STATUS OF THE NEW L_i^R FIT AT $O(p^6)$

Johan Bijnens

Lund University

bijnens@thep.lu.se
http://www.thep.lu.se/∼bijnens

Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Ximo

Dedicated to

Ximo Prades 1963-2010

Friend and collaborator

Postdoc 93-95 with me in Copenhagen we have worked together ever since

Ximo

We have worked together on $g-2$, $\Delta I = 1/2$, B_K, $\varepsilon'_K/\varepsilon$, Quark models and ENJL, electromagnetic effects,…and were working on rare kaon decays and $g-2$.

1. J. Bijnens and J. Prades, Mod. Phys. Lett. A **22** (2007) 767 [arXiv:hep-ph/0702170].
2. J. Bijnens and J. Prades, Acta Phys. Polon. B **38** (2007) 2819 [arXiv:hep-ph/0701240].
3. J. Bijnens, E. Gamiz and J. Prades, JHEP **0603** (2006) 048 [arXiv:hep-ph/0601197].
4. J. Prades, J. Bijnens and E. Gamiz, arXiv:hep-ph/0501177.
5. J. Bijnens, E. Gamiz and J. Prades, Nucl. Phys. Proc. Suppl. **133** (2004) 245 [arXiv:hep-ph/0309216].
6. J. Bijnens, E. Gamiz, E. Lipartia and J. Prades, JHEP **0304** (2003) 055 [arXiv:hep-ph/0304222].
7. E. Gamiz, J. Prades and J. Bijnens, Nucl. Phys. Proc. Suppl. **121** (2003) 195 [arXiv:hep-ph/0209089].
8. J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B **626** (2002) 410 [arXiv:hep-ph/0112255].

ximo

9. J. Bijnens, E. Gamiz and J. Prades, JHEP **0110** (2001) 009 [arXiv:hep-ph/0108240].
10. J. Bijnens and J. Prades, Nucl. Phys. Proc. Suppl. **96** (2001) 354 [arXiv:hep-ph/0010008].
11. J. Bijnens and J. Prades, arXiv:hep-ph/0009156.
12. J. Bijnens and J. Prades, arXiv:hep-ph/0009155.
13. J. Bijnens and J. Prades, JHEP **0006** (2000) 035 [arXiv:hep-ph/0005189].
14. J. Bijnens and J. Prades, JHEP **0001** (2000) 002 [arXiv:hep-ph/9911392].
15. J. Bijnens and J. Prades, JHEP **9901** (1999) 023 [arXiv:hep-ph/9811472].
16. J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B **521** (1998) 305 [arXiv:hep-ph/9801326].
17. J. Bijnens and J. Prades, Nucl. Phys. B **490** (1997) 239 [arXiv:hep-ph/9610360].
18. J. Bijnens, A. Fayyazuddin and J. Prades, Phys. Lett. B **379** (1996) 209 [arXiv:hep-ph/9512374].
19. J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388].

ximo

20. J. Bijnens, E. Pallante and J. Prades, Phys. Rev. Lett. **75** (1995) 1447 [Erratum-ibid. **75** (1995) 3781] [arXiv:hep-ph/9505251].
21. J. Bijnens and J. Prades, Nucl. Phys. B **444** (1995) 523 [arXiv:hep-ph/9502363].
22. J. Bijnens, J. Prades and E. de Rafael, Phys. Lett. B **348** (1995) 226 [arXiv:hep-ph/9411285].
23. J. Bijnens and J. Prades, Phys. Lett. B **342** (1995) 331 [arXiv:hep-ph/9409255].
24. J. Bijnens and J. Prades, Nucl. Phys. Proc. Suppl. **39BC** (1995) 245 [arXiv:hep-ph/9409231].
25. J. Bijnens and J. Prades, Z. Phys. C **64** (1994) 475 [arXiv:hep-ph/9403233].
26. J. Bijnens and J. Prades, Phys. Lett. B **320** (1994) 130 [arXiv:hep-ph/9310355].

