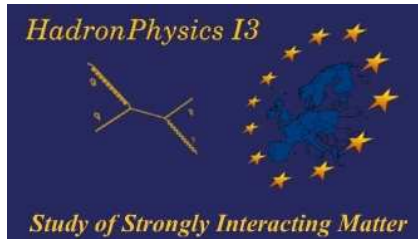




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Lund University: High Energy Theory

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Overview

- Who are we?
- Recent papers
- What do we do?
- Colour effects at small x
- Partially Quenched Chiral Perturbation Theory
 - Chiral Perturbation Theory
 - What is partially quenched?
 - ChPT and Lattice QCD
 - Very long expressions (and why)
 - Some results as well

Who are we

● Faculty

- Bijnens Johan
- Gustafson Gösta
- Lönnblad Leif
- Sjöstrand Torbjörn (at CERN)
- Svensson Bengt E Y (retired)

● PhD Students

- Avsar Emil
- Danielsson Niclas (19/12/06)
- Flensburg Christoffer*
- Ghorbani Karim
- Lavesson Nils
- Sjödahl Malin (29/9/06 → Manchester)

Who are we

- HEP-EST EU school on LHC physics (joint theory-experiment)
 - Nele Boelaert (BE)
 - Lisa Carloni (IT)
 - Richard Corke (UK)
 - Alexandru Dobrin (RO)
 - Christoffer Flensburg (SE) (faculty LU TP)
 - Jacob Groth-Jensen (DK)
 - Wei-Na Ji (CN)
 - Jie Lu (CN)
- Integrate theory and experiment
- Use PBL as a learning tool at the graduate level
- <http://www.hep.lu.se/Lund-HEP/>

Recent past students

\geq 2003

- Sandipan Mohanty
- Fredrik Söderberg
- Fredrik Borg
- Pierre Dhonte
- Peter Skands (Fermilab)

Recent papers

- J. Bijnens and N. Danielsson, “The eta mass and NNLO three-flavor partially quenched chiral perturbation theory,”
- J. Bijnens, “Chiral perturbation theory beyond one loop,”
- J. Bijnens and K. Ghorbani, “Finite volume dependence of the quark-antiquark vacuum expectation value,”
- **J. Bijnens, N. Danielsson and T. A. Lahde, “Three-flavor partially quenched chiral perturbation theory at NNLO for meson masses and decay constants,”**
- J. Bijnens, E. Gamiz and J. Prades, “The $B(K)$ kaon parameter in the chiral limit,”
- T. A. Lahde, J. Bijnens and N. Danielsson, “Partially quenched chiral perturbation theory to NNLO,”
- J. Bijnens, “eta and eta’ decays and what can we learn from them?,”

Recent papers

- E. Avsar, G. Gustafson and L. Lonnblad, “Small-x Dipole Evolution Beyond the Large- N_c Limit,”
- L. Lönnblad and M. Sjö Dahl, “Classical and non-classical ADD-phenomenology with high-E(T) jet observables at collider experiments,”
- L. Lonnblad, “ThePEG, Pythia7, herwig++ and Ariadne,”
- J. R. Andersen *et al.* [Small x Collaboration], “Small x phenomenology: Summary of the 3rd Lund small x workshop in 2004,”
- S. Hoche, F. Krauss, N. Lavesson, L. Lonnblad, M. Mangano, A. Schalicke and S. Schumann, “Matching parton showers and matrix elements,”
- L. Lönnblad, M. Sjö Dahl and T. Åkesson, “QCD-suppression by black hole production at the LHC,”

Recent papers

- N. Lavesson and L. Lonnblad, “W + jets matrix elements and the dipole cascade,”
- E. Avsar, G. Gustafson and L. Lonnblad, “Energy conservation and saturation in small-x evolution,”
- M. Sjö Dahl and G. Gustafson, “Gravitational scattering in the ADD-model at high and low energies,”
- S. Alekhin *et al.*, “HERA and the LHC - A workshop on the implications of HERA for LHC physics”
- T. Q. W. Group *et al.*, “Tevatron-for-LHC report of the QCD working group,”
- W. M. Yao *et al.* [Particle Data Group], “Review of particle physics,”
- T. Sjostrand, S. Mrenna and P. Skands, “PYTHIA 6.4 physics and manual.”

What do we do

PHENOMENOLOGY and the STRONG interaction

What do we do

PHENOMENOLOGY and the STRONG interaction

We do this at low (JB) and high (GG,LL,TS) energies

What do we do

PHENOMENOLOGY and the STRONG interaction

We do this at low (JB) and high (GG,LL,TS) energies

At high energies:

- Matching matrix elements and parton showers
- Updating the Lund Monte Carlos to C++
- Extra large dimension signatures
- Signatures for [Beyond the Standard Model](#) in the presence of large QCD backgrounds

What do we do

At high energies:

- Small x : resumming logarithms in x and Q^2 . Cascade models with applications to DIS and high energy pp collisions.
- LHC physics: Combine formulations in momentum space and transverse coordinate space in analyses of final state properties, including saturation effects and correlations.
- ...

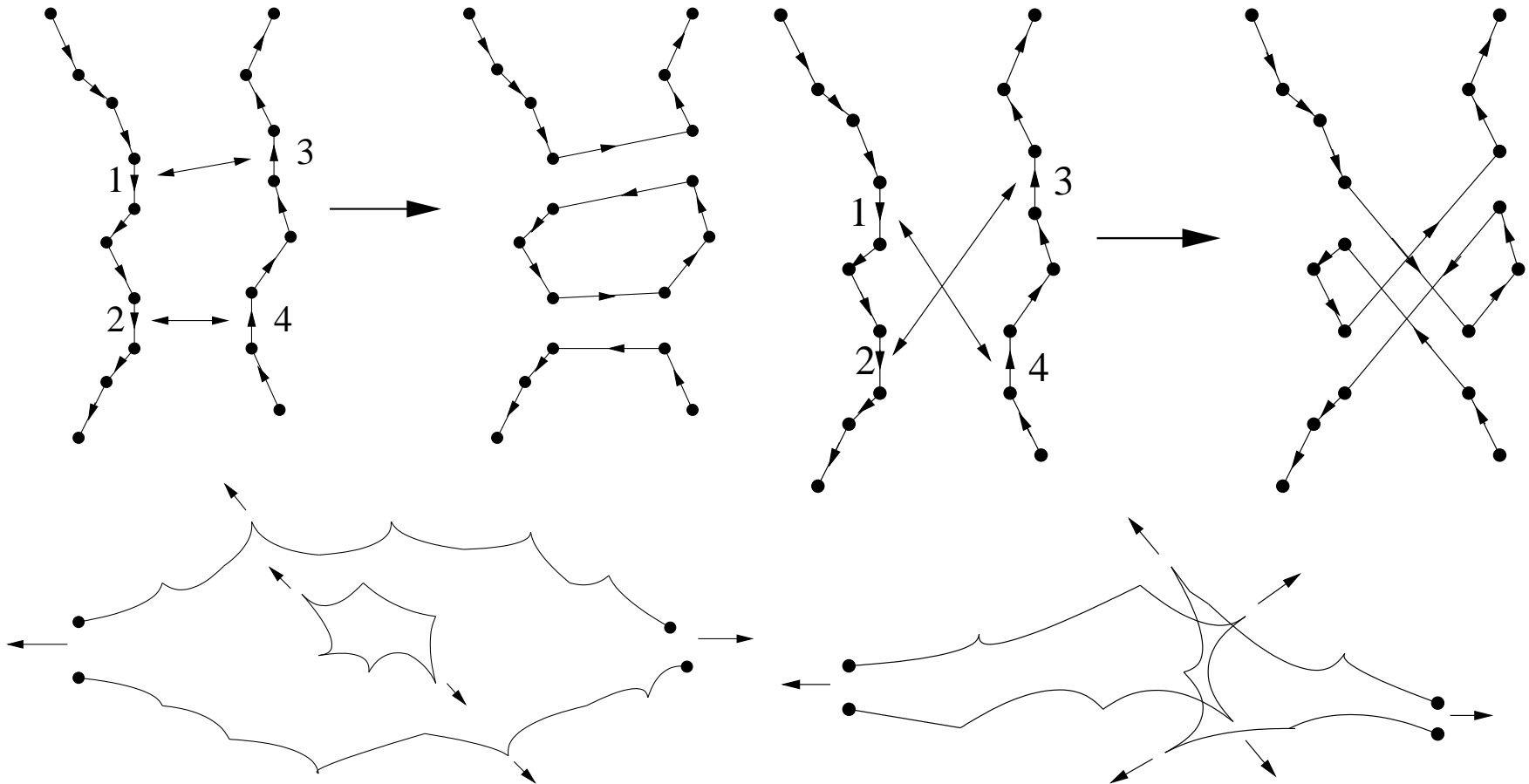
What do we do

At low energies:

- Flavour Physics
- Using Effective Field Theory, especially Chiral Perturbation Theory for
 - $\pi\pi$, πK scattering
 - $\eta \rightarrow 3\pi$: talk Karim Ghorbani
 - V_{us} from $K \rightarrow \pi e\nu$.
 - Connecting Lattice QCD to the real world: Chiral Extrapolations
 - ...
- CP violation in Kaon decays

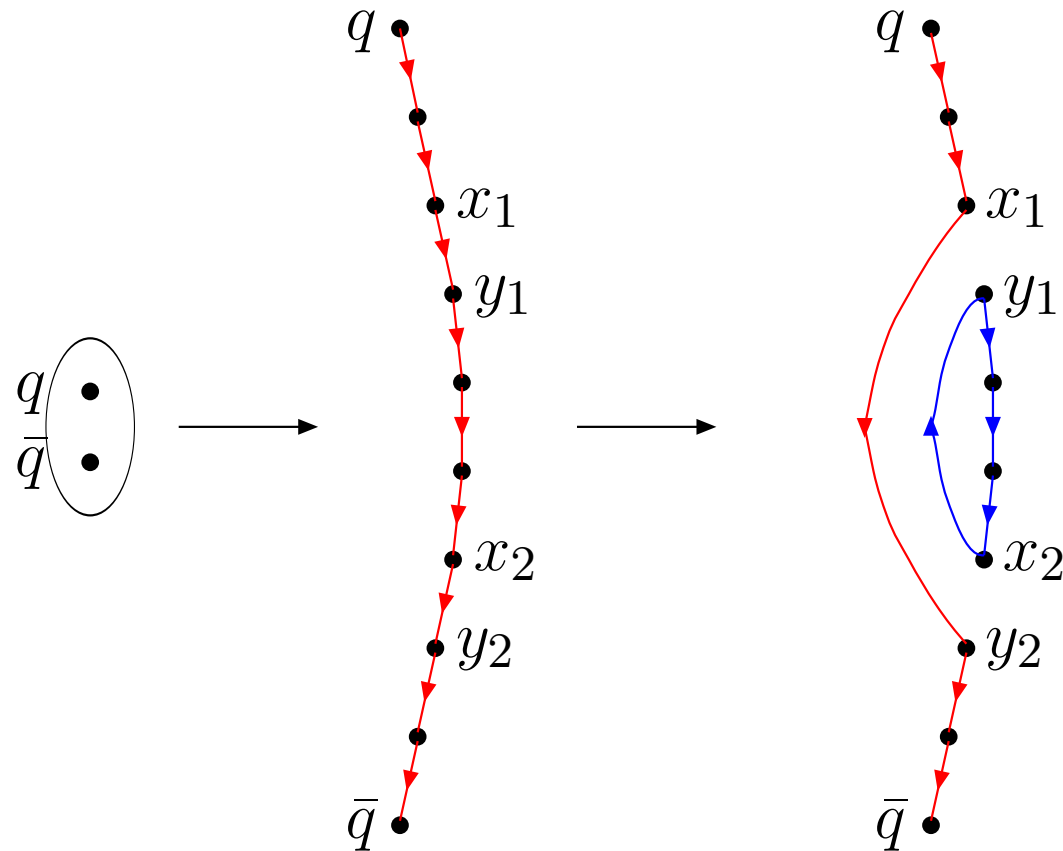
Colour effects at small x

E. Avsar, G. Gustafson and L. Lonnblad, "Small- x Dipole Evolution Beyond the Large- N_c Limit,"



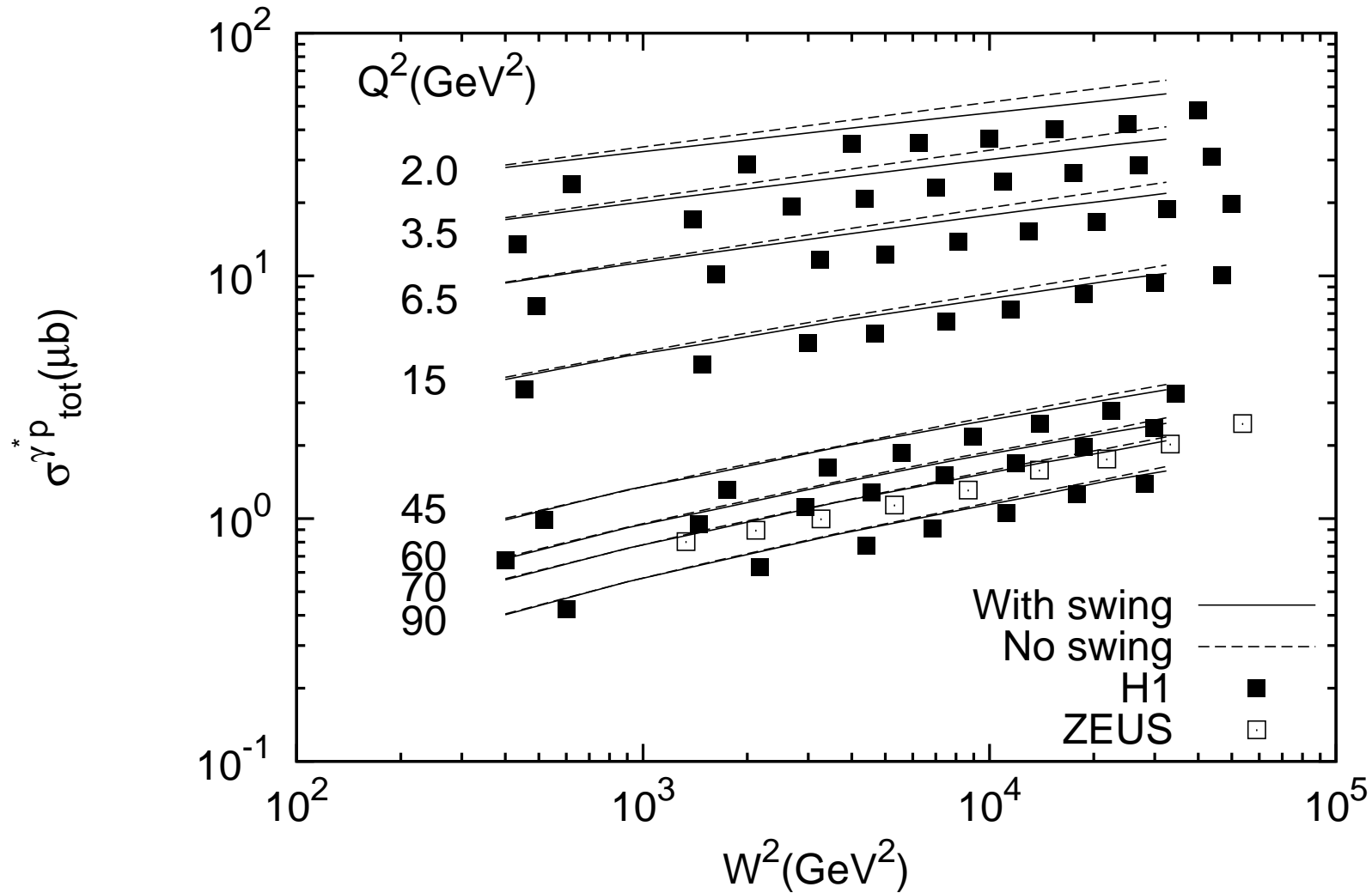
Colour effects at small x

Must allow also for collisions inside one chain to be Lorentz invariant: a swing

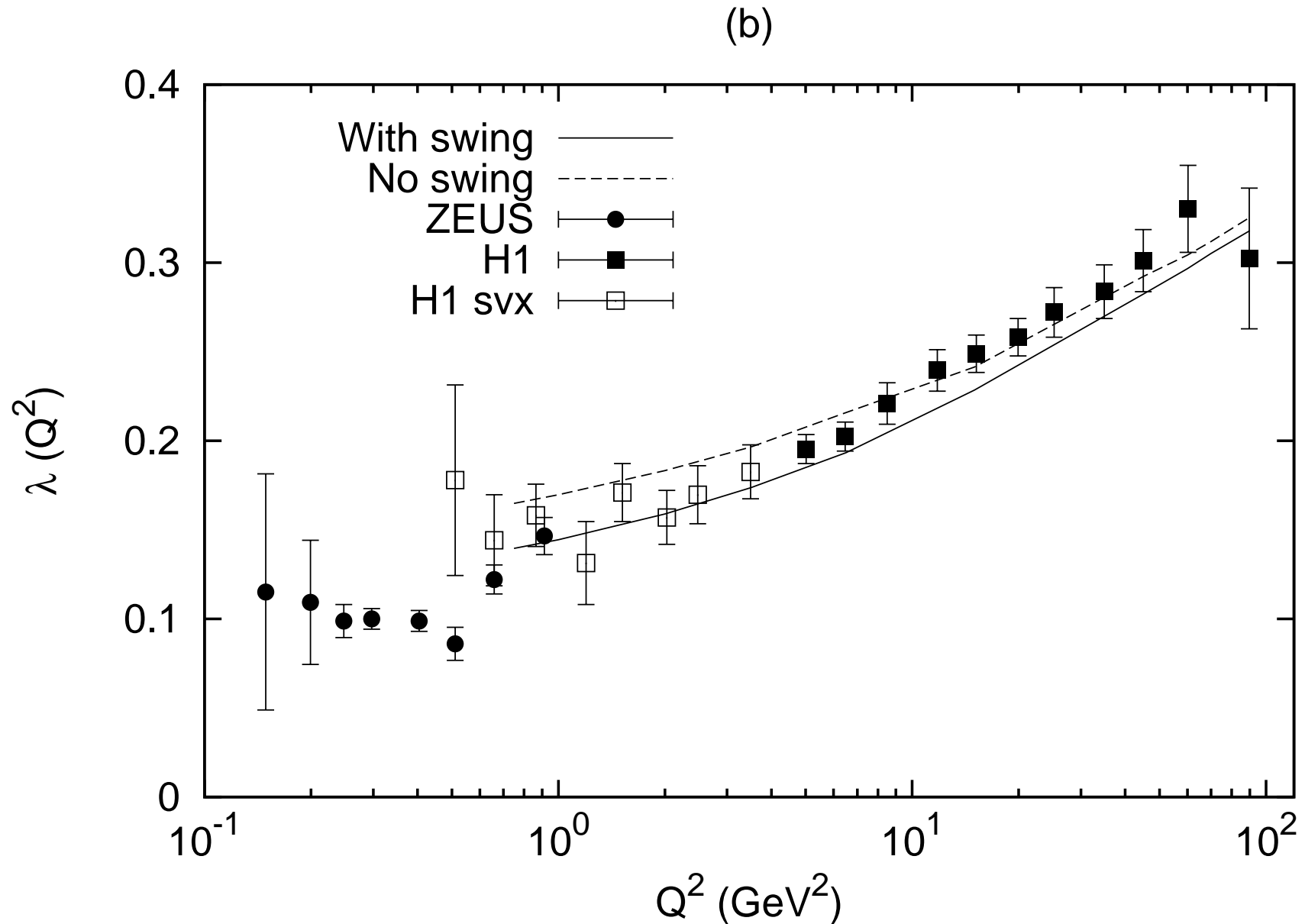


Colour effects at small x: $\gamma^* p$

(a)



Colour effects at small x: γ^*p slope



Chiral Perturbation Theory

Chiral Symmetry:

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But
$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken \implies 8 massless degrees of freedom
and interaction vanishes at zero momentum

Chiral Perturbation Theory

Make an Effective Field Theory with:

Degrees of freedom: Goldstone Bosons from Chiral
Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher
depending on the channel

Chiral Perturbation Theory

Make an Effective Field Theory with:

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Perturbation Theory with mesons, expansion in momenta

used for **The physics of pions, kaons and eta**

Nonrenormalizable: new parameters (Low Energy Constants (LEC)) at every new order

Review paper: JB, LU TP 06-16 hep-ph/0604043

What is Partially Quenched?

In Lattice gauge theory one calculates

$$\langle 0 | (\bar{u} \gamma_5 d)(x) (\bar{d} \gamma_5 u)(0) | \rangle$$
$$= \frac{\int [dq][d\bar{q}][dG] (\bar{u} \gamma_5 d)(x) (\bar{d} \gamma_5 u)(0) e^{i \int d^4 y \mathcal{L}_{\text{QCD}}}}{\int [dq][d\bar{q}][dG] e^{i \int d^4 y \mathcal{L}_{\text{QCD}}}}$$

