Lund University: High Energy Theory

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Overview

- Who are we?
- Recent papers
- What do we do?
- Colour effects at small x
- Partially Quenched Chiral Perturbation Theory
  - Chiral Perturbation Theory
  - What is partially quenched?
  - ChPT and Lattice QCD
  - Very long expressions (and why)
  - Some results as well

Who are we

- Faculty
  - Bijnens Johan
  - Gustafson Gösta
  - Lönnblad Leif
  - Sjöstrand Torbjörn (at CERN)
  - Svensson Bengt E Y (retired)
- PhD Students
  - Avsar Emil
  - Danielsson Niclas (19/12/06)
  - Flensburg Christoffer*
  - Ghorbani Karim
  - Lavesson Nils
  - Sjödahl Malin (29/9/06 $\longrightarrow$ Manchester)

Who are we

- HEP-EST EU school on LHC physics (joint theory-experiment)
  - Nele Boelaert (BE)
  - Lisa Carloni (IT)
  - Richard Corke (UK)
  - Alexandru Dobrin (RO)
  - Christoffer Flensburg (SE) (faculty LU TP)
  - Jacob Groth-Jensen (DK)
  - Wei-Na Ji (CN)
  - Jie Lu (CN)
- Integrate theory and experiment
- Use PBL as a learning tool at the graduate level
- `http://www.hep.lu.se/Lund-HEP/`

Recent past students

≥ 2003

- Sandipan Mohanty
- Fredrik Söderberg
- Fredrik Borg
- Pierre Dhonte
- Peter Skands (Fermilab)

Recent papers

- J. Bijnens and N. Danielsson, “The eta mass and NNLO three-flavor partially quenched chiral perturbation theory,”
- J. Bijnens, “Chiral perturbation theory beyond one loop,”
- J. Bijnens and K. Ghorbani, “Finite volume dependence of the quark-antiquark vacuum expectation value,”
- **J. Bijnens, N. Danielsson and T. A. Lahde, “Three-flavor partially quenched chiral perturbation theory at NNLO for meson masses and decay constants,”**
- J. Bijnens, E. Gamiz and J. Prades, “The B(K) kaon parameter in the chiral limit,”
- T. A. Lahde, J. Bijnens and N. Danielsson, “Partially quenched chiral perturbation theory to NNLO,”
- J. Bijnens, “eta and eta’ decays and what can we learn from them?,”

Recent papers

- **E. Avsar, G. Gustafson and L. Lonnblad, “Small-x Dipole Evolution Beyond the Large-N_c Limit,”**
- L. Lönnblad and M. Sjödahl, “Classical and non-classical ADD-phenomenology with high-E(T) jet observables at collider experiments,”
- L. Lonnblad, “ThePEG, Pythia7, herwig++ and Ariadne,”
- J. R. Andersen *et al.* [Small x Collaboration], “Small x phenomenology: Summary of the 3rd Lund small x workshop in 2004,”
- S. Hoche, F. Krauss, N. Lavesson, L. Lonnblad, M. Mangano, A. Schalicke and S. Schumann, “Matching parton showers and matrix elements,”
- L. Lönnblad, M. Sjödahl and T. Åkesson, “QCD-supression by black hole production at the LHC,”

Recent papers

- N. Lavesson and L. Lonnblad, “W + jets matrix elements and the dipole cascade,”
- E. Avsar, G. Gustafson and L. Lonnblad, “Energy conservation and saturation in small-x evolution,”
- M. Sjödahl and G. Gustafson, “Gravitational scattering in the ADD-model at high and low energies,”
- S. Alekhin *et al.*, “HERA and the LHC - A workshop on the implications of HERA for LHC physics”
- T. Q. W. Group *et al.*, “Tevatron-for-LHC report of the QCD working group,”
- W. M. Yao *et al.* [Particle Data Group], “Review of particle physics,”
- T. Sjostrand, S. Mrenna and P. Skands, “PYTHIA 6.4 physics and manual.”

What do we do

PHENOMENOLOGY and the STRONG interaction

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We do this at low (JB) and high (GG,LL,TS) energies

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We do this at low (JB) and high (GG,LL,TS) energies

At high energies:

- Matching matrix elements and parton showers
- Updating the Lund Monte Carlos to C++
- Extra large dimension signatures
- Signatures for Beyond the Standard Model in the presence of large QCD backgrounds

What do we do

At high energies:

- Small x: resumming logarithms in x and Q^2. Cascade models with applications to DIS and high energy pp collisions.
- LHC physics: Combine formulations in momentum space and transverse coordinate space in analyses of final state properties, including saturation effects and correlations.
- ...

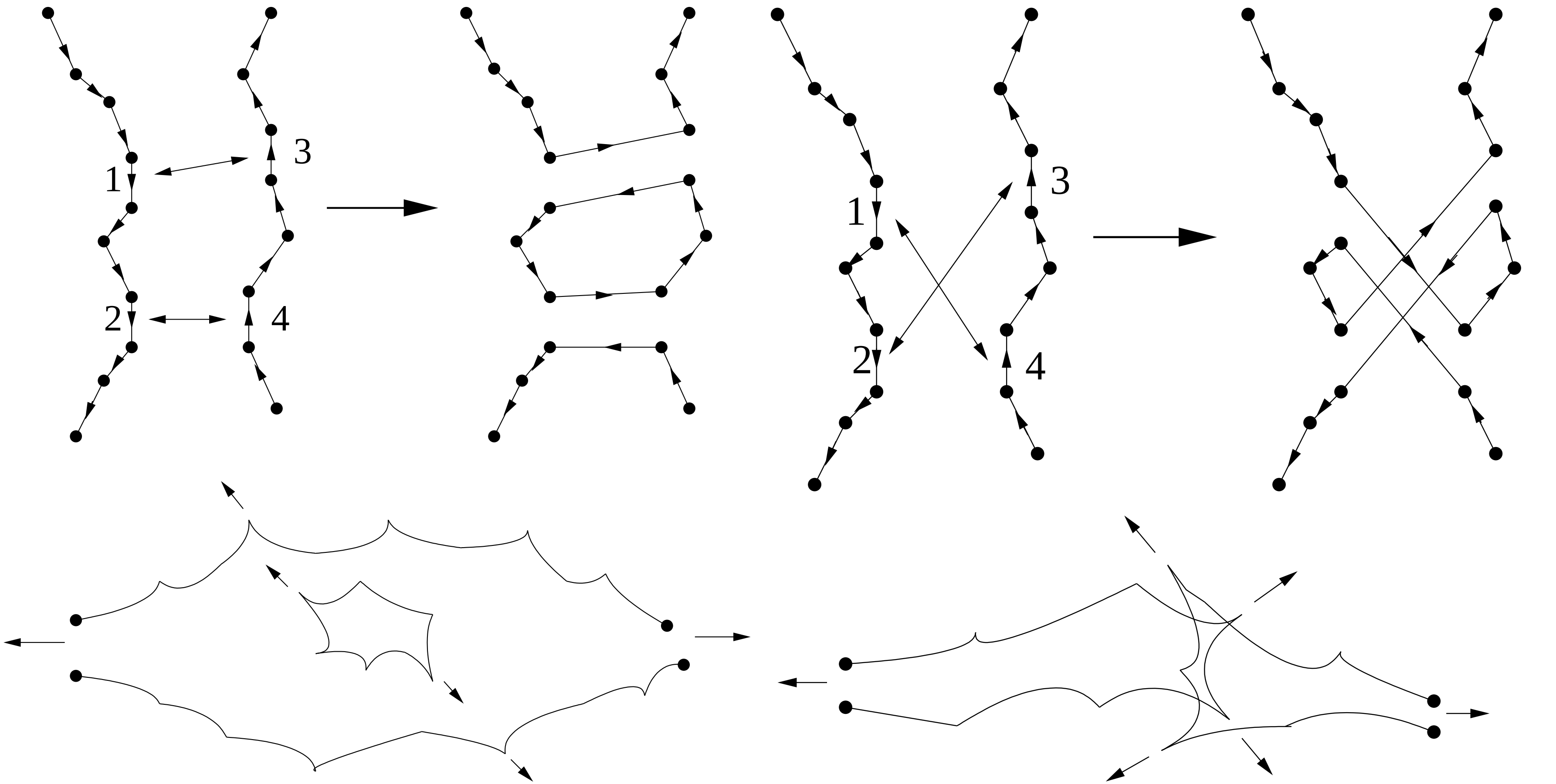
What do we do

At low energies:

