CHIRAL PERTURBATION THEORY AT TWO LOOP ORDER

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Overview

- Effective Field Theory
- Chiral Perturbation Theory
- some comments about Lagrangians/divergences
- list of existing calculations
- some general comments
- some results of the usual calculation
- Partially Quenched Chiral Perturbation Theory
- Some two-loop results
- What when not diagonal: the eta mass and PQChPT

Effective Field Theory

Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples:

- Solid state physics: conductors: neglect the empty bands above the partially filled one
- Atomic physics: Blue sky: neglect atomic structure

Power Counting

- gap in the spectrum $\Longrightarrow$ separation of scales
- with the lower degrees of freedom, build the most general effective Lagrangian

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$\Longrightarrow$ Need some ordering principle: power counting

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  - Where did my predictivity go ?

$\Longrightarrow$ Need some ordering principle: power counting

- Taylor series expansion does not work (convergence radius is zero)
- Continuum of excitation states need to be taken into account

Why Field Theory ?

- Only known way to combine QM and special relativity
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Drawbacks

- Many parameters (but finite number at any order)
  any model has few parameters but model-space is large
- expansion: it might not converge or only badly

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Advantages

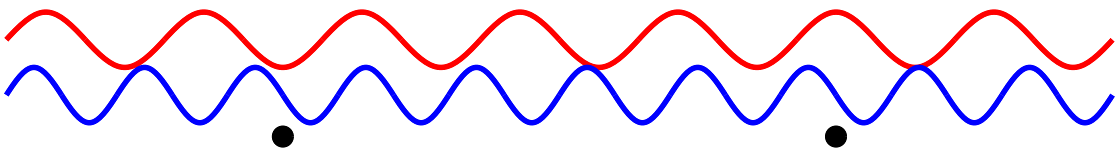
- Calculations are (relatively) simple
- It is general: model-independent
- Theory $\Longrightarrow$ errors can be estimated
- Systematic: ALL effects at a given order can be included
- Even if no convergence: classification of models often useful

Why is the sky blue ?

System: Photons of visible light and neutral atoms

Length scales: a few 1000 Å versus 1 Å

Atomic excitations suppressed by $\approx 10^{-3}$



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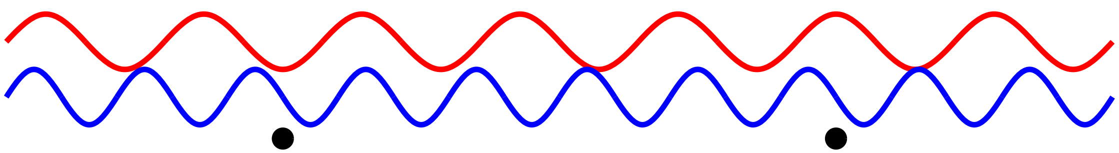

$$\mathcal{L}_A = \Phi_v^\dagger \partial_t \Phi_v + \dots \qquad \mathcal{L}_{\gamma A} = G F_{\mu\nu}^2 \Phi_v^\dagger \Phi_v + \dots$$

Units with $\hbar = c = 1$: G energy dimension -3:

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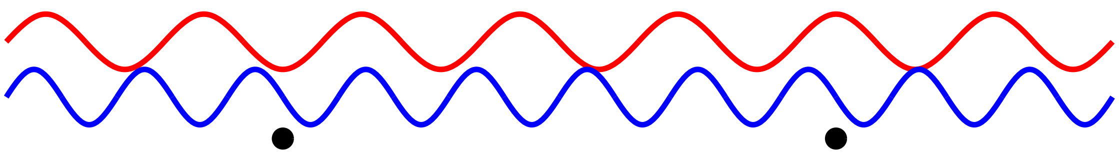

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$$\boxed{\sigma \approx G^2 E_\gamma^4}$$

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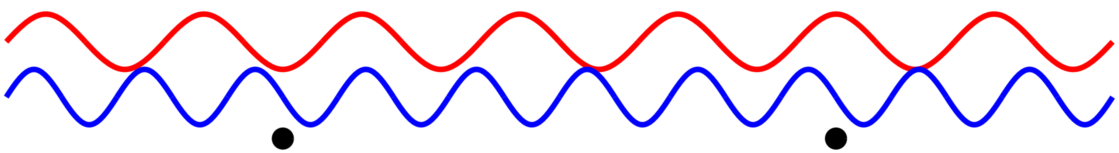

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blue light scatters a lot more than red $\begin{cases} \Longrightarrow \text{red sunsets} \\ \Longrightarrow \text{blue sky} \end{cases}$

Higher orders suppressed by 1 Å$/\lambda_\gamma$.

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

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Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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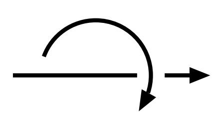
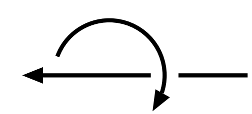
Can also see that via



$v < c,\ m_q \neq 0 \Longrightarrow$

$v = c,\ m_q = 0 \not\Longrightarrow$



Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

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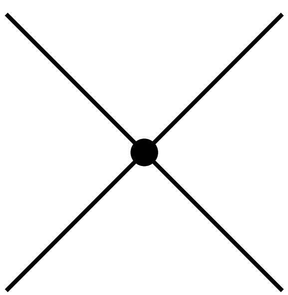
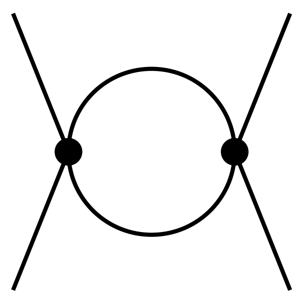
Power counting in momenta:



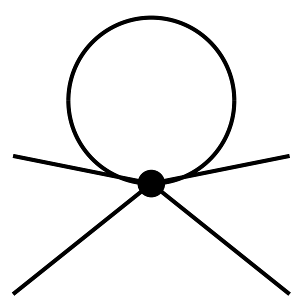
p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$

$\int d^4p$ $\quad p^4$



$(p^2)\,(1/p^2)\,p^4 = p^4$

Two Loop: Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

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Note
- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour is a special case of 3+3 PQ:

  $\hat{L}_i^r, K_i^r \rightarrow L_i^r, C_i^r$

Constructing Lagrangians

Easy part:

- Construct a complete set of needed basic objects
- Often useful to make everything transform under H alone (conserved part)
- Construct all terms up to the order you want

Tricky part: finding a minimal basis

- Field redefinitions or equations of motion
- Partial integration
- Cayley-Hamilton identities
- Bianchi identities, $\det\chi$ is also chirally invariant

Final check: can I determine all from experiment?

Constructing Lagrangians

Infinities:
the general divergence structure can be derived using heat kernel methods and/or background field methods.

Very useful for checks on calculations

Constructing Lagrangians

Question: do I always have to do all that?

- If only one process: can stop if enough terms found for all analytical dependences
- full Lagrangian: when doing relations with many observables
- often $SU(3)_V$ gives the needed relations but sometimes surprising cross relations show up (e.g. $f_+(0)$ in $K_{\ell 3}$).

Integrals, Divergences, Subtractions

- one-loop: Passarino-Veltman, but need also the order $d-4$ part.
- two-loop: sunset (i.e. two-point) integrals: dispersive method
- two-loop: vertex (i.e. three-point) integrals: Ghinculov-Van der Bij-Yao
- The last need two and three parameter numerical integrals: (very) slow.
- Subtraction: modified modified minimal subtraction (tadpoles are just a logarithm at NLO)

Three Flavours at Two Loop

Review paper: JB, LU TP 06-16 hep-ph/0604043

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$\Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVK}$	Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera	
$\Pi_{VV\rho\omega}$	Maltman	
$\Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta$	Kambor, Golowich; Amorós, JB, Talavera	
Π_{SS}	Moussallam	L_4^r, L_6^r
$\Pi_{VVK}, \Pi_{AAK}, F_K, m_K$	Amorós, JB, Talavera	
$K_{\ell 4}$, $\langle \overline{q}q \rangle$	Amorós, JB, Talavera	L_1^r, L_2^r, L_3^r
$F_M, m_M, \langle \overline{q}q \rangle$ $(m_u \neq m_d)$	Amorós, JB, Talavera	$L_{5,7,8}^r, m_u/m_d$
$F_{V\pi}, F_{VK^+}, F_{VK^0}$	Post, Schilcher; JB, Talavera	L_9^r
$K_{\ell 3}$	Post, Schilcher; JB, Talavera	V_{us}
$F_{S\pi}, F_{SK}$ (includes σ-terms)	JB, Dhonte	L_4^r, L_6^r
$K, \pi \to \ell\nu\gamma$	Geng, Ho, Wu	L_{10}^r
$\pi\pi$	JB,Dhonte,Talavera	
πK	JB,Dhonte,Talavera	
m_M and F_M PQChPT	JB,Danielsson,Lähde	
Finite Volume	JB,Colangelo,Ghorbani,Haefeli	

General Strategy and some comments

- Find enough inputs from experiment
- C_i^r:
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not σ)
  - quark masses: which scalars? unrealistically large estimates
- in p^6 physical or lowest order masses: thresholds in right place requires physical

General Strategy and some comments

Inputs:

$K_{\ell 4}$: $F(0)$, $G(0)$, λ E865 BNL

$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$ em with Dashen violation

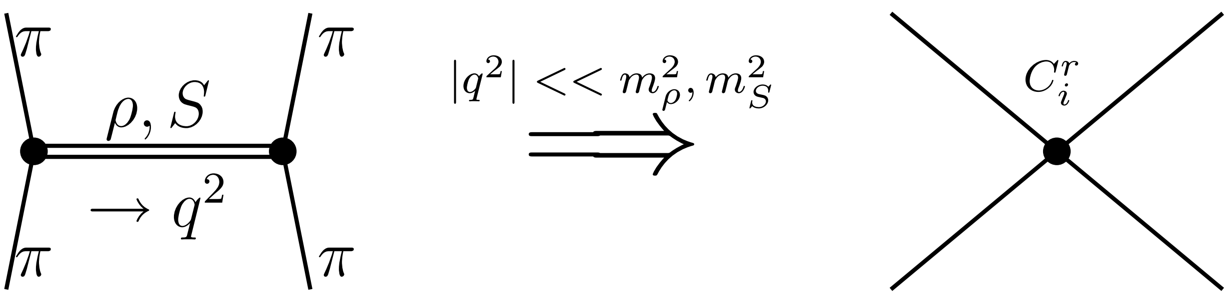
F_{π^+}

F_{K^+}/F_{π^+}

$m_s/\hat{m}$ 24 (26) $\hat{m} = (m_u + m_d)/2$

L_4^r, L_6^r

C_i^r from single resonance approximation



General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

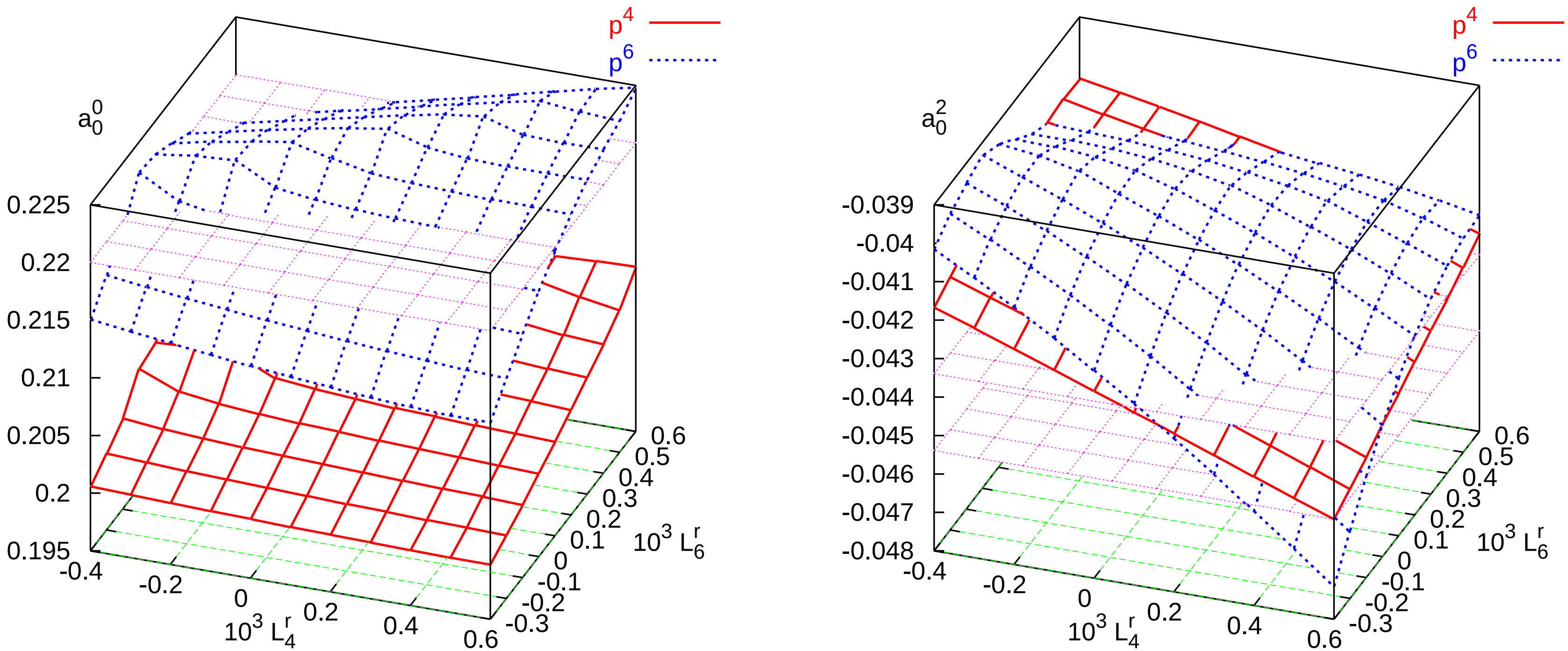
- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying C_i^r factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and πK thresholds

General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	0.009,$\equiv 0$	$-$0.138,0.009	$-$0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	0.075,$\equiv 0$	$-$0.149,0.094	$-$0.096,0.201
m_η^2: p^4, p^6	$-$0.052,0.318	0.013,$\equiv 0$	$-$0.197,0.073	$-$0.151,0.197
m_u/m_d	0.45$\pm$0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}$: p^4, p^6	0.169,0.051	0.22,$\equiv 0$	0.153,0.067	0.159,0.061

- $m_u = 0$ always very far from the fits
- F_0: pion decay constant in the chiral limit

$\pi\pi$

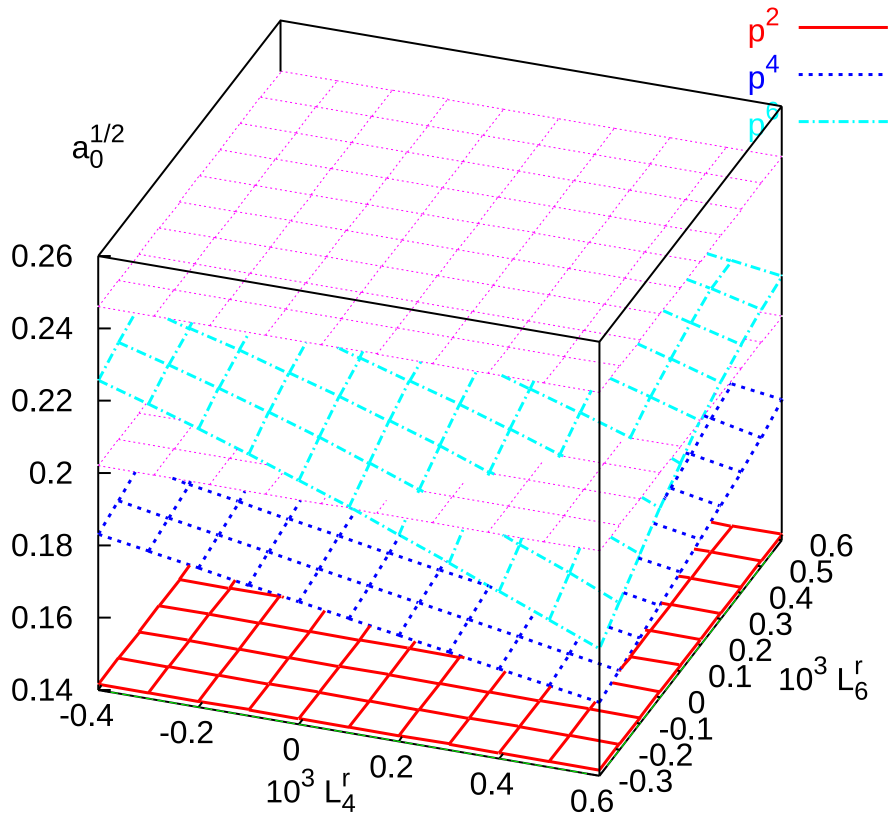


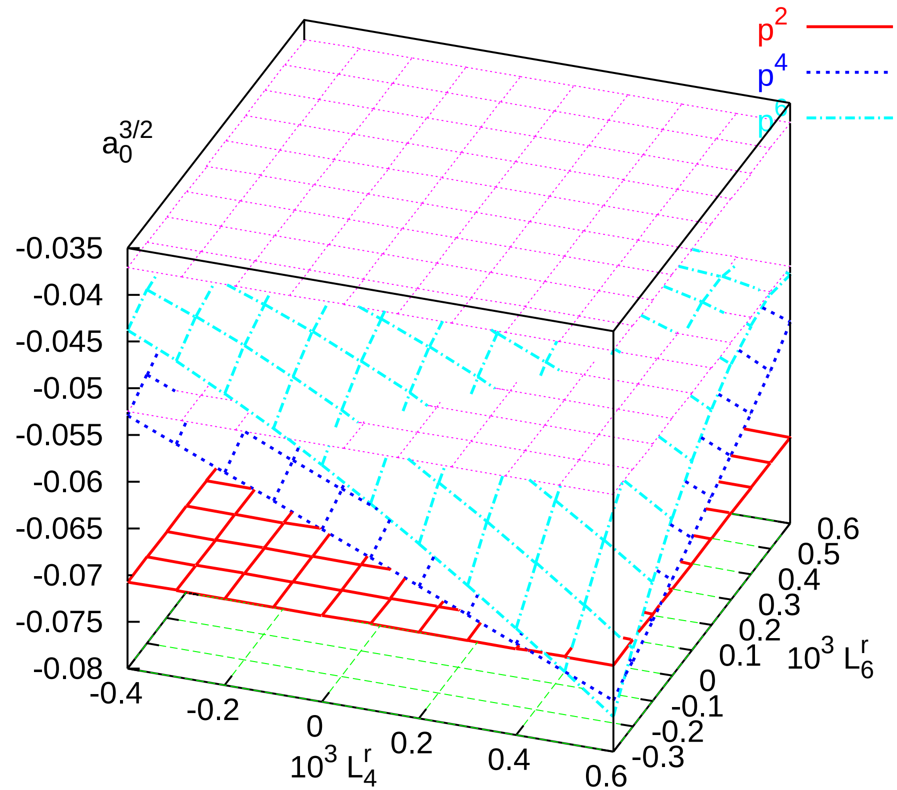
$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159$ $a_0^2 = -0.0454$ at order p^2