Overview

- Work done with Ilaria Jemos
- LECs
- Numerical programs fully rewritten
- Old estimate of the C_i^r
- Old fit 10
- Relations to test ChPT
- new inputs and results: talks: JB CD09, IJ Euroflavour2009
- put in more inputs but go beyond old C_i^r
- some preliminary results and a cautionary remark

Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour	
p^2	F, B	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2
p^6	c_i^r	52+4	C_i^r	90+4

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

- All infinities known
- 53 $\rightarrow$ 52 arXiv:0705.0576 [hep-ph]

Two-loops two flavour, LEC

Review paper on Two-Loops: JB, hep-ph/0604043 Prog. Part. Nucl. Phys. 58 (2007) 521

$\bar{l}_1$ to $\bar{l}_4$: ChPT at order p^6 and the Roy equation analysis in $\pi\pi$ and F_S Colangelo, Gasser and Leutwyler, *Nucl. Phys.* B 603 (2001) 125 [hep-ph/0103088]

$$\bar{l}_1 = -0.4 \pm 0.6\,, \qquad \bar{l}_2 = 4.3 \pm 0.1\,,$$
$$\bar{l}_3 = 2.9 \pm 2.4\,, \qquad \bar{l}_4 = 4.4 \pm 0.2\,.$$

$\bar{l}_5$ and $\bar{l}_6$: from F_V and $\pi \to \ell\nu\gamma$ JB,(Colangelo,)Talavera and from $\Pi_V - \Pi_A$ González-Alonso, Pich, Prades,

$$\bar{l}_5 = 12.24 \pm 0.21\,, \qquad \bar{l}_6 - \bar{l}_5 = 3.0 \pm 0.3\,,$$
$$\bar{l}_6 = 16.0 \pm 0.5 \pm 0.7\,.$$

$l_7 \sim 5 \cdot 10^{-3}$ from π^0-η mixing Gasser, Leutwyler 1984

Two-loop Three-flavour

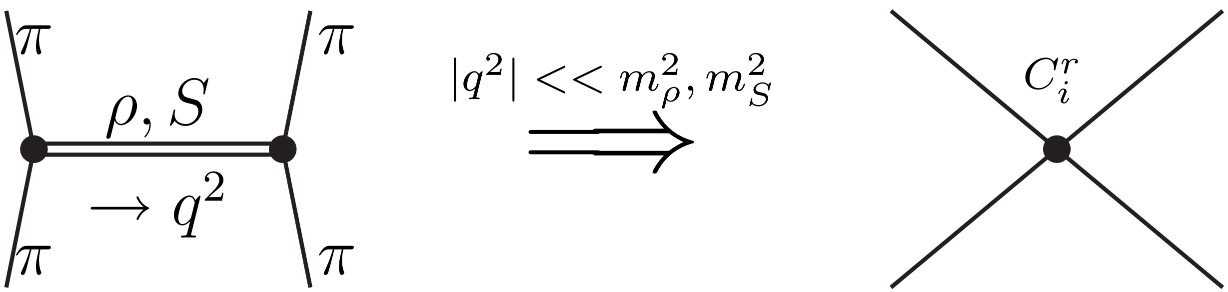
Review paper on Two-Loops: JB, hep-ph/0604043 Prog. Part. Nucl. Phys. 58 (2007) 521

In progress: changing numerical programs to C++

- Two-loop integrals numerics (somewhat) improved
- Consistent interface
- Use classes to make handling many sets of LECs easier

C_i^r simple estimates

Most analysis use:
C_i^r from (single) resonance approximation



Motivated by large N_c: large effort goes in this

Ananthanarayan, JB, Cirigliano, Donoghue, Ecker, Gamiz, Golterman, Kaiser, Kampf, Knecht, Moussallam, Peris, Pich, Prades, Portoles, de Rafael,. . .

