SOME (THEORY) CHALLENGES FOR PANDA

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Overview

Challenge: provide good predictions for: panda

Overview

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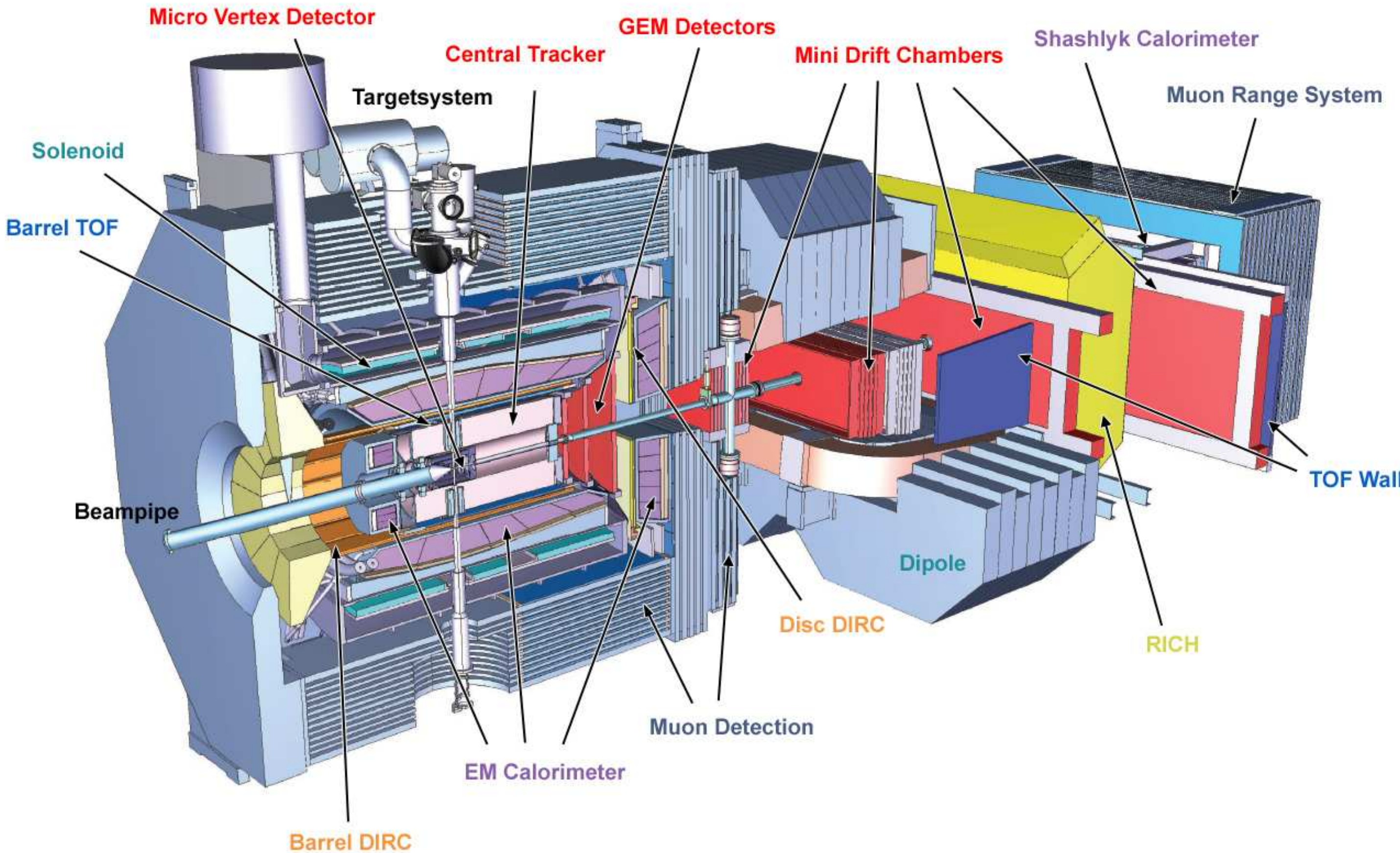
Or what can this here do?



