HADRONIC LIGHT-BY-LIGHT: EXTENDED NAMBU-JONA-LASINIO AND CHIRAL QUARK MODELS

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Overview

- Ximo and our $g-2$ related work
- What do we calculate and some general statements
- ENJL
- A note on MV short-distance and the quark loop
- Quark-loop: irreducible four point-functions
- Scalar exchange
- Pseudo-scalar exchange
- Axial vector exchange
- π and K loop
- Summary
- More recent and some comments

In memoriam: Joaquim Prades

Dedicated to

Ximo Prades 1963-2010

Friend and collaborator

Postdoc 93-95
with me in Copenhagen
we have worked together ever since

Ximo

We have worked together on $g-2$, $\Delta I = 1/2$, B_K, $\varepsilon'_K/\varepsilon$, Quark models and ENJL, electromagnetic effects,…and were working on rare kaon decays and $g-2$.

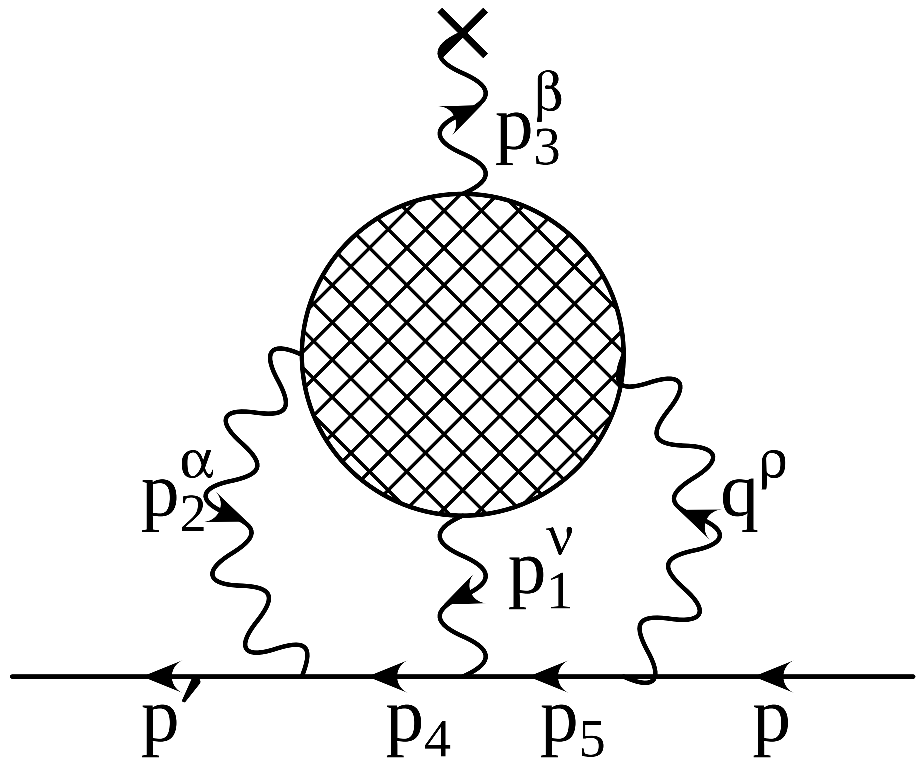
1. J. Bijnens and J. Prades, “The hadronic light-by-light contribution to the muon anomalous magnetic moment: Where do we stand?,” Mod. Phys. Lett. A **22** (2007) 767 [arXiv:hep-ph/0702170].
2. J. Bijnens, E. Gamiz, E. Lipartia and J. Prades, “QCD short-distance constraints and hadronic approximations,” JHEP **0304** (2003) 055 [arXiv:hep-ph/0304222].
3. J. Bijnens, E. Pallante and J. Prades, “Comment on the pion pole part of the light-by-light contribution to the muon g-2,” Nucl. Phys. B **626** (2002) 410 [arXiv:hep-ph/0112255].
4. J. Bijnens, E. Pallante and J. Prades, “Analysis of the Hadronic Light-by-Light Contributions to the Muon $g-2$,” Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388].
5. J. Bijnens, E. Pallante and J. Prades, “Hadronic light by light contributions to the muon g-2 in the large N(c) limit,” Phys. Rev. Lett. **75** (1995) 1447 [Erratum-ibid. **75** (1995) 3781] [arXiv:hep-ph/9505251].

Ximo

7. J. Bijnens and J. Prades, “Two and three point functions in the extended NJL model,” Z. Phys. C **64** (1994) 475 [arXiv:hep-ph/9403233].

8. J. Bijnens and J. Prades, “Anomalies, VMD and the NJL model,” Phys. Lett. B **320** (1994) 130 [arXiv:hep-ph/9310355].

Our object



- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks

A separation proposal: a start

E. de Rafael, "Hadronic contributions to the muon g-2 and low-energy QCD," Phys. Lett. **B322** (1994) 239-246. [hep-ph/9311316].