- Flavour Physics
- Using Effective Field Theory, especially Chiral Perturbation Theory for
  - $\pi\pi$, πK scattering
  - $\eta \to 3\pi$: talk Karim Ghorbani
  - V_{us} from $K \to \pi e \nu$.
  - Connecting Lattice QCD to the real world: Chiral Extrapolations
  - ...
- CP violation in Kaon decays

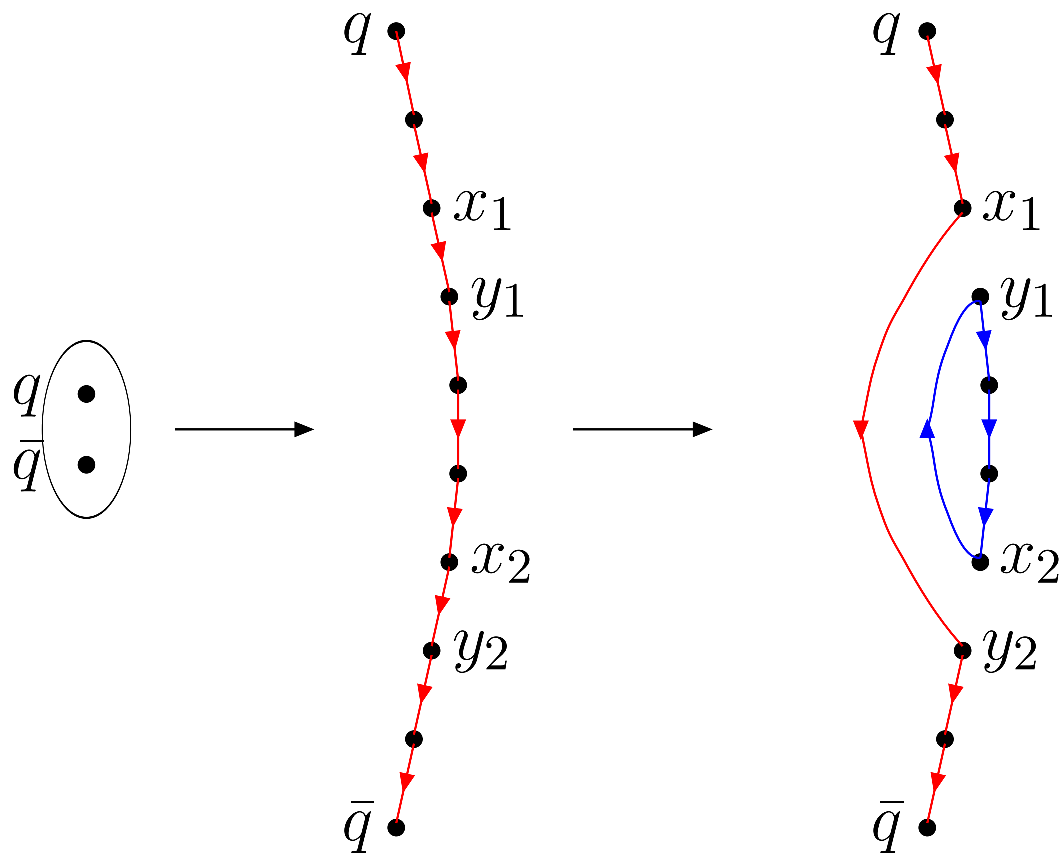
Colour effects at small x

E. Avsar, G. Gustafson and L. Lonnblad, “Small-x Dipole Evolution Beyond the Large-N_c Limit,”

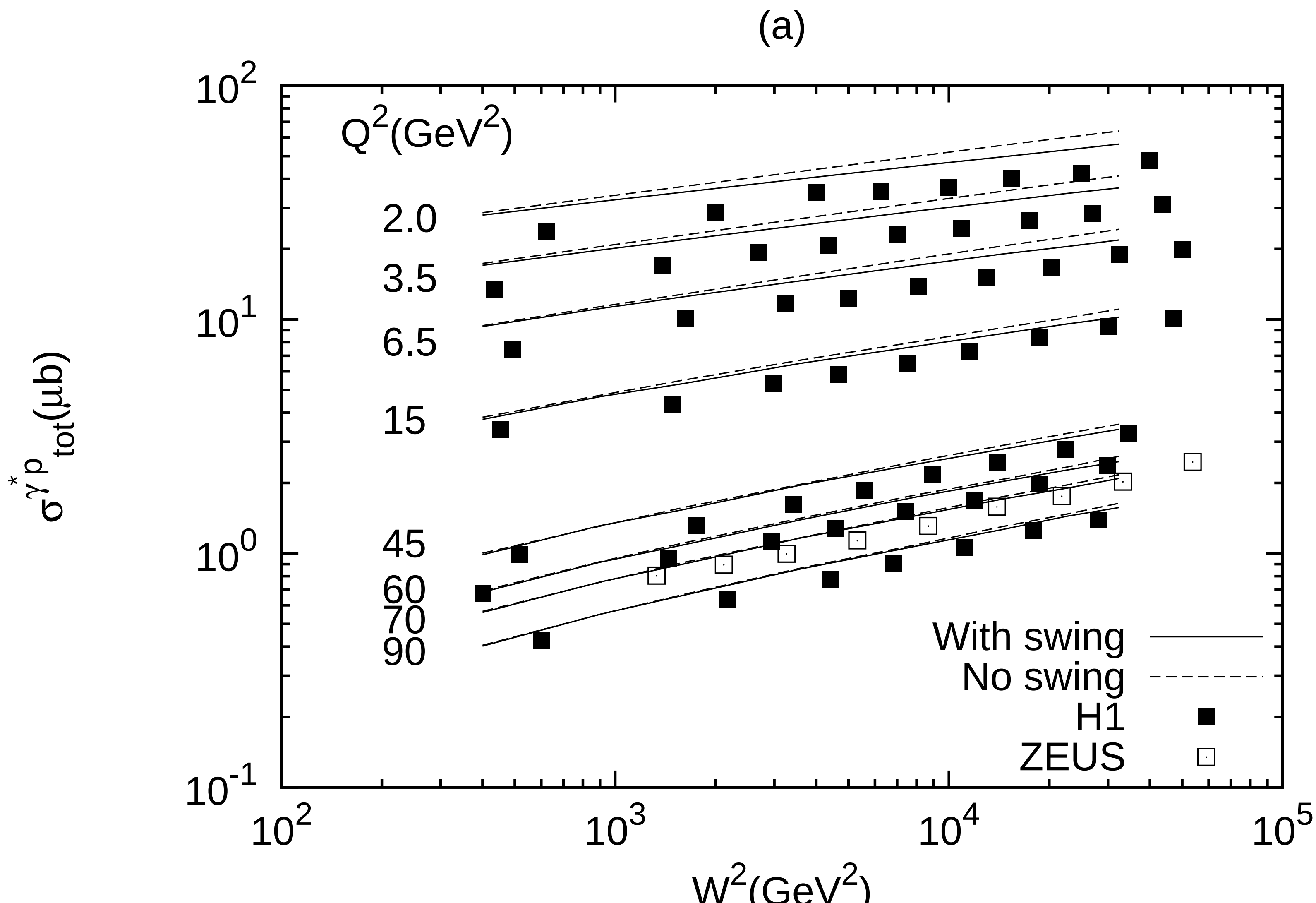


Colour effects at small x

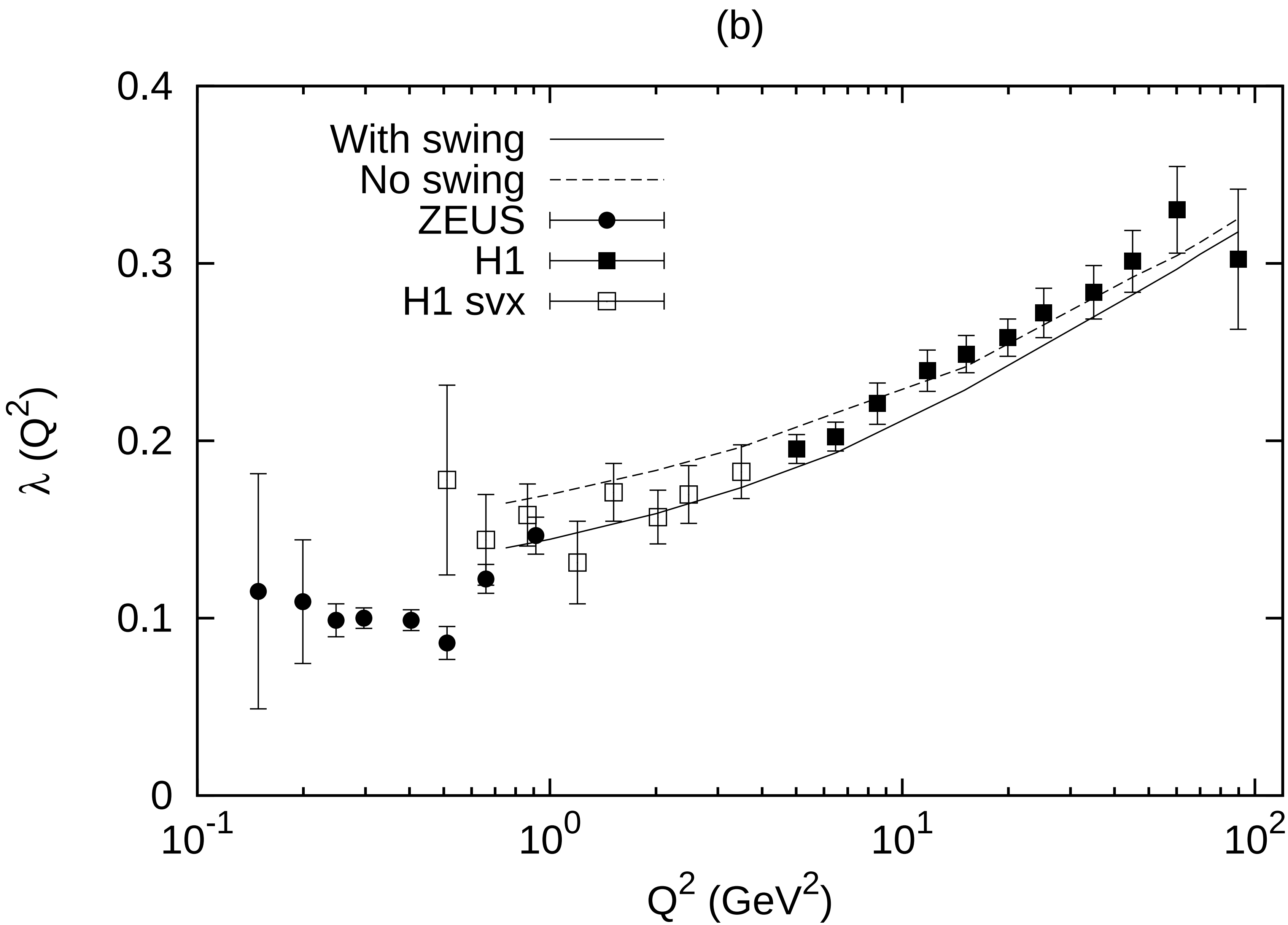
Must allow also for collisions inside one chain to be Lorentz invariant: a swing



Colour effects at small x: $\gamma^* p$



Colour effects at small x: $\gamma^* p$ slope



Chiral Perturbation Theory

Chiral Symmetry:

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

Chiral Perturbation Theory

Make an Effective Field Theory with:

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

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Perturbation Theory with mesons, expansion in momenta

used for The physics of pions, kaons and eta

Nonrenormalizable: new parameters (Low Energy Constants (LEC)) at every new order

Review paper: JB, LU TP 06-16 hep-ph/0604043

What is Partially Quenched?

In Lattice gauge theory one calculates

$$\langle 0|(\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)|\rangle$$

$$= \frac{\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}}}{\int [dq][d\overline{q}][dG]e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}}}$$

for Euclidean separations x

Integrals also performed after rotation to Euclidean

(note that I use Minkowski notation throughout)

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y \mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}} (\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x) \det (\not{D}_G)_{\mathrm{QCD}}$$

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}\,(\not{D}^u_G)^{-1}(x,0)(\not{D}^d_G)^{-1}(0,x)\,\det\,(\not{D}_G)_{\mathrm{QCD}}$$

$\int [dG]$ done via importance sampling

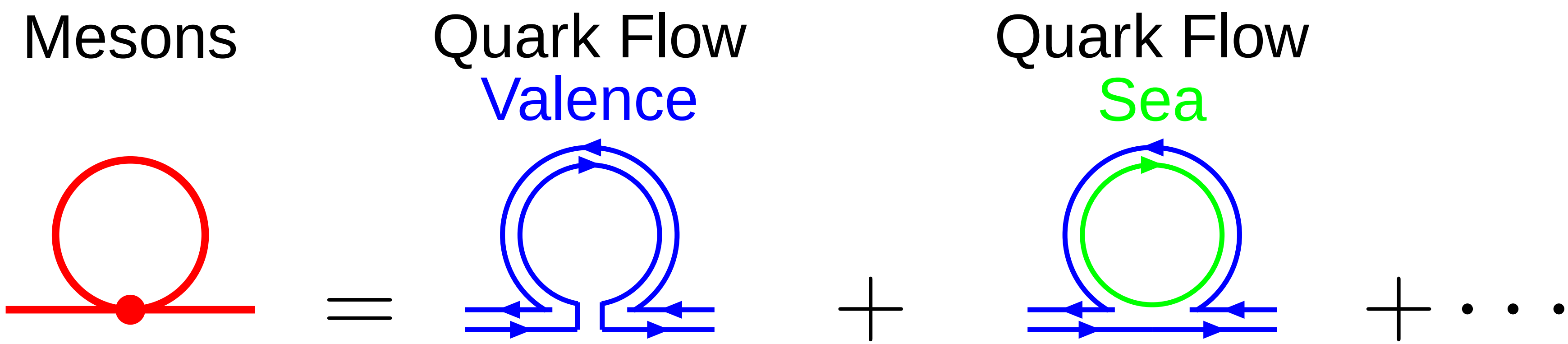
- Quenched: get distribution from $e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}$ only
- Unquenched: include $\det\,(\not{D}_G)_{\mathrm{QCD}}$ VERY expensive
- Partially quenched: $(\not{D}^u_G)^{-1}(x,0)(\not{D}^d_G)^{-1}(0,x)$
  DIFFERENT Quarks then in $\det\,(\not{D}_G)_{\mathrm{QCD}}$

What is Partially Quenched?

Why do this?

- Is not Quenched: Real QCD is continuous limit from Partially Quenched
- More handles to turn:
  - Allows more systematic studies by varying parameters
  - Sometimes allows to disentangle things from different observables: Physical results from unphysical calculations
- $\det\left(\not{D}_G\right)_{\mathrm{QCD}}$: Sea quarks
- $(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)$: Valence Quarks

ChPT and Lattice QCD

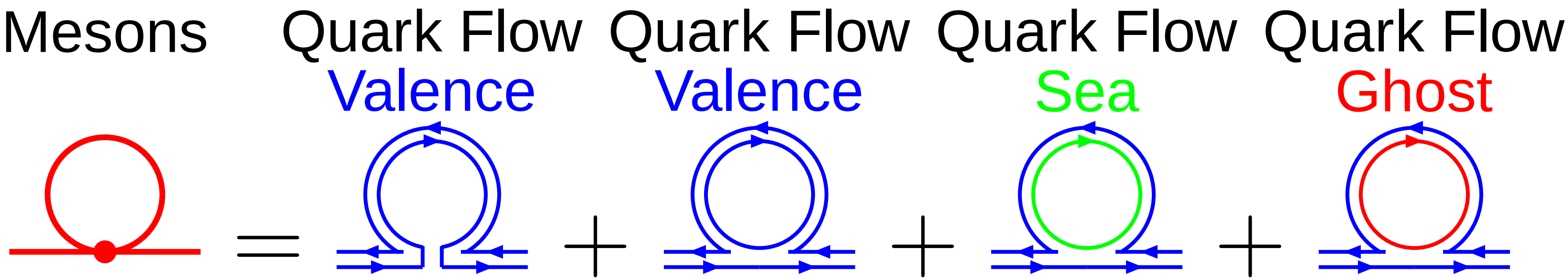


Valence and Sea treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

PQChPT at Two Loops: General

Add ghost quarks: remove the unwanted free valence loops



Possible problem: QCD $\Longrightarrow$ ChPT relies heavily on unitarity

Symmetry group becomes $SU(n_v + n_s|n_v) \times SU(n_v + n_s|n_v)$ (approximately)