PANDA physics program

- **Hadron Spectroscopy:** Search for exotic particles and measurement of hadron properties
- **Hadrons in Matter:** Study in-medium effects of hadronic particles
- **Nucleon Structure:** Generalized parton distribution, Drell-Yan processes and time-like form factor of the proton
- **Hypernuclei:** Measurement of nuclear properties with an additional strangeness degree of freedom

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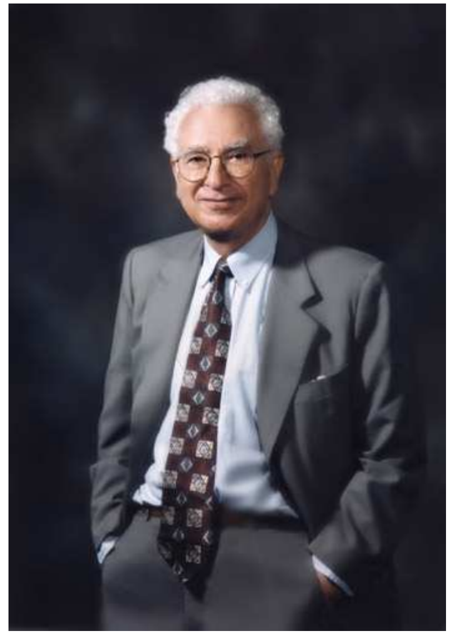
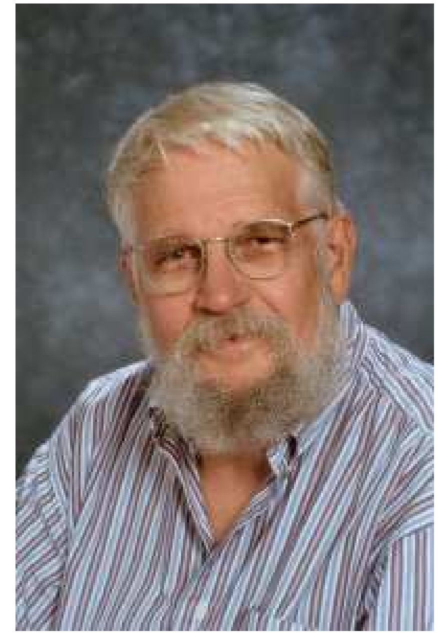
All strong interaction $\Longrightarrow$ Theory known: QCD

Quantum Chromodynamics

Harald Fritzsch



Murray Gell-Mann



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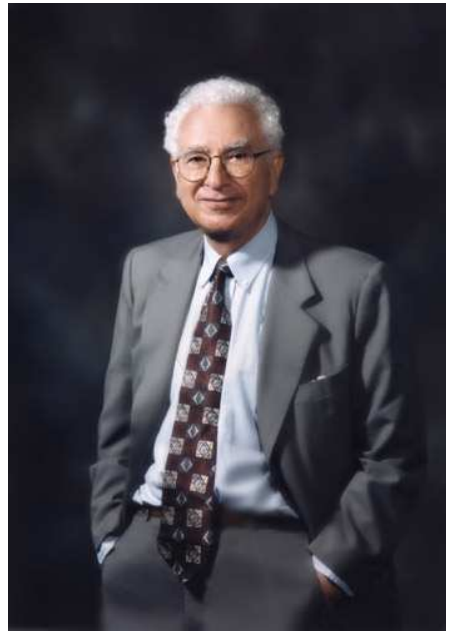

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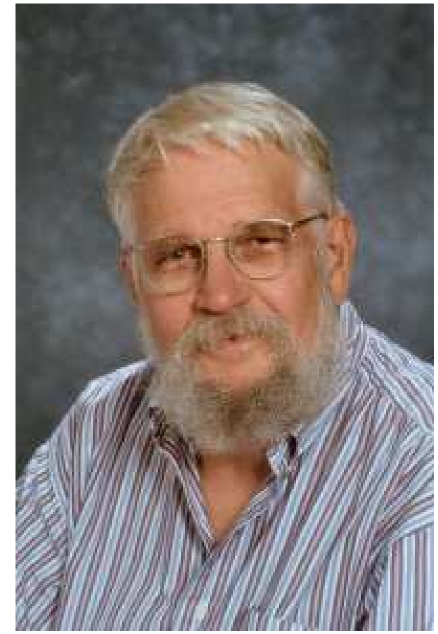

Heiri Leutwyler

$$\boxed{-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \sum_q \bar{q} i \gamma_\mu \left(\partial^\mu - i\frac{g_S}{2}\lambda^a G^{a\mu}\right) q - m_q \bar{q} q}$$