- Use ChPT p counting and large N_c
- p^4, order 1: pion-loop
- p^8, order N_c: quark-loop and heavier exchanges
- p^6, order N_c: pion exchange

Does not fully solve the problem
only short-distance quark-loop is really p^8
but it's a start

A separation proposal: a start

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- p^4, order 1: pion-loop
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- p^6, order N_c: pion exchange

- Hayakawa, Kinoshita, Sanda: meson models, pion loop using hidden local symmetry, quark-loop with VMD, calculation in Minkowski space
- JB, Pallante, Prades: This talk but try using as much as possible a consistent model-approach, calculation in Euclidean space

Papers: BPP and HKS

- JB, E. Pallante and J. Prades
  - “Comment on the pion pole part of the light-by-light contribution to the muon g-2,” Nucl. Phys. B **626** (2002) 410 [arXiv:hep-ph/0112255].
  - “Analysis of the Hadronic Light-by-Light Contributions to the Muon $g-2$,” Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388].
  - “Hadronic light by light contributions to the muon g-2 in the large N(c) limit,” Phys. Rev. Lett. **75** (1995) 1447 [Erratum-ibid. **75** (1995) 3781] [arXiv:hep-ph/9505251].
- Hayakawa, Kinoshita, (Sanda)
  - “Pseudoscalar pole terms in the hadronic light by light scattering contribution to muon g - 2,” Phys. Rev. **D57** (1998) 465-477. [hep-ph/9708227], Erratum-ibid.D66 (2002) 019902[hep-ph/0112102].
  - “Hadronic light by light scattering contribution to muon g-2,” Phys. Rev. **D54** (1996) 3137-3153. [hep-ph/9601310].
  - “Hadronic light by light scattering effect on muon g-2,” Phys. Rev. Lett. **75** (1995) 790-793. [hep-ph/9503463].

Differences

- HK(S)
  - Purely hadronic exchanges
  - quark-loop with hadronic VMD
  - Studied dependence on everything on m_V
- BPP
  - Use the ENJL as an overall model to have a similar uncertainty on all low-energy parts
  - repair some of the worst short-comings
  - Add the short-distance quark-loop
  - Large study of cut-off dependence

Differences

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  - Large study of cut-off dependence
- Sign mistake
  - HKS: Euclidean versus Minkowski $\varepsilon^{\mu\nu\alpha\beta}$
  - BPP: notes all correct sign, program had wrong sign, probably minus sign from fermion loop not removed

The overall

$$a_\mu^{\mathrm{light-by-light}} = \frac{1}{48 m_\mu} \mathrm{tr}[(\not{p} + m_\mu) M^{\lambda\beta}(0)\,(\not{p} + m_\mu)[\gamma_\lambda, \gamma_\beta]]\,.$$

$$M^{\lambda\beta}(p_3) = |e|^6 \int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4} \frac{1}{q^2\, p_1^2\, p_2^2 (p_4^2 - m_\mu^2)\,(p_5^2 - m_\mu^2)}$$
$$\times \left[\frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right] \gamma_\alpha (\not{p}_4 + m_\mu) \gamma_\nu (\not{p}_5 + m_\mu) \gamma_\rho\,.$$

- We used: $\Pi^{\rho\nu\alpha\lambda}(p_1,p_2,p_3) = -p_{3\beta} \frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}$.
- Can calculate at $p_3 = 0$ but must take derivative
- derivative makes in quark-loop each permutation finite
- Four point function of $V_i^\mu(x) \equiv \sum_i Q_i\ [\bar{q}_i(x)\gamma^\mu q_i(x)]$

$$\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3) \equiv$$
$$i^3 \int d^4x \int d^4y \int d^4z\, e^{i(p_1\cdot x + p_2\cdot y + p_3\cdot z)} \langle 0| T\left(V_a^\rho(0) V_b^\nu(x) V_c^\alpha(y) V_d^\beta(z)\right)|0\rangle$$

General properties

$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$:

- In general 138 Lorentz structures (but only 32 contribute to $g-2$)
- Using $q_\rho \Pi^{\rho\nu\alpha\beta} = p_{1\nu}\Pi^{\rho\nu\alpha\beta} = p_{2\alpha}\Pi^{\rho\nu\alpha\beta} = p_{3\beta}\Pi^{\rho\nu\alpha\beta} = 0$
  43 gauge invariant structures
- Bose symmetry relates some of them

$\int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4}$

- 8 dimensional integral, three trivial,
- 5 remain: $p_1^2, p_2^2, p_1 \cdot p_2, p_1 \cdot p_\mu, p_2 \cdot p_\mu$
- Rotate to Euclidean space:
  - Easier separation of long and short-distance
  - Artefacts (confinement) in models smeared out.