Beyond tree level: $R\chi T$ Cata, Peris, Pich, Portoles, Rosell, . . .

C_i^r

$$\mathcal{L}_V = -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}m_V^2\langle V_\mu V^\mu\rangle - \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle$$
$$-\frac{ig_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu,u^\nu]\rangle + f_\chi\langle V_\mu[u^\mu,\chi_-]\rangle$$
$$\mathcal{L}_A = -\frac{1}{4}\langle A_{\mu\nu}A^{\mu\nu}\rangle + \frac{1}{2}m_A^2\langle A_\mu A^\mu\rangle - \frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle$$
$$\mathcal{L}_S = \frac{1}{2}\langle\nabla^\mu S\nabla_\mu S - M_S^2 S^2\rangle + c_d\langle S u^\mu u_\mu\rangle + c_m\langle S\chi_+\rangle$$
$$\mathcal{L}_{\eta'} = \frac{1}{2}\partial_\mu P_1\partial^\mu P_1 - \frac{1}{2}M_{\eta'}^2 P_1^2 + i\tilde{d}_m P_1\langle\chi_-\rangle\,.$$

$f_V = 0.20$, $f_\chi = -0.025$, $g_V = 0.09$, $c_m = 42$ MeV, $c_d = 32$ MeV, $\tilde{d}_m = 20$ MeV,

$m_V = m_\rho = 0.77$ GeV, $m_A = m_{a_1} = 1.23$ GeV, $m_S = 0.98$ GeV, $m_{P_1} = 0.958$ GeV

f_V, g_V, f_χ, f_A: experiment

c_m and c_d from resonance saturation at $\mathcal{O}(p^4)$

C_i^r

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

The old fit 10

- Existing fit of the L_i at NNLO (fit 10) Amoros, Bijnens, Talavera, Nucl. Phys. B 602 (2001) 87 [hep-ph/0101127]
- INPUT:
  1. masses: $m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$
  2. $F_\pi = 92.4\,\mathrm{MeV}$
  3. $K_{\ell 4}$: f_s, f'_s, g_s, g'_s (linear fit in E865) L_1^r, L_2^r, L_3^r
  4. $F_K/F_\pi = 1.22 \pm 0.01$ L_5^r
  5. $m_s/\hat{m} = 24$ L_5^r, L_7^r, L_8^r
  6. $L_4^r \equiv L_6^r \equiv 0$ ($1/N_c$ suppressed)
  7. L_9^r, L_{10}^r Separate fit done
- C_i^r from resonance saturation as show above $\mu \equiv 0.77\,\mathrm{Gev}$. $\mu = 0.5, 1\,\mathrm{Gev}$ within errors.

Fit 10: output

fit10: with isospin breaking corrections
fit10 iso: no isospin breaking (new programs)

	fit 10	fit 10 iso
$10^3 L_1^r$	0.43	0.40 ± 0.12
$10^3 L_2^r$	0.73	0.76 ± 0.12
$10^3 L_3^r$	-2.35	-2.40 ± 0.37
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$
$10^3 L_5^r$	0.97	0.97 ± 0.11
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$
$10^3 L_7^r$	-0.31	-0.30 ± 0.15
$10^3 L_8^r$	0.60	0.61 ± 0.20
χ^2 (dof)	- -	0.28 (1)

Convergence using fit10 iso
(similar for fit10)

	p^2	p^4	p^6
m_π^2	0.753	0.006	0.241
m_K^2	0.702	0.008	0.290
m_η^2	0.748	-0.046	0.298

- $L_4^r \equiv L_6^r \equiv 0$ and no estimates of the C_i^r appearing in the masses $\rightarrow$ maybe source of bad convergence
- F_K/F_π convergence is good

Relations at NNLO

Yes: JB, Jemos 2009, Systematic search for relations between observables that do not depend on the C_i^r.
Included:

- m_M^2 and F_M for π, K, η.
- 11 $\pi\pi$ threshold parameters
- 14 πK threshold parameters
- 6 $\eta \to 3\pi$ decay parameters,
- 10 observables in $K_{\ell 4}$
- 18 in the scalar formfactors
- 11 in the vectorformfactors
- Total: 76

We found 35 relations

Relations: ChPT (sort of) works

- We did numerics for $\pi\pi$, πK and $K_{\ell 4}$: 13 relations
- The two involving a_3^- significantly did not work well
- The relation with $K_{\ell 4}$ also did not work:

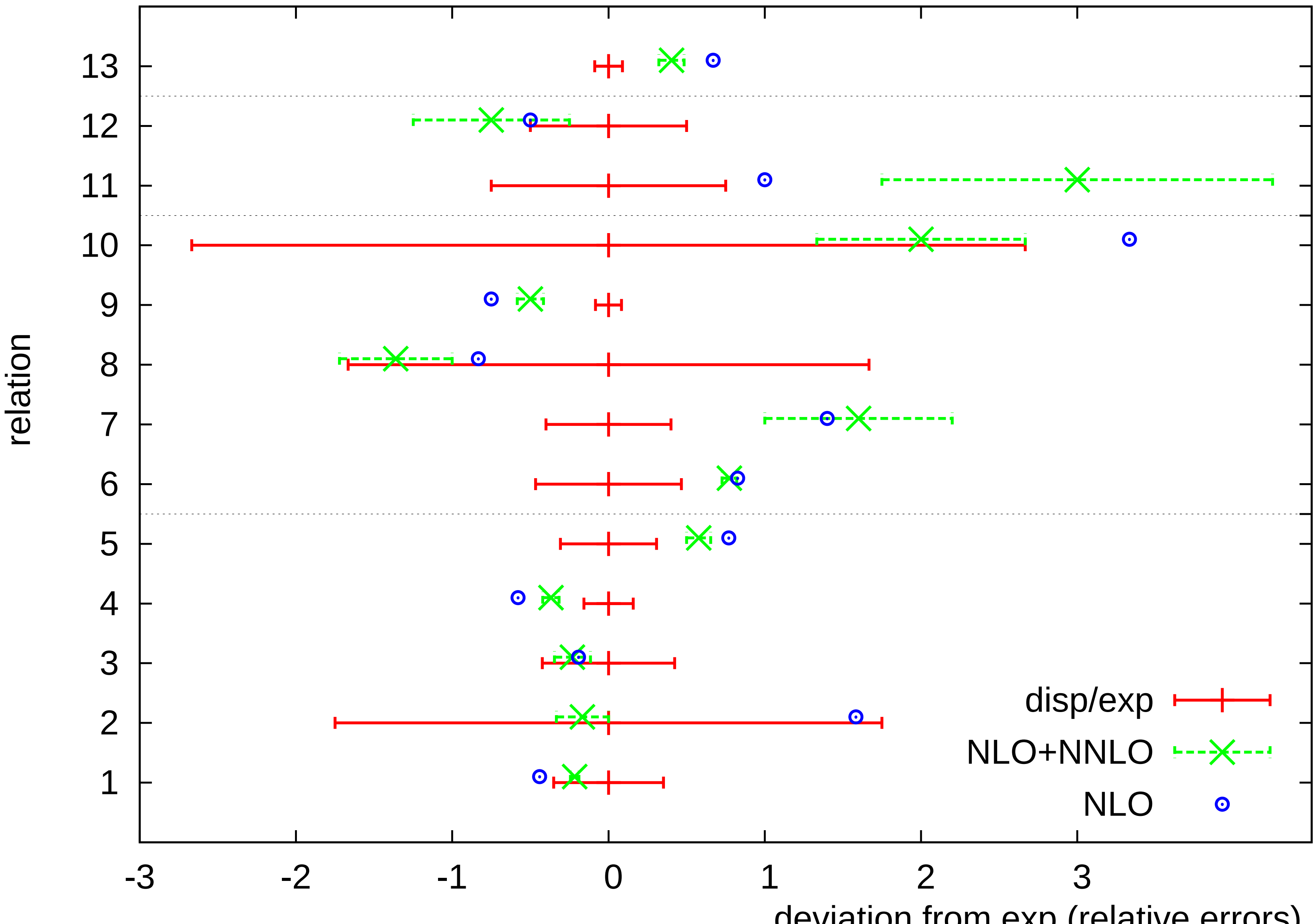
$$\sqrt{2}\,[f_s'']_{C_i} = \frac{32\pi\rho F_\pi}{1+\rho}\left[\frac{35}{6}\left(2+\rho+2\rho^2\right)\left[a_3^+\right]_{C_i} - \frac{5}{4}\left[a_2^+ + 2\rho b_2^+\right]_{C_i}\right]$$