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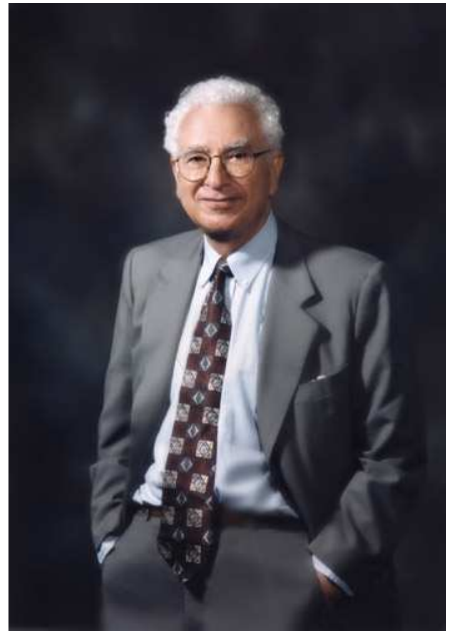

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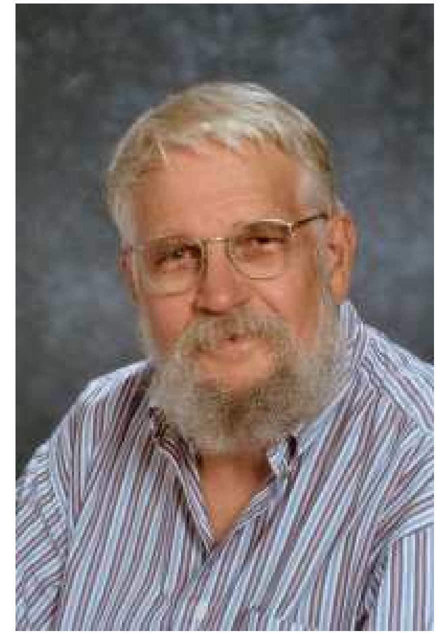

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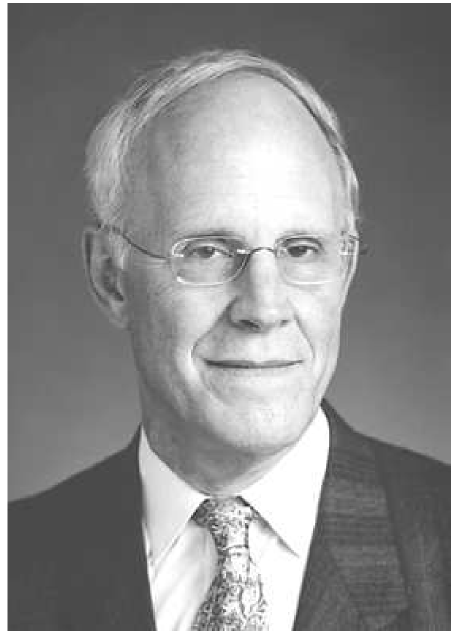
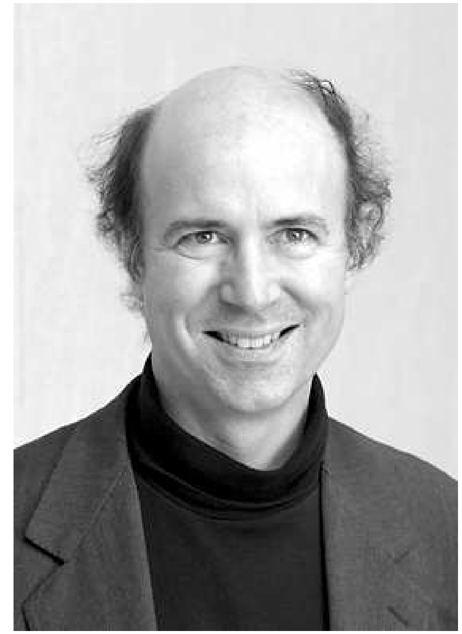
Looks rather simple: why is this so difficult?