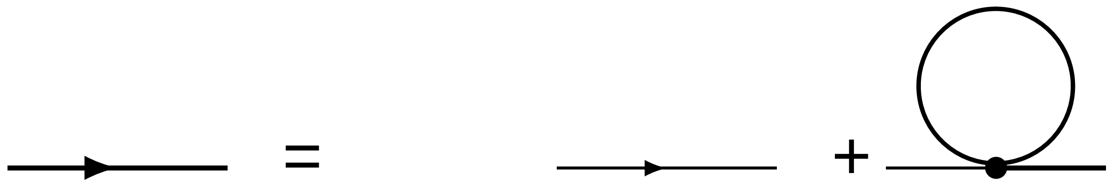
ENJL: our main model

$$\mathcal{L}_{\mathrm{ENJL}} = \overline{q}^{\alpha}\left\{i\gamma^{\mu}\left(\partial_{\mu} - iv_{\mu} - ia_{\mu}\gamma_5\right) - \left(\mathcal{M} + s - ip\gamma_5\right)\right\} q^{\alpha} + 2\, g_S\, \left(\overline{q}^{\alpha}_R q^{\beta}_L\right)\left(\overline{q}^{\beta}_L q^{\alpha}_R\right)$$
$$- g_V \left[\left(\overline{q}^{\alpha}_L \gamma^{\mu} q^{\beta}_L\right)\left(\overline{q}^{\beta}_L \gamma_{\mu} q^{\alpha}_L\right) + \left(\overline{q}^{\alpha}_R \gamma^{\mu} q^{\beta}_R\right)\left(\overline{q}^{\beta}_R \gamma_{\mu} q^{\alpha}_R\right)\right]$$

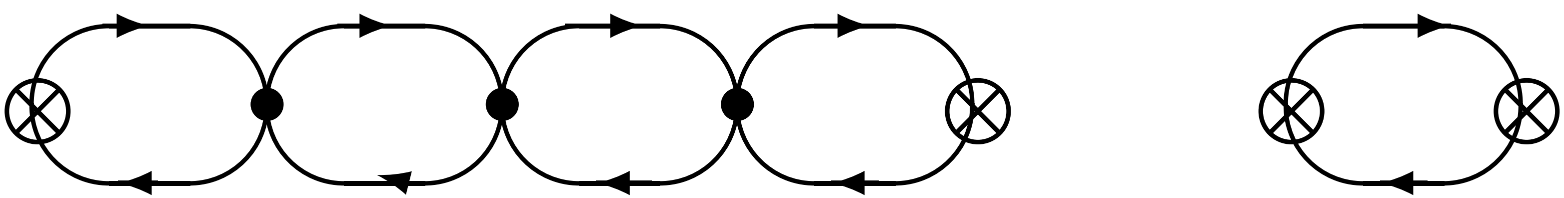
- $\overline{q} \equiv \left(\overline{u}, \overline{d}, \overline{s}\right)$
- v_μ, a_μ, s, p: external vector, axial-vector, scalar and pseudoscalar matrix sources
- $\mathcal{M}$ is the quark-mass matrix.
- $g_V \equiv \frac{8\pi^2 G_V(\Lambda)}{N_c \Lambda^2}$, $g_S \equiv \frac{4\pi^2 G_S(\Lambda)}{N_c \Lambda^2}$.
- G_V, G_S are dimensionless and valid up to Λ
- No confinement but has good pion, vector meson and OK axial vector-meson phenomenology

ENJL: our main model

- (this) ENJL JB, Bruno, de Rafael, Nucl. Phys. B390 (1993) 501 [hep-ph/9206236]; JB, Phys. Rep. 265 (1996) 369 [hep-ph/9502335] (review)
- Gap equation: chiral symmetry spontaneously broken



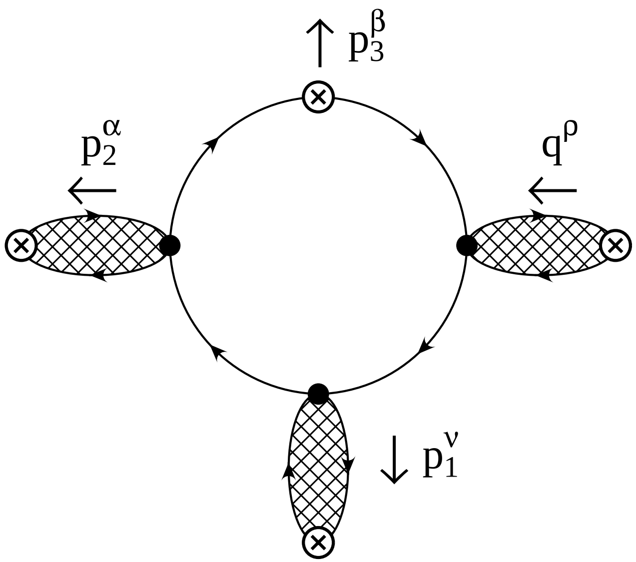
- Generates poles, i.e. mesons via bubble resummation



ENJL: our main model

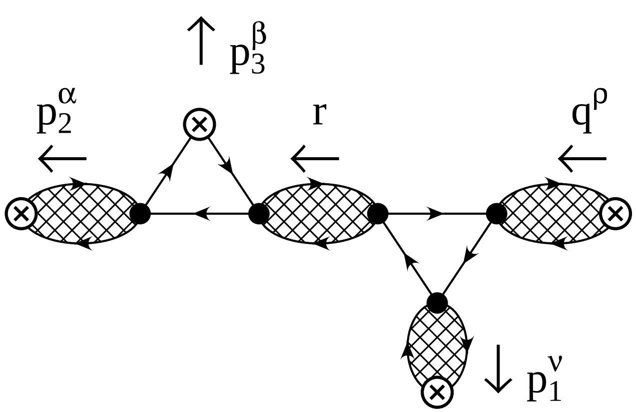
- Can be thought of as a very simple rainbow and ladder approximation in the DSE equation with constant kernels for the one-gluon exchange
- Parameters fit via F_π, L_i^r, vector meson properties,...
- $G_S = 1.216$, $G_V = 1.263$, $\Lambda = 1.16$ GeV
- has $M_Q = 263$ MeV
- Has a number of decent matchings to short-distance, e.g. $\Pi_V - \Pi_A$ but fails in others.
- Generates always VMD in external legs (but with a twist)
- Hook together general processes by one-loop vertices and bubble-chain propagators