PQChPT at Two Loops: General

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

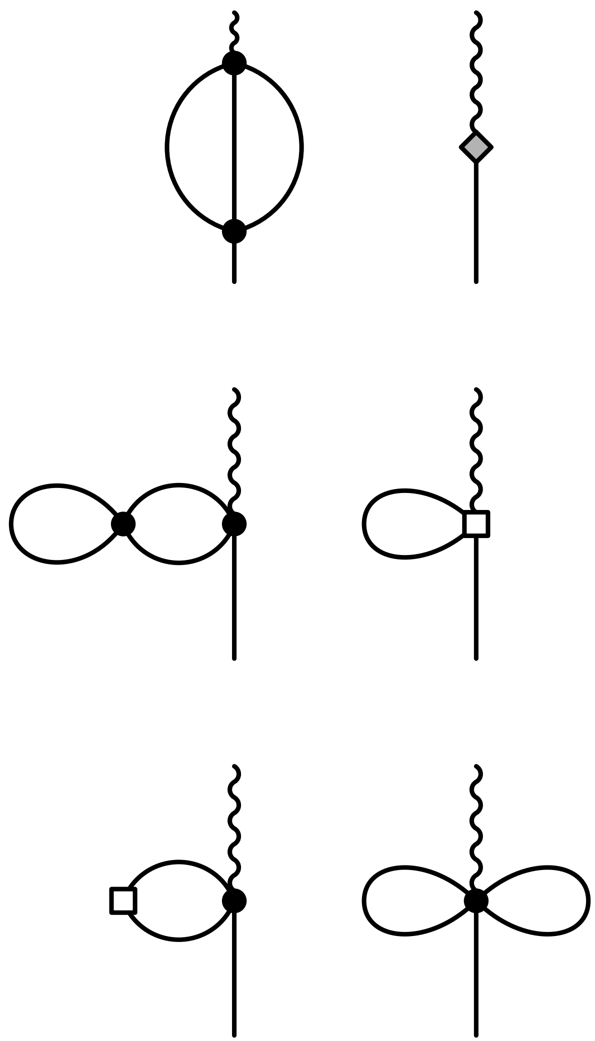
Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT
$\Longrightarrow$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

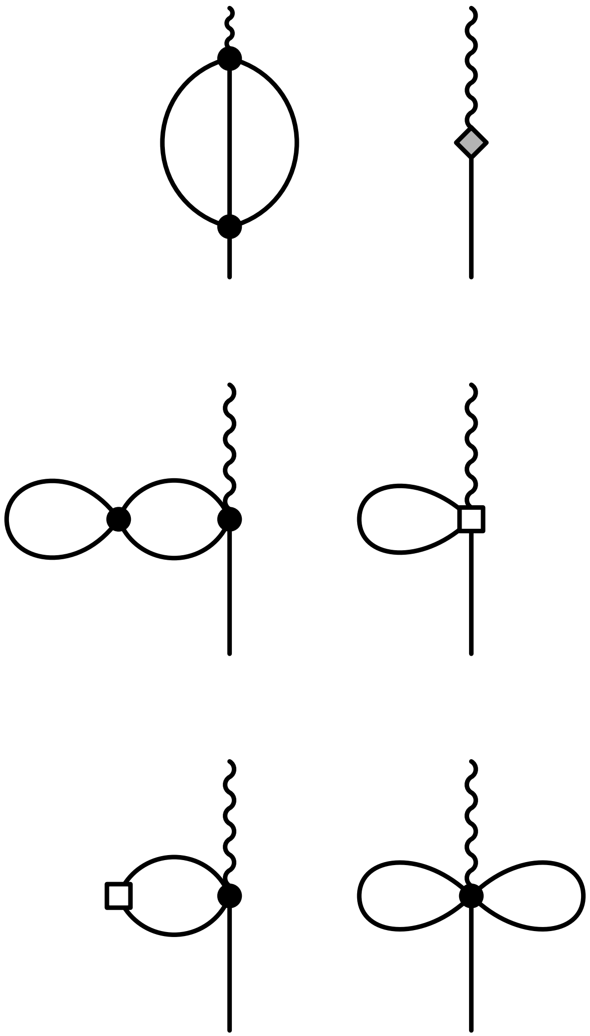
E.g. $L_1^r = L_0^{r(3pq)}/2 + L_1^{r(3pq)}$