πK



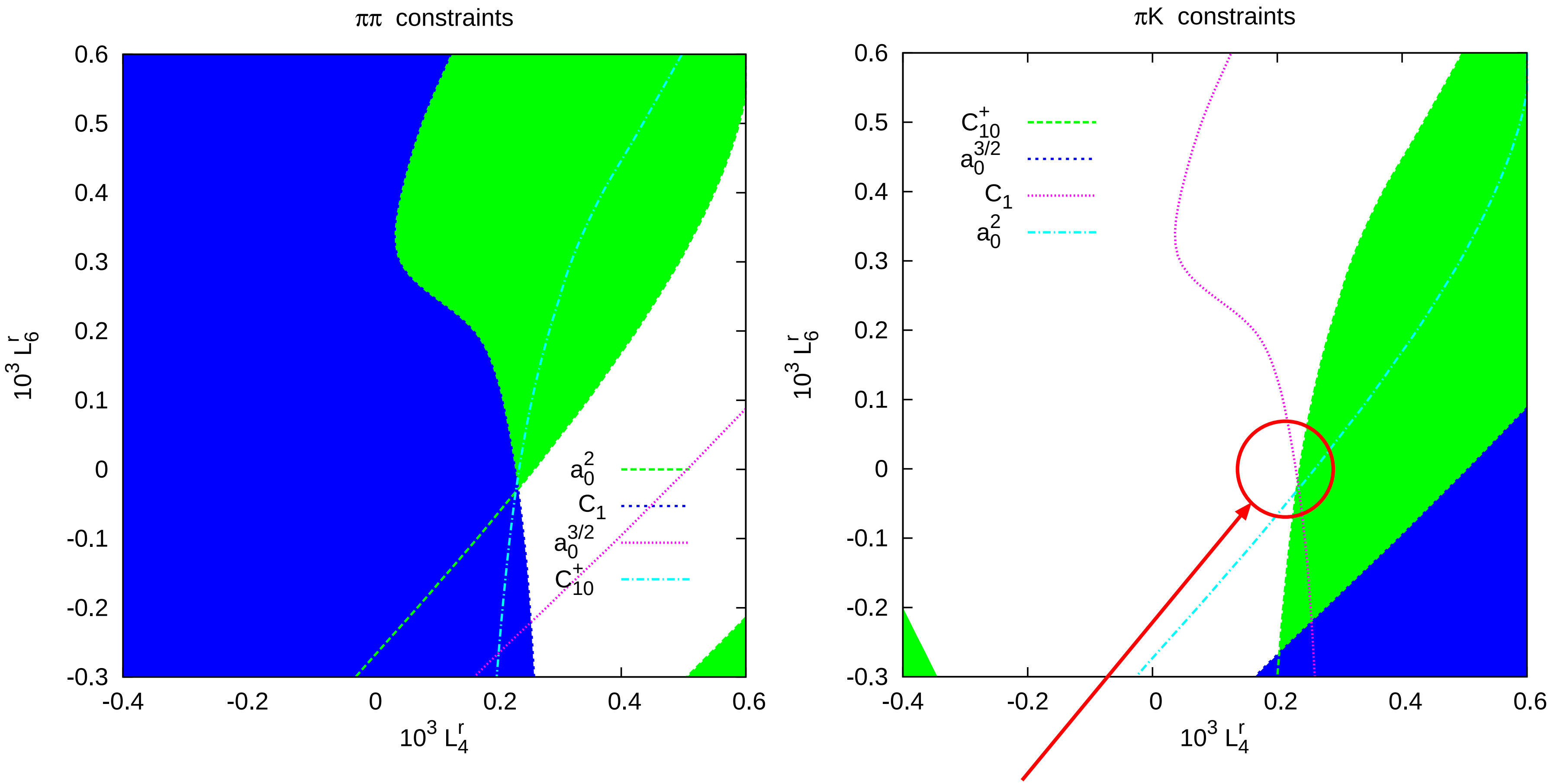


$a_0^{1/2} = 0.224 \pm 0.022$, $a_{3/2}^2 = -0.0448 \pm 0.0077$

Büttiker, Descotes-Genon, Moussallam

$a_0^{1/2} = 0.142$ $a_0^2 = -0.0708$ at order p^2

$\pi\pi$ and πK



preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization: $$f_{+,0}(t) = f_+(0)\left(1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right)$$

$|V_{us}|$:

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$|V_{us}|$:

- Know theoretically $f_+(0) = 1 + \cdots$
  - Short distance correction to G_F from G_μ Marciano-Sirlin
  - Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
  - Isospin Breaking Leutwyler-Roos $\dfrac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$ In Progress

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- Know experimentally $f_+(0)$
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  - Parametrize form-factor: is linear enough for $f_+(t)$?

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PDG2002: $|V_{ud}| = 0.9734 \pm 0.0008$, $|V_{us}| = 0.2196 \pm 0.0026$
$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) = 0.9957 \pm 0.0019$

$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

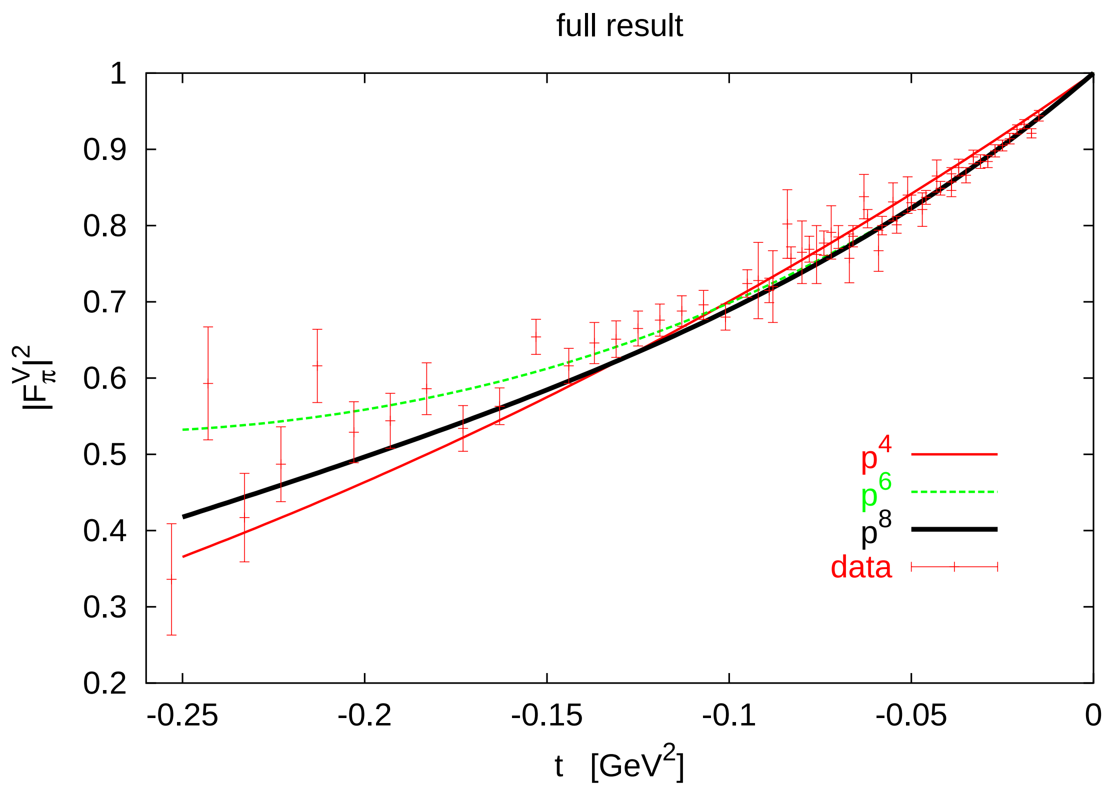
$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4}(-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

Pion electromagnetic Form factor:
JB, Talavera

$L_9^r = 0.00593 \pm 0.00043$

$-4C_{88}^r + 4C_{90}^r = 0.00022 \pm 0.00002$

VMD: $R_{+1}^{K\pi} \approx -4\,10^{-5}$ GeV^2



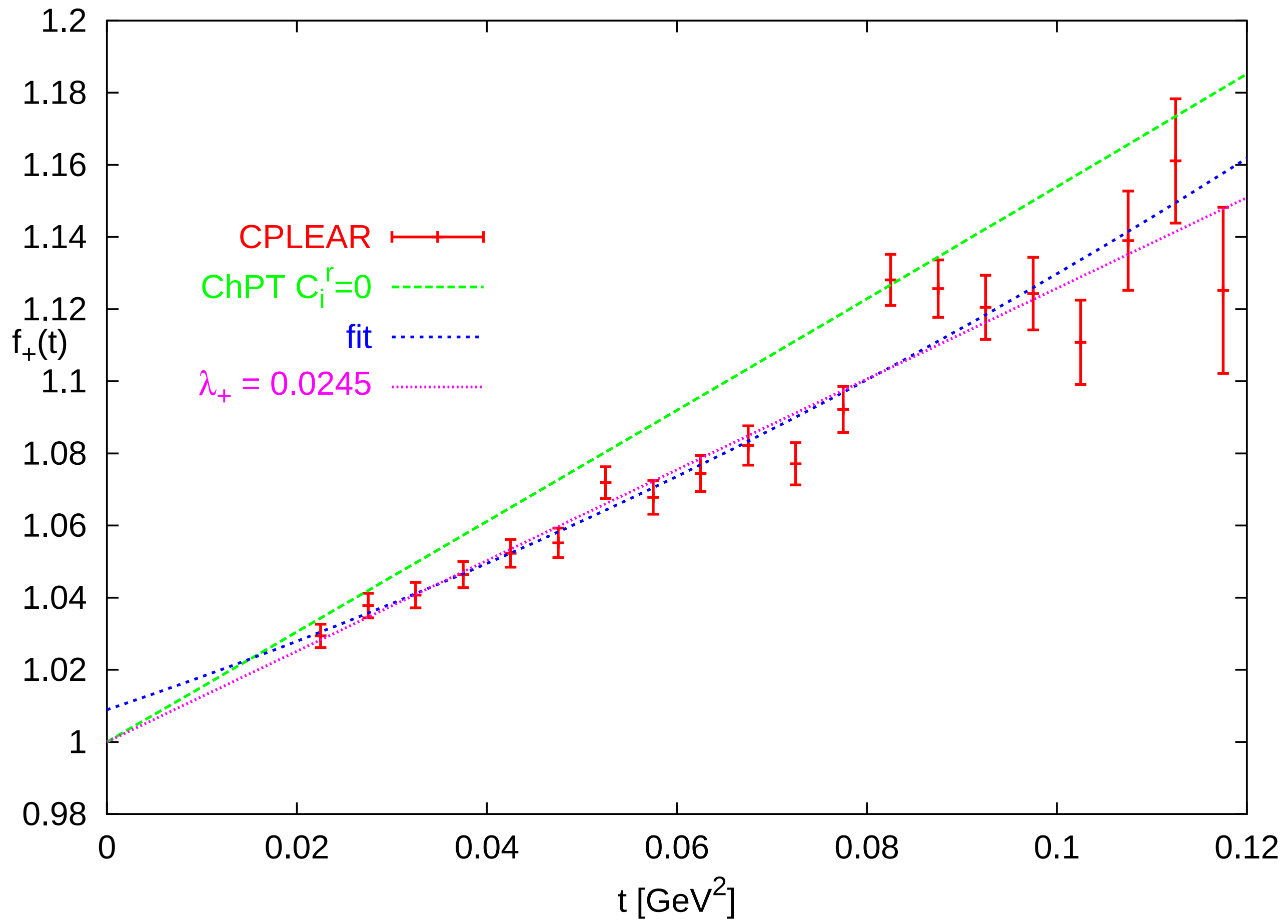
ChPT fit to $f_+(t)$

$\Rightarrow R_{+1}^{K\pi} =$
$-(4.7 \pm 0.5)\, 10^{-5}$ GeV^2

$\left(c_+ = 3.2 \text{ GeV}^{-4}\right)$
fixed by ChPT

$\Rightarrow a_+ = 1.009 \pm 0.004$

$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$



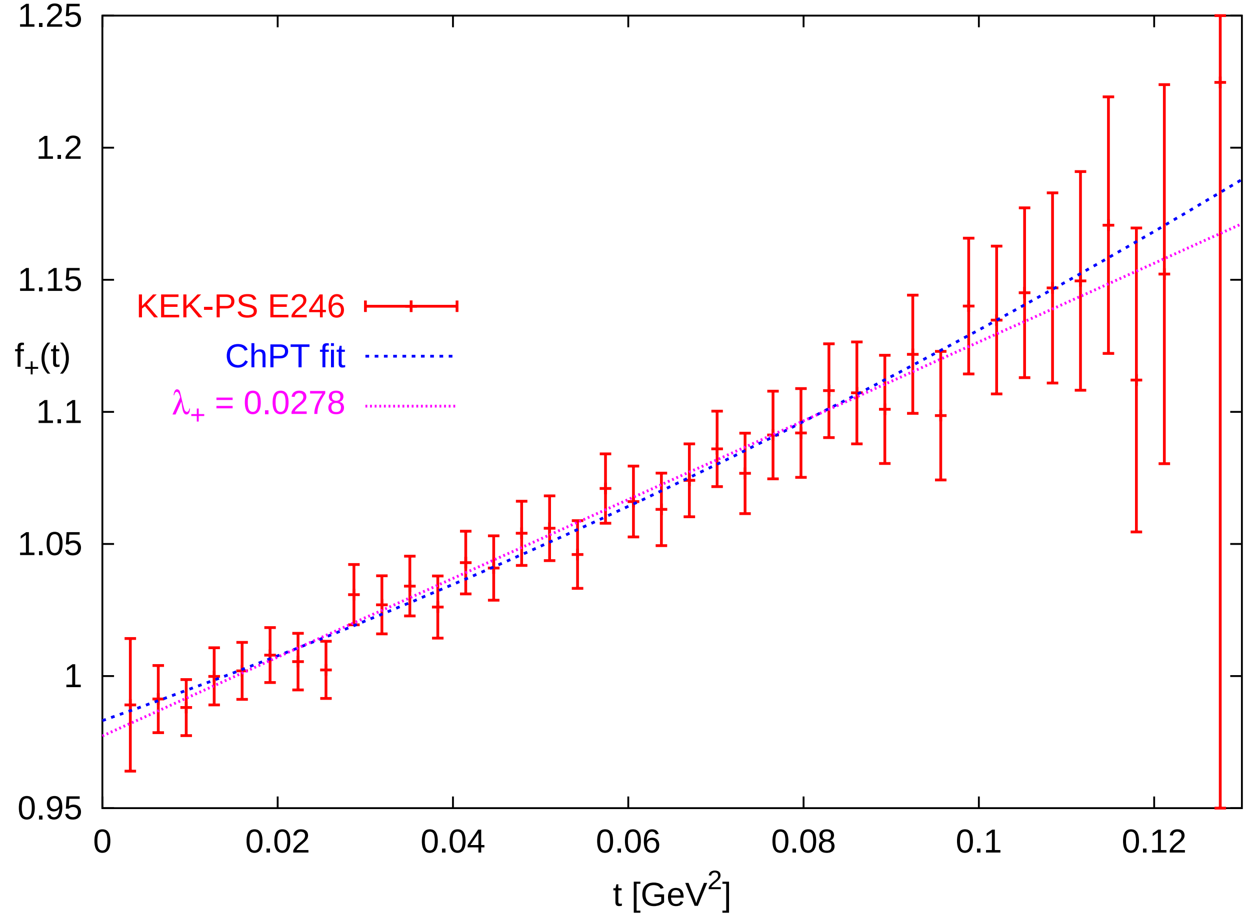
ChPT fit to $f_+(t)$

$\Rightarrow R^{K\pi}_{+1} =$
$-2.5\ 10^{-5}$ GeV^2

$\left(c_+ = 3.2 \text{ GeV}^{-4}\right)$
fixed by ChPT

$\Rightarrow a_+ = 1.006$

$\Rightarrow \lambda_+ = 0.0214 \pm 0.0018$



$f_0(t)$

Main Result:

$$
\begin{aligned}
f_0(t) \quad = \quad & 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 \\
& +8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
& -\frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

$f_0(t)$

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$$
\begin{aligned}
f_0(t) \quad = \quad & 1 - \frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 \\
& +8\frac{t}{F_\pi^4}(2C_{12}^r + C_{34}^r)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
& -\frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6

$\Longrightarrow$

All needed parameters can be determined experimentally

$f_0(t)$

Main Result:

$$
\begin{aligned}
f_0(t) \quad = \quad & 1 - \frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 \\
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& - \frac{8}{F_\pi^4} t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
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$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6

$\Longrightarrow$

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.$$

$$\Delta(0) = -0.0227\,(p^4) + 0.0113\,(p^6 \text{ pure loop}) + 0.0033\,(p^6 L_i^r)$$

V_{us} present status

- More Theory: Dispersion theory relates slopes and curvature in $f_0(t)$ Jamin, Oller, Pich

  $C^r_{12} + C^r_{34} = 3.2 \pm 1.5\ 10^{-6} \Longrightarrow f_+(0)_{p^6} = 0.002 \pm 0.009$.

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- More Experiment:
  - 2003: E865 $K^+_{\ell 3}$ branching ratio: strong increase
  - 2004: KTeV $K^0_{\ell 3}$ branching ratio: strong increase
  - 2004: formfactor data KTeV (K^0) and ISTRA+ (K^+):
    - Curvature seen, reasonable agreement with ChPT
    - λ_0: old discrepancies gone?

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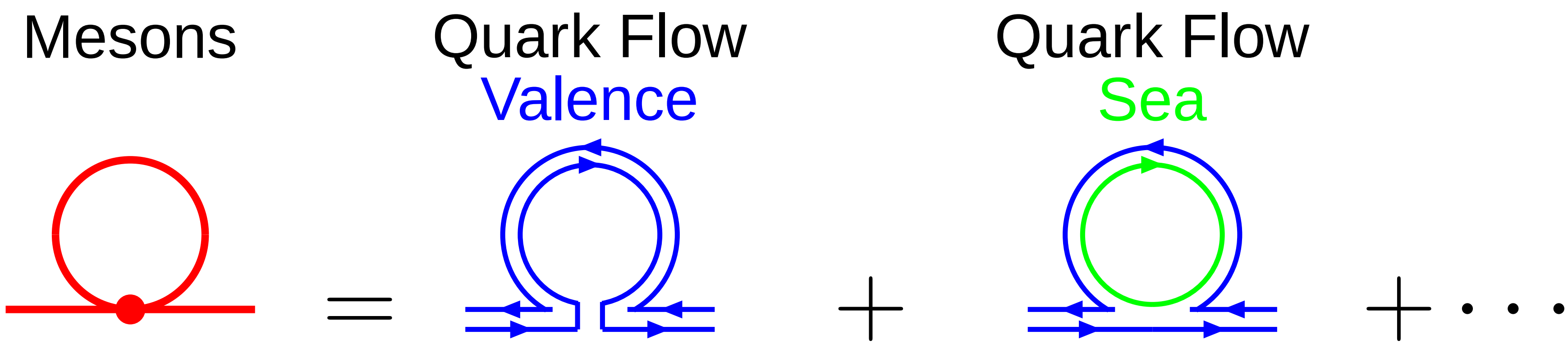
    - λ_0: old discrepancies gone?
- Expect both more theory and experiment (KLOE,NA48)

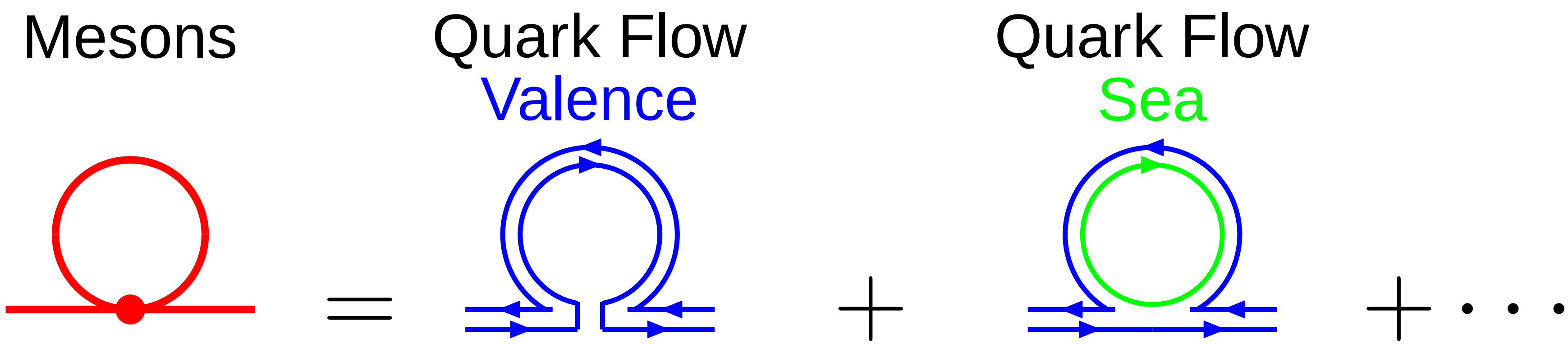
the unitarity problem might be on the way out

ChPT and Lattice QCD



Valence is easy to deal with in lattice QCD
Sea is very difficult

ChPT and Lattice QCD



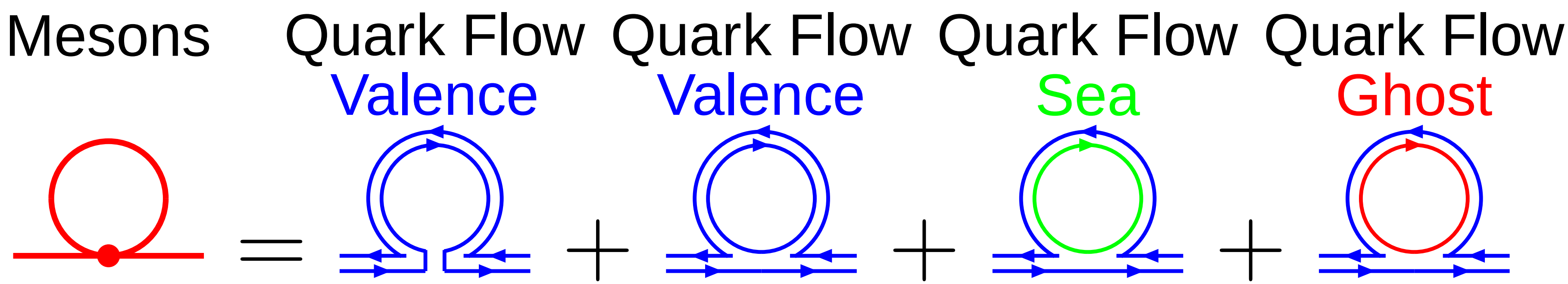
Valence is easy to deal with in lattice QCD
Sea is very difficult

They can be treated separately: i.e. different quark masses
Partially Quenched ChPT (PQChPT)
One Loop or p^4: Bernard, Golterman, Pallante, Sharpe, Shoresh,...

Vary valence $\Longrightarrow$ more quantities calculable, more studies

PQChPT at Two Loops: General

Add ghost quarks: remove the unwanted free valence loops



Possible problem: QCD $\Longrightarrow$ ChPT relies heavily on unitarity

Partially quenched: at least one dynamical sea quark
$\Longrightarrow$ Φ_0 is heavy: remove from PQChPT

Symmetry group becomes $SU(n_v + n_s|n_v) \times SU(n_v + n_s|n_v)$ (approximately)

PQChPT at Two Loops: General

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT $\Longrightarrow$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

E.g. $L_1^r = L_0^{r(3pq)}/2 + L_1^{r(3pq)}$

PQChPT at Two Loop

Subject started:

valence equal mass, 3 sea equal mass:

$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0406017

Other mass combinations:

F_{π^+}: JB,Lähde, hep-lat/0501014

F_{π^+}, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004

$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0602003

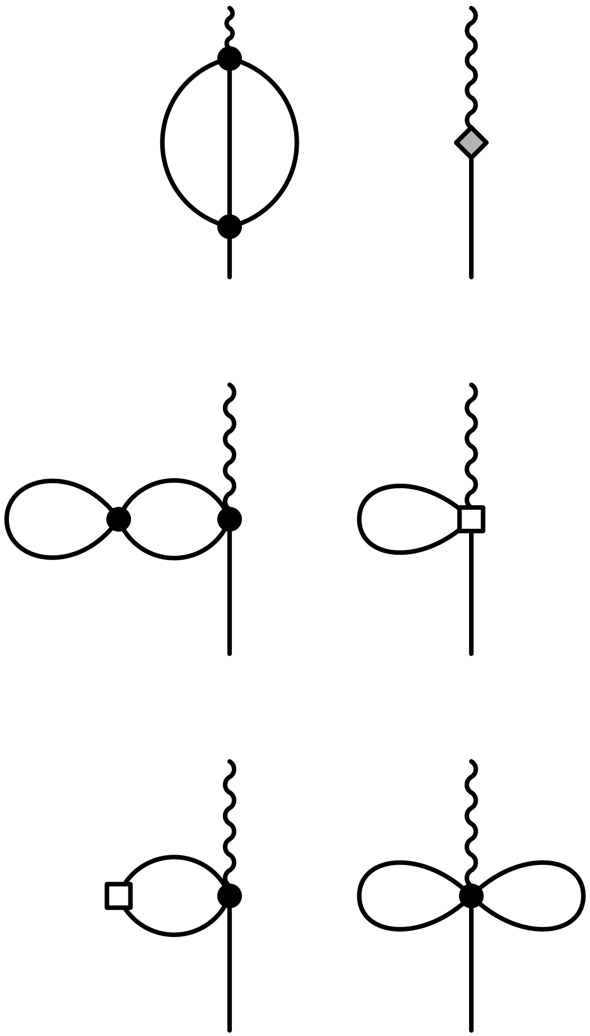
m^2_{η}: JB,Danielsson,Lähde, hep-lat/0606017

Actual Calculations:

- heavy use of FORM Vermaseren
- use PQ without super Φ_0 in super-symmetric formalism
- Main problem: sheer size of the expressions

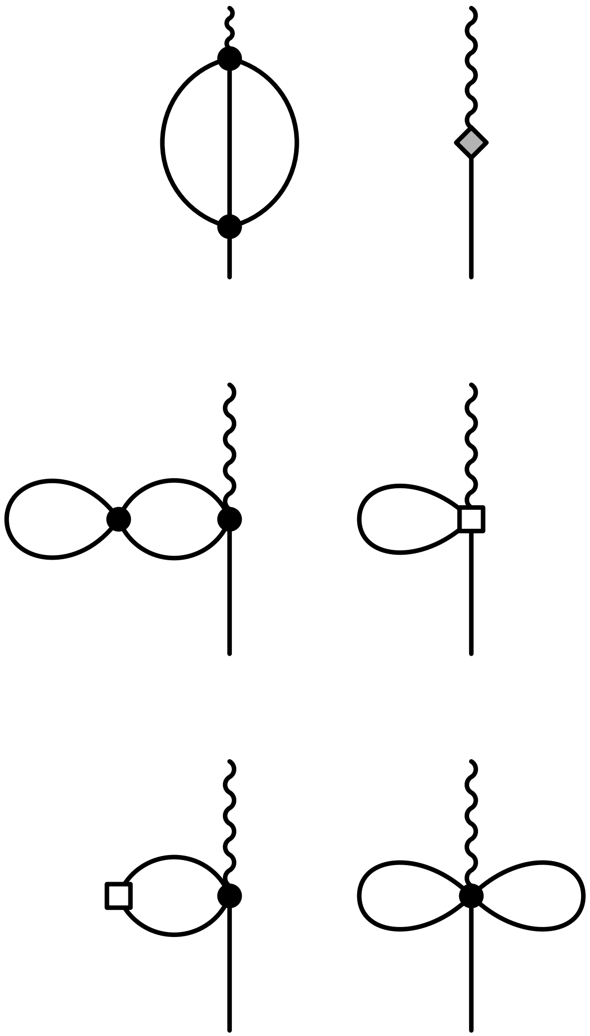
Iso breaking from lattice data: a and L extrapolations needed

Long Expressions



$\Longrightarrow$

Long Expressions



$\Longrightarrow$

$$
\begin{aligned}
\delta^{(6)22}_{\rm loops} = {} & \pi_{16}\, L_0^r\, [4/9\,\chi_\eta\chi_4 - 1/2\,\chi_1\chi_3 + \chi_{13}^2 - 13/3\,\bar\chi_1\chi_{13} - 35/18\,\bar\chi_2] \,-\, 2\,\pi_{16}\, L_1^r\,\chi_{13}^2 \\
& - \pi_{16}\, L_2^r\, [11/3\,\chi_\eta\chi_4 + \chi_{13}^2 + 13/3\,\bar\chi_2] + \pi_{16}\, L_3^r\, [4/9\,\chi_\eta\chi_4 - 7/12\,\chi_1\chi_3 + 11/6\,\chi_{13}^2 - 17/6\,\bar\chi_1\chi_{13} - 43/36\,\bar\chi_2] \\
& + \pi_{16}^2\, [-15/64\,\chi_\eta\chi_4 - 59/384\,\chi_1\chi_3 + 65/384\,\chi_{13}^2 - 1/2\,\bar\chi_1\chi_{13} - 43/128\,\bar\chi_2] \,-\, 48\, L_4^r L_5^r\,\bar\chi_1\chi_{13} \,-\, 72\, L_4^{r2}\,\bar\chi_1^2 \\
& - 8\, L_5^{r2}\,\chi_{13}^2 \,+\, \bar A(\chi_p)\,\pi_{16}\, [-1/24\,\chi_p + 1/48\,\bar\chi_1 - 1/8\,\bar\chi_1\, R^p_{q\eta} + 1/16\,\bar\chi_1\, R^c_p - 1/48\, R^p_{q\eta}\,\chi_p - 1/16\, R^p_{q\eta}\,\chi_q \\
& + 1/48\, R^\eta_{pp}\,\chi_\eta + 1/16\, R^c_p\,\chi_{13}] \,+\, \bar A(\chi_p)\, L_0^r\, [8/3\, R^p_{q\eta}\,\chi_p + 2/3\, R^c_p\,\chi_p + 2/3\, R^d_p] \,+\, \bar A(\chi_p)\, L_3^r\, [2/3\, R^p_{q\eta}\,\chi_p \\
& + 5/3\, R^c_p\,\chi_p + 5/3\, R^d_p] \,+\, \bar A(\chi_p)\, L_4^r\, [-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0} - 2\,\bar\chi_1\, R^p_{q\eta} + 3\,\bar\chi_1\, R^c_p] \,+\, \bar A(\chi_p)\, L_5^r\, [-2/3\,\bar\chi^{pp}_{\eta\eta 1} - R^p_{q\eta}\,\chi_p \\
& + 1/3\, R^p_{q\eta}\,\chi_q + 1/2\, R^c_p\,\chi_p - 1/6\, R^c_p\,\chi_q] \,+\, \bar A(\chi_p)^2\, [1/16 + 1/72\,(R^p_{q\eta})^2 - 1/72\, R^p_{q\eta} R^c_p + 1/288\,(R^c_p)^2] \\
& + \bar A(\chi_p)\bar A(\chi_{ps})\, [-1/36\, R^p_{q\eta} - 5/72\, R^p_{s\eta} + 7/144\, R^c_p] \,-\, \bar A(\chi_p)\bar A(\chi_{qs})\, [1/36\, R^p_{q\eta} + 1/24\, R^p_{s\eta} + 1/48\, R^c_p] \\
& + \bar A(\chi_p)\bar A(\chi_\eta)\, [-1/72\, R^p_{q\eta} R^v_{\eta 13} + 1/144\, R^c_p R^v_{\eta 13}] \,+\, 1/8\,\bar A(\chi_p)\bar A(\chi_{13}) \,+\, 1/12\,\bar A(\chi_p)\bar A(\chi_{46})\, R^\eta_{pp} \\
& + \bar A(\chi_p)\bar B(\chi_p,\chi_p;0)\, [1/4\,\chi_p - 1/18\, R^p_{q\eta} R^c_p\,\chi_p - 1/72\, R^p_{q\eta} R^d_p + 1/18\,(R^c_p)^2\,\chi_p + 1/144\, R^c_p R^d_p] \\
& + \bar A(\chi_p)\bar B(\chi_p,\chi_\eta;0)\, [1/18\, R^\eta_{pp} R^c_p\,\chi_p - 1/18\, R^\eta_{13} R^c_p\,\chi_p] \,+\, \bar A(\chi_p)\bar B(\chi_q,\chi_q;0)\, [-1/72\, R^p_{q\eta} R^d_q + 1/144\, R^c_p R^d_q] \\
& - 1/12\,\bar A(\chi_p)\bar B(\chi_{ps},\chi_{ps};0)\, R^p_{s\eta}\,\chi_{ps} \,-\, 1/18\,\bar A(\chi_p)\bar B(\chi_1,\chi_3;0)\, R^q_{p\eta} R^c_p\,\chi_p \\
& + 1/18\,\bar A(\chi_p)\bar C(\chi_p,\chi_p,\chi_p;0)\, R^c_p R^d_p\,\chi_p \,+\, \bar A(\chi_p;\varepsilon)\,\pi_{16}\, [1/8\,\bar\chi_1 R^p_{q\eta} - 1/16\bar\chi_1\, R^c_p - 1/16\, R^c_p\,\chi_p - 1/16\, R^d_p] \\
& + \bar A(\chi_{ps})\,\pi_{16}\, [1/16\,\chi_{ps} - 3/16\,\chi_{qs} - 3/16\,\bar\chi_1] \,-\, 2\,\bar A(\chi_{ps})\, L_0^r\,\chi_{ps} \,-\, 5\,\bar A(\chi_{ps})\, L_3^r\,\chi_{ps} \,-\, 3\,\bar A(\chi_{ps})\, L_4^r\,\bar\chi_1 \\
& + \bar A(\chi_{ps})\, L_5^r\,\chi_{13} \,+\, \bar A(\chi_{ps})\bar A(\chi_\eta)\, [7/144\, R^\eta_{pp} - 5/72\, R^\eta_{ps} - 1/48\, R^\eta_{qq} + 5/72\, R^\eta_{qs} - 1/36\, R^\eta_{13}] \\
& + \bar A(\chi_{ps})\bar B(\chi_p,\chi_p;0)\, [1/24\, R^p_{s\eta}\,\chi_p - 5/24\, R^p_{s\eta}\,\chi_{ps}] \,+\, \bar A(\chi_{ps})\bar B(\chi_p,\chi_\eta;0)\, [-1/18\, R^\eta_{ps} R^z_{qp\eta}\,\chi_p \\
& - 1/9\, R^\eta_{ps} R^z_{qp\eta}\,\chi_{ps}] \,-\, 1/48\,\bar A(\chi_{ps})\bar B(\chi_q,\chi_q;0)\, R^d_q \,+\, 1/18\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0)\, R^q_{s\eta}\,\chi_s \\
& + 1/9\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0,k)\, R^q_{s\eta} \,+\, 3/16\,\bar A(\chi_{ps};\varepsilon)\,\pi_{16}\, [\chi_s + \bar\chi_1] \,-\, 1/8\,\bar A(\chi_{p4})^2 \,-\, 1/8\,\bar A(\chi_{p4})\bar A(\chi_{p6}) \\
& + 1/8\,\bar A(\chi_{p4})\bar A(\chi_{q6}) \,-\, 1/32\,\bar A(\chi_{p6})^2 \,+\, \bar A(\chi_\eta)\,\pi_{16}\, [1/16\,\bar\chi_1\, R^v_{\eta 13} - 1/48\, R^v_{\eta 13}\,\chi_\eta + 1/16\, R^v_{\eta 13}\,\chi_{13}] \\
& + \bar A(\chi_\eta)\, L_0^r\, [4R^\eta_{13}\,\chi_\eta + 2/3\, R^v_{\eta 13}\,\chi_\eta] \,-\, 8\,\bar A(\chi_\eta)\, L_1^r\,\chi_\eta \,-\, 2\,\bar A(\chi_\eta)\, L_2^r\,\chi_\eta \,+\, \bar A(\chi_\eta)\, L_3^r\, [4R^\eta_{13}\,\chi_\eta + 5/3\, R^v_{\eta 13}\chi_\eta] \\
& + \bar A(\chi_\eta)\, L_4^r\, [4\,\chi_\eta + \bar\chi_1\, R^v_{\eta 13}] \,-\, \bar A(\chi_\eta)\, L_5^r\, [1/6\, R^\eta_{pp}\,\chi_q + R^\eta_{13}\,\chi_{13} + 1/6\, R^v_{\eta 13}\,\chi_\eta] \,+\, 1/288\,\bar A(\chi_\eta)^2\,(R^v_{\eta 13})^2 \\
& + 1/12\,\bar A(\chi_\eta)\bar A(\chi_{46})\, R^v_{\eta 13} \,+\, \bar A(\chi_\eta)\bar B(\chi_p,\chi_p;0)\, [-1/36\,\bar\chi^{pp\eta}_{\eta\eta 1} - 1/18\, R^p_{q\eta} R^\eta_{pp}\,\chi_p + 1/18\, R^\eta_{pp} R^c_p\,\chi_p \\
& + 1/144\, R^d_p R^v_{\eta 13}] \,+\, \bar A(\chi_\eta)\bar B(\chi_p,\chi_\eta;0)\, [-1/18\,\bar\chi^{\eta p\eta}_{p\eta 1} + 1/18\,\bar\chi^{\eta p\eta}_{q\eta 1} + 1/18\,(R^\eta_{pp})^2 R^z_{qp\eta}\,\chi_p] \\
& - 1/12\,\bar A(\chi_\eta)\bar B(\chi_{ps},\chi_{ps};0)\, R^\eta_{ps}\,\chi_{ps} \,-\, \bar A(\chi_\eta)\bar B(\chi_\eta,\chi_\eta;0)\, [1/216\, R^v_{\eta 13}\,\chi_4 + 1/27\, R^v_{\eta 13}\,\chi_6] \\
& - 1/18\,\bar A(\chi_\eta)\bar B(\chi_1,\chi_3;0)\, R^1_{\eta\eta} R^3_{\eta\eta}\,\chi_\eta \,+\, 1/18\,\bar A(\chi_\eta)\bar C(\chi_p,\chi_p,\chi_p;0)\, R^\eta_{pp} R^d_p\,\chi_p \,+\, \bar A(\chi_\eta;\varepsilon)\,\pi_{16}\, [1/8\,\chi_\eta \\
& - 1/16\,\bar\chi_1\, R^v_{\eta 13} - 1/8\, R^\eta_{13}\,\chi_\eta - 1/16\, R^v_{\eta 13}\,\chi_\eta] \,+\, \bar A(\chi_1)\bar A(\chi_3)\, [-1/72\, R^p_{q\eta} R^c_q + 1/36\, R^1_{3\eta} R^3_{1\eta} + 1/144\, R^c_1 R^c_3] \\
& - 4\,\bar A(\chi_{13})\, L_1^r\,\chi_{13} \,-\, 10\,\bar A(\chi_{13})\, L_2^r\,\chi_{13} \,+\, 1/8\,\bar A(\chi_{13})^2 \,-\, 1/2\,\bar A(\chi_{13})\bar B(\chi_1,\chi_3;0,k) \\
& + 1/4\,\bar A(\chi_{13};\varepsilon)\,\pi_{16}\,\chi_{13} \,+\, 1/4\,\bar A(\chi_{14})\bar A(\chi_{34}) \,+\, 1/16\,\bar A(\chi_{16})\bar A(\chi_{36}) \,-\, 24\,\bar A(\chi_4)\, L_1^r\,\chi_4 \,-\, 6\,\bar A(\chi_4)\, L_2^r\,\chi_4 \\
& + 12\,\bar A(\chi_4)\, L_4^r\,\chi_4 \,+\, 1/12\,\bar A(\chi_4)\bar B(\chi_p,\chi_p;0)\,(R^p_{4\eta})^2\,\chi_4 \,+\, 1/6\,\bar A(\chi_4)\bar B(\chi_p,\chi_\eta;0)\, [R^p_{4\eta} R^\eta_{p4}\,\chi_4 - R^p_{4\eta} R^\eta_{q4}\,\chi_4] \\
& - 1/24\,\bar A(\chi_4)\bar B(\chi_\eta,\chi_\eta;0)\, R^v_{\eta 13}\,\chi_4 \,-\, 1/6\,\bar A(\chi_4)\bar B(\chi_1,\chi_3;0)\, R^1_{4\eta} R^3_{4\eta}\,\chi_4 \,+\, 3/8\,\bar A(\chi_4;\varepsilon)\,\pi_{16}\,\chi_4 \\
& - 32\,\bar A(\chi_{46})\, L_1^r\,\chi_{46} \,-\, 8\,\bar A(\chi_{46})\, L_2^r\,\chi_{46} \,+\, 16\,\bar A(\chi_{46})\, L_4^r\,\chi_{46} \,+\, \bar A(\chi_{46})\bar B(\chi_p,\chi_p;0)\, [1/9\,\chi_{46} + 1/12\, R^\eta_{pp}\,\chi_p \\
& + 1/36\, R^\eta_{pp}\,\chi_4 + 1/9\, R^\eta_{p4}\,\chi_6] \,+\, \bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0)\, [-1/18\, R^\eta_{pp}\,\chi_4 - 1/9\, R^\eta_{p4}\,\chi_6 + 1/9\, R^\eta_{q4}\,\chi_6 + 1/18\, R^\eta_{13}\,\chi_4] \\
& - 1/6\,\bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0,k)\, [R^\eta_{pp} - R^\eta_{13}] \,+\, 1/9\,\bar A(\chi_{46})\bar B(\chi_\eta,\chi_\eta;0)\, R^v_{\eta 13}\,\chi_{46} \,-\, \bar A(\chi_{46})\bar B(\chi_1,\chi_3;0)\, [2/9\,\chi_{46} \\
& + 1/9\, R^\eta_{p4}\,\chi_6 + 1/18\, R^\eta_{13}\,\chi_4] \,-\, 1/6\,\bar A(\chi_{46})\bar B(\chi_1,\chi_3;0,k)\, R^\eta_{13} \,+\, 1/2\,\bar A(\chi_{46};\varepsilon)\,\pi_{16}\,\chi_{46} \\
& + \bar B(\chi_p,\chi_p;0)\,\pi_{16}\, [1/16\,\bar\chi_1\, R^d_p + 1/96\, R^d_p\,\chi_p + 1/32\, R^d_p\,\chi_q] \,+\, 2/3\,\bar B(\chi_p,\chi_p;0)\, L_0^r\, R^d_p\,\chi_p \\
& + 5/3\,\bar B(\chi_p,\chi_p;0)\, L_3^r\, R^d_p\,\chi_p \,+\, \bar B(\chi_p,\chi_p;0)\, L_4^r\, [-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0}\chi_p - 4\,\bar\chi_1 R^p_{q\eta}\,\chi_p + 4\,\bar\chi_1 R^c_p\,\chi_p + 3\,\bar\chi_1 R^d_p] \\
& + \bar B(\chi_p,\chi_p;0)\, L_5^r\, [-2/3\bar\chi^{pp}_{\eta\eta 1}\chi_p - 4/3\, R^p_{q\eta}\,\chi_p^2 + 4/3\, R^c_p\,\chi_p^2 + 1/2\, R^d_p\,\chi_p - 1/6\, R^d_p\,\chi_q] \\
& + \bar B(\chi_p,\chi_p;0)\, L_6^r\, [4\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 1} + 8\,\bar\chi_1 R^p_{q\eta}\,\chi_p - 8\,\bar\chi_1 R^c_p\,\chi_p] \,+\, 4\,\bar B(\chi_p,\chi_p;0)\, L_7^r\,(R^d_p)^2 \\
& + \bar B(\chi_p,\chi_p;0)\, L_8^r\, [4/3\,\bar\chi^{pp}_{\eta\eta 2} + 8/3\, R^p_{q\eta}\,\chi_p^2 - 8/3\, R^c_p\,\chi_p^2] \,+\, \bar B(\chi_p,\chi_p;0)^2\, [-1/18\, R^p_{q\eta} R^d_p\,\chi_p + 1/18\, R^c_p R^d_p\,\chi_p \\
& + 1/288\,(R^d_p)^2] \,+\, 1/18\,\bar B(\chi_p,\chi_p;0)\bar B(\chi_p,\chi_\eta;0)\, [R^\eta_{pp} R^d_p\,\chi_p - R^\eta_{13} R^d_p\,\chi_p]
\end{aligned}
$$

plus several more pages

Why so long expressions

- Many different quark and meson masses (χ_{ij})
- Charged propagators: $-i\, G^c_{ij}(k) = \frac{\epsilon_j}{k^2-\chi_{ij}+i\varepsilon} \quad (i \neq j)$
- Neutral propagators: $G^n_{ij}(k) = G^c_{ij}(k)\,\delta_{ij} - \frac{1}{n_{\mathrm{sea}}}\, G^q_{ij}(k)$

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$$-i\,G^q_{ii}(k) = \frac{R^d_i}{(k^2-\chi_i+i\varepsilon)^2} + \frac{R^c_i}{k^2-\chi_i+i\varepsilon} + \frac{R^\pi_{\eta ii}}{k^2-\chi_\pi+i\varepsilon} + \frac{R^\eta_{\pi ii}}{k^2-\chi_\eta+i\varepsilon}$$

$$R^i_{jkl} = R^z_{i456jkl}, \qquad R^d_i = R^z_{i456\pi\eta},$$

$$R^c_i = R^i_{4\pi\eta} + R^i_{5\pi\eta} + R^i_{6\pi\eta} - R^i_{\pi\eta\eta} - R^i_{\pi\pi\eta}$$

$$R^z_{ab} = \chi_a - \chi_b, \qquad R^z_{abc} = \frac{\chi_a-\chi_b}{\chi_a-\chi_c}, \qquad R^z_{abcd} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)}{\chi_a-\chi_d}$$

$$R^z_{abcdefg} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)(\chi_a-\chi_d)}{(\chi_a-\chi_e)(\chi_a-\chi_f)(\chi_a-\chi_g)}$$

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$$R^c_i = R^i_{4\pi\eta} + R^i_{5\pi\eta} + R^i_{6\pi\eta} - R^i_{\pi\eta\eta} - R^i_{\pi\pi\eta}$$

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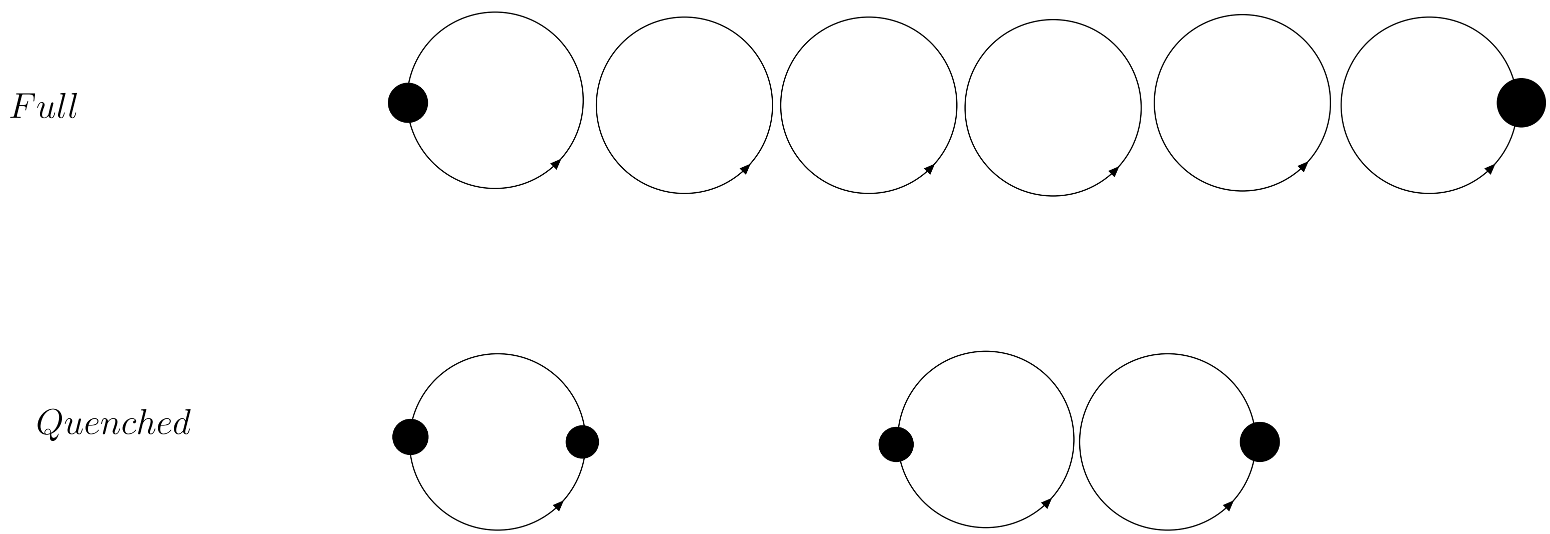
- Relations $\Longrightarrow$ order of magnitude smaller

Double poles ?



So no resummation at the quark level:

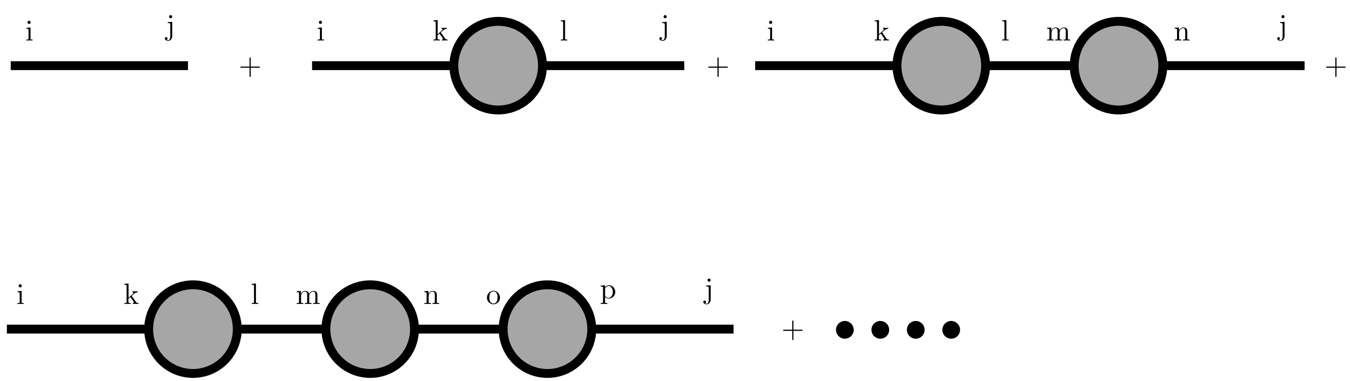
naively a double pole

Same follows from inverting the lowest order kinetic terms

Additional problem

Many neutral states: diagonalizing lowest order very difficult

Use flavour basis instead but lowest order propagator as a matrix not invertible



$$G^n_{ij} = G^0_{ij} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lj} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lm}(-i)\Sigma_{mn}G^0_{nj} + \cdots$$
$$= G^0(1 + i\Sigma G^0)^{-1}$$

Additional problem

can be done if Σ and G^0 diagonal: usual resummation

$$\frac{1}{p^2 - m^2} \to \frac{1}{p^2 - m^2 + \Sigma(p^2)}$$

so works in the charged or off-diagonal or $\overline{q}q'$ sector

masses and decay constants, all mass cases worked out for two and three flavours

PQChPT at Two Loop

Use lowest order mass squared: $\chi_i = 2B_0 m_i = m_M^{2(0)}$

Remember: $\chi_i \approx 0.3\ GeV^2 \approx (550\ MeV)^2 \sim$ border ChPT

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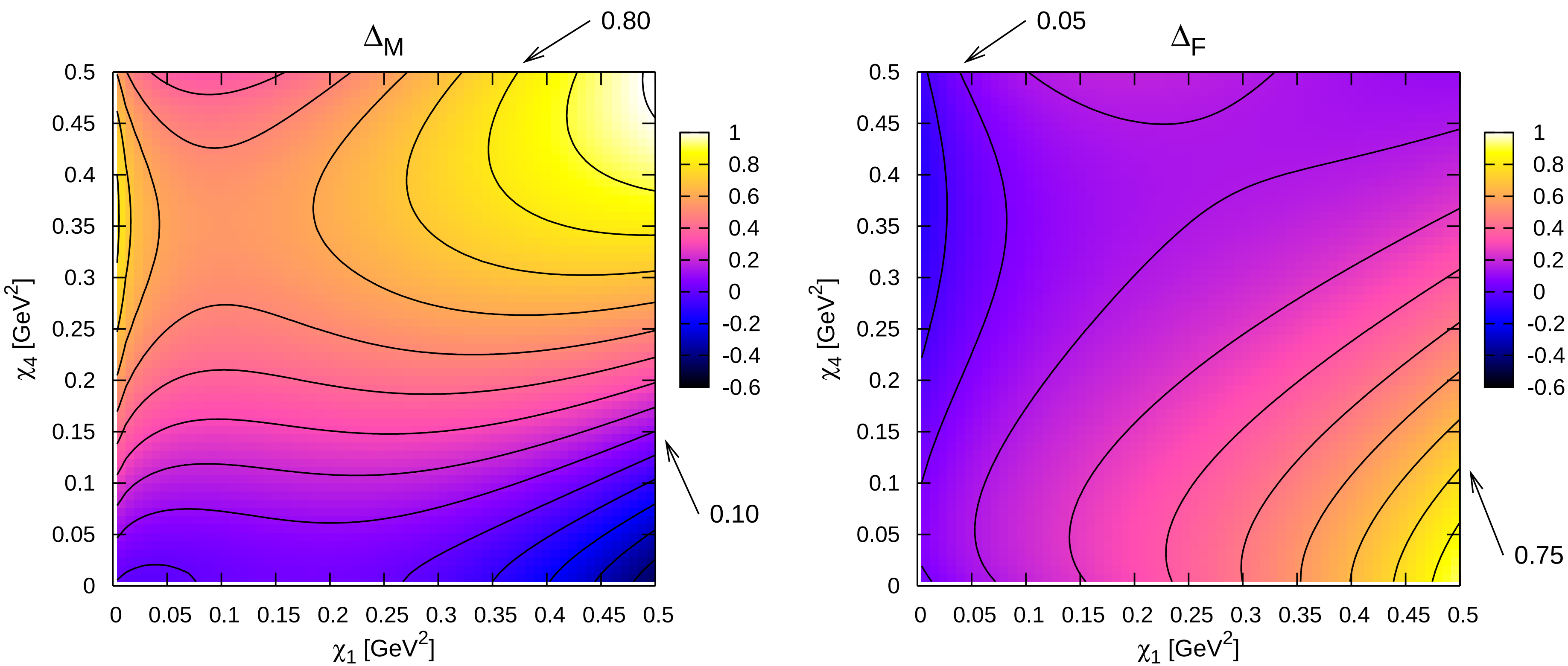

Relative corrections: mass^2 decay constant

When not diagonal ?

$$G^0 = i \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & -\alpha \end{pmatrix} - i\delta \begin{pmatrix} \tilde\alpha \\ \tilde\beta \\ \tilde\alpha \end{pmatrix} \begin{pmatrix} \tilde\alpha^T & \tilde\beta^T & \tilde\alpha^T \end{pmatrix} .$$

$$\begin{aligned}
\alpha &= \mathrm{diag}\left(\alpha_1, \ldots, \alpha_{n_\mathrm{val}}\right), \beta = \mathrm{diag}\left(\beta_1, \ldots, \beta_{n_\mathrm{sea}}\right) , \\
\tilde\alpha^T &= \left(\alpha_1, \ldots, \alpha_{n_\mathrm{val}}\right), \tilde\beta^T = \left(\beta_1, \ldots, \beta_{n_\mathrm{sea}}\right) , \\
\alpha_i &= 1/\left(p^2 - \chi_i\right), \beta_i = 1/\left(p^2 - \chi_{n_\mathrm{val}+i}\right) , \\
\delta &= \frac{1}{\sum_{j=1,n_\mathrm{sea}} \beta_j}, .
\end{aligned}$$

When not diagonal ?

$$\Sigma = \begin{pmatrix} R+S & T^T & -R \\ T & W & -T \\ -R & -T^T & R-S \end{pmatrix} . \quad G = \sum_{n=0,\infty} \left(G^0(-i)\Sigma\right)^n G^0 .$$

$$G^0(-i)\Sigma = A + B + C + D ,$$

$$A = \begin{pmatrix} \alpha S & 0 & 0 \\ 0 & \beta W & 0 \\ 0 & 0 & \alpha S \end{pmatrix} \quad B = \begin{pmatrix} \alpha R & 0 & -\alpha R \\ 0 & 0 & 0 \\ \alpha R & 0 & -\alpha R \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & \alpha T^T & 0 \\ \beta T & 0 & -\beta T \\ 0 & \alpha T^T & 0 \end{pmatrix} \quad D = \begin{pmatrix} -\delta\tilde{\alpha} \\ -\delta\tilde{\beta} \\ -\delta\tilde{\alpha} \end{pmatrix} \begin{pmatrix} \tilde{\gamma} + tilde\epsilon - \tilde{\gamma} \end{pmatrix} .$$

When not diagonal ?

Define $\bar{A} \equiv \sum_{n=0,\infty} A^n$

$$\sum_{n=0,\infty} \left(-iG^0\Sigma\right)^n = \sum_{n=1,\infty} \left((\bar{A}+\bar{A}C\bar{A})D\right)^n (\bar{A}+\bar{A}C\bar{A})$$

$$+\bar{A}+\bar{A}B\bar{A}+\bar{A}C\bar{A}+\bar{A}C\bar{A}C\bar{A}\,.$$

$$\bar{A} = \begin{pmatrix} \bar{\alpha} & 0 & 0 \\ 0 & \bar{\beta} & 0 \\ 0 & 0 & \bar{\alpha} \end{pmatrix} = \begin{pmatrix} \frac{1}{1-\alpha S} & 0 & 0 \\ 0 & \frac{1}{1-\beta W} & 0 \\ 0 & 0 & \frac{1}{1-\alpha S} \end{pmatrix}\,.$$

$$\left((\bar{A}+\bar{A}C\bar{A})D\right)^2 = \left((\bar{A}+\bar{A}C\bar{A})D\right)\left(-\delta\tilde{\beta}^T W\bar{\beta}\tilde{\beta}\right)\,.$$

When not diagonal ? Final

$$G = i\begin{pmatrix} r+s & t^T & r \\ t & w & t \\ r & t^T & r-s \end{pmatrix} - i\bar\delta \begin{pmatrix} \tilde a \\ \tilde b \\ \tilde a \end{pmatrix} \begin{pmatrix} \tilde a^T & \tilde b^T & \tilde a^T \end{pmatrix}.$$

$$\begin{aligned} s &= \bar\alpha\alpha\,, \quad w = \bar\beta\beta\,, \quad r = \bar\alpha\alpha R\bar\alpha\alpha + \bar\alpha\alpha T^T\bar\beta\beta T\bar\alpha\alpha\,, \\ t &= \bar\beta\beta T\bar\alpha\alpha\,, \quad \tilde a = \bar\alpha\tilde\alpha + \bar\alpha T^T\bar\beta\tilde\beta\,, \quad \tilde b = \bar\beta\tilde\beta\,, \quad \bar\delta = \frac{\delta}{1+\delta\tilde\beta^T W\bar\beta\tilde\beta} \end{aligned}$$

All poles and double poles get shifted consistently

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BUT valence poles are either charged or sea-quark only

neutrals $\Longrightarrow$ neutrals more difficult (L_7^r and η)

The eta mass from PQChPT

Expand around the double pole term

$$G^n_{ij} = \frac{-i\mathcal{Z}\mathcal{D}}{(p^2 - M^2_{ch})^2} + \cdots .$$

Measure on the lattice via (Sharpe-Shoresh)

$$R_0(t) \equiv \frac{\langle \pi_{ii}(t, \vec{p} = 0)\pi_{jj}(x = 0)\rangle}{\langle \pi_{ij}(t, \vec{p} = 0)\pi_{ji}(x = 0)\rangle} \quad R_0(t \to \infty) = \frac{\mathcal{D}t}{2M_{ij}} ,$$

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Conclusions

- 3 flavour ChPT at 2 loops
  - many calculations done
  - things seem to work but convergence is fairly slow
  - “kinematical” and “vector” C_i^r seem to be OK
  - L_4^r, L_6^r nonzero but reasonable for large N_c
  - $\eta \to 3\pi$: isospin breaking part needed to p^6, compare with dispersive (see $K_{\ell 4}$), status: infinities have canceled
  - iso breaking in $K_{\ell 3}$: partly programmed
- PQChPT at 2 loops: subject beginning, first phase finished