Separation of contributions



(a)

- Quark loop with external bubble-chains
- $\approx$ Quark-loop with VMD

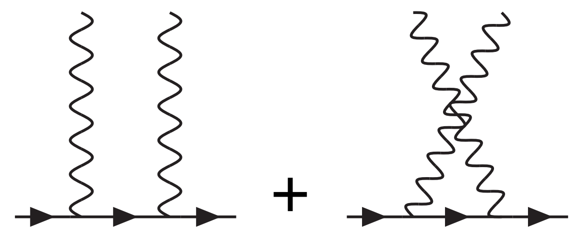


(b)

- Also internal bubble chain
- $\approx$ meson exchange
- Note that vertices have structure
- Off-shell effect in model included

MV short-distance and quark-loop

- K. Melnikov, A. Vainshtein, Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment revisited, Phys. Rev. **D70** (2004) 113006. [hep-ph/0312226]
- take $p_1^2 \approx p_2^2 \gg q^2$: Leading term in OPE of two vector currents is proportional to axial current
- These come from



- Are these part of the quark-loop? See also in Dorokhov,Broniowski, phys.Rev. D78(2008)07301
- Which momentum regimes important studied: JB and J. Prades, The hadronic light-by-light contribution to the muon anomalous magnetic moment: Where do we stand?, Mod. Phys. Lett. A **22** (2007) 767 [hep-ph/0702170]
- Ximo noticed: large N_c MV $\approx$ large N_c BPP

Pure quark loop

Cut-off Λ (GeV)	$a_\mu \times 10^7$ Electron Loop	$a_\mu \times 10^9$ Muon Loop	$a_\mu \times 10^9$ Constituent Quark Loop
0.5	2.41(8)	2.41(3)	0.395(4)
0.7	2.60(10)	3.09(7)	0.705(9)
1.0	2.59(7)	3.76(9)	1.10(2)
2.0	2.60(6)	4.54(9)	1.81(5)
4.0	2.75(9)	4.60(11)	2.27(7)
8.0	2.57(6)	4.84(13)	2.58(7)
Known Results	2.6252(4)	4.65	2.37(16)

M_Q : 300 MeV

now all known analytically

Us: 5+(3-1) integrals

extra are Feynman parameters

Slow convergence:

- electron: all at 500 MeV
- Muon: only half at 500 MeV, at 1 GeV still 20% missing
- 300 MeV quark: at 2 GeV still 25% missing

ENJL quark-loop

$$\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3) = \overline{\Pi}(p_1,p_2,p_3)\,\mathcal{V}^{\rho\nu\alpha\beta}(p_1,p_2,p_3)$$

$$\mathcal{V}^{abcd\rho\nu\alpha\beta}(p_1,p_2,p_3) = \left(\frac{g^{a\rho}M_V^2(-q^2)-q^a q^\rho}{M_V^2(-q^2)-q^2}\right)\left(\frac{g^{b\nu}M_V^2(-p_1^2)-p_1^b p_1^\nu}{M_V^2(-p_1^2)-p_1^2}\right) \times\left(\frac{g^{c\alpha}M_V^2(-p_2^2)-p_2^c p_2^\alpha}{M_V^2(-p_2^2)-p_2^2}\right)\left(\frac{g^{d\beta}M_V^2(-p_3^2)-p_3^d p_3^\beta}{M_V^2(-p_3^2)-p_3^2}\right)$$

- Barred = one-loop but need to add all permutations
- the extra terms with $q^\alpha q^\rho,\ldots$ vanish because of one-loop gauge invariance
- remainder amounts to $\times\dfrac{M_V(q^2)}{M_V(q^2)-q^2}$ on each leg

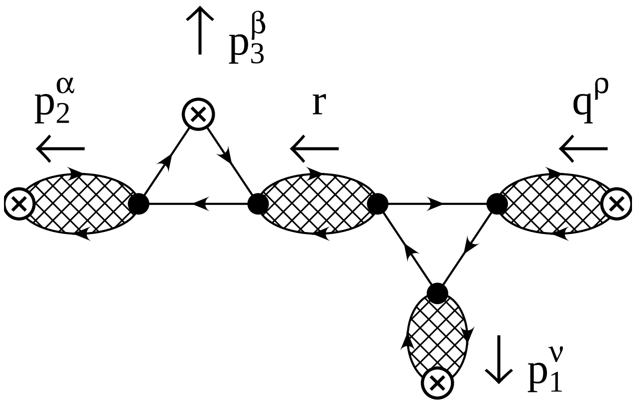
ENJL quark-loop

Cut-off Λ GeV	$a_\mu \times 10^{10}$ Quark-loop VMD	$a_\mu \times 10^{10}$ Quark-loop ENJL	$a_\mu \times 10^{10}$ Quark-loop masscut	$a_\mu \times 10^{10}$ sum ENL+masscut
0.5	0.48	0.78	2.46	3.2
0.7	0.72	1.14	1.13	2.3
1.0	0.87	1.44	0.59	2.0
2.0	0.98	1.78	0.13	1.9
4.0	0.98	1.98	0.03	2.0
8.0	0.98	2.00	.005	2.0