Long Expressions



$\Longrightarrow$

Long Expressions



$\Longrightarrow$

$$
\begin{aligned}
\delta^{(6)22}_{\text{loops}} &= \pi_{16}\, L^r_0\, \left[4/9\,\chi_\eta\chi_4 - 1/2\,\chi_1\chi_3 + \chi_{13}^2 - 13/3\,\bar\chi_1\chi_{13} - 35/18\,\bar\chi_2\right] \,-\, 2\,\pi_{16}\, L^r_1\,\chi_{13}^2 \\
&- \pi_{16}\, L^r_2\, \left[11/3\,\chi_\eta\chi_4 + \chi_{13}^2 + 13/3\,\bar\chi_2\right] + \pi_{16}\, L^r_3\, \left[4/9\,\chi_\eta\chi_4 - 7/12\,\chi_1\chi_3 + 11/6\,\chi_{13}^2 - 17/6\,\bar\chi_1\chi_{13} - 43/36\,\bar\chi_2\right] \\
&+ \pi_{16}^2\, \left[-15/64\,\chi_\eta\chi_4 - 59/384\,\chi_1\chi_3 + 65/384\,\chi_{13}^2 - 1/2\,\bar\chi_1\chi_{13} - 43/128\,\bar\chi_2\right] \,-\, 48\, L^r_4 L^r_5\,\bar\chi_1\chi_{13} \,-\, 72\, L^{r2}_4\,\bar\chi_1^2 \\
&- 8\, L^{r2}_5\,\chi_{13}^2 \,+\, \bar A(\chi_p)\,\pi_{16}\, \left[-1/24\,\chi_p + 1/48\,\bar\chi_1 - 1/8\,\bar\chi_1\, R^p_{q\eta} + 1/16\,\bar\chi_1\, R^c_p - 1/48\, R^p_{q\eta}\,\chi_p - 1/16\, R^p_{q\eta}\,\chi_q\right. \\
&+ \left. 1/48\, R^\eta_{pp}\,\chi_\eta + 1/16\, R^c_p\,\chi_{13}\right] \,+\, \bar A(\chi_p)\, L^r_0\, \left[8/3\, R^p_{q\eta}\,\chi_p + 2/3\, R^c_p\,\chi_p + 2/3\, R^d_p\right] \,+\, \bar A(\chi_p)\, L^r_3\, \left[2/3\, R^p_{q\eta}\,\chi_p\right. \\
&+ \left. 5/3\, R^c_p\,\chi_p + 5/3\, R^d_p\right] \,+\, \bar A(\chi_p)\, L^r_4\, \left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0} - 2\,\bar\chi_1\, R^p_{q\eta} + 3\,\bar\chi_1\, R^c_p\right] \,+\, \bar A(\chi_p)\, L^r_5\, \left[-2/3\,\bar\chi^{pp}_{\eta\eta 1} - R^p_{q\eta}\,\chi_p\right. \\
&+ \left. 1/3\, R^p_{q\eta}\,\chi_q + 1/2\, R^c_p\,\chi_p - 1/6\, R^c_p\,\chi_q\right] \,+\, \bar A(\chi_p)^2\, \left[1/16 + 1/72\,(R^p_{q\eta})^2 - 1/72\, R^p_{q\eta} R^c_p + 1/288\,(R^c_p)^2\right] \\
&+ \bar A(\chi_p)\bar A(\chi_{ps})\, \left[-1/36\, R^p_{q\eta} - 5/72\, R^p_{s\eta} + 7/144\, R^c_p\right] \,-\, \bar A(\chi_p)\bar A(\chi_{qs})\, \left[1/36\, R^p_{q\eta} + 1/24\, R^p_{s\eta} + 1/48\, R^c_p\right] \\
&+ \bar A(\chi_p)\bar A(\chi_\eta)\, \left[-1/72\, R^p_{q\eta} R^v_{\eta 13} + 1/144\, R^c_p R^v_{\eta 13}\right] \,+\, 1/8\,\bar A(\chi_p)\bar A(\chi_{13}) \,+\, 1/12\,\bar A(\chi_p)\bar A(\chi_{46})\, R^\eta_{pp} \\
&+ \bar A(\chi_p)\bar B(\chi_p,\chi_p;0)\, \left[1/4\,\chi_p - 1/18\, R^p_{q\eta} R^c_p\,\chi_p - 1/72\, R^p_{q\eta} R^d_p + 1/18\,(R^c_p)^2\,\chi_p + 1/144\, R^c_p R^d_p\right] \\
&+ \bar A(\chi_p)\bar B(\chi_p,\chi_\eta;0)\, \left[1/18\, R^\eta_{pp} R^c_p\,\chi_p - 1/18\, R^\eta_{13} R^c_p\,\chi_p\right] \,+\, \bar A(\chi_p)\bar B(\chi_q,\chi_q;0)\, \left[-1/72\, R^p_{q\eta} R^d_q + 1/144\, R^c_p R^d_q\right] \\
&- 1/12\,\bar A(\chi_p)\bar B(\chi_{ps},\chi_{ps};0)\, R^p_{s\eta}\,\chi_{ps} \,-\, 1/18\,\bar A(\chi_p)\bar B(\chi_1,\chi_3;0)\, R^q_{p\eta} R^c_p\,\chi_p \\
&+ 1/18\,\bar A(\chi_p)\bar C(\chi_p,\chi_p,\chi_p;0)\, R^c_p R^d_p\,\chi_p \,+\, \bar A(\chi_p;\varepsilon)\,\pi_{16}\, \left[1/8\,\bar\chi_1 R^p_{q\eta} - 1/16\bar\chi_1\, R^c_p - 1/16\, R^c_p\,\chi_p - 1/16\, R^d_p\right] \\
&+ \bar A(\chi_{ps})\,\pi_{16}\, \left[1/16\,\chi_{ps} - 3/16\,\chi_{qs} - 3/16\,\bar\chi_1\right] \,-\, 2\,\bar A(\chi_{ps})\, L^r_0\,\chi_{ps} \,-\, 5\,\bar A(\chi_{ps})\, L^r_3\,\chi_{ps} \,-\, 3\,\bar A(\chi_{ps})\, L^r_4\,\bar\chi_1 \\
&+ \bar A(\chi_{ps})\, L^r_5\,\chi_{13} \,+\, \bar A(\chi_{ps})\bar A(\chi_\eta)\, \left[7/144\, R^\eta_{pp} - 5/72\, R^\eta_{ps} - 1/48\, R^\eta_{qq} + 5/72\, R^\eta_{qs} - 1/36\, R^\eta_{13}\right] \\
&+ \bar A(\chi_{ps})\bar B(\chi_p,\chi_p;0)\, \left[1/24\, R^p_{s\eta}\,\chi_p - 5/24\, R^p_{s\eta}\,\chi_{ps}\right] \,+\, \bar A(\chi_{ps})\bar B(\chi_p,\chi_\eta;0)\, \left[-1/18\, R^\eta_{ps} R^z_{qp\eta}\,\chi_p\right. \\
&- \left. 1/9\, R^\eta_{ps} R^z_{qp\eta}\,\chi_{ps}\right] \,-\, 1/48\,\bar A(\chi_{ps})\bar B(\chi_q,\chi_q;0)\, R^d_q \,+\, 1/18\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0)\, R^q_{s\eta}\,\chi_s \\
&+ 1/9\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0,k)\, R^q_{s\eta} \,+\, 3/16\,\bar A(\chi_{ps};\varepsilon)\,\pi_{16}\, \left[\chi_s + \bar\chi_1\right] \,-\, 1/8\,\bar A(\chi_{p4})^2 \,-\, 1/8\,\bar A(\chi_{p4})\bar A(\chi_{p6}) \\
&+ 1/8\,\bar A(\chi_{p4})\bar A(\chi_{q6}) \,-\, 1/32\,\bar A(\chi_{p6})^2 \,+\, \bar A(\chi_\eta)\,\pi_{16}\, \left[1/16\,\bar\chi_1\, R^v_{\eta 13} - 1/48\, R^v_{\eta 13}\,\chi_\eta + 1/16\, R^v_{\eta 13}\,\chi_{13}\right] \\
&+ \bar A(\chi_\eta)\, L^r_0\, \left[4 R^\eta_{13}\,\chi_\eta + 2/3\, R^v_{\eta 13}\,\chi_\eta\right] \,-\, 8\,\bar A(\chi_\eta)\, L^r_1\,\chi_\eta \,-\, 2\,\bar A(\chi_\eta)\, L^r_2\,\chi_\eta \,+\, \bar A(\chi_\eta)\, L^r_3\, \left[4 R^\eta_{13}\,\chi_\eta + 5/3\, R^v_{\eta 13}\chi_\eta\right] \\
