\dagger \rangle^2 \\ & + L_7 \langle \chi^\dagger U - \chi U^\dagger \rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger \rangle \\ & - i L_9 \langle F_{\mu\nu}^R D^\mu U D^\nu U^\dagger + F_{\mu\nu}^L D^\mu U^\dagger D^\nu U \rangle \\ & + L_{10} \langle U^\dagger F_{\mu\nu}^R U F^{L\mu\nu} \rangle + H_1 \langle F_{\mu\nu}^R F^{R\mu\nu} + F_{\mu\nu}^L F^{L\mu\nu} \rangle + H_2 \langle \chi^\dagger \chi \rangle\end{aligned}$$

L_i : Low-energy-constants (LECs)

H_i : Values depend on definition of currents/densities

These absorb the divergences of loop diagrams: $L_i \rightarrow L_i^r$

Renormalization: order by order in the powercounting

Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	\hat{L}_i^r, \hat{H}_i^r	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2 : Weinberg 1966

p^4 : Gasser, Leutwyler 84,85

p^6 : JB, Colangelo, Ecker 99,00

- ▣ replica method \implies PQ obtained from N_F flavour
- ▣ All infinities known
- ▣ 3 flavour special case of 3+3 PQ: $\hat{L}_i^r, K_i^r \rightarrow L_i^r, C_i^r$
- ▣ 53 \rightarrow 52 [arXiv:0705.0576 \[hep-ph\]](https://arxiv.org/abs/0705.0576)

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms

$$m_{\pi}^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2} \log \frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu) \right] + \dots$$

$$M^2 = 2B\hat{m}$$

$B \neq B_0, F \neq F_0$ (two versus three-flavour)

LECs and μ

$$l_3^r(\mu)$$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i^r(\mu) - \log \frac{M_\pi^2}{\mu^2}.$$

Independent of the scale μ .

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

μ :

- m_π, m_K : chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$ large N_c arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exists
- Express orders in terms of physical masses and quantities (F_π , F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering
- Relative sizes of order p^2 , p^2 , p^4 , ... can vary considerably

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- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering
- Relative sizes of order p^2 , p^2 , p^4 , ... can vary considerably
- I prefer physical masses
- Thresholds correct
- Chiral logs are from physical particles propagating

LECs

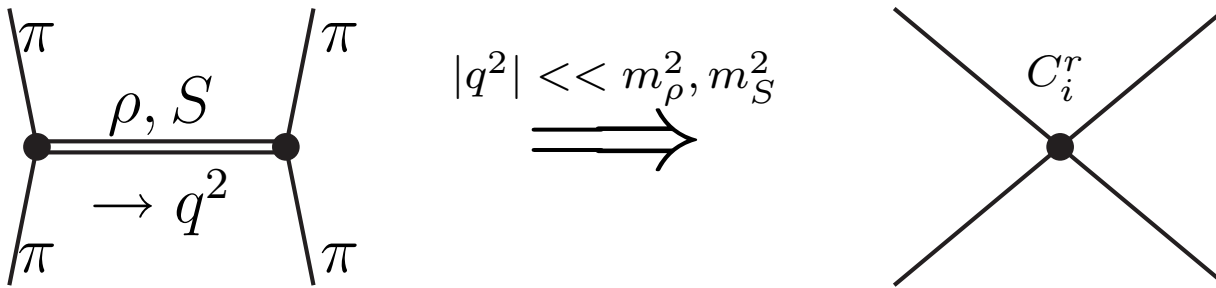
Some combinations of order p^6 LECs are known as well: curvature of the scalar and vector formfactor, two more combinations from $\pi\pi$ scattering (implicit in b_5 and b_6)

General observation:

- Obtainable from kinematical dependences: known
- Only via quark-mass dependence: poorly known

Most analysis use:

C_i^r from (single) resonance approximation



Motivated by large N_c : large effort goes in this

Ananthanarayan, JB, Cirigliano, Donoghue, Ecker, Gamiz, Golterman, Kaiser, Knecht, Peris, Pich, Prades, Portoles, de Rafael, . . .

$$\begin{aligned}
 \mathcal{L}_V &= -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}m_V^2\langle V_\mu V^\mu\rangle - \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle \\
 &\quad - \frac{ig_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + f_\chi\langle V_\mu[u^\mu, \chi_-]\rangle \\
 \mathcal{L}_A &= -\frac{1}{4}\langle A_{\mu\nu}A^{\mu\nu}\rangle + \frac{1}{2}m_A^2\langle A_\mu A^\mu\rangle - \frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle \\
 \mathcal{L}_S &= \frac{1}{2}\langle \nabla^\mu S \nabla_\mu S - M_S^2 S^2\rangle + c_d\langle S u^\mu u_\mu\rangle + c_m\langle S \chi_+\rangle \\
 \mathcal{L}_{\eta'} &= \frac{1}{2}\partial_\mu P_1 \partial^\mu P_1 - \frac{1}{2}M_{\eta'}^2 P_1^2 + i\tilde{d}_m P_1 \langle \chi_-\rangle.
 \end{aligned}$$

$$f_V = 0.20, \quad f_\chi = -0.025, \quad g_V = 0.09, \quad c_m = 42 \text{ MeV}, \quad c_d = 32 \text{ MeV}, \quad \tilde{d}_m = 20 \text{ MeV},$$

$$m_V = m_\rho = 0.77 \text{ GeV}, \quad m_A = m_{a_1} = 1.23 \text{ GeV}, \quad m_S = 0.98 \text{ GeV}, \quad m_{P_1} = 0.958 \text{ GeV}$$

f_V, g_V, f_χ, f_A : experiment

c_m and c_d from resonance saturation at $\mathcal{O}(p^4)$

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

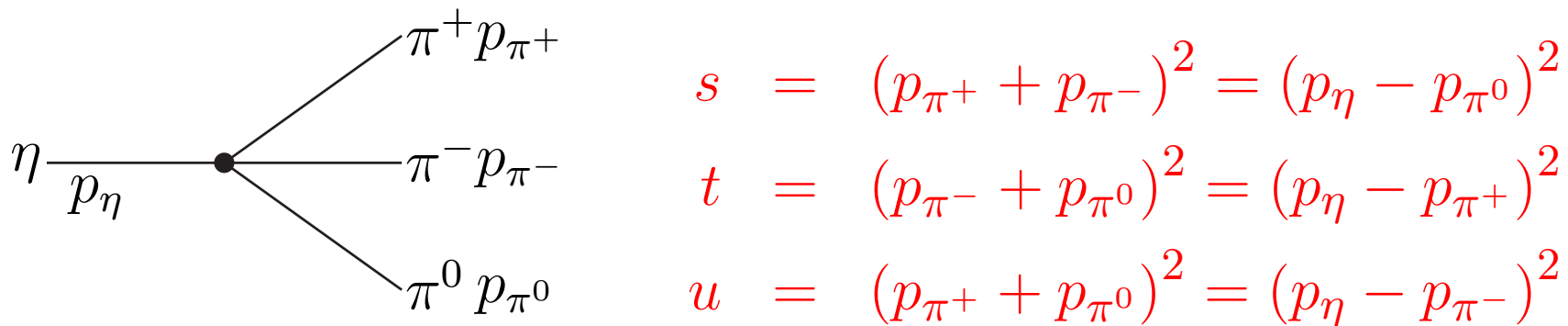
What we do/did about it:

- Vary resonance estimate by factor of two
- Vary the scale μ at which it applies: 600-900 MeV
- Check the estimates for the measured ones
- Again: kinematic can be had, quark-mass dependence difficult

$\eta \rightarrow 3\pi$

Reviews: JB, Gasser, Phys.Scripta T99(2002)34 [hep-ph/0202242]