Very stable

- ENJL cuts off slower than pure VMD
- masscut: $M_Q = \Lambda$ to have short-distance and no problem with momentum regions
- Quite stable in region 1-4 GeV

ENJL: scalar



$$\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3) = \overline{\Pi}^{VVS}_{ab}(p_1,r) g_S \left(1+g_S\Pi^S(r)\right)\overline{\Pi}^{SVV}_{cd}(p_2,p_3)\mathcal{V}^{abcd\rho\nu\alpha\beta}(p_1,p_2,p_3)$$
$$+\text{permutations}$$

$$g_S\left(1+g_S\Pi_S\right) = \frac{g_A(q^2)(2M_Q)^2}{2f^2(q^2)}\frac{1}{M_S^2(q^2)-q^2}$$

$\mathcal{V}^{abcd\rho\nu\alpha\beta}$ was ENJL VMD legs

In ENJL only scalar+quark-loop properly chiral invariant

ENJL: scalar

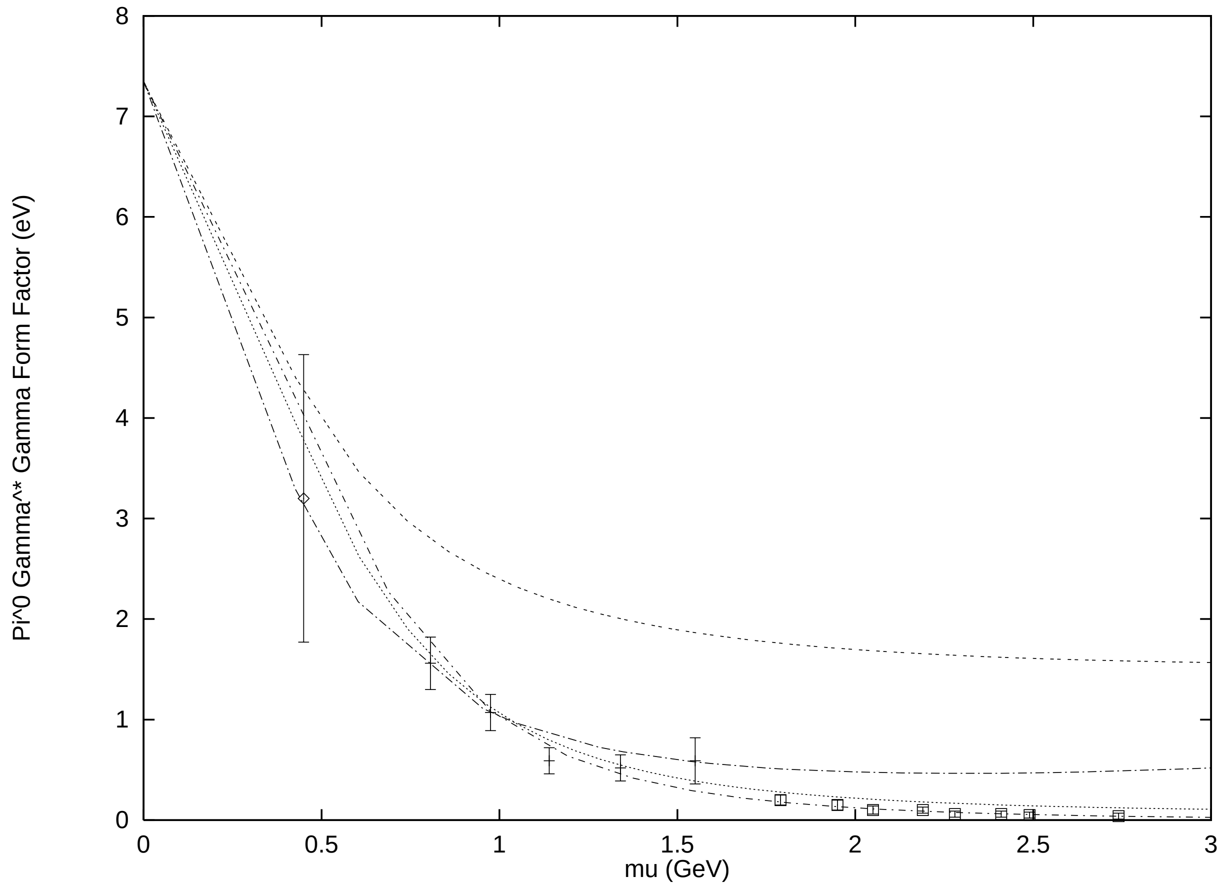
Cut-off Λ GeV	$a_\mu \times 10^{10}$ Quark-loop VMD	$a_\mu \times 10^{10}$ Quark-loop ENJL	$a_\mu \times 10^{10}$ Scalar Exchange
0.5	0.48	0.78	$-$0.22
0.7	0.72	1.14	$-$0.46
1.0	0.87	1.44	$-$0.60
2.0	0.98	1.78	$-$0.68
4.0	0.98	1.98	$-$0.68
8.0	0.98	2.00	$-$0.68

- Note: ENJL+scalar similar to Quark-loop VMD
- $M_S \approx 620$ MeV so certainly an overestimate for real scalars
- Is why ENJL has $\pi\pi$-scattering very well
- If scalar is σ: related to pion loop part?