for Euclidean separations x

Integrals also performed after rotation to Euclidean
(note that I use Minkowski notation throughout)

What is Partially Quenched?

$$\int [dq][d\bar{q}][dG] (\bar{u}\gamma_5 d)(x) (\bar{d}\gamma_5 u)(0) e^{i \int d^4 y \mathcal{L}_{\text{QCD}}} \propto$$

$$\int [dG] e^{i \int d^4 x (-1/4) G_{\mu\nu} G^{\mu\nu}} (\not{D}_G^u)^{-1}(x, 0) (\not{D}_G^d)^{-1}(0, x) \det(\not{D}_G)_{\text{QCD}}$$

What is Partially Quenched?

$$\int [dq][d\bar{q}][dG] (\bar{u}\gamma_5 d)(x) (\bar{d}\gamma_5 u)(0) e^{i \int d^4 y \mathcal{L}_{\text{QCD}}} \propto$$

$$\int [dG] e^{i \int d^4 x (-1/4) G_{\mu\nu} G^{\mu\nu}} (\not{D}_G^u)^{-1}(x, 0) (\not{D}_G^d)^{-1}(0, x) \det(\not{D}_G)_{\text{QCD}}$$

$\int [dG]$ done via importance sampling

- Quenched: get distribution from $e^{i \int d^4 x (-1/4) G_{\mu\nu} G^{\mu\nu}}$ only
- Unquenched: include $\det(\not{D}_G)_{\text{QCD}}$ VERY expensive
- Partially quenched: $(\not{D}_G^u)^{-1}(x, 0) (\not{D}_G^d)^{-1}(0, x)$
DIFFERENT Quarks then in $\det(\not{D}_G)_{\text{QCD}}$

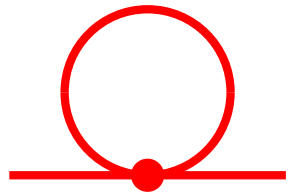
What is Partially Quenched?

Why do this?

- Is not Quenched: Real QCD is continuous limit from Partially Quenched
- More handles to turn:
 - Allows more systematic studies by varying parameters
 - Sometimes allows to disentangle things from different observables: **Physical results from unphysical calculations**
- $\det(\not{D}_G)_{\text{QCD}}$: Sea quarks
- $(\not{D}_G^u)^{-1}(x, 0)(\not{D}_G^d)^{-1}(0, x)$: Valence Quarks

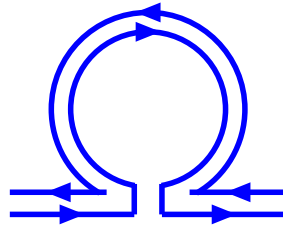
ChPT and Lattice QCD

Mesons



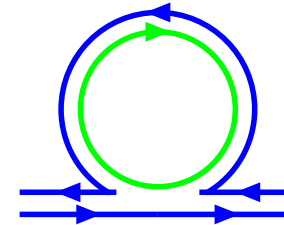
=

Quark Flow
Valence



+

Quark Flow
Sea



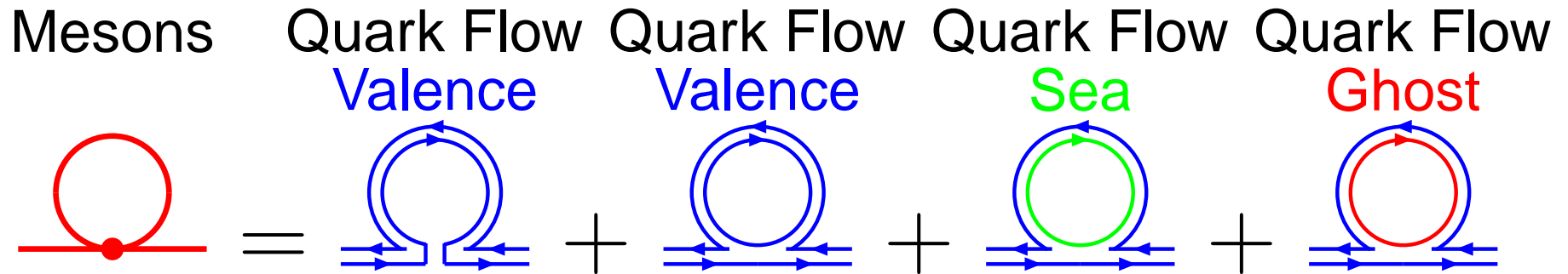
+ ...