&+ \bar A(\chi_\eta)\, L^r_4\, \left[4\,\chi_\eta + \bar\chi_1\, R^v_{\eta 13}\right] \,-\, \bar A(\chi_\eta)\, L^r_5\, \left[1/6\, R^\eta_{pp}\,\chi_q + R^\eta_{13}\,\chi_{13} + 1/6\, R^v_{\eta 13}\,\chi_\eta\right] \,+\, 1/288\,\bar A(\chi_\eta)^2\,(R^v_{\eta 13})^2 \\
&+ 1/12\,\bar A(\chi_\eta)\bar A(\chi_{46})\, R^v_{\eta 13} \,+\, \bar A(\chi_\eta)\bar B(\chi_p,\chi_p;0)\, \left[-1/36\,\bar\chi^{pp\eta}_{\eta\eta 1} - 1/18\, R^p_{q\eta} R^\eta_{pp}\,\chi_p + 1/18\, R^\eta_{pp} R^c_p\,\chi_p\right. \\
&+ \left. 1/144\, R^d_p R^v_{\eta 13}\right] \,+\, \bar A(\chi_\eta)\bar B(\chi_p,\chi_\eta;0)\, \left[-1/18\,\bar\chi^{\eta p\eta}_{p\eta 1} + 1/18\,\bar\chi^{\eta p\eta}_{q\eta 1} + 1/18\,(R^\eta_{pp})^2 R^z_{qp\eta}\,\chi_p\right] \\
&- 1/12\,\bar A(\chi_\eta)\bar B(\chi_{ps},\chi_{ps};0)\, R^\eta_{ps}\,\chi_{ps} \,-\, \bar A(\chi_\eta)\bar B(\chi_\eta,\chi_\eta;0)\, \left[1/216\, R^v_{\eta 13}\,\chi_4 + 1/27\, R^v_{\eta 13}\,\chi_6\right] \\
&- 1/18\,\bar A(\chi_\eta)\bar B(\chi_1,\chi_3;0)\, R^1_{\eta\eta} R^3_{\eta\eta}\,\chi_\eta \,+\, 1/18\,\bar A(\chi_\eta)\bar C(\chi_p,\chi_p,\chi_p;0)\, R^\eta_{pp} R^d_p\,\chi_p \,+\, \bar A(\chi_\eta;\varepsilon)\,\pi_{16}\, \left[1/8\,\chi_\eta\right. \\
&- \left. 1/16\,\bar\chi_1\, R^v_{\eta 13} - 1/8\, R^\eta_{13}\,\chi_\eta - 1/16\, R^v_{\eta 13}\,\chi_\eta\right] \,+\, \bar A(\chi_1)\bar A(\chi_3)\, \left[-1/72\, R^p_{q\eta} R^c_q + 1/36\, R^1_{3\eta} R^3_{1\eta} + 1/144\, R^c_1 R^c_3\right] \\
&- 4\,\bar A(\chi_{13})\, L^r_1\,\chi_{13} \,-\, 10\,\bar A(\chi_{13})\, L^r_2\,\chi_{13} \,+\, 1/8\,\bar A(\chi_{13})^2 \,-\, 1/2\,\bar A(\chi_{13})\bar B(\chi_1,\chi_3;0,k) \\
&+ 1/4\,\bar A(\chi_{13};\varepsilon)\,\pi_{16}\,\chi_{13} \,+\, 1/4\,\bar A(\chi_{14})\bar A(\chi_{34}) \,+\, 1/16\,\bar A(\chi_{16})\bar A(\chi_{36}) \,-\, 24\,\bar A(\chi_4)\, L^r_1\,\chi_4 \,-\, 6\,\bar A(\chi_4)\, L^r_2\,\chi_4 \\
&+ 12\,\bar A(\chi_4)\, L^r_4\,\chi_4 \,+\, 1/12\,\bar A(\chi_4)\bar B(\chi_p,\chi_p;0)\,(R^p_{4\eta})^2\,\chi_4 \,+\, 1/6\,\bar A(\chi_4)\bar B(\chi_p,\chi_\eta;0)\, \left[R^p_{4\eta} R^\eta_{p4}\,\chi_4 - R^p_{4\eta} R^\eta_{q4}\,\chi_4\right] \\
&- 1/24\,\bar A(\chi_4)\bar B(\chi_\eta,\chi_\eta;0)\, R^v_{\eta 13}\,\chi_4 \,-\, 1/6\,\bar A(\chi_4)\bar B(\chi_1,\chi_3;0)\, R^1_{4\eta} R^3_{4\eta}\,\chi_4 \,+\, 3/8\,\bar A(\chi_4;\varepsilon)\,\pi_{16}\,\chi_4 \\
&- 32\,\bar A(\chi_{46})\, L^r_1\,\chi_{46} \,-\, 8\,\bar A(\chi_{46})\, L^r_2\,\chi_{46} \,+\, 16\,\bar A(\chi_{46})\, L^r_4\,\chi_{46} \,+\, \bar A(\chi_{46})\bar B(\chi_p,\chi_p;0)\, \left[1/9\,\chi_{46} + 1/12\, R^\eta_{pp}\,\chi_p\right. \\
&+ \left. 1/36\, R^\eta_{pp}\,\chi_4 + 1/9\, R^\eta_{p4}\,\chi_6\right] \,+\, \bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0)\, \left[-1/18\, R^\eta_{pp}\,\chi_4 - 1/9\, R^\eta_{p4}\,\chi_6 + 1/9\, R^\eta_{q4}\,\chi_6 + 1/18\, R^\eta_{13}\,\chi_4\right] \\
&- 1/6\,\bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0,k)\, \left[R^\eta_{pp} - R^\eta_{13}\right] \,+\, 1/9\,\bar A(\chi_{46})\bar B(\chi_\eta,\chi_\eta;0)\, R^v_{\eta 13}\,\chi_{46} \,-\, \bar A(\chi_{46})\bar B(\chi_1,\chi_3;0)\, \left[2/9\,\chi_{46}\right. \\
&+ \left. 1/9\, R^\eta_{p4}\,\chi_6 + 1/18\, R^\eta_{13}\,\chi_4\right] \,-\, 1/6\,\bar A(\chi_{46})\bar B(\chi_1,\chi_3;0,k)\, R^\eta_{13} \,+\, 1/2\,\bar A(\chi_{46};\varepsilon)\,\pi_{16}\,\chi_{46} \\
&+ \bar B(\chi_p,\chi_p;0)\,\pi_{16}\, \left[1/16\,\bar\chi_1\, R^d_p + 1/96\, R^d_p\,\chi_p + 1/32\, R^d_p\,\chi_q\right] \,+\, 2/3\,\bar B(\chi_p,\chi_p;0)\, L^r_0\, R^d_p\,\chi_p \\
&+ 5/3\,\bar B(\chi_p,\chi_p;0)\, L^r_3\, R^d_p\,\chi_p \,+\, \bar B(\chi_p,\chi_p;0)\, L^r_4\, \left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0}\chi_p - 4\,\bar\chi_1 R^p_{q\eta}\,\chi_p + 4\,\bar\chi_1 R^c_p\,\chi_p + 3\,\bar\chi_1 R^d_p\right] \\
&+ \bar B(\chi_p,\chi_p;0)\, L^r_5\, \left[-2/3\bar\chi^{pp}_{\eta\eta 1}\chi_p - 4/3\, R^p_{q\eta}\,\chi_p^2 + 4/3\, R^c_p\,\chi_p^2 + 1/2\, R^d_p\,\chi_p - 1/6\, R^d_p\,\chi_q\right] \\
&+ \bar B(\chi_p,\chi_p;0)\, L^r_6\, \left[4\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 1} + 8\,\bar\chi_1 R^p_{q\eta}\,\chi_p - 8\,\bar\chi_1 R^c_p\,\chi_p\right] \,+\, 4\,\bar B(\chi_p,\chi_p;0)\, L^r_7\,(R^d_p)^2 \\
&+ \bar B(\chi_p,\chi_p;0)\, L^r_8\, \left[4/3\,\bar\chi^{pp}_{\eta\eta 2} + 8/3\, R^p_{q\eta}\,\chi_p^2 - 8/3\, R^c_p\,\chi_p^2\right] \,+\, \bar B(\chi_p,\chi_p;0)^2\, \left[-1/18\, R^p_{q\eta} R^d_p\,\chi_p + 1/18\, R^c_p R^d_p\,\chi_p\right. \\
&+ \left. 1/288\,(R^d_p)^2\right] \,+\, 1/18\,\bar B(\chi_p,\chi_p;0)\bar B(\chi_p,\chi_\eta;0)\, \left[R^\eta_{pp} R^d_p\,\chi_p - R^\eta_{13} R^d_p\,\chi_p\right]
\end{aligned}
$$