ENJL: pseudo-scalar

- ENJL needs added pieces to get anomaly correct
- Many quark models with cut-off need this: JB, Prades, 1993
- Pure ENJL does not reproduce CLEO $P\gamma\gamma^*$ data
  - Tamper with the ENJL VMD factor (transverse VMD)
- Pure ENJL is large N_c: 9 Goldstone bosons:
  3 neutral: π^0, $\pi_1 \sim \bar{u}u + \bar{d}d$, $\pi_2 \sim \bar{s}s$.
  - Our solutions: take ENJL for π^0 and keep ratio π^0, η and η' using double VMD+pointlike propagator
  - All large N_c models face this problem

Fitting CLEO data



The $\pi^0\gamma^*\gamma$ form factor. At high energies the curves from top to bottom are: ENJL, ENJL-VMD, Point-like-VMD and Transverse-VMD. Pointlike is a straight line at the top

ENJL: pseudo-scalar

$$\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3) = \Big[\overline{\Pi}^{VVP}(p_1,r)\left(1+g_S\Pi^P(r)\right)\overline{\Pi}^{PVV}(p_2,p_3)$$

$$-g_V\overline{\Pi}^{VVP}(p_1,r)\Pi^{P\mu}(r)\overline{\Pi}^{AVV}_\mu(p_2,p_3) - g_V\ \overline{\Pi}^{VVA}_\mu(p_1,r)\Pi^{P\mu}(-r)\overline{\Pi}^{PVV}(p_2,p_3)\Big]$$

$$\times\quad g_S\mathcal{V}^{\rho\nu\alpha\beta}(p_1,p_2,p_3)+\cdots$$

BLUE: extra anomaly pieces

$$\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3) =$$

$$g_S\overline{\Pi}^{VVP}(p_1,r)\left(1+g_S\Pi^P(r)-4g_VM_i\Pi^P_M(r^2)\right)\overline{\Pi}^{PVV}(p_2,p_3)\,\mathcal{V}^{\rho\nu\alpha\beta}(p_1,p_2,p_3)$$

$$+2M_Qg_Sg_V\Pi^P_M(r^2)\left[\overline{\Pi}^{VVP}(p_1,r)\left\{\overline{\Pi}^{PVV}(p_2,p_3)\,|^{M_i=M_Q}_{p_2^2=p_3^2=r^2=0}\right\}\right.$$

$$\times\left(\frac{g^{a\rho}M_V^2(-q^2)-q^aq^\rho}{M_V^2(-q^2)-q^2}\right)\left(\frac{g^{b\nu}M_V^2(-p_1^2)-p_1^cp_1^\nu}{M_V^2(-p_1^2)-p_1^2}\right)g^{c\alpha}g^{d\beta}$$

$$+\left\{\overline{\Pi}^{VVP}(p_1,r)\,|^{M_i=M_Q}_{p_1^2=r^2=q^2=0}\right\}\overline{\Pi}^{PVV}(p_2,p_3)$$

$$\left.\times\ g^{a\rho}g^{b\nu}\left(\frac{g^{c\alpha}M_V^2(-p_2^2)-p_2^cp_2^\alpha}{M_V^2(-p_2^2)-p_2^2}\right)\left(\frac{g^{d\beta}M_V^2(-p_3^2)-p_3^dp_3^\beta}{M_V^2(-p_3^2)-p_3^2}\right)\right]+\cdots,$$

π^0 exchange

Cut-off μ (GeV)	$a_\mu \times 10^{10}$ Point-like	$a_\mu \times 10^{10}$ ENJL–VMD	$a_\mu \times 10^{10}$ Point-Like-VMD	$a_\mu \times 10^{10}$ Transverse-VMD	$a_\mu \times 10^{10}$ Transverse-VMD
0.5	4.92(2)	3.29(2)	3.46(2)	3.60(3)	3.53(2)
0.7	7.68(4)	4.24(4)	4.49(3)	4.73(4)	4.57(4)
1.0	11.15(7)	4.90(5)	5.18(3)	5.61(6)	5.29(5)
2.0	21.3(2)	5.63(8)	5.62(5)	6.39(9)	5.89(8)
4.0	32.7(5)	6.22(17)	5.58(5)	6.59(16)	6.02(10)

π^0 exchange contributions with various parametrizations of the $\pi^0\gamma^*\gamma^*$ vertex that fit the data for the $\pi^0\gamma\gamma^*$ vertex.
All in reasonable agreement