Valence and Sea treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

PQChPT at Two Loops: General

Add ghost quarks: remove the unwanted free valence loops



Possible problem: **QCD** \implies **ChPT** relies heavily on unitarity

Symmetry group becomes $SU(n_v + n_s | n_v) \times SU(n_v + n_s | n_v)$
(approximately)

PQChPT at Two Loops: General

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

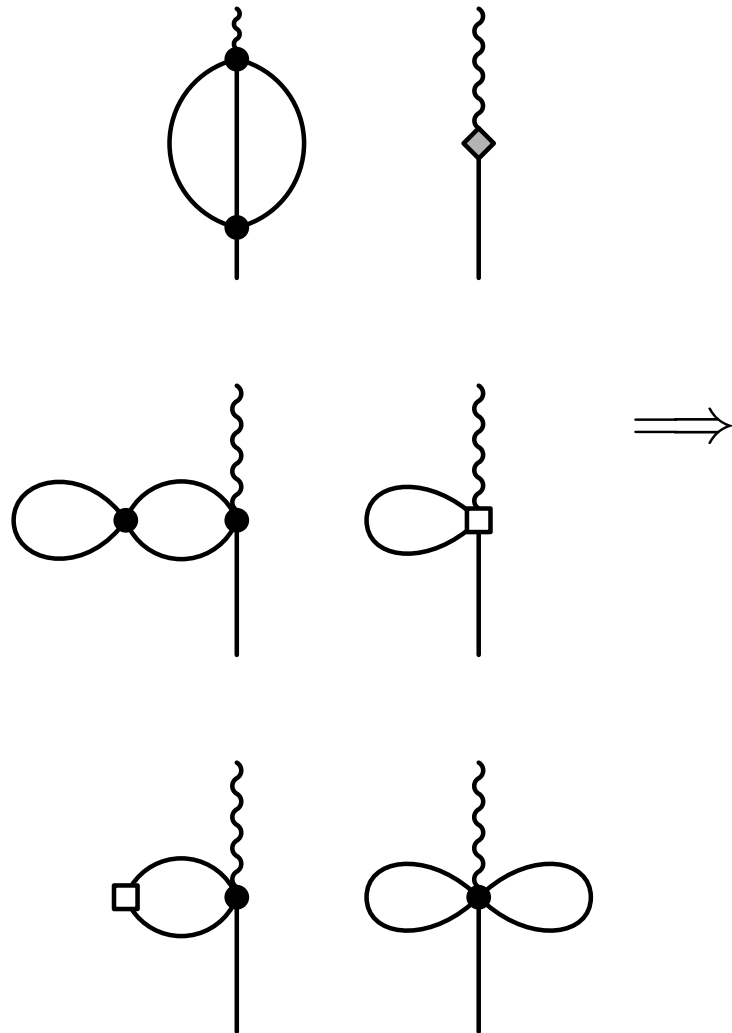
Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

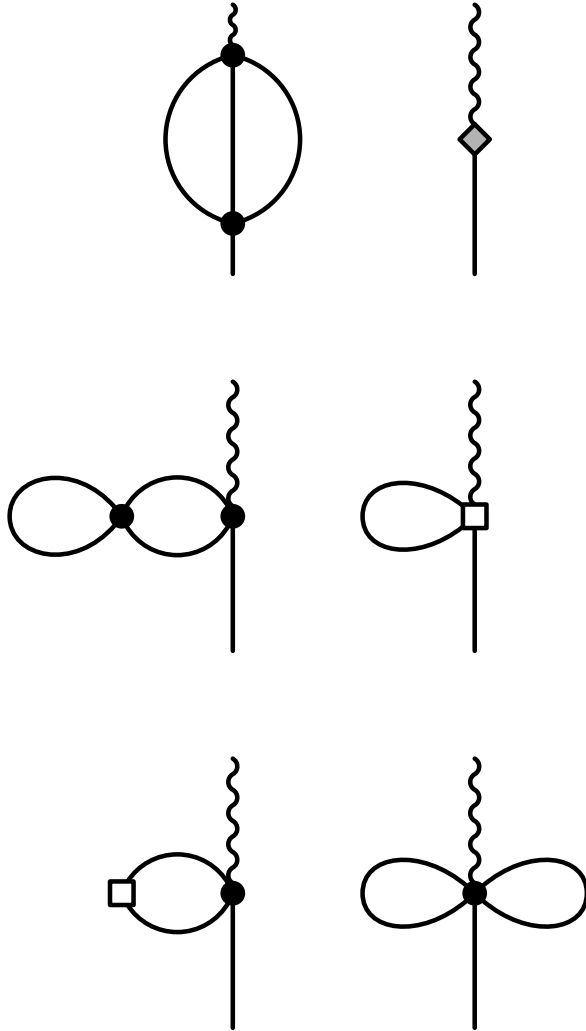
Very important note: ChPT is a limit of PQChPT
 \implies LECs from ChPT are linear combinations of LECs of PQChPT with the **same** number of sea quarks.