Quantum Chromodynamics

David Politzer



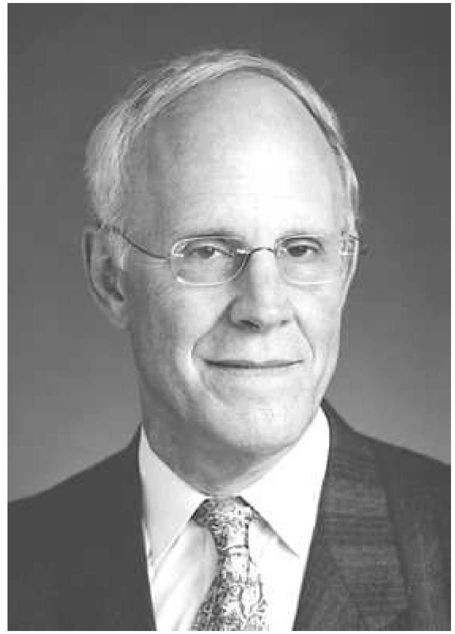
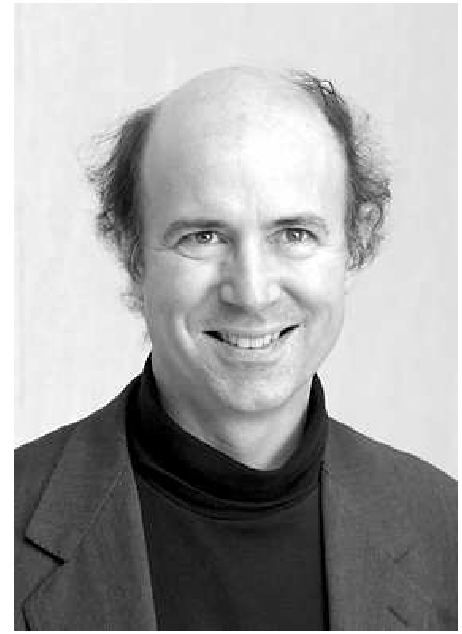
David Gross



Frank Wilczek

Nobel Prize 2004: "for the discovery of asymptotic freedom in the theory of the strong interaction"

Quantum Chromodynamics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Nobel Prize 2004: "for the discovery of asymptotic freedom in the theory of the strong interaction"

Easy or perturbative at short distances
Difficult and nonperturbative at long distances

$$\alpha_S \sim \frac{1}{\log(q^2)}$$

PANDA

PANDA: detector for the nonperturbative strong interaction

- Hypernuclei: see talk by E. Epelbaum
- Hadrons in Matter: similar but not discussed here

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Methods to use:

- Perturbative QCD
- Lattice QCD (thanks to Thomas DeGrand)
- Effective Field Theory
  - Introduction
  - NRQCD and friends (many thanks to Antonio Vairo)
  - Hard Pion Chiral Perturbation Theory
- Models, QCD sum rules, ...

Perturbative QCD

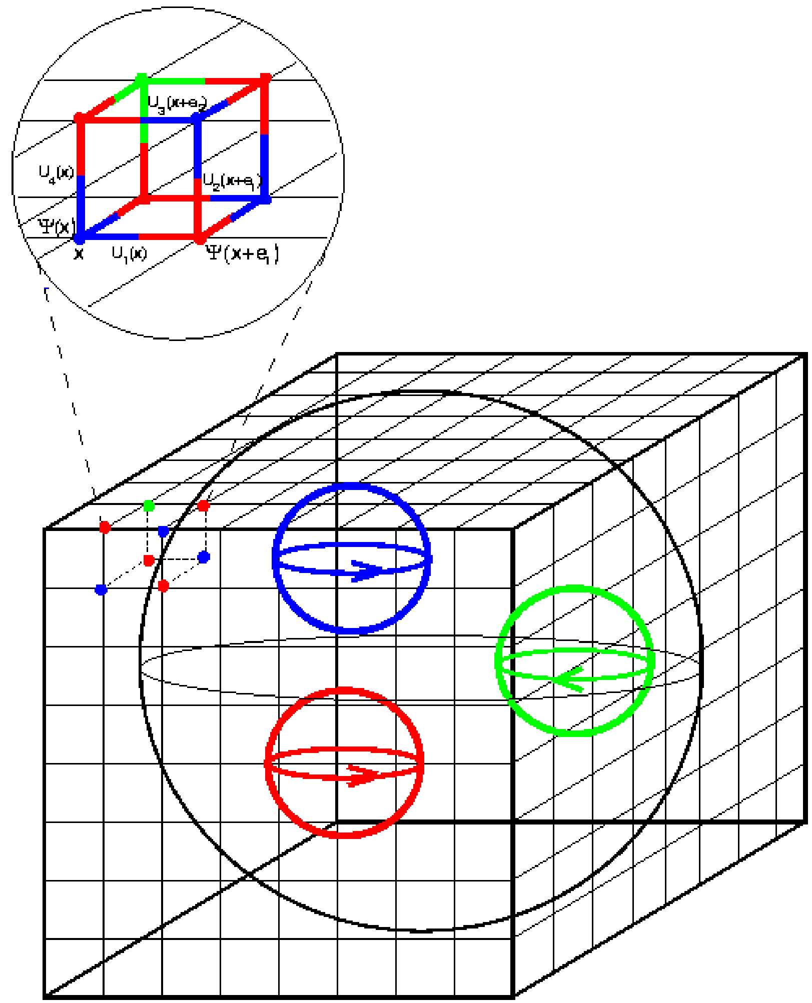
- In PANDA physics essentially only in first step
- Hard part of a process which than leads to various parton distributions
- Input into lattice QCD and/or Effective Field Theory