Pseudoscalar exchange

Cut-off μ (GeV)	$a_\mu \times 10^{10}$ ENJL	$a_\mu \times 10^{10}$ Point-Like–VMD (π^0)	$a_\mu \times 10^{10}$ η	$a_\mu \times 10^{10}$ η'
0.4	2.84(2)	2.70(1)	0.425(1)	0.266(1)
0.5	3.70(3)	3.46(2)	0.616(2)	0.399(2)
0.7	5.04(4)	4.49(3)	0.923(3)	0.631(2)
1.0	6.44(7)	5.18(3)	1.180(4)	0.847(3)
2.0	8.83(17)	5.62(5)	1.37(1)	1.03(1)
4.0	10.51(37)	5.58(5)	1.38(1)	1.04(1)

The pseudoscalar exchange contribution to a_μ for the ENJL (large N_c) and the point-like Wess-Zumino vertex, damped with two vector propagators for the π^0, η and η'.

Axial-vector exchange exchange

Cut-off Λ (GeV)	$a_\mu \times 10^{10}$ from Axial-Vector Exchange $\mathcal{O}(N_c)$
0.5	0.05(0.01)
0.7	0.07(0.01)
1.0	0.13(0.01)
2.0	0.24(0.02)
4.0	0.59(0.07)

There is some pseudo-scalar exchange piece here as well, off-shell not quite clear what is what.

π and K-loop

- The $\pi\pi\gamma^*$ vertex is alwys done using VMD
- $\pi\pi\gamma^*\gamma^*$ vertex two choices:
  - Hidden local symmetry model (only one γ has VMD
  - Full VMD
  - Both are chirally symmetric
  - Check if they live up to MV short distance (Full VMD does, HLS not checked yet)
  - The HLS model used has problems with π^+-π^0 mass difference (due not having an a_1)
- Final numbers quite different: -0.045 and -0.19
- For us stopped at 1 GeV but within 10% of higher Λ

π and K-loop

Cut-off GeV	$10^{10}a_\mu$ π bare	$10^{10}a_\mu$ π VMD	$10^{10}a_\mu$ π ENJL	$10^{10}a_\mu$ K ENJL
0.5	$-$1.71(7)	$-$1.16(3)	$-$1.20(0.03)	$-$0.020(0.001)
0.6	$-$2.03(8)	$-$1.41(4)	$-$1.42(0.03)	$-$0.026(0.001)
0.7	$-$2.41(9)	$-$1.46(4)	$-$1.56(0.03)	$-$0.034(0.001)
0.8	$-$2.64(9)	$-$1.57(6)	$-$1.67(0.04)	$-$0.042(0.001)
1.0	$-$2.97(12)	$-$1.59(15)	$-$1.81(0.05)	$-$0.048(0.002)
2.0	$-$3.82(18)	$-$1.70(7)	$-$2.16(0.06)	$-$0.087(0.005)
4.0	$-$4.12(18)	$-$1.66(6)	$-$2.18(0.07)	$-$0.099(0.005)

- We ran HLS but those data I don’t find anymore
- note the suppression by the propagators

Summary: ENJL

$$
\begin{aligned}
a_\mu^{\mathrm{LbL}} &= (2.1 \pm 0.3) \cdot 10^{-10}[\text{quark-loop}] \\
&+ (-0.68 \pm 0.2) \cdot 10^{-10}[\text{scalar}] \\
&+ (8.5 \pm 1.3) \cdot 10^{-10}[\text{pseudoscalar}] \\
&+ (0.25 \pm 0.1) \cdot 10^{-10}[\text{axial-vector}] \\
&+ (-1.9 \pm 1.3) \cdot 10^{-10}[\pi K\text{-loop}] \\
&= (8.3 \pm 3.2) \cdot 10^{-10}.
\end{aligned}
$$

Since then

- Constraints from experiment: J. Bijnens and F. Persson, hep-ph/hep-ph/0106130 Studying three formfactors $P\gamma^*\gamma^*$ in $P \to \ell^+\ell^-\ell'^+\ell'^-$, $e^+e^- \to e^+e^-P$ exact tree level and for $g-2$ (but beware sign):
  - Conclusion: possible but VERY difficult
  - Two γ^* off-shell not so important for our choice of form-factor
- Sign mistake found and a more analytical evaluation of π^0 exchange M. Knecht, A. Nyffeler, Hadronic light by light corrections to the muon g-2: The Pion pole contribution, Phys. Rev. **D65**(2002)073034, [hep-ph/0111058]
- Leading chiral logarithm: M. Knecht *et al.*, Hadronic light by light scattering contribution to the muon g-2: An Effective field theory approach, Phys. Rev. Lett. **88**(2002)071802[hep-ph/0111059]; Ramsey-Musolf, M. Wise, Hadronic light by light contribution to muon g-2 in ChPT,Phys.Rev.Lett.**89**(2002)041601[hep-ph/0201297]