$$\text{E.g. } L_1^r = L_0^{r(3pq)} / 2 + L_1^{r(3pq)}$$

Long Expressions



Long Expressions



$$\begin{aligned}
 \delta_{\text{loops}}^{(6)22} = & \pi_{16} L_0^2 [4/9 \chi_7 \chi_4 - 1/2 \chi_1 \chi_3 + \chi_{13}^2 - 13/3 \bar{\chi}_1 \chi_{13} - 35/18 \bar{\chi}_2] - 2 \pi_{16} L_1^2 \chi_{13}^2 \\
 & - \pi_{16} L_2^2 [11/3 \chi_7 \chi_4 + \chi_{13}^2 + 13/3 \bar{\chi}_2] + \pi_{16} L_3^2 [4/9 \chi_7 \chi_4 - 7/12 \chi_1 \chi_3 + 11/6 \chi_{13}^2 - 17/6 \bar{\chi}_1 \chi_{13} - 43/36 \bar{\chi}_2] \\
 & + \pi_{16}^2 [-15/64 \chi_7 \chi_4 - 59/384 \chi_1 \chi_3 + 65/384 \chi_{13}^2 - 1/2 \bar{\chi}_1 \chi_{13} - 43/128 \bar{\chi}_2] - 48 L_4^2 L_5^2 \bar{\chi}_1 \chi_{13} - 72 L_4^2 \bar{\chi}_2^2 \\
 & - 8 L_5^2 \chi_{13}^2 + \bar{A}(\chi_p) \pi_{16} [-1/24 \chi_p + 1/48 \bar{\chi}_1 - 1/8 \bar{\chi}_1 R_{\eta\eta}^p + 1/16 \bar{\chi}_1 R_p^c - 1/48 R_{\eta\eta}^p \chi_p - 1/16 R_{\eta\eta}^p \chi_q \\
 & + 1/48 R_{pp}^p \chi_\eta + 1/16 R_p^c \chi_{13}] + \bar{A}(\chi_p) L_0^2 [8/3 R_{\eta\eta}^p \chi_p + 2/3 R_p^c \chi_p + 2/3 R_{\eta\eta}^p] + \bar{A}(\chi_p) L_5^2 [2/3 R_{\eta\eta}^p \chi_p \\
 & + 5/3 R_p^c \chi_p + 5/3 R_{\eta\eta}^p] + \bar{A}(\chi_p) L_4^2 [-2 \bar{\chi}_1 \bar{\chi}_{\eta\eta}^{pp} - 2 \bar{\chi}_1 R_{\eta\eta}^p + 3 \bar{\chi}_1 R_p^c] + \bar{A}(\chi_p) L_5^2 [-2/3 \bar{\chi}_{\eta\eta}^{pp} - R_{\eta\eta}^p \chi_p \\
 & + 1/3 R_{\eta\eta}^p \chi_q + 1/2 R_p^c \chi_p - 1/6 R_p^c \chi_q] + \bar{A}(\chi_p)^2 [1/16 + 1/72 (R_{\eta\eta}^p)^2 - 1/72 R_{\eta\eta}^p R_p^c + 1/288 (R_p^c)^2] \\
 & + \bar{A}(\chi_p) \bar{A}(\chi_{ps}) [-1/36 R_{\eta\eta}^p - 5/72 R_{\eta\eta}^p + 7/144 R_p^c] - \bar{A}(\chi_p) \bar{A}(\chi_{qs}) [1/36 R_{\eta\eta}^p + 1/24 R_{\eta\eta}^p + 1/48 R_p^c] \\
 & + \bar{A}(\chi_p) \bar{A}(\chi_\eta) [-1/72 R_{\eta\eta}^p R_{\eta\eta}^p + 1/144 R_p^c R_{\eta\eta}^p] + 1/8 \bar{A}(\chi_p) \bar{A}(\chi_{13}) + 1/12 \bar{A}(\chi_p) \bar{A}(\chi_{46}) R_{pp}^p \\
 & + \bar{A}(\chi_p) \bar{B}(\chi_p, \chi_p; 0) [1/4 \chi_p - 1/18 R_{\eta\eta}^p R_p^c \chi_p - 1/72 R_{\eta\eta}^p R_p^d + 1/18 (R_p^c)^2 \chi_p + 1/144 R_p^c R_p^d] \\
 & + \bar{A}(\chi_p) \bar{B}(\chi_p, \chi_\eta; 0) [1/18 R_{\eta\eta}^p R_p^c \chi_p - 1/18 R_{13}^p R_p^c \chi_p] + \bar{A}(\chi_p) \bar{B}(\chi_q, \chi_q; 0) [-1/72 R_{\eta\eta}^p R_p^d + 1/144 R_p^c R_p^d] \\
 & - 1/12 \bar{A}(\chi_p) \bar{B}(\chi_{ps}, \chi_{ps}; 0) R_{\eta\eta}^p \chi_{ps} - 1/18 \bar{A}(\chi_p) \bar{B}(\chi_1, \chi_3; 0) R_{\eta\eta}^p R_p^c \chi_p \\
 & + 1/18 \bar{A}(\chi_p) \bar{C}(\chi_p, \chi_p, \chi_p; 0) R_p^d R_p^d \chi_p + \bar{A}(\chi_p; \varepsilon) \pi_{16} [1/8 \bar{\chi}_1 R_{\eta\eta}^p - 1/16 \bar{\chi}_1 R_p^c - 1/16 R_p^d] \\
 & + \bar{A}(\chi_{ps}) \pi_{16} [1/16 \chi_{ps} - 3/16 \chi_{qs} - 3/16 \bar{\chi}_1] - 2 \bar{A}(\chi_{ps}) L_0^2 \chi_{ps} - 5 \bar{A}(\chi_{ps}) L_3^2 \chi_{ps} - 3 \bar{A}(\chi_{ps}) L_4^2 \bar{\chi}_1 \\
 & + \bar{A}(\chi_{ps}) L_5^2 \chi_{13} + \bar{A}(\chi_{ps}) \bar{A}(\chi_\eta) [7/144 R_{\eta\eta}^p - 5/72 R_{\eta\eta}^p - 1/48 R_{\eta\eta}^p + 5/72 R_{\eta\eta}^p - 1/36 R_{13}^p] \\
 & + \bar{A}(\chi_p) \bar{B}(\chi_p, \chi_p; 0) [1/24 R_{\eta\eta}^p \chi_p - 5/24 R_{\eta\eta}^p \chi_{ps}] + \bar{A}(\chi_{ps}) \bar{B}(\chi_p, \chi_\eta; 0) [-1/18 R_{\eta\eta}^p R_{\eta\eta}^p \chi_p \\
 & - 1/9 R_{\eta\eta}^p R_{\eta\eta}^p \chi_{ps}] - 1/48 \bar{A}(\chi_{ps}) \bar{B}(\chi_q, \chi_q; 0) R_p^d + 1/18 \bar{A}(\chi_{ps}) \bar{B}(\chi_1, \chi_3; 0) R_{\eta\eta}^p \chi_s \\
 & + 1/9 \bar{A}(\chi_{ps}) \bar{B}(\chi_1, \chi_3; 0, k) R_{\eta\eta}^p + 3/16 \bar{A}(\chi_{ps}; \varepsilon) \pi_{16} [\chi_s + \bar{\chi}_1] - 1/8 \bar{A}(\chi_{p4})^2 - 1/8 \bar{A}(\chi_{p4}) \bar{A}(\chi_{p6}) \\
 & + 1/8 \bar{A}(\chi_{p4}) \bar{A}(\chi_{46}) - 1/32 \bar{A}(\chi_{p6})^2 + \bar{A}(\chi_\eta) \pi_{16} [1/16 \bar{\chi}_1 R_{\eta\eta}^p + 1/48 R_{\eta\eta}^p \chi_\eta + 1/16 R_{\eta\eta}^p \chi_{13}] \\
 & + \bar{A}(\chi_\eta) L_0^2 [4 R_{\eta\eta}^p \chi_\eta + 2/3 R_{\eta\eta}^p \chi_\eta] - 8 \bar{A}(\chi_\eta) L_1^2 \chi_\eta - 2 \bar{A}(\chi_\eta) L_2^2 \chi_\eta + \bar{A}(\chi_\eta) L_3^2 [4 R_{\eta\eta}^p \chi_\eta + 5/3 R_{\eta\eta}^p \chi_\eta] \\
 & + \bar{A}(\chi_\eta) L_4^2 [4 \chi_\eta + \bar{\chi}_1 R_{\eta\eta}^p] - \bar{A}(\chi_\eta) L_5^2 [1/6 R_{\eta\eta}^p \chi_q + R_{13}^p \chi_{13} + 1/6 R_{\eta\eta}^p \chi_\eta] + 1/288 \bar{A}(\chi_\eta)^2 (R_{\eta\eta}^p)^2 \\
 & + 1/12 \bar{A}(\chi_\eta) \bar{A}(\chi_{46}) R_{\eta\eta}^p + \bar{A}(\chi_\eta) \bar{B}(\chi_p, \chi_p; 0) [-1/36 \bar{\chi}_{\eta\eta}^{pp} - 1/18 R_{\eta\eta}^p R_{\eta\eta}^p \chi_p + 1/18 R_{\eta\eta}^p R_p^c \chi_p \\
 & + 1/144 R_{\eta\eta}^p R_{13}^p] + \bar{A}(\chi_\eta) \bar{B}(\chi_p, \chi_p; 0) [-1/18 \bar{\chi}_{\eta\eta}^{pp} + 1/18 \bar{\chi}_{\eta\eta}^{pp} + 1/18 (R_{\eta\eta}^p)^2 R_{\eta\eta}^p \chi_p] \\
 & - 1/12 \bar{A}(\chi_\eta) \bar{B}(\chi_{ps}, \chi_{ps}; 0) R_{\eta\eta}^p \chi_{ps} - \bar{A}(\chi_\eta) \bar{B}(\chi_\eta, \chi_\eta; 0) [1/216 R_{\eta\eta}^p \chi_4 + 1/27 R_{\eta\eta}^p \chi_6] \\
 & - 1/18 \bar{A}(\chi_\eta) \bar{B}(\chi_1, \chi_3; 0) R_{\eta\eta}^p R_{\eta\eta}^p \chi_\eta + 1/18 \bar{A}(\chi_\eta) \bar{C}(\chi_p, \chi_p, \chi_p; 0) R_{\eta\eta}^p R_p^d \chi_p + \bar{A}(\chi_\eta; \varepsilon) \pi_{16} [1/8 \chi_\eta \\
 & - 1/16 \bar{\chi}_1 R_{\eta\eta}^p - 1/8 R_{13}^p \chi_\eta - 1/16 R_{\eta\eta}^p \chi_\eta] + \bar{A}(\chi_1) \bar{A}(\chi_3) [-1/72 R_{\eta\eta}^p R_p^c + 1/36 R_{13}^p R_{\eta\eta}^p + 1/144 R_p^c R_{13}^p] \\
 & - 4 \bar{A}(\chi_{13}) L_1^2 \chi_{13} - 10 \bar{A}(\chi_{13}) L_2^2 \chi_{13} + 1/8 \bar{A}(\chi_{13})^2 - 1/2 \bar{A}(\chi_{13}) \bar{B}(\chi_1, \chi_3; 0, k) \\
 & + 1/4 \bar{A}(\chi_{13}; \varepsilon) \pi_{16} \chi_{13} + 1/4 \bar{A}(\chi_{14}) \bar{A}(\chi_{34}) + 1/16 \bar{A}(\chi_{16}) \bar{A}(\chi_{36}) - 24 \bar{A}(\chi_4) L_1^2 \chi_4 - 6 \bar{A}(\chi_4) L_2^2 \chi_4 \\
 & + 12 \bar{A}(\chi_4) L_4^2 \chi_4 + 1/12 \bar{A}(\chi_4) \bar{B}(\chi_p, \chi_p; 0) (R_{4\eta}^p)^2 \chi_4 + 1/6 \bar{A}(\chi_4) \bar{B}(\chi_p, \chi_\eta; 0) [R_{\eta\eta}^p R_{13}^p \chi_4 - R_{\eta\eta}^p R_{13}^p \chi_4] \\
 & - 1/24 \bar{A}(\chi_4) \bar{B}(\chi_\eta, \chi_\eta; 0) R_{\eta\eta}^p \chi_4 - 1/6 \bar{A}(\chi_4) \bar{B}(\chi_1, \chi_3; 0) R_{13}^p R_{4\eta}^p \chi_4 + 3/8 \bar{A}(\chi_4; \varepsilon) \pi_{16} \chi_4 \\
 & - 32 \bar{A}(\chi_{46}) L_1^2 \chi_{46} - 8 \bar{A}(\chi_{46}) L_2^2 \chi_{46} + 16 \bar{A}(\chi_{46}) L_4^2 \chi_{46} + \bar{A}(\chi_{46}) \bar{B}(\chi_p, \chi_p; 0) [1/9 \chi_{46} + 1/12 R_{\eta\eta}^p \chi_p \\
 & + 1/36 R_{\eta\eta}^p \chi_4 + 1/9 R_{13}^p \chi_6] + \bar{A}(\chi_{46}) \bar{B}(\chi_p, \chi_\eta; 0) [-1/18 R_{\eta\eta}^p \chi_4 - 1/9 R_{\eta\eta}^p \chi_6 + 1/9 R_{\eta\eta}^p \chi_6 + 1/18 R_{13}^p \chi_4] \\
 & - 1/6 \bar{A}(\chi_{46}) \bar{B}(\chi_p, \chi_\eta; 0, k) [R_{\eta\eta}^p - R_{13}^p] + 1/9 \bar{A}(\chi_{46}) \bar{B}(\chi_\eta, \chi_\eta; 0) R_{\eta\eta}^p \chi_{46} - \bar{A}(\chi_{46}) \bar{B}(\chi_1, \chi_3; 0) [2/9 \chi_{46} \\
 & + 1/9 R_{\eta\eta}^p \chi_6 + 1/18 R_{13}^p \chi_4] - 1/6 \bar{A}(\chi_{46}) \bar{B}(\chi_1, \chi_3; 0, k) R_{\eta\eta}^p + 1/2 \bar{A}(\chi_{46}; \varepsilon) \pi_{16} \chi_{46} \\
 & + \bar{B}(\chi_p, \chi_p; 0) \pi_{16} [1/16 \bar{\chi}_1 R_p^d + 1/96 R_p^d \chi_p + 1/32 R_p^d \chi_q] + 2/3 \bar{B}(\chi_p, \chi_p; 0) L_0^2 R_p^d \chi_p \\
 & + 5/3 \bar{B}(\chi_p, \chi_p; 0) L_3^2 R_p^d \chi_p + \bar{B}(\chi_p, \chi_p; 0) L_4^2 [-2 \bar{\chi}_1 \bar{\chi}_{\eta\eta}^{pp} \chi_p - 4 \bar{\chi}_1 R_{\eta\eta}^p \chi_p + 4 \bar{\chi}_1 R_p^c \chi_p + 3 \bar{\chi}_1 R_p^d] \\
 & + \bar{B}(\chi_p, \chi_p; 0) L_5^2 [-2/3 \bar{\chi}_{\eta\eta}^{pp} \chi_p - 4/3 R_{\eta\eta}^p \chi_p^2 + 4/3 R_p^c \chi_p^2 + 1/2 R_p^d \chi_p - 1/6 R_p^d \chi_q] \\
 & + \bar{B}(\chi_p, \chi_p; 0) L_6^2 [4 \bar{\chi}_1 \bar{\chi}_{\eta\eta}^{pp} + 8 \bar{\chi}_1 R_{\eta\eta}^p \chi_p - 8 \bar{\chi}_1 R_p^c \chi_p] + 4 \bar{B}(\chi_p, \chi_p; 0) L_7^2 (R_p^d)^2 \\
 & + \bar{B}(\chi_p, \chi_p; 0) L_8^2 [4/3 \bar{\chi}_{\eta\eta}^{pp} + 8/3 R_{\eta\eta}^p \chi_p^2 - 8/3 R_p^c \chi_p^2] + \bar{B}(\chi_p, \chi_p; 0)^2 [-1/18 R_{\eta\eta}^p R_p^d \chi_p + 1/18 R_p^c R_p^d \chi_p \\
 & + 1/288 (R_p^d)^2] + 1/18 \bar{B}(\chi_p, \chi_p; 0) \bar{B}(\chi_\eta, \chi_\eta; 0) [R_{\eta\eta}^p R_p^d \chi_p - R_{13}^p R_p^d \chi_p]
 \end{aligned}$$