Lattice QCD

- Brute force: do full functional integral numerically
- discretize space-time
- quarks and gluons: $8 \times 2 + 3 \times 4$ d.o.f. per point (naively)
- Do the resulting (very high dimensional) integral numerically
- Large field with many successes

Lattice QCD

- evaluate Feynman path integral numerically
  $\int [dG][dq][d\overline{q}]e^{i\int d^4x\mathcal{L}_{QCD}}$
- Discretize space-time
- Rotate to Euclidean
  $t \longrightarrow i\tau$
- do integral with Monte Carlo



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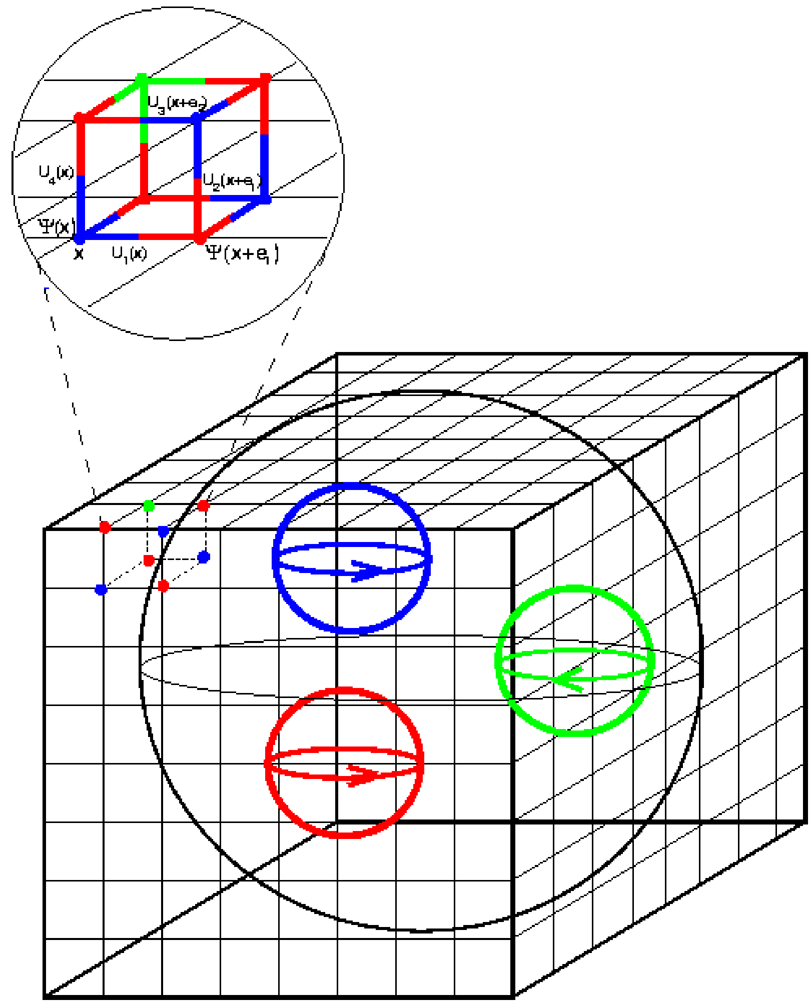
- Direct from QCD
- Precision can be improved systematically



Lattice QCD

- Rotation to Euclidean
- Lattice results need to be continued to Minkowski (in general)
- Static properties: can be calculated directly
- Dynamic properties: tend to be difficult
- Need to extrapolate to small enough (light) quark masses

Lattice QCD

- Calculate correlators:
$$\langle O_1(x_1)\ldots O_n(x_n)\rangle = \int [dG][dq][d\overline{q}]O_1(x_1)\ldots O_n(x_n)e^{-S_{QCD}}$$
- x_i are in Euclidean

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- Two-point function $\langle O_1(\tau_1)O_2(0)\rangle = \sum_n g_n e^{-E_n\tau}$