	Roy-Steiner NA48	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	remainder
LHS	-0.73 ± 0.10	-0.23	0.00	-0.15	-0.05	-0.29 ± 0.10
RHS	0.50 ± 0.07	0.19	0.00	0.10	0.03	0.18 ± 0.07

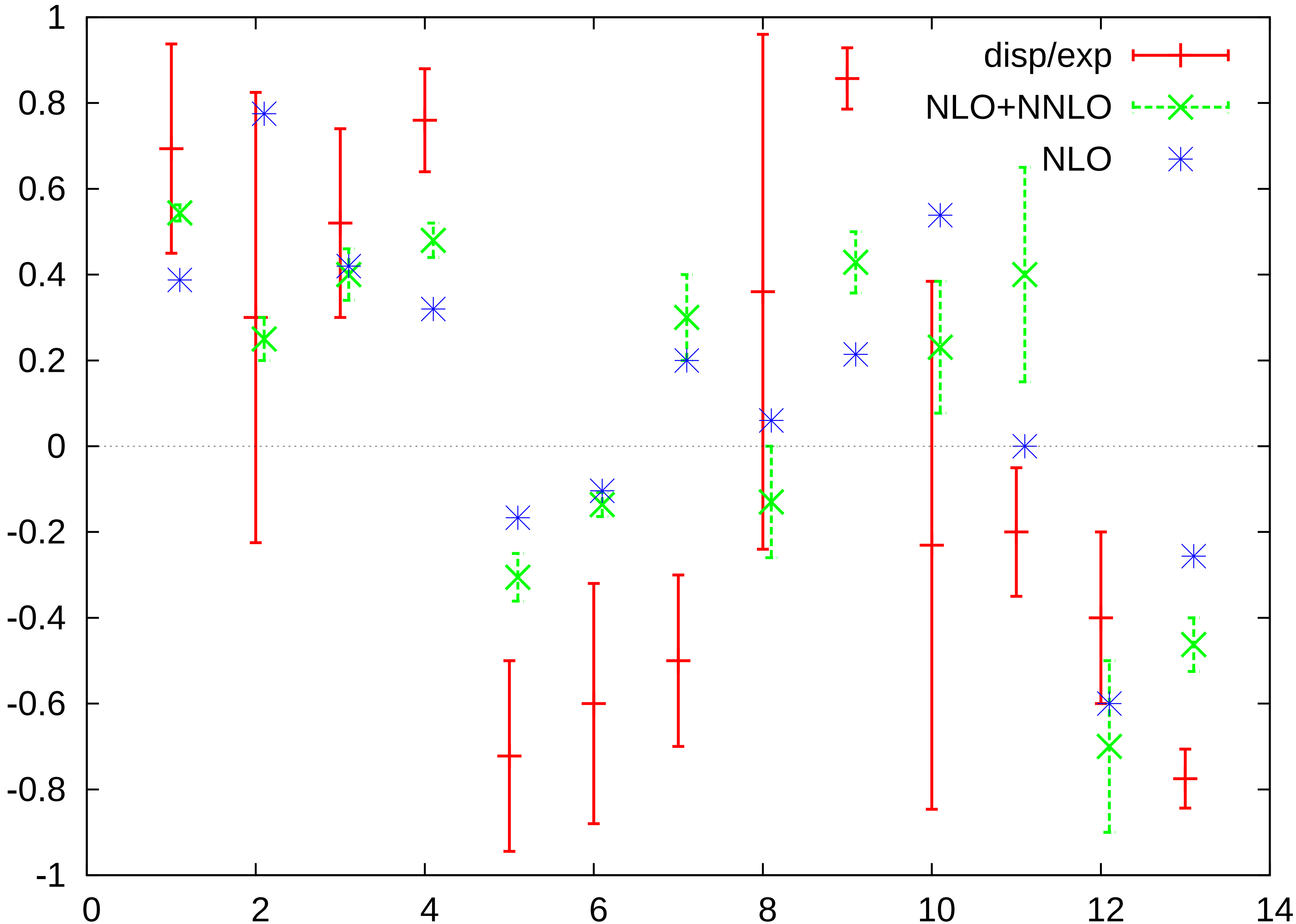
πK-scattering lengths and curvature in F in $K_{\ell 4}$