plus several more pages

Why so long expressions

- Many different quark and meson masses (χ_{ij})
- Charged propagators: $-i G_{ij}^c(k) = \frac{\epsilon_j}{k^2 - \chi_{ij} + i\epsilon} \quad (i \neq j)$
- Neutral propagators: $G_{ij}^n(k) = G_{ij}^c(k) \delta_{ij} - \frac{1}{n_{\text{sea}}} G_{ij}^q(k)$

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$$-i G_{ii}^q(k) = \frac{R_i^d}{(k^2 - \chi_i + i\epsilon)^2} + \frac{R_i^c}{k^2 - \chi_i + i\epsilon} + \frac{R_{\eta ii}^\pi}{k^2 - \chi_\pi + i\epsilon} + \frac{R_{\pi ii}^\eta}{k^2 - \chi_\eta + i\epsilon}$$

$$R_{jkl}^i = R_{i456jkl}^z, \quad R_i^d = R_{i456\pi\eta}^z,$$

$$R_i^c = R_{4\pi\eta}^i + R_{5\pi\eta}^i + R_{6\pi\eta}^i - R_{\pi\eta\eta}^i - R_{\pi\pi\eta}^i$$

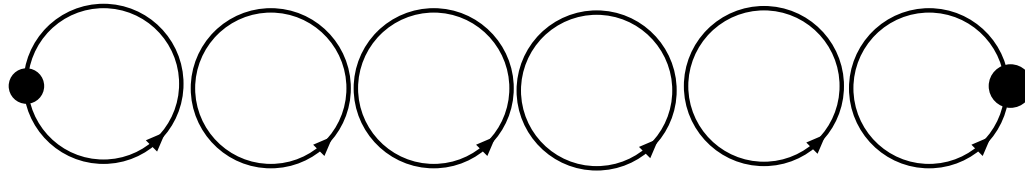
$$R_{ab}^z = \chi_a - \chi_b, \quad R_{abc}^z = \frac{\chi_a - \chi_b}{\chi_a - \chi_c}, \quad R_{abcd}^z = \frac{(\chi_a - \chi_b)(\chi_a - \chi_c)}{\chi_a - \chi_d}$$

$$R_{abcdefg}^z = \frac{(\chi_a - \chi_b)(\chi_a - \chi_c)(\chi_a - \chi_d)}{(\chi_a - \chi_e)(\chi_a - \chi_f)(\chi_a - \chi_g)}$$

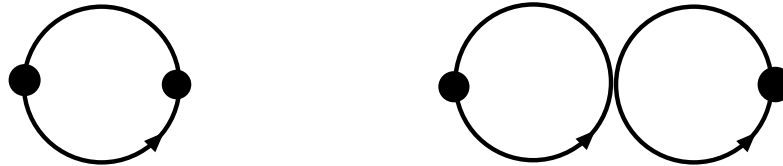
Double poles ?