  E_n all states that couple to the O_i
- Lowest state easy
- Excited states difficult

Lattice QCD

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- Lowest state easy
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- Maiani-Testa theorem: always lowest order state: decays also difficult $\Longrightarrow$
  $\phi\pi\pi$ coupling would be with $E_{\pi\pi} = 2m_\pi$
- Interactions: indirectly via finite volume dependence

Lattice QCD

- Light Dynamical fermions: $\int [dq][d\overline{q}] \longrightarrow det \not{D}$
- Global quantity
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- Larger hadrons: need bigger lattice

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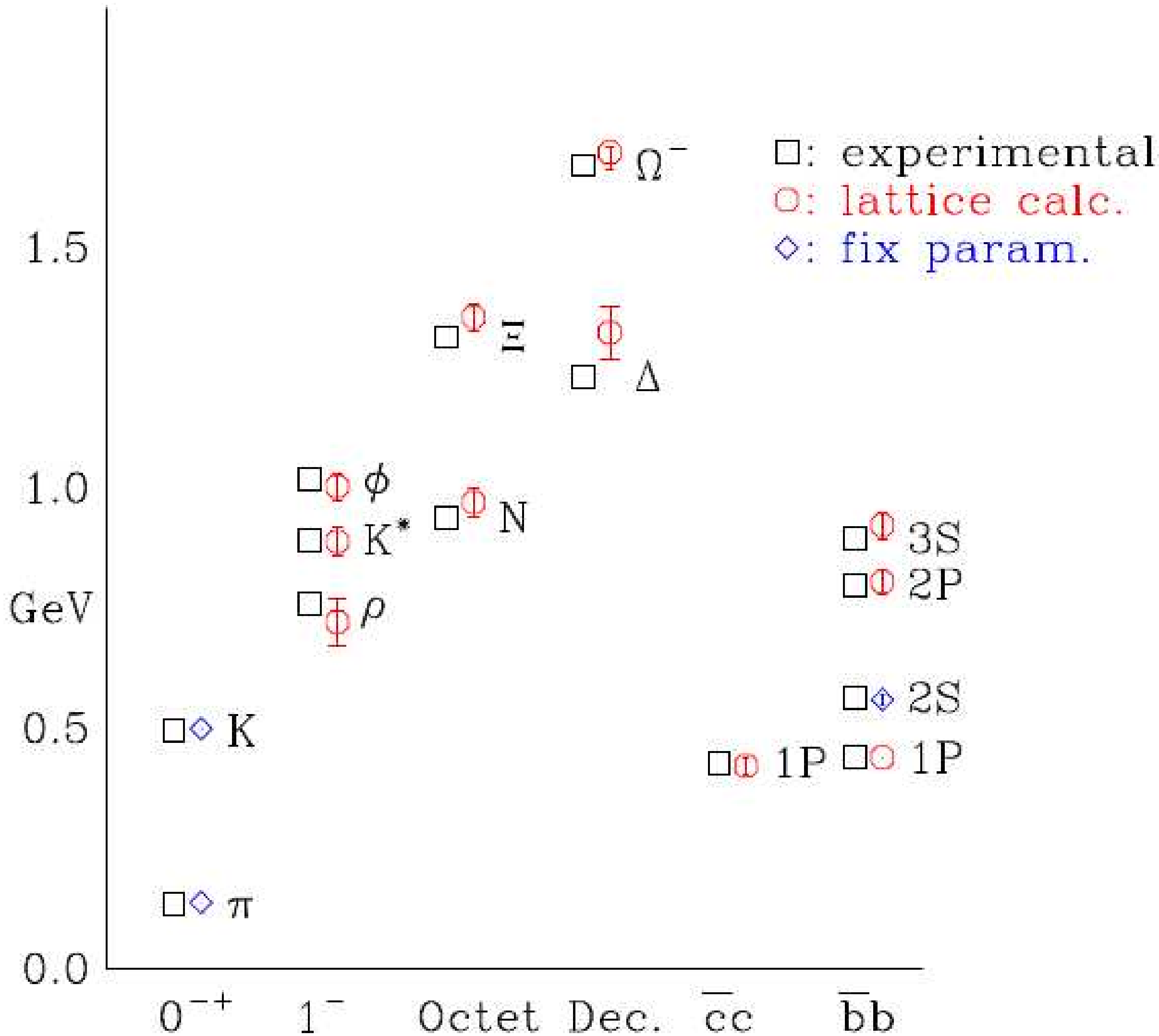