JB, Acta Phys. Slov. 56(2005)305 [hep-ph/0511076]



$$s + t + u = m_{\eta}^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0.$$

$$\langle \pi^0 \pi^+ \pi^- \text{out} | \eta \rangle = i (2\pi)^4 \delta^4 (p_{\eta} - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}) A(s, t, u).$$

$$\langle \pi^0 \pi^0 \pi^0 \text{out} | \eta \rangle = i (2\pi)^4 \delta^4 (p_{\eta} - p_1 - p_2 - p_3) \bar{A}(s_1, s_2, s_3)$$

$$\bar{A}(s_1, s_2, s_3) = A(s_1, s_2, s_3) + A(s_2, s_3, s_1) + A(s_3, s_1, s_2),$$

$\eta \rightarrow 3\pi$: Lowest order (LO)

Pions are in $I = 1$ state $\implies A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \rightarrow \pi^+ \pi^- \pi^0 \gamma$ needs to be included directly

$\eta \rightarrow 3\pi$: Lowest order (LO)

Pions are in $I = 1$ state $\implies A \sim (m_u - m_d)$ or α_{em}

$$\text{ChPT:Cronin 67: } A(s, t, u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2} \left\{ 1 + \frac{3(s - s_0)}{m_\eta^2 - m_\pi^2} \right\}$$

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$$\text{with } Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \text{ or } R \equiv \frac{m_s - \hat{m}}{m_d - m_u} \quad \hat{m} = \frac{1}{2}(m_u + m_d)$$

$$A(s, t, u) = \frac{1}{Q^2} \frac{m_K^2}{m_\pi^2} (m_\pi^2 - m_K^2) \frac{\mathcal{M}(s, t, u)}{3\sqrt{3}F_\pi^2},$$

$$A(s, t, u) = \frac{\sqrt{3}}{4R} M(s, t, u)$$

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$$A(s, t, u) = \frac{\sqrt{3}}{4R} M(s, t, u)$$

$$\text{LO: } \mathcal{M}(s, t, u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2} \quad M(s, t, u) = \frac{1}{F_\pi^2} \left(\frac{4}{3}m_\pi^2 - s \right)$$

$\eta \rightarrow 3\pi$ beyond p^4 : p^2 and p^4

$\Gamma(\eta \rightarrow 3\pi) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

$Q \approx 24$ gives lowest order $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 66$ eV.

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Other Source from $m_{K^+}^2 - m_{K^0}^2 \sim Q^{-2} \implies Q = 20.0 \pm 1.5$

Lowest order prediction $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 140 \text{ eV}$.

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At order p^4 Gasser-Leutwyler 1985:
$$\frac{\int dLIPS |A_2 + A_4|^2}{\int dLIPS |A_2|^2} = 2.4,$$

(*LIPS*=Lorentz invariant phase-space)

Major source: large *S*-wave final state rescattering

Experiment: 295 ± 17 eV (PDG 2006)

$\eta \rightarrow 3\pi$ beyond p^4 : Dispersive

Try to resum the S -wave rescattering:

Anisovich-Leutwyler (AL), Kambor, Wiesendanger, Wyler (KWW)

Different method but similar approximations

Here: simplified version of AL

Up to p^8 : No absorptive parts from $\ell \geq 2$

$\implies M(s, t, u) =$

$$M_0(s) + (s - u)M_1(t) + (s - t)M_1(t) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

M_I : “roughly” contributions with isospin 0,1,2

$\eta \rightarrow 3\pi$ beyond p^4 : Dispersive

3 body dispersive: difficult: keep only 2 body cuts

start from $\pi\eta \rightarrow \pi\pi$ ($m_\eta^2 < 3m_\pi^2$) standard dispersive analysis

analytically continue to physical m_η^2 .

$$M_I(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}M_I(s')}{s' - s - i\varepsilon}$$

$$\text{Im}M_I(s') \longrightarrow \text{disc}M_I(s) = \frac{1}{2i} (M_I(s + i\varepsilon) - M_I(s - i\varepsilon))$$

$$M_0(s) = a_0 + b_0s + c_0s^2 + \frac{s^3}{\pi} \int \frac{ds'}{s'^3} \frac{\text{disc}M_0(s')}{s' - s - i\varepsilon},$$

$$M_1(s) = a_1 + b_1s + \frac{s^2}{\pi} \int \frac{ds'}{s'^2} \frac{\text{disc}M_1(s')}{s' - s - i\varepsilon},$$

$$M_2(s) = a_2 + b_2s + c_2s^2 + \frac{s^3}{\pi} \int \frac{ds'}{s'^3} \frac{\text{disc}M_2(s')}{s' - s - i\varepsilon}.$$

$\eta \rightarrow 3\pi$ beyond p^4

- Technical complications in solving
- **Only 4 relevant constants:**

$$M(s, t, u) = a + bs + cs^2 - d(s^2 + tu)$$

$$M_0(s) + \frac{4}{3}M_2(s) \quad sM_1(s) + M_2(s) + s^2 \frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)}$$

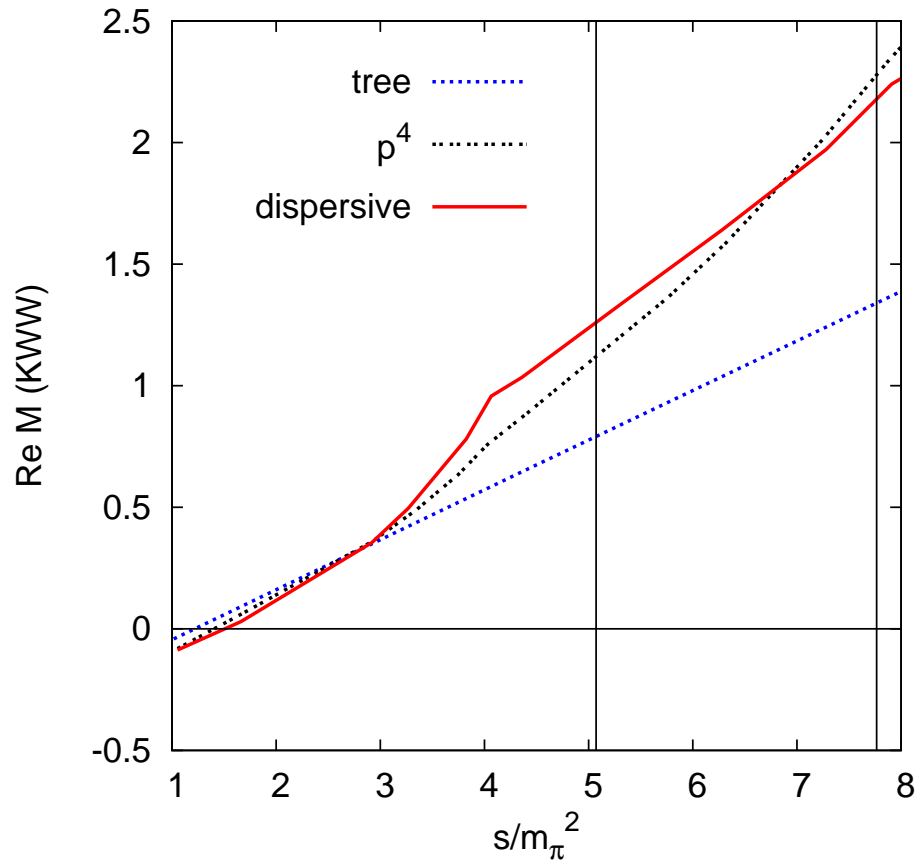
converge better

$$c = c_0 + \frac{4}{3}c_2 = \frac{1}{\pi} \int \frac{ds'}{s'^3} \left\{ \text{disc}M_0(s') + \frac{4}{3}\text{disc}M_2(s') \right\},$$