Think quark lines and add gluons everywhere

Full



Quenched



So no resummation at the quark level:

naively a **double pole**

Same follows from inverting the lowest order kinetic terms

PQChPT at Two Loop

Masses and decay constants: all possible mass cases worked out for two and three flavours of sea quarks for the charged/off-diagonal case to NNLO.

hep-lat/0406017, hep-lat/0501014, hep-lat/0506004, hep-lat/0602003 all published in Phys. Rev. D

Earlier work at NLO:

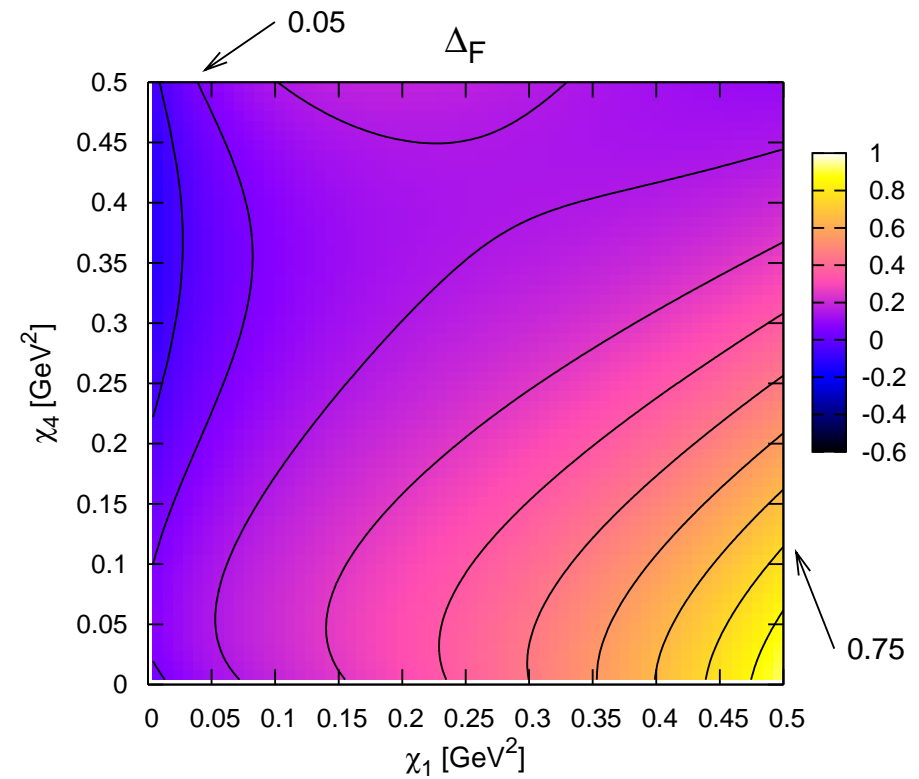
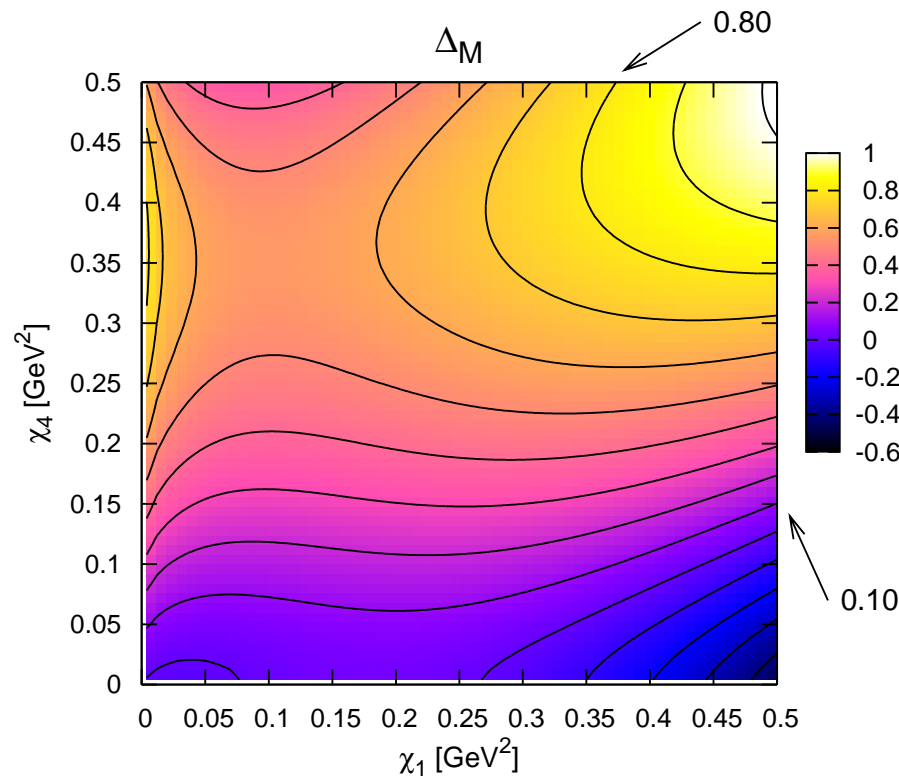
Bernard-Golterman-Sharpe-Shoresh-...

review/lectures: S. Sharpe [hep-lat/0607016](#)

Results: $\chi_1 = \chi_2, \chi_4 = \chi_5 = \chi_6$

Use lowest order mass squared: $\chi_i = 2B_0 m_i = m_M^{2(0)}$

Remember: $\chi_i \approx 0.3 \text{ GeV}^2 \approx (550 \text{ MeV})^2 \sim \text{border ChPT}$