Resonance p^6 contribution both sides +0.05

Relations: ChPT (sort of) works



Relations: ChPT (sort of) works



Fit: Inputs

Fit: Amoros, JB Talavera 2001

$K_{\ell 4}$: $F(0)$, $G(0)$, λ_F, λ_G	E865 BNL $\Longrightarrow$ NA48
$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$	em with Dashen violation
F_{π^+}	$92.4 \Longrightarrow 92.2 \pm 0.05$ MeV
F_{K^+}/F_{π^+}	$1.22 \pm 0.01 \Longrightarrow 1.193 \pm 0.002 \pm 0.006 \pm 0.001$
$m_s/\hat{m}$	24 (26) (27.8 lattice)
L^r_4, L^r_6	free

Many more calculations done: include those as well;
Comprehensive new fit in progress: preliminary results, see below

New fitting results

	fit 10 iso	NA48	F_K/F_π	Scatt	All	All ($C_i^r=0$)
$10^3 L_1^r$	0.40 ± 0.12	0.98	0.97	0.97	0.98 ± 0.11	0.75
$10^3 L_2^r$	0.76 ± 0.12	0.78	0.79	0.79	0.59 ± 0.21	0.09
$10^3 L_3^r$	-2.40 ± 0.37	-3.14	-3.12	-3.14	-3.08 ± 0.46	-1.49
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.71 ± 0.67	0.78
$10^3 L_5^r$	0.97 ± 0.11	0.93	0.72	0.56	0.56 ± 0.11	0.67
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.15 ± 0.71	0.18
$10^3 L_7^r$	-0.30 ± 0.15	-0.30	-0.26	-0.23	-0.22 ± 0.15	-0.24
$10^3 L_8^r$	0.61 ± 0.20	0.59	0.48	0.44	0.38 ± 0.18	0.39
χ^2 (dof)	0.25 (1)	0.17 (1)	0.19 (1)	5.38 (5)	1.44 (4)	1.51 (4)

- NA48: use NA48 formfactors but E865 normalization
- F_K/F_π also change this to 1.193
- Scatt: add a_0^0, a_0^2, $a_0^{1/2}$ and $a_0^{3/2}$, $\chi^2 = 5.04$ from a_0^2
- All: add pion scalar radius 0.61 ± 0.04: $\chi^2 = 61$!! for $L_4^r = L_6^r = 0$
- All results preliminary
- JB CD09. IJ Euroflavour2009 similar but many more things tried.