plus several more pages

Why so long expressions

- Many different quark and meson masses (χ_{ij})
- Charged propagators: $-i\,G^c_{ij}(k) = \frac{\epsilon_j}{k^2-\chi_{ij}+i\varepsilon} \quad (i \neq j)$
- Neutral propagators: $G^n_{ij}(k) = G^c_{ij}(k)\,\delta_{ij} - \frac{1}{n_{\text{sea}}}\,G^q_{ij}(k)$

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$$-i\,G^q_{ii}(k) = \frac{R^d_i}{(k^2-\chi_i+i\varepsilon)^2} + \frac{R^c_i}{k^2-\chi_i+i\varepsilon} + \frac{R^\pi_{\eta ii}}{k^2-\chi_\pi+i\varepsilon} + \frac{R^\eta_{\pi ii}}{k^2-\chi_\eta+i\varepsilon}$$

$$R^i_{jkl} = R^z_{i456jkl}, \qquad R^d_i = R^z_{i456\pi\eta},$$

$$R^c_i = R^i_{4\pi\eta} + R^i_{5\pi\eta} + R^i_{6\pi\eta} - R^i_{\pi\eta\eta} - R^i_{\pi\pi\eta}$$

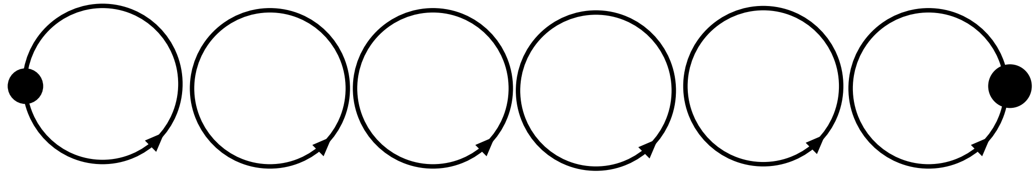
$$R^z_{ab} = \chi_a - \chi_b, \qquad R^z_{abc} = \frac{\chi_a-\chi_b}{\chi_a-\chi_c}, \qquad R^z_{abcd} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)}{\chi_a-\chi_d}$$

$$R^z_{abcdefg} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)(\chi_a-\chi_d)}{(\chi_a-\chi_e)(\chi_a-\chi_f)(\chi_a-\chi_g)}$$

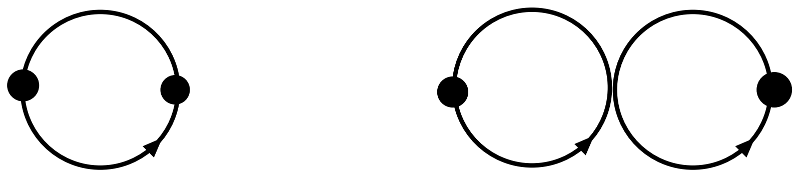
Double poles ?