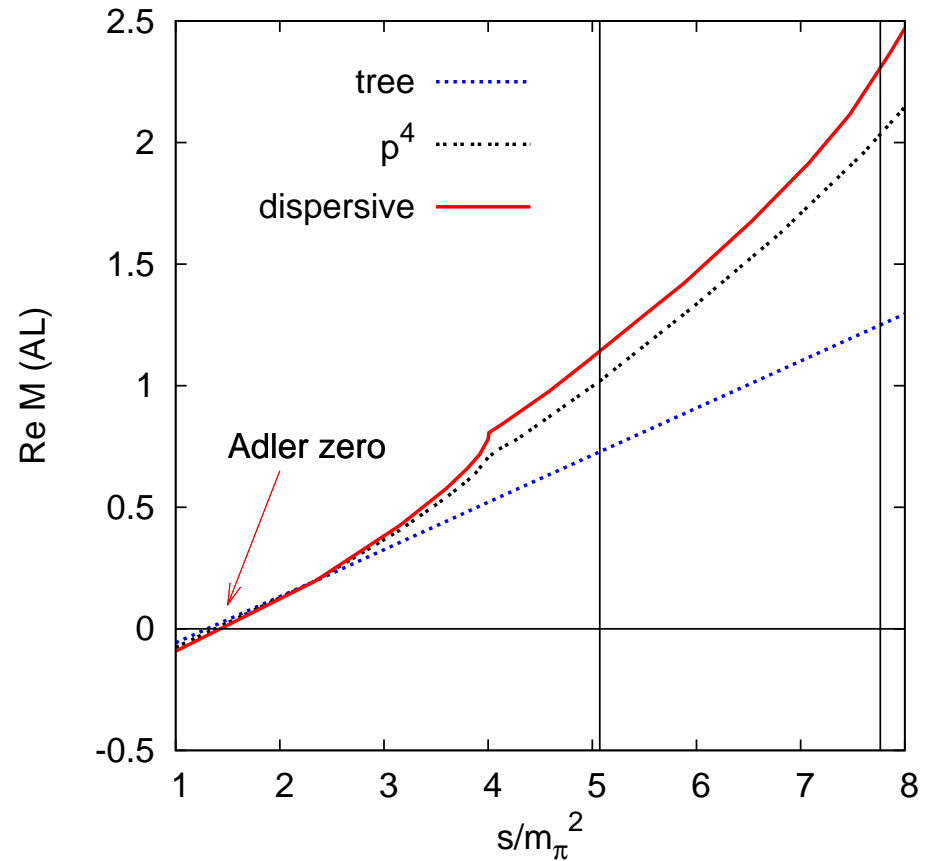
$$d = -\frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)} + \frac{1}{\pi} \int \frac{ds'}{s'^3} \{s' \text{disc}M_1(s') + \text{disc}M_2(s')\}$$

Fix a, b by matching to tree level or p^4 amplitude

$\eta \rightarrow 3\pi$ beyond p^4

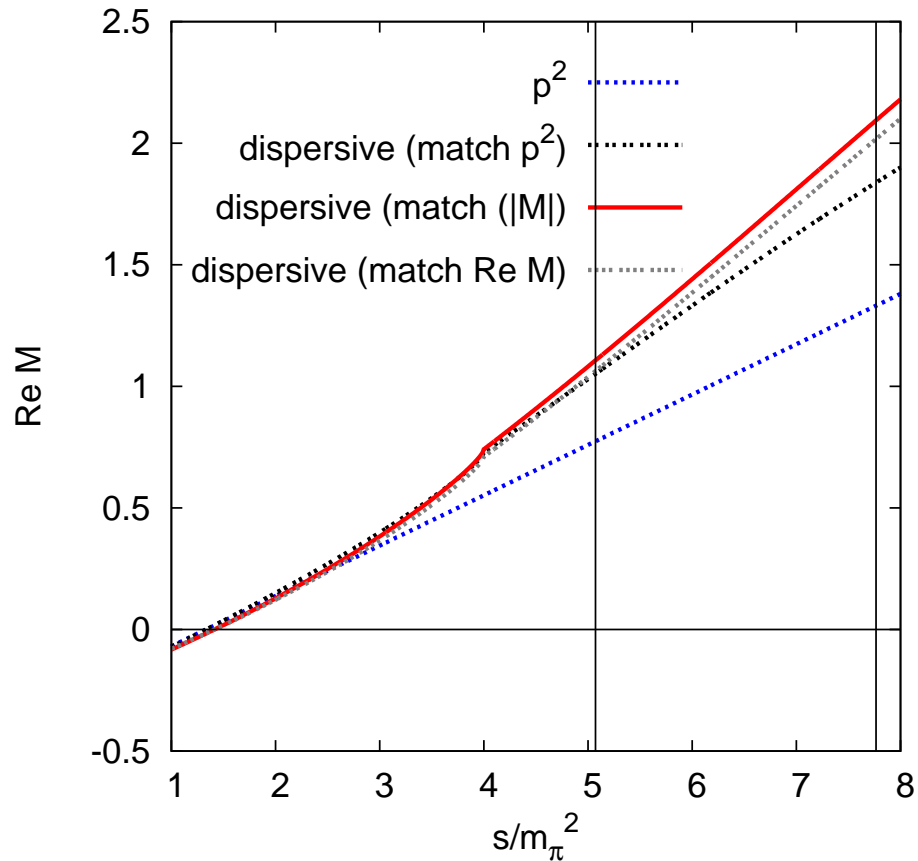


Along $s = u$ KWW



Along $s = u$ AL

$\eta \rightarrow 3\pi$ beyond p^4



Along $s = u$ BG

Very simplified analysis

JB, Gasser 2002

looks more like AL

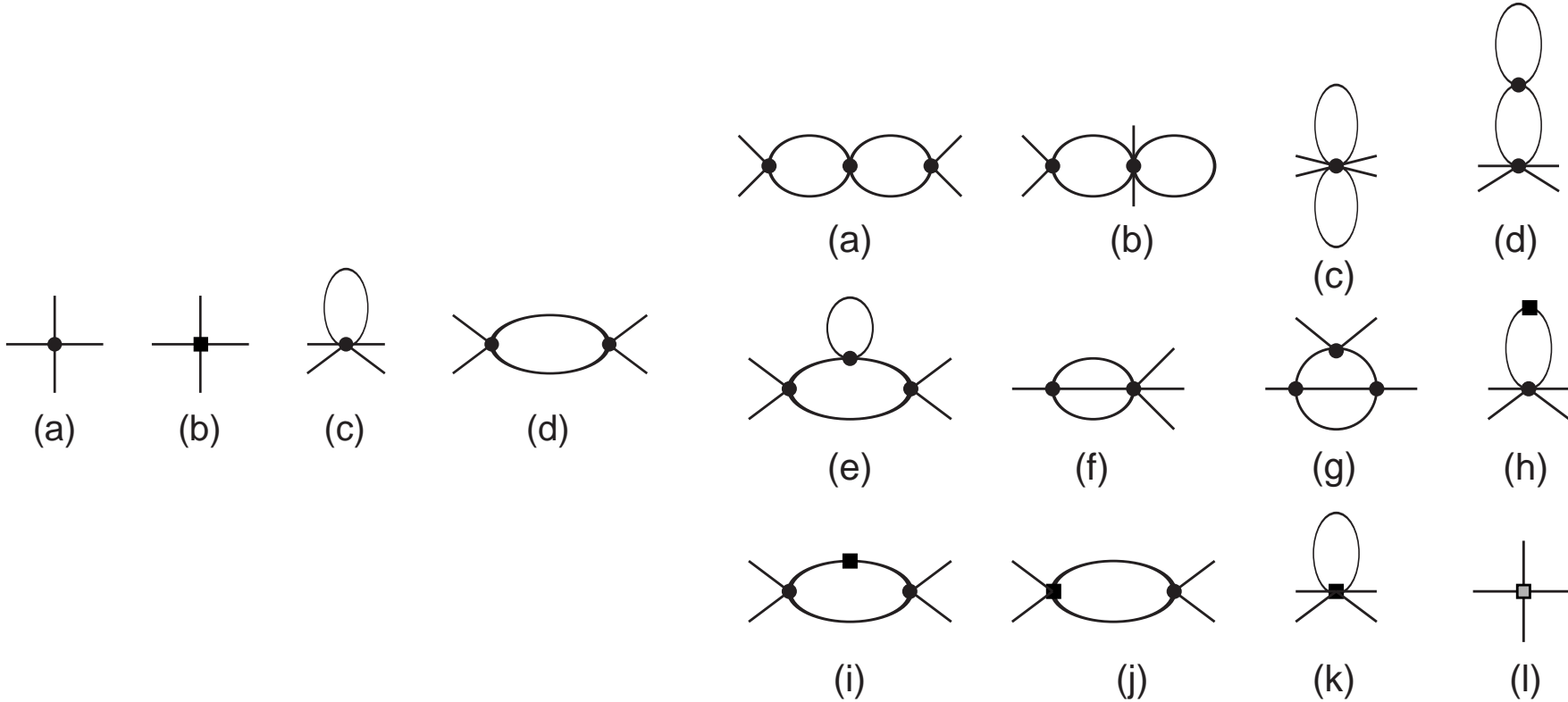
Two Loop Calculation: why

- In $K_{\ell 4}$ dispersive gave about half of p^6 in amplitude
- Same order in ChPT as masses for consistency check on m_u/m_d
- Check size of 3 pion dispersive part
- At order p^4 unitarity about half of correction
- Technology exists:
 - Two-loops: Amorós, JB, Dhonte, Talavera, . . .
 - Dealing with the mixing π^0 - η :
Amorós, JB, Dhonte, Talavera 01

Two Loop Calculation: why

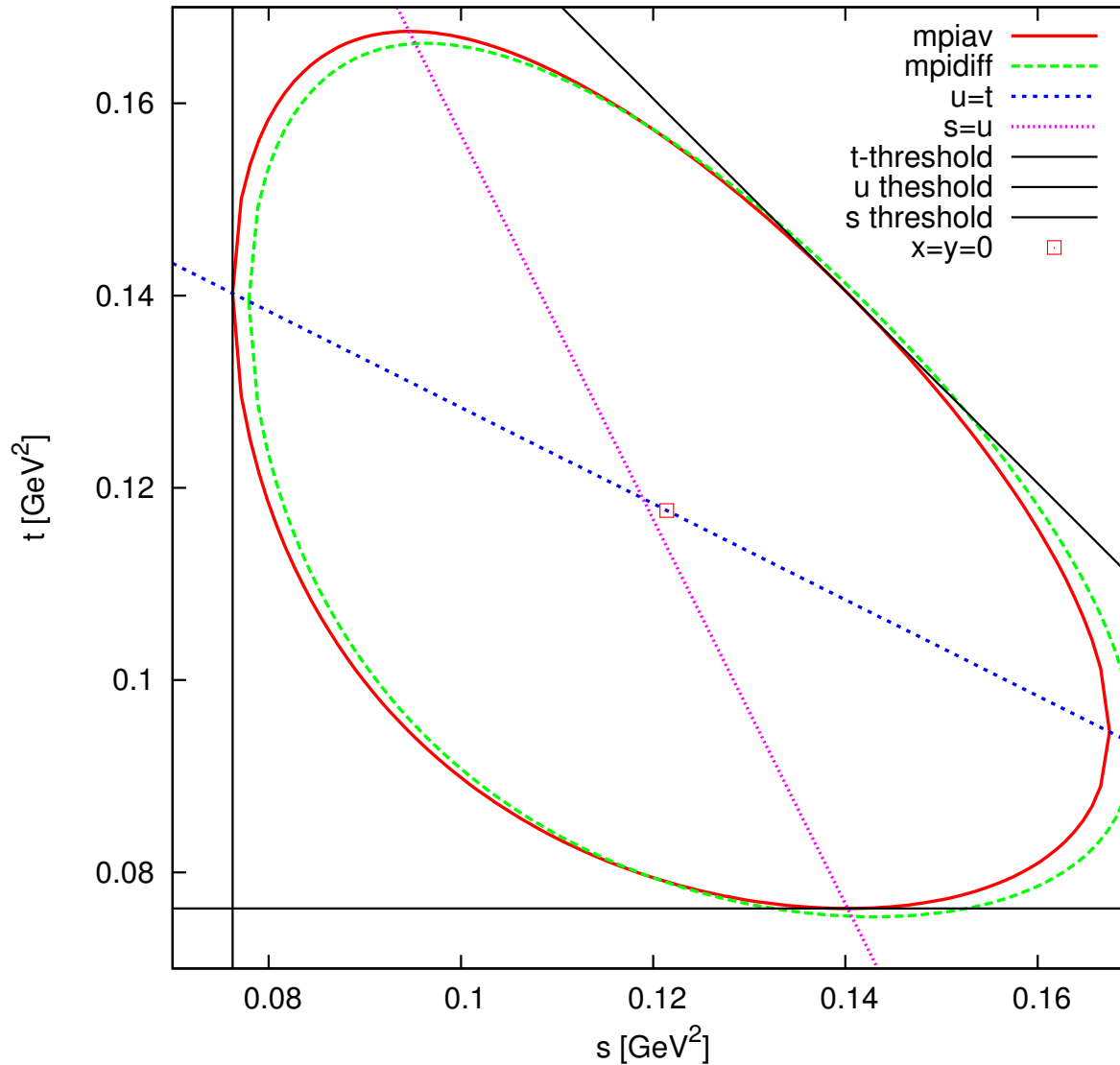
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 - Dealing with the mixing π^0 - η :
Amorós, JB, Dhonte, Talavera 01
- **Done:** JB, Ghorbani, arXiv:0709.0230 [hep-ph]
 - Dealing with the mixing π^0 - η : extended to $\eta \rightarrow 3\pi$

Diagrams



- Include mixing, renormalize, pull out factor $\frac{\sqrt{3}}{4R}$, ...
- Two independent calculations (comparison major amount of work)