But then

	p^4	p^6	2-loop	L_i^r	C_i^r
F_π/F_0	0.301	0.081	-0.102	0.183	0
F_K/F_0	0.429	0.185	-0.092	0.278	0
F_K/F_π	0.127	0.066			
m_π^2/m_π^2	-0.188	0.006	0.252	-0.246	0
m_K^2/m_K^2	-0.194	0.095	0.357	-0.261	0
m_η^2/m_η^2	-0.237	0.061	0.528	-0.402	-0.065

- Ugly convergence
- $\bar{l}_i$ from L_i^r requires $1/N_c$ suppressed C_i^r

more input

- $\bar{l}_1, \ldots, \bar{l}_4$
- Curvature in scalar form factor (c_S^π)
- Insist on not too large p^6 corrections in'
  - F_π, F_K and F_K/F_π (5% is $\chi^2 = 1$)
  - masses (10% is $\chi^2 = 1$)
- Very large χ^2
- Need to do something more with C_i^r
- $\bar{l}_1, \ldots, \bar{l}_4$ need $1/N_c$ suppressed C_i^r

Searching for C_i^r

Procedure:

- Random walk in the C_i^r (discussion with Ecker/Gasser)
- Start with a set of C_i^r
- Minimize by varying L_i^r
- random step by $0.01(1/3)/(16\pi^2)^2 r_i$
  $-1 < r_i < 1$
- Minimize by varying L_i^r
- If χ^2 decreases accept, else accept with small probability with a "temperature"
- next step

Results

Many good fits possible with small total χ^2, too many

	all			
$10^3 L_1^r$	0.98 ± 0.11	1.29	1.01	1.02 ± 0.09
$10^3 L_2^r$	0.59 ± 0.21	1.39	0.90	0.51 ± 0.04
$10^3 L_3^r$	-3.08 ± 0.46	-5.67	-4.82	-4.07 ± 0.29
$10^3 L_4^r$	0.71 ± 0.67	0.19	0.06	0.25 ± 0.09
$10^3 L_5^r$	0.56 ± 0.11	1.46	1.35	1.38 ± 0.09
$10^3 L_6^r$	0.15 ± 0.71	0.05	0.08	0.14 ± 0.13
$10^3 L_7^r$	-0.22 ± 0.15	-0.37	-0.32	-0.32 ± 0.12
$10^3 L_8^r$	0.38 ± 0.18	0.66	0.63	0.61 ± 0.14
χ^2 (dof)		0.55	0.34	0.48

Remarks

- Errors are including the requirement on small NNLO corrections
- C_i^r patterns rather more different than the L_i^r
- Large N_c pattern of the C_i^r seems to remain
- But see no easy way to prefer one set above another

A cautionary remark

- $F_\pi / F_0 = 1 + \pi_4 + \pi_6$
- $F_K / F_0 = 1 + k_4 + k_6$
- $F_K / F_\pi = 1 + k_4 - \pi_4 + k_6 - \pi_6 + (\pi_4)^2 - k_4 \pi_4$
- Require small corrections to all 3 at p^6

A cautionary remark

- $F_\pi / F_0 = 1 + \pi_4 + \pi_6$
- $F_K / F_0 = 1 + k_4 + k_6$
- $F_K / F_\pi = 1 + k_4 - \pi_4 + k_6 - \pi_6 + (\pi_4)^2 - k_4 \pi_4$
- Require small corrections to all 3 at p^6
- $\pi_6 = k_6 = \pi_4 = 0$

  $$k_4 = \frac{F_K}{F_\pi}$$

  $$F_0 = F_\pi$$
- See also comments in resummed ChPT talk by Toucas
- Always problem with curvature in F formfactor in $K_{\ell 4}$.

Conclusions

- Whole formalism for a new fit set up
- Relations: ChPT seems to sort of work also for three flavour
- New fit needed: new data
- Naive resonance saturation has problems with $\bar{l}_i$ versus L_i^r
- Naive resonance saturation has problems with corrections to the masses
- Can find good fits with “reasonable looking” L_i^r
- But no good feeling which set of C_i^r one should take