Think quark lines and add gluons everywhere

Full



Quenched



So no resummation at the quark level:

naively a double pole

Same follows from inverting the lowest order kinetic terms

PQChPT at Two Loop

Masses and decay constants: all possible mass cases worked out for two and three flavours of sea quarks for the charged/off-diagonal case to NNLO.
hep-lat/0406017,hep-lat/0501014,hep-lat/0506004,hep-lat/0602003 all published in Phys. Rev. D

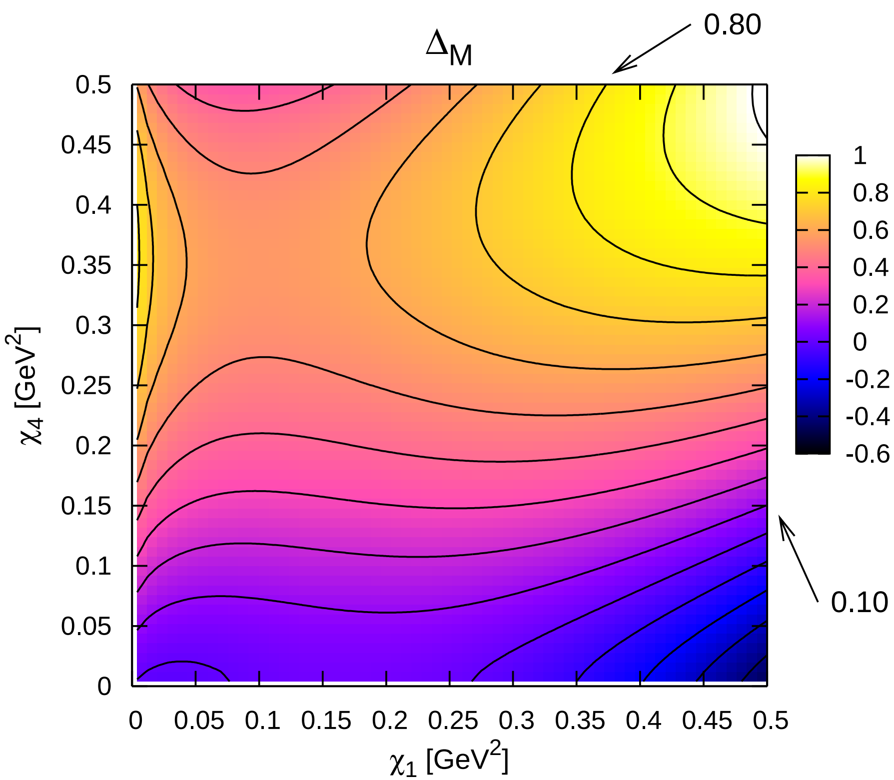
Earlier work at NLO:
Bernard-Golterman-Sharpe-Shoresh-$\cdots$

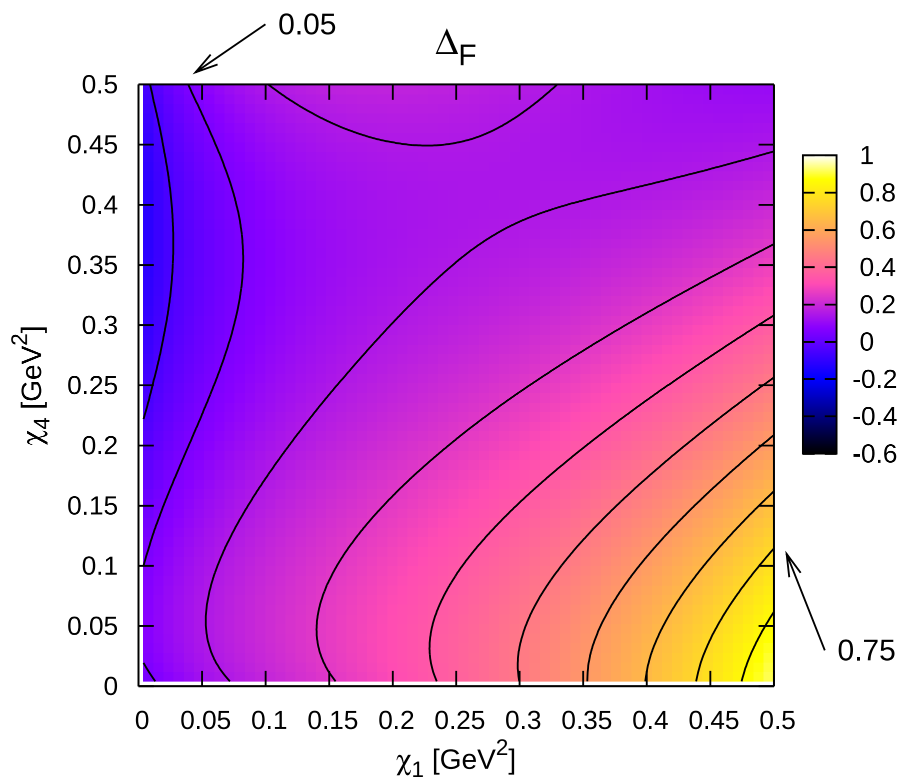
review/lectures: S. Sharpe hep-lat/0607016