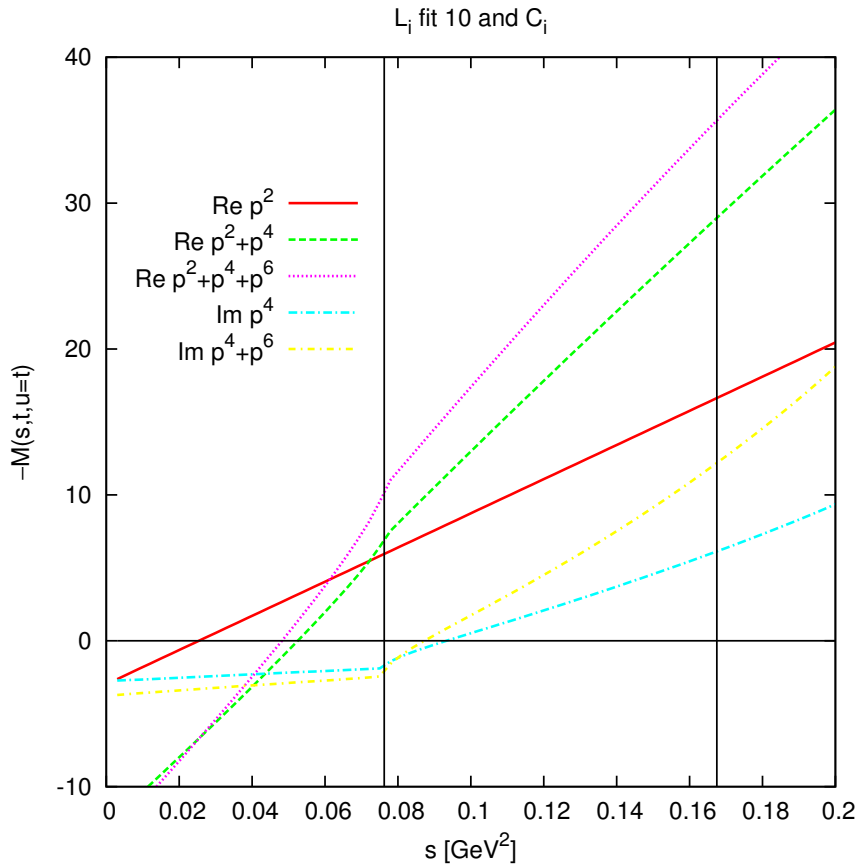
Dalitzplot



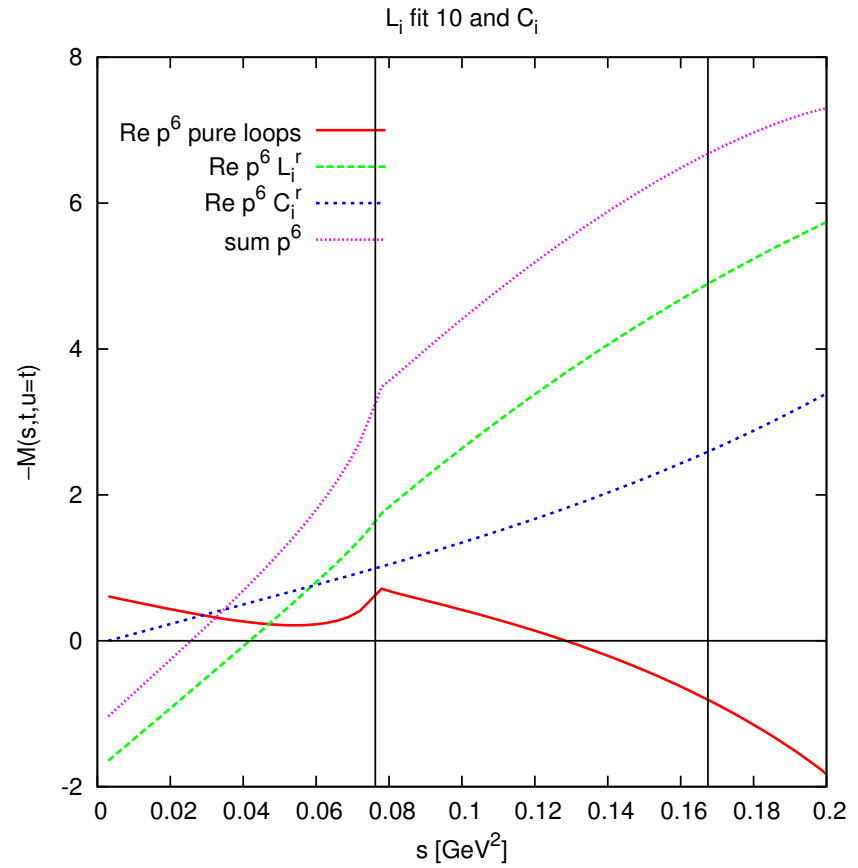
x variation:
vertical

y variation:
parallel to $t = u$

$\eta \rightarrow 3\pi: M(s, t = u)$

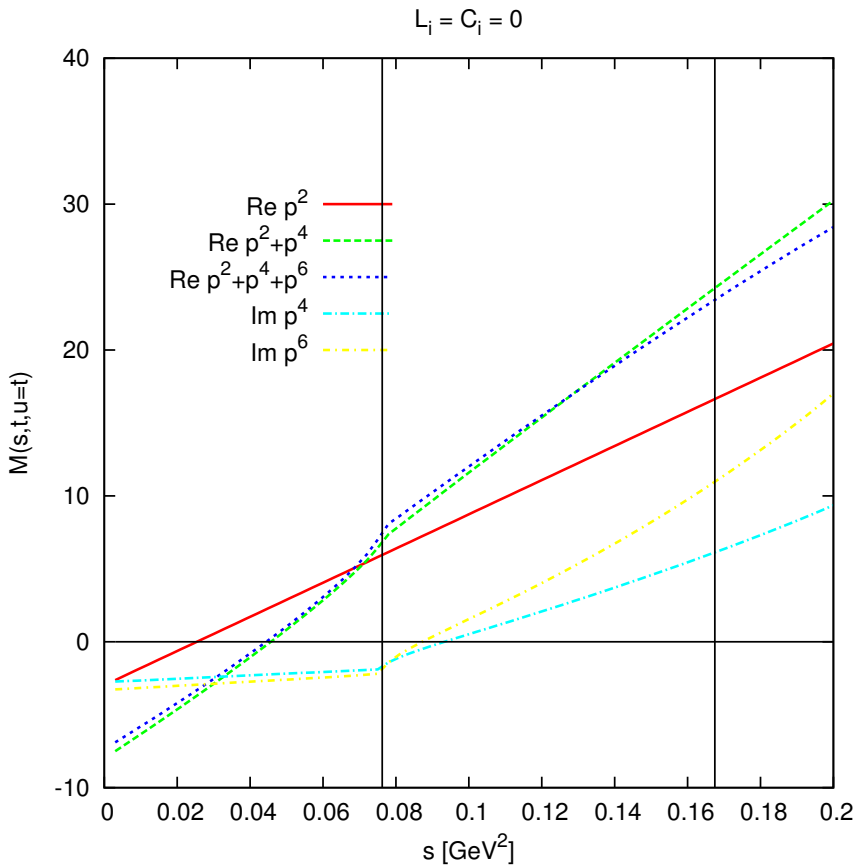


Along $t = u$

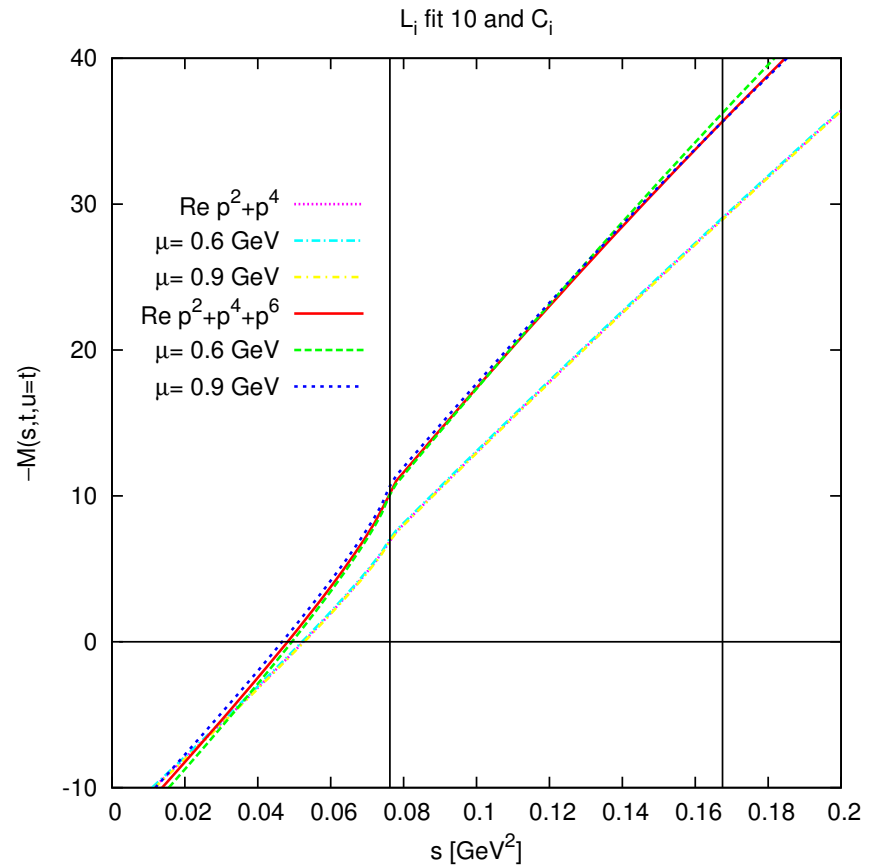


Along $t = u$ parts

$\eta \rightarrow 3\pi: M(s, t = u)$

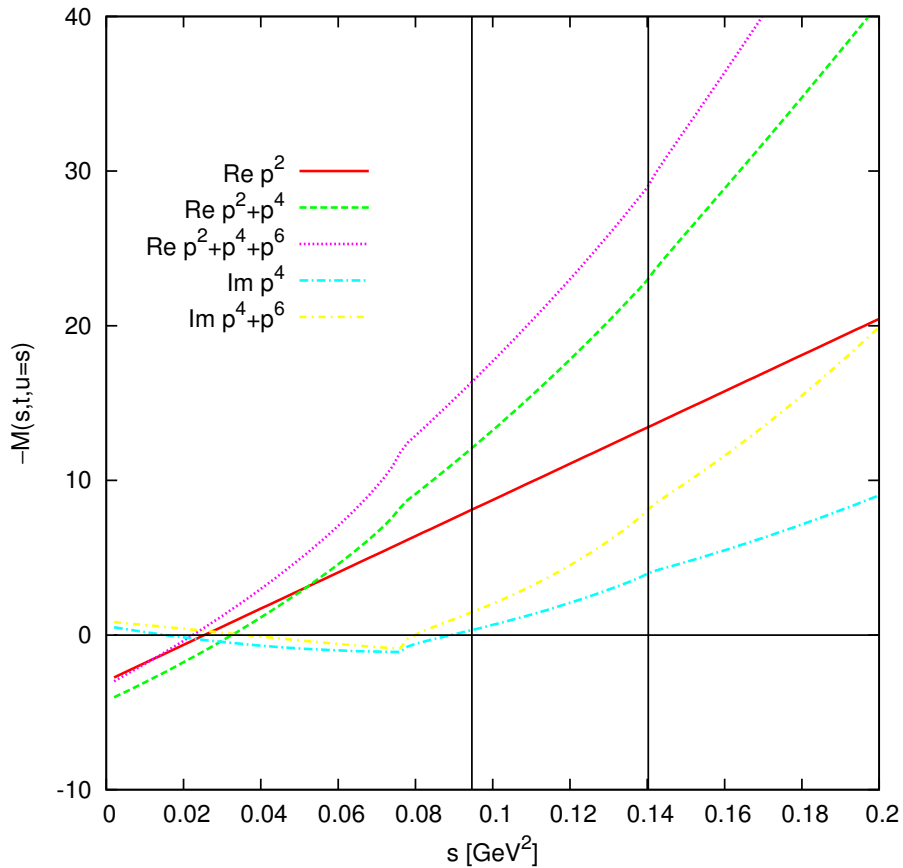


Along $t = u$
 $L_i^r = C_i^r = 0$



Along $t = u: \mu$ dependence
 I.e. where $C_i^r(\mu)$ estimated

$\eta \rightarrow 3\pi: M(s = u, t)$



Along $s = u$

Shape agrees with AL

Correction larger:
20-30% in amplitude

Dalitz plot

$$x = \sqrt{3} \frac{T_+ - T_-}{Q_\eta} = \frac{\sqrt{3}}{2m_\eta Q_\eta} (u - t)$$

$$y = \frac{3T_0}{Q_\eta} - 1 = \frac{3((m_\eta - m_{\pi^0})^2 - s)}{2m_\eta Q_\eta} - 1 \stackrel{\text{iso}}{=} \frac{3}{2m_\eta Q_\eta} (s_0 - s)$$

$$Q_\eta = m_\eta - 2m_{\pi^+} - m_{\pi^0}$$

T^i is the kinetic energy of pion π^i

$$z = \frac{2}{3} \sum_{i=1,3} \left(\frac{3E_i - m_\eta}{m_\eta - 3m_\pi^0} \right)^2 \quad E_i \text{ is the energy of pion } \pi^i$$

$$|M|^2 = A_0^2 (1 + ay + by^2 + dx^2 + fy^3 + gx^2y + \dots)$$

$$|\overline{M}|^2 = \overline{A}_0^2 (1 + 2\alpha z + \dots)$$

Experiment: charged

Exp.	a	b	d
KLOE	$-1.090 \pm 0.005^{+0.008}_{-0.019}$	$0.124 \pm 0.006 \pm 0.010$	$0.057 \pm 0.006^{+0.007}_{-0.016}$
Crystal Barrel	-1.22 ± 0.07	0.22 ± 0.11	0.06 ± 0.04 (input)
Layter et al.	-1.08 ± 0.014	0.034 ± 0.027	0.046 ± 0.031
Gormley et al.	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04

KLOE has: $f = 0.14 \pm 0.01 \pm 0.02$.

Crystal Barrel: d input, but a and b insensitive to d

Theory: charged

	A_0^2	a	b	d	f
LO	120	-1.039	0.270	0.000	0.000
NLO	314	-1.371	0.452	0.053	0.027
NLO ($L_i^r = 0$)	235	-1.263	0.407	0.050	0.015
NNLO	538	-1.271	0.394	0.055	0.025
NNLOp (y from T^0)	574	-1.229	0.366	0.052	0.023
NNLOq (incl $(x, y)^4$)	535	-1.257	0.397	0.076	0.004
NNLO ($\mu = 0.6$ GeV)	543	-1.300	0.415	0.055	0.024
NNLO ($\mu = 0.9$ GeV)	548	-1.241	0.374	0.054	0.025
NNLO ($C_i^r = 0$)	465	-1.297	0.404	0.058	0.032
NNLO ($L_i^r = C_i^r = 0$)	251	-1.241	0.424	0.050	0.007
dispersive (KWW)	—	-1.33	0.26	0.10	—
tree dispersive	—	-1.10	0.33	0.001	—
absolute dispersive	—	-1.21	0.33	0.04	—
error	18	0.075	0.102	0.057	0.160

NLO to
NNLO:
Little
change

Error on
 $|M(s, t, u)|^2$:
 $|M^{(6)} M(s, t, u)|$

Experiment: neutral

Exp.	α
KLOE 2007	$-0.027 \pm 0.004^{+0.004}_{-0.006}$
KLOE (prel)	$-0.014 \pm 0.005 \pm 0.004$
Crystal Ball	-0.031 ± 0.004
WASA/CELSIUS	$-0.026 \pm 0.010 \pm 0.010$
Crystal Barrel	$-0.052 \pm 0.017 \pm 0.010$
GAMS2000	-0.022 ± 0.023
SND	$-0.010 \pm 0.021 \pm 0.010$

	\overline{A}_0^2	α
LO	1090	0.000
NLO	2810	0.013
NLO ($L_i^r = 0$)	2100	0.016
NNLO	4790	0.013
NNLOq	4790	0.014
NNLO ($C_i^r = 0$)	4140	0.011
NNLO ($L_i^r = C_i^r = 0$)	2220	0.016
dispersive (KWW)	—	—(0.007—0.014)
tree dispersive	—	—0.0065
absolute dispersive	—	—0.007
Borasoy	—	—0.031
error	160	0.032

Note: NNLO ChPT gets a_0^0 in $\pi\pi$ correct

α is difficult

Expand amplitudes and isospin:

$$M(s, t, u) = A \left(1 + \tilde{a}(s - s_0) + \tilde{b}(s - s_0)^2 + \tilde{d}(u - t)^2 + \dots \right)$$

$$\overline{M}(s, t, u) = A \left(3 + (\tilde{b} + 3\tilde{d}) \left((s - s_0)^2 + (t - s_0)^2 + (u - s_0)^2 \right) + \dots \right)$$

Gives relations ($R_\eta = (2m_\eta Q_\eta)/3$)

$$a = -2R_\eta \operatorname{Re}(\tilde{a}), \quad b = R_\eta^2 \left(|\tilde{a}|^2 + 2\operatorname{Re}(\tilde{b}) \right), \quad d = 6R_\eta^2 \operatorname{Re}(\tilde{d}).$$

$$\alpha = \frac{1}{2} R_\eta^2 \operatorname{Re}(\tilde{b} + 3\tilde{d}) = \frac{1}{4} (d + b - R_\eta^2 |\tilde{a}|^2) \leq \frac{1}{4} \left(d + b - \frac{1}{4} a^2 \right)$$

equality if $\operatorname{Im}(\tilde{a}) = 0$

Large cancellation in α , overestimate of b likely the problem

r and decay rates

$$\sin \epsilon = \frac{\sqrt{3}}{4R} + \mathcal{O}(\epsilon^2)$$

$$\begin{aligned} \Gamma(\eta \rightarrow \pi^+ \pi^- \pi^0) &= \sin^2 \epsilon \cdot 0.572 \text{ MeV} && \text{LO,} \\ & \sin^2 \epsilon \cdot 1.59 \text{ MeV} && \text{NLO,} \\ & \sin^2 \epsilon \cdot 2.68 \text{ MeV} && \text{NNLO,} \\ & \sin^2 \epsilon \cdot 2.33 \text{ MeV} && \text{NNLO } C_i^r = 0, \\ \Gamma(\eta \rightarrow \pi^0 \pi^0 \pi^0) &= \sin^2 \epsilon \cdot 0.884 \text{ MeV} && \text{LO,} \\ & \sin^2 \epsilon \cdot 2.31 \text{ MeV} && \text{NLO,} \\ & \sin^2 \epsilon \cdot 3.94 \text{ MeV} && \text{NNLO,} \\ & \sin^2 \epsilon \cdot 3.40 \text{ MeV} && \text{NNLO } C_i^r = 0. \end{aligned}$$

r and decay rates

$$r \equiv \frac{\Gamma(\eta \rightarrow \pi^0 \pi^0 \pi^0)}{\Gamma(\eta \rightarrow \pi^+ \pi^- \pi^0)}$$

$$r_{\text{LO}} = 1.54$$

$$r_{\text{NLO}} = 1.46$$

$$r_{\text{NNLO}} = 1.47$$

$$r_{\text{NNLO } C_i^r=0} = 1.46$$

PDG 2006

$$r = 1.49 \pm 0.06 \quad \text{our average .}$$

$$r = 1.43 \pm 0.04 \quad \text{our fit ,}$$

Good agreement

R and Q

	LO	NLO	NNLO	NNLO ($C_i^r = 0$)
$R(\eta)$	19.1	31.8	42.2	38.7
R (Dashen)	44	44	37	—
R (Dashen-violation)	36	37	32	—
$Q(\eta)$	15.6	20.1	23.2	22.2
Q (Dashen)	24	24	22	—
Q (Dashen-violation)	22	22	20	—

LO from $R = \frac{m_{K^0}^2 + m_{K^+}^2 - 2m_{\pi^0}^2}{2(m_{K^0}^2 - m_{K^+}^2)}$ (QCD part only)

NLO and NNLO from masses: Amorós, JB, Talavera 2001

$$Q^2 = \frac{m_s + \hat{m}}{2\hat{m}} R = 12.7R \quad (m_s/\hat{m} = 24.4)$$

$$\eta' \rightarrow \eta\pi\pi \text{ and } \eta' \rightarrow 3\pi$$

η' decays: add a ϕ_0 degree of freedom to the usual ChPT

Witten, DiVecchia, Veneziano, Schechter, Rosenzweig,...

Write the most general $U(3)_L \times U(3)_R$ invariant Lagrangian as a function of:

$$\frac{\sqrt{2}\phi_0}{F} + \theta \quad \text{and} \quad U = e^{i\sqrt{2}\phi_0/F} e^{i\sqrt{2}M/F} \quad \text{with} \quad U \rightarrow g_R U g_L^\dagger$$

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Most general lagrangian: very many terms: $F_i \left(\frac{\sqrt{2}\phi_0}{F} + \theta \right)$

Order p^2, m_q : **5 functions** (Gasser-Leutwyler)

Order $p^4, m_q p^2, m_q^2$: **57 functions** (Herrera-Siklody et al.)

$$\eta' \rightarrow \eta\pi\pi \text{ and } \eta' \rightarrow 3\pi$$

Need something more

$$\eta' \rightarrow \eta\pi\pi \text{ and } \eta' \rightarrow 3\pi$$

Need something more

Large number of colours: large N_c

In this limit:

$$\begin{array}{l} \partial_\mu A^{0\mu} = m_q P + \omega \\ \mathcal{O}(N_c) \quad \mathcal{O}(N_c) \quad \mathcal{O}(1) \end{array}$$

$$\eta' \rightarrow \eta\pi\pi \text{ and } \eta' \rightarrow 3\pi$$

Need something more

Large number of colours: large N_c

In this limit:

$$\begin{array}{ccc} \partial_\mu A^{0\mu} & = & m_q P + \omega \\ \mathcal{O}(N_c) & & \mathcal{O}(N_c) \quad \mathcal{O}(1) \end{array}$$

So we CAN treat ω as a perturbation

\implies Basis of *all* the large N_c Chiral lagrangian predictions for η'

$$\eta' \rightarrow \eta\pi\pi \text{ and } \eta' \rightarrow 3\pi$$

BUT some problems remain:

- There are large $\pi\pi$ rescatterings possible in the S -wave channel ($1/N_c$ suppressed but sizable) \implies “ σ ”
- ρ and ω are present in the final states \implies obvious need to go beyond ChPT

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Some attempts at resummation exist: e.g. Beisert, Borasoy, Nissler

$$\eta' \rightarrow \eta\pi\pi \text{ and } \eta' \rightarrow 3\pi$$

Both decays have different sources in the Chiral Lagrangian

$$\mathcal{L} = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger \rangle + \frac{F^2}{4} \langle \chi U^\dagger + U \chi^\dagger \rangle - \frac{1}{2} m_0^2 \phi_0^2$$

(1) (2)

$\eta' \rightarrow \eta\pi\pi$ and $\eta' \rightarrow 3\pi$

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(1) (2)

$$\eta' \rightarrow \eta\pi\pi: \quad (1) \quad A = \frac{m_\pi^2}{6F} \left(\sqrt{2} \cos(2\theta) - \sin(2\theta) \right)$$
$$\eta' \rightarrow \pi\pi\pi: \quad (2) \quad A \sim \frac{m_u - m_d}{F_\pi}$$

$\eta' \rightarrow \eta\pi\pi$ and $\eta' \rightarrow 3\pi$

Both decays have different sources in the Chiral Lagrangian

$$\mathcal{L} = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger \rangle + \frac{F^2}{4} \langle \chi U^\dagger + U \chi^\dagger \rangle - \frac{1}{2} m_0^2 \phi_0^2$$

(1) (2)

$$\eta' \rightarrow \eta\pi\pi: \quad (1) \quad A = \frac{m_\pi^2}{6F} \left(\sqrt{2} \cos(2\theta) - \sin(2\theta) \right)$$

$$\eta' \rightarrow \pi\pi\pi: \quad (2) \quad A \sim \frac{m_u - m_d}{F_\pi}$$

	$\eta' \rightarrow \eta\pi^0\pi^0$	$\eta' \rightarrow \eta\pi^+\pi^-$	$\eta' \rightarrow \pi^0\pi^0\pi^0$	$\eta' \rightarrow \pi^0\pi^+\pi^-$
	1.0 keV	1.9 keV	455 eV	405 eV
Exp	42 ± 6 keV	90 ± 10 keV	315 ± 77 eV	≤ 10 keV

$\eta' \rightarrow \eta\pi\pi$ and $\eta' \rightarrow 3\pi$: so different?

$\eta' \rightarrow \eta\pi\pi$: no isospin breaking needed, m_π^2 disappears at higher orders \Rightarrow large enhancements possible

$\eta' \rightarrow \pi\pi\pi$: isospin breaking needed, $m_u - m_d$ remains present and fixes the order of magnitude

Borasoy, Meissner, Nissler, [hep-ph/0609010](https://arxiv.org/abs/hep-ph/0609010) discuss in more detail what is known

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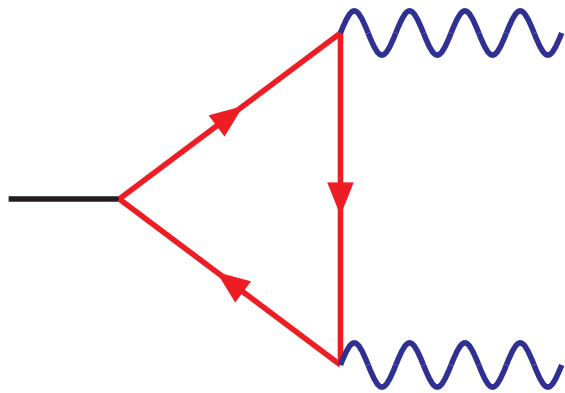
$\eta' \rightarrow \eta\pi\pi$ at NLO in $1/N_c$ can add $\eta'\eta\partial_\mu\pi\partial^\mu\pi$: fixes rate but not slope

Study distributions to uncover effects from a_0, σ, f_0, \dots and improve BR precision to check isospin prediction for ratio 2

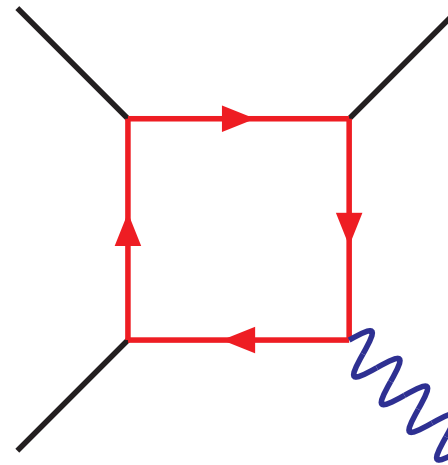
$\eta' \rightarrow \pi\pi\pi$ Detect charged decay mode, study distributions

Anomalies

The famous triangle graph (Now with photons)



but also



and ...

Wess-Zumino-Witten term, Adler-Bardeen theorem,
Anomaly, and much more

$$\partial_\mu A^{0\mu} = 2\sqrt{N_f}\omega + F_{\mu\nu}\tilde{F}^{\mu\nu} + \dots$$

anomaly with the other interactions

Anomalies: What processes

List of experiments *obviously* incomplete

$$\pi^0 \rightarrow \gamma\gamma \quad \text{Primex, } e^+e^-$$

$$\eta \rightarrow \gamma\gamma \quad \text{Primex, } e^+e^-$$

$$\eta' \rightarrow \gamma\gamma \quad e^+e^-$$

$$\gamma\pi^0\pi^+\pi^- \quad \text{Primakoff}$$

$$\eta \rightarrow \pi^+\pi^-\gamma^{(*)} \quad \text{WASA, KLOE}$$

$$\eta' \rightarrow \pi^+\pi^-\gamma^{(*)} \quad \text{WASA}$$

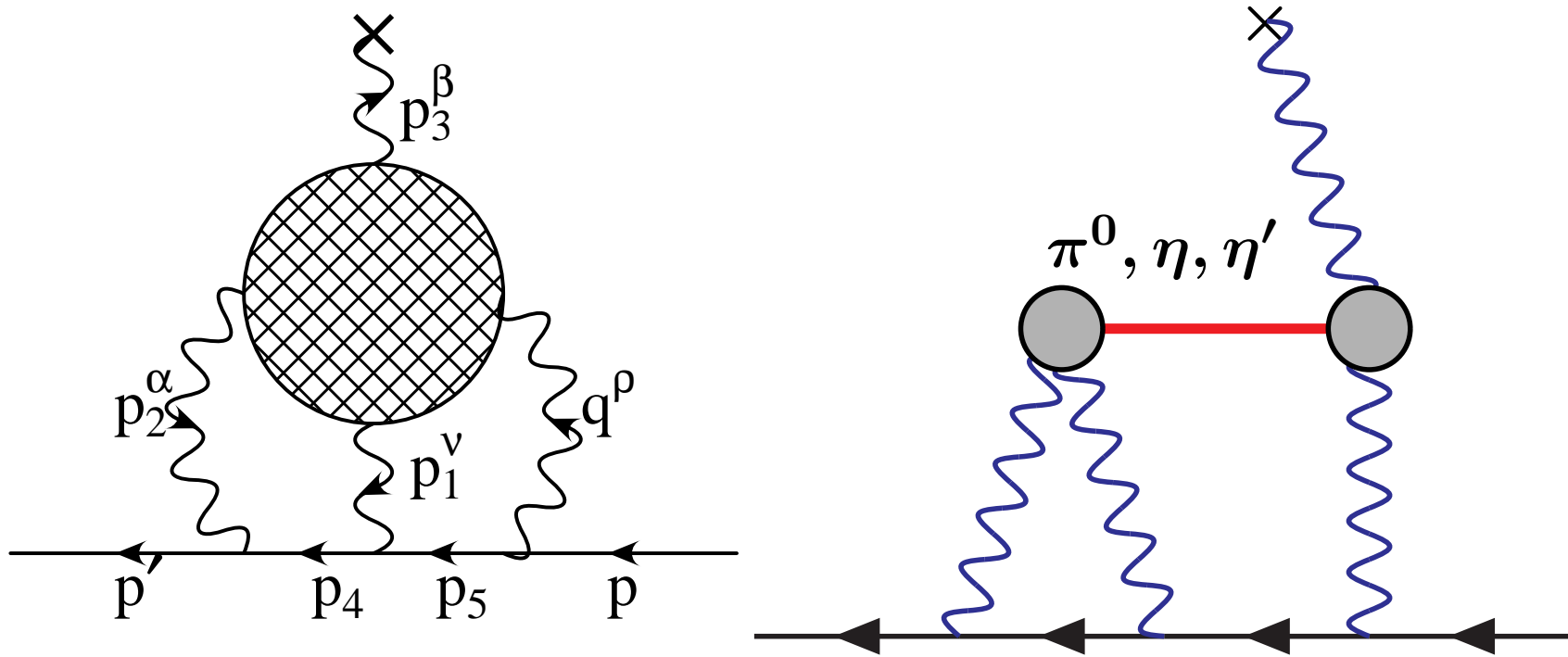
$$\eta \rightarrow \gamma^{(*)}\gamma^{(*)} \quad \text{WASA, KLOE, CLEO}c \quad \text{g-2}$$

$$\eta' \rightarrow \gamma^{(*)}\gamma^{(*)} \quad \text{WASA, KLOE, CLEO}c \quad \text{g-2}$$

$$\eta' \rightarrow \rho^0\gamma \quad \text{WASA}$$

$\gamma^{(*)}$ stands for (if allowed) $\gamma, e^+e^-, \mu^+\mu^-$ or an off-shell photon in tagged $\gamma\gamma$ collisions

Muon g-2



Conclusions

- $\eta \rightarrow 3\pi$
 - Precision physics: Q or R
 - $NNLO$ -calculation of $\eta \rightarrow 3\pi$ performed
 - Some puzzling differences vs NLO+dispersive
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- Glue important for η' mass: glue also in η' decays or is that *merely* final state interactions
- Need the mechanisms behind η, η' decays
- High quality distributions are a must: strong constraints on theory