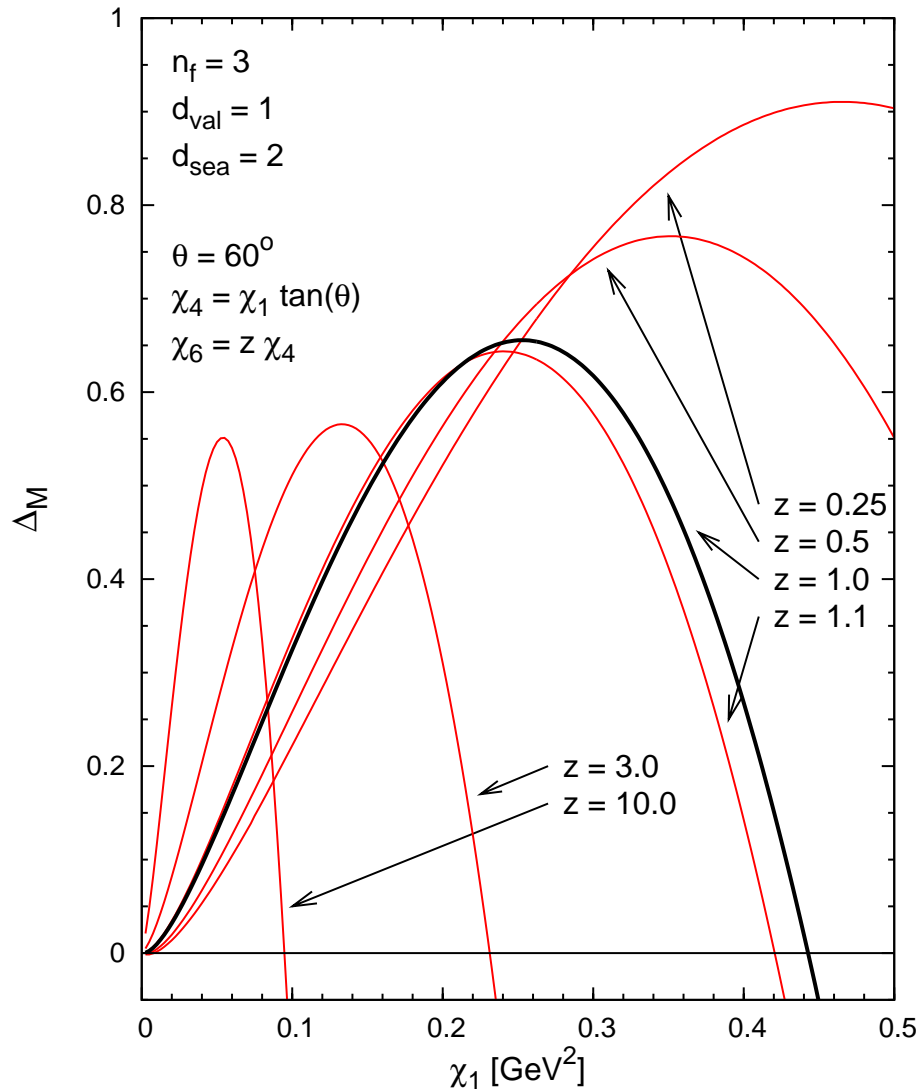


Relative corrections: mass²

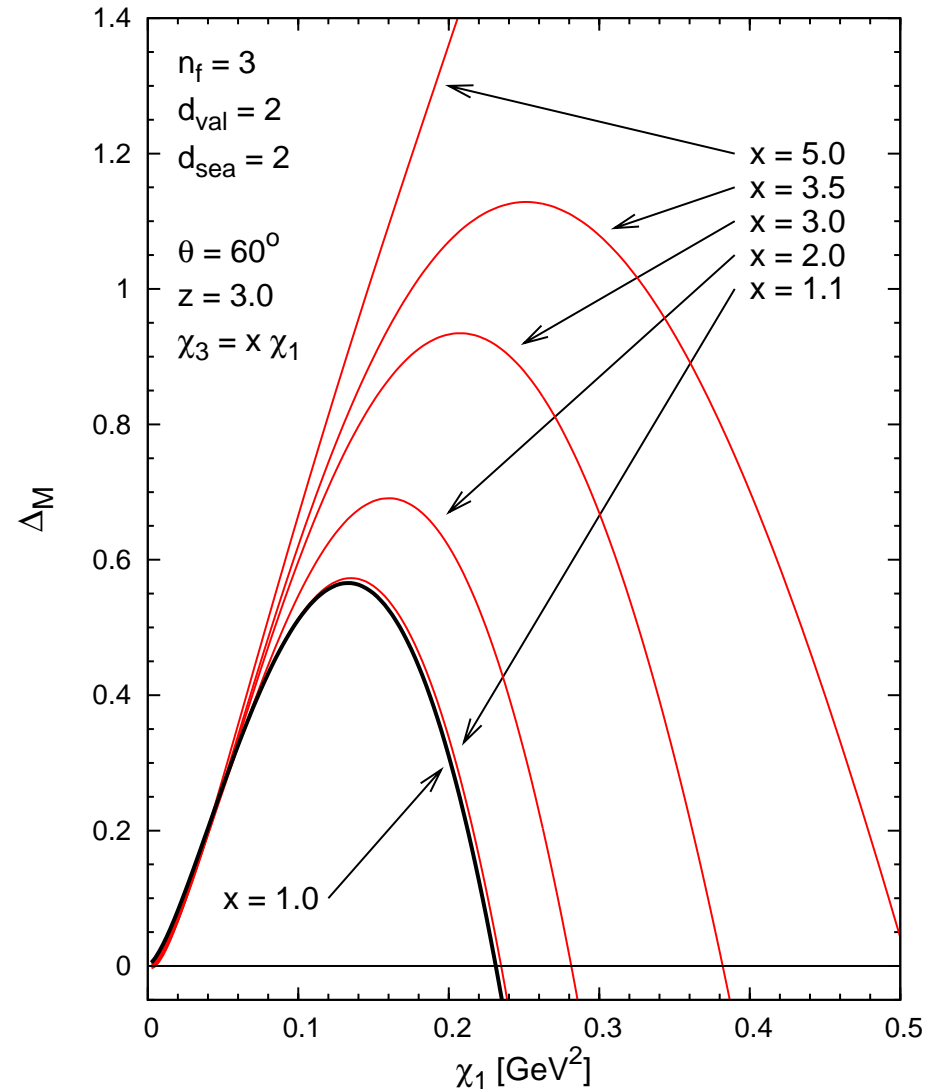
decay constant

mass²: $\chi_4 = \chi_5 \neq \chi_6$

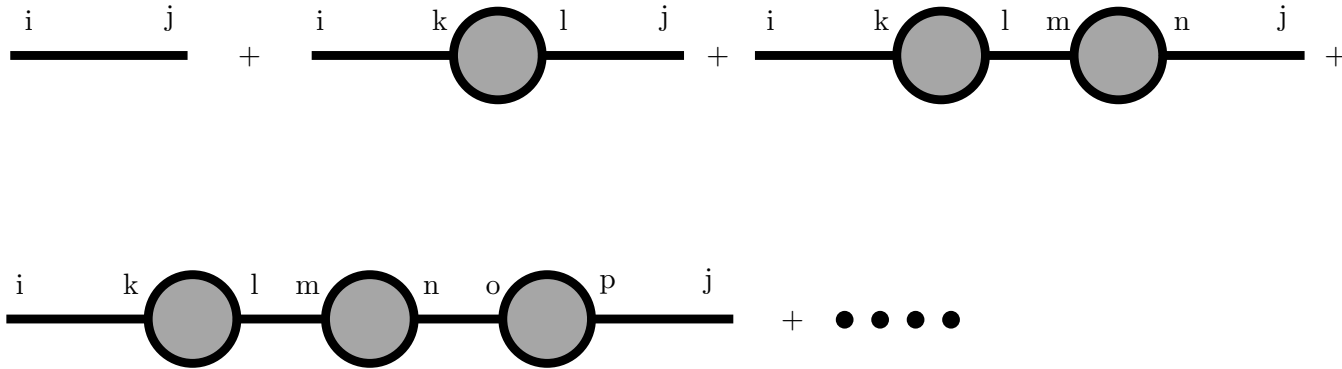
$\chi_1 = \chi_2$



$\chi_1 \neq \chi_2$



Neutral masses

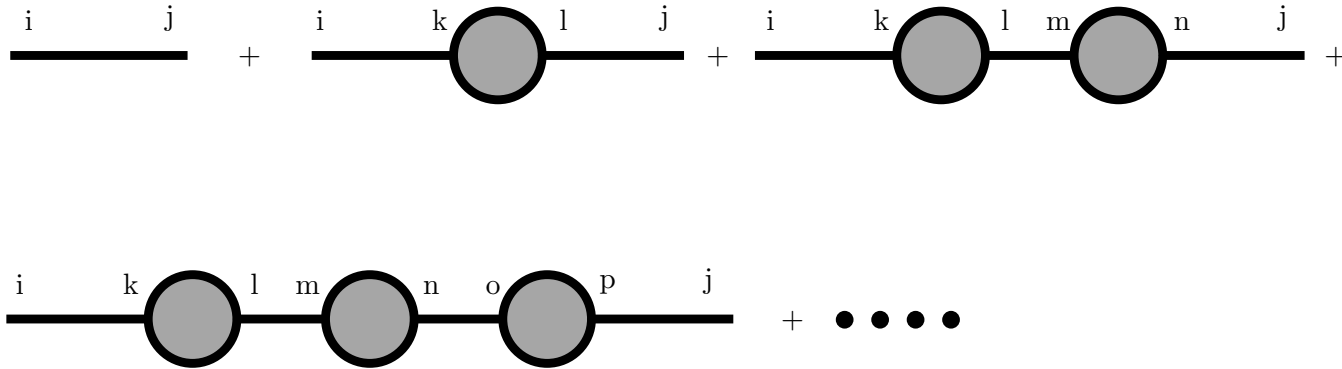


$$G_{ij}^n = G_{ij}^0 + G_{ik}^0(-i)\Sigma_{kl}G_{lj}^0 + G_{ik}^0(-i)\Sigma_{kl}G_{lm}^0(-i)\Sigma_{mn}G_{nj}^0 + \dots$$

ij means: from $\bar{q}_i q_i$ to $\bar{q}_j q_j$ meson

Full resummation done by JB, Danielsson [hep-lat/0606017](https://arxiv.org/abs/hep-lat/0606017)
 i.e. full propagator from one-particle-irreducible diagrams

Neutral masses



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Actually useful: residue of double pole allows to get at **all** LECs needed for the neutral masses

Conclusions

The **Lund High Energy Theory Group** is working on **the strong interaction and beyond** in many aspects and expects to continue this in the foreseeable future.