Results: $\chi_1 = \chi_2, \chi_4 = \chi_5 = \chi_6$

Use lowest order mass squared: $\chi_i = 2B_0 m_i = m_M^{2(0)}$

Remember: $\chi_i \approx 0.3\ GeV^2 \approx (550\ MeV)^2 \sim$ border ChPT

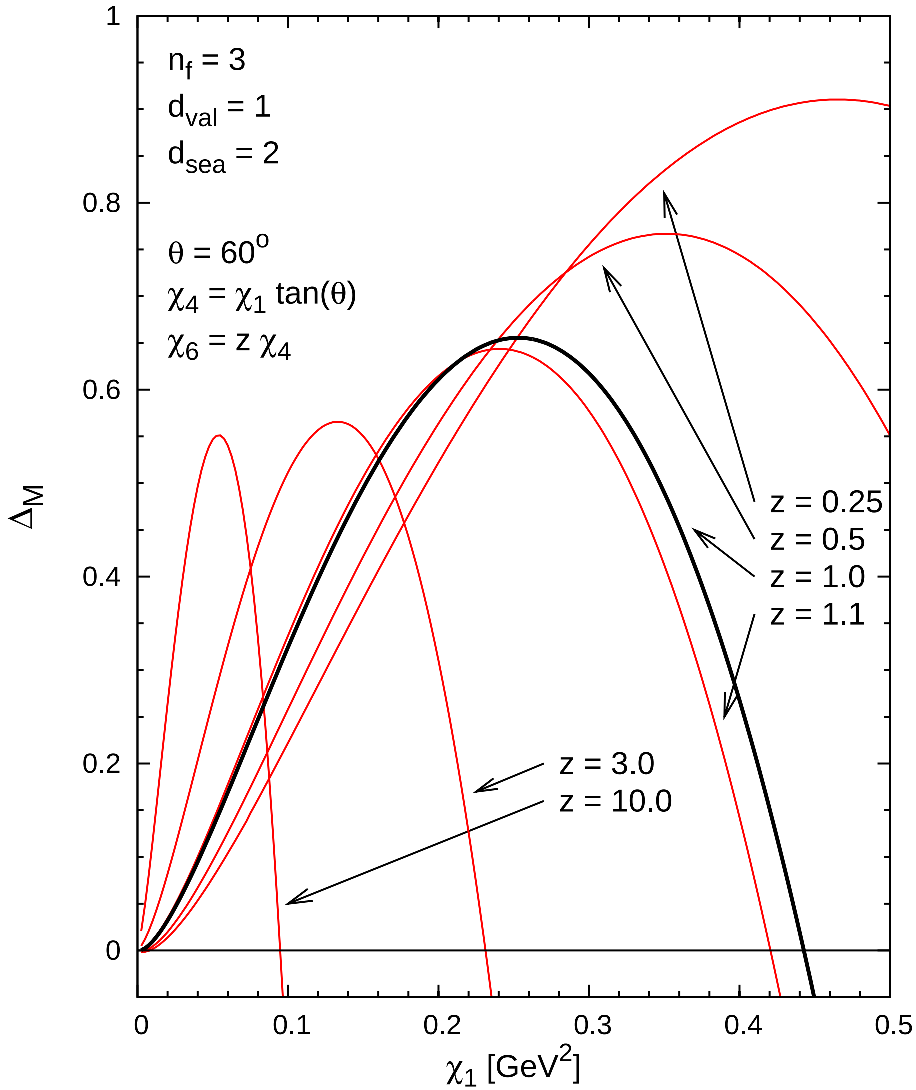




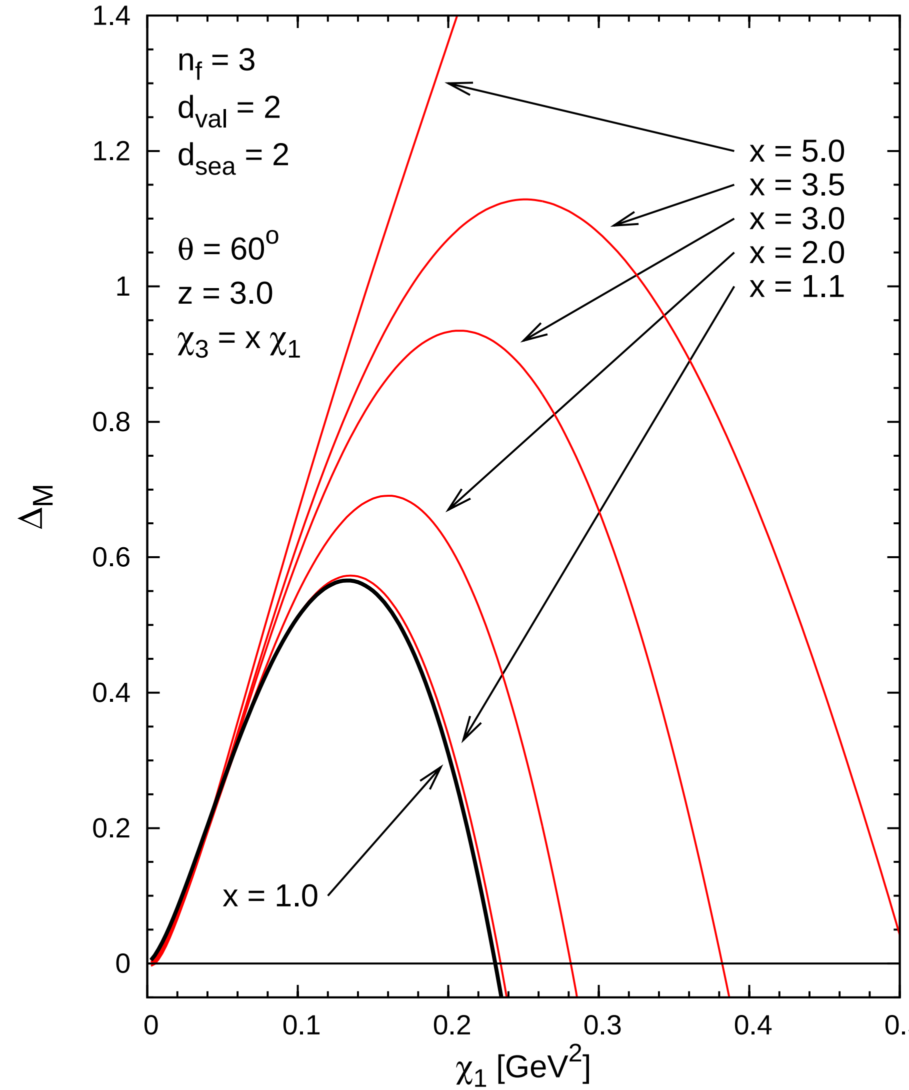
Relative corrections: mass^2 decay constant

mass^2: $\chi_4 = \chi_5 \neq \chi_6$

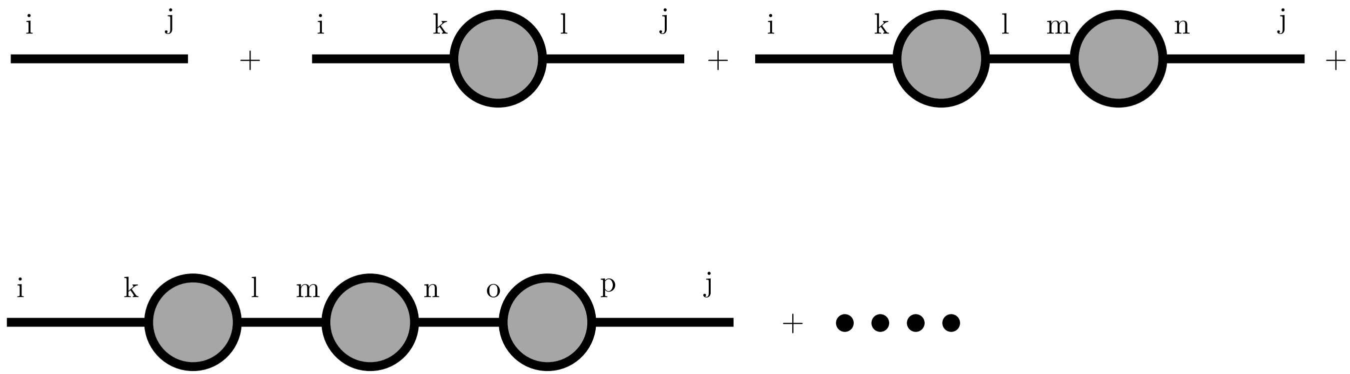
$\chi_1 = \chi_2$



$\chi_1 \neq \chi_2$



Neutral masses

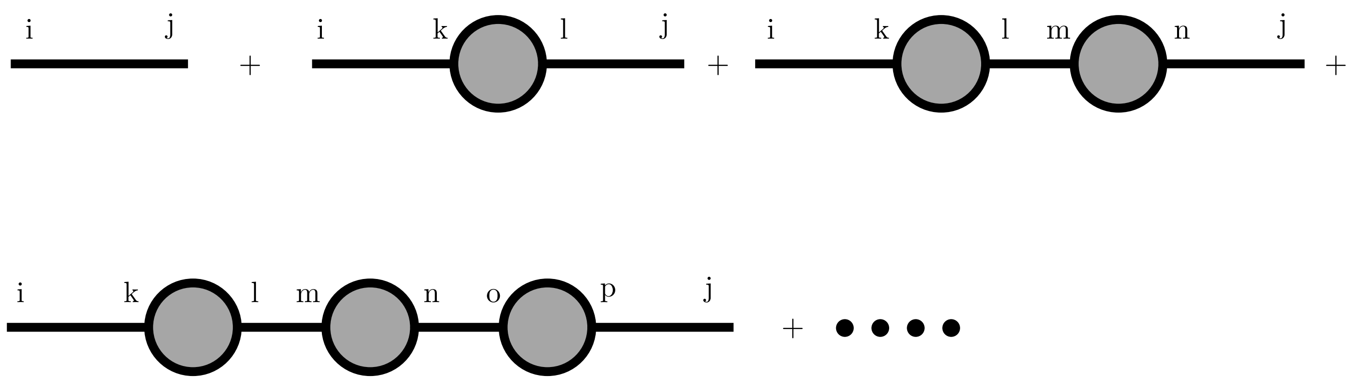


$$G^n_{ij} = G^0_{ij} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lj} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lm}(-i)\Sigma_{mn}G^0_{nj} + \cdots$$

ij means: from $\bar{q}_i q_i$ to $\bar{q}_j q_j$ meson

Full resummation done by JB,Danielsson hep-lat/0606017
i.e. full propagator from one-particle-irreducible diagrams

Neutral masses



$$G^n_{ij} = G^0_{ij} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lj} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lm}(-i)\Sigma_{mn}G^0_{nj} + \cdots$$

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Actually useful: residue of double pole allows to get at all LECs needed for the neutral masses

Conclusions

The **Lund High Energy Theory Group** is working on **the strong interaction and beyond** in many aspects and expects to continue this in the foreseeable future.