Since then

- More short-distance constraints: see later talks by Vainshtein and Nyffeler
- Chiral nonlocal quark-model (like nonlocal ENJL):
  - A. E. Dorokhov, W. Broniowski, Pion pole light-by-light contribution to g-2 of the muon in a nonlocal chiral quark model, Phys. Rev. **D78** (2008) 073011. [arXiv:0805.0760 [hep-ph]]

$$\mathcal{L} = \overline{q}^{\alpha}(x)\left\{i\gamma^{\mu}\left(\partial_{\mu} - iv_{\mu} - ia_{\mu}\gamma_5\right) - \left(\mathcal{M} + s - ip\gamma_5\right)\right\}q^{\alpha}(x)$$

$$+\frac{1}{2}G_P\left(\Pi_{i=1,4}\int d^4x_i f(x_i)\right)\left(\overline{q}^{\alpha}(x-x_1)\Gamma q^{\alpha}(x+x_3)\right)\left(\overline{q}^{\beta}(x-x_2)\Gamma q^{\beta}(x+x_4)\right)$$

  - $\Gamma\otimes\Gamma = 1\otimes 1 - \gamma_5\tau^a\otimes\gamma_5\tau^a$ and $f(p) = e^{-p^2/\Lambda^2}$
  - Path ordered external fields part not written
  - $a_{\mu}^{\pi^0} = 6.27\times 10^{-10}$, very much like π^0 exchange with double VMD

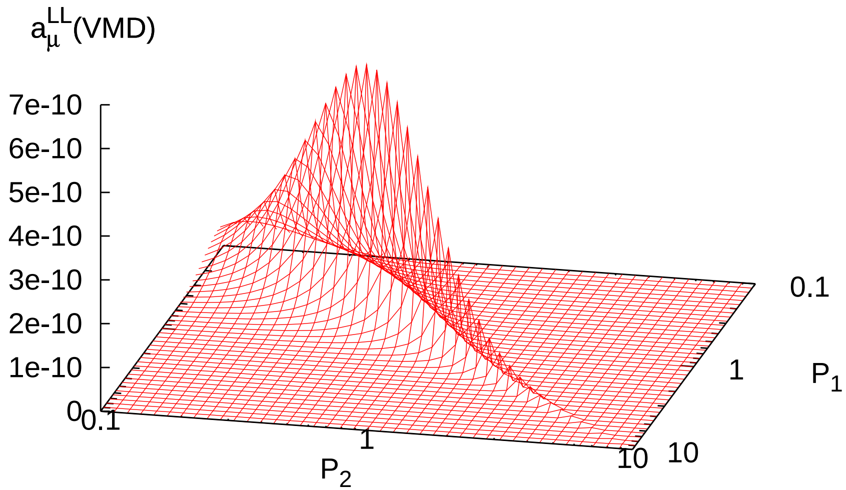
Since then

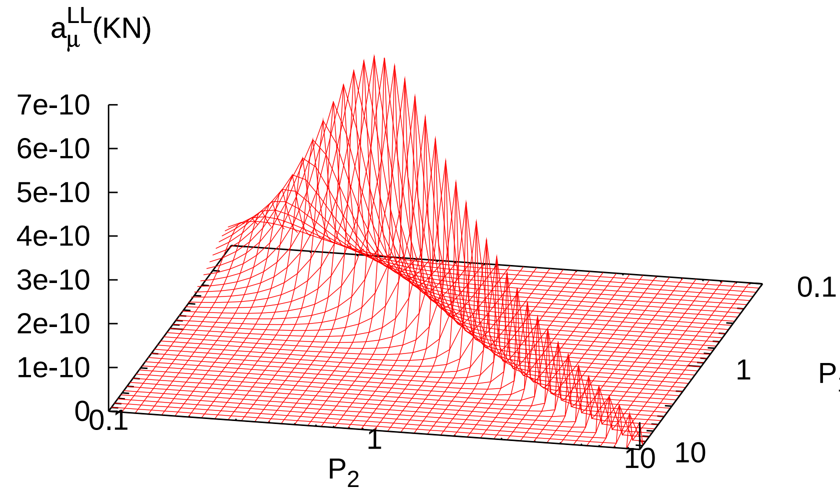
- The ENJL model can certainly be improved: Nonlocal model, so far only π^0-exchange DSE approach, first results see talk by Williams, published only π^0 exchange. Both only $SU(2)$ as well so far.
- Variations on hadronic models: AdS/QCD: talk by Cata
- Ximo and I were working on getting a more consistent matching between the different regimes

Since then

For that we need to look at more figures like:

$a_\mu = \int dl_1 dl_2 a_\mu^{LL}$ with $l_i = \log(P_i/GeV)$





Checking which momentum regions do what (but would need three dimensional)

Some funny comments to end with

- Need a new overall evaluation with consistent approach.
- More resonances models should be tried, AdS/QCD is one approach, RχT (Valencia *et al.*) possible,. . .
- But note that short-distance matching here must be done in many channels and there are theorems JB,Gamiz,Lipartia,Prades that with only a few resonances this requires compromises
- Pion loop needs a bit more study, why is HLS smaller than double VMD. HLS model with ρ and a_1?
- More general theorems on the signs of contributions like scalar negative, pseudo-scalar positive?
- Cancellations between higher mass chiral partners?
- Can (hard pion) ChPT help (also for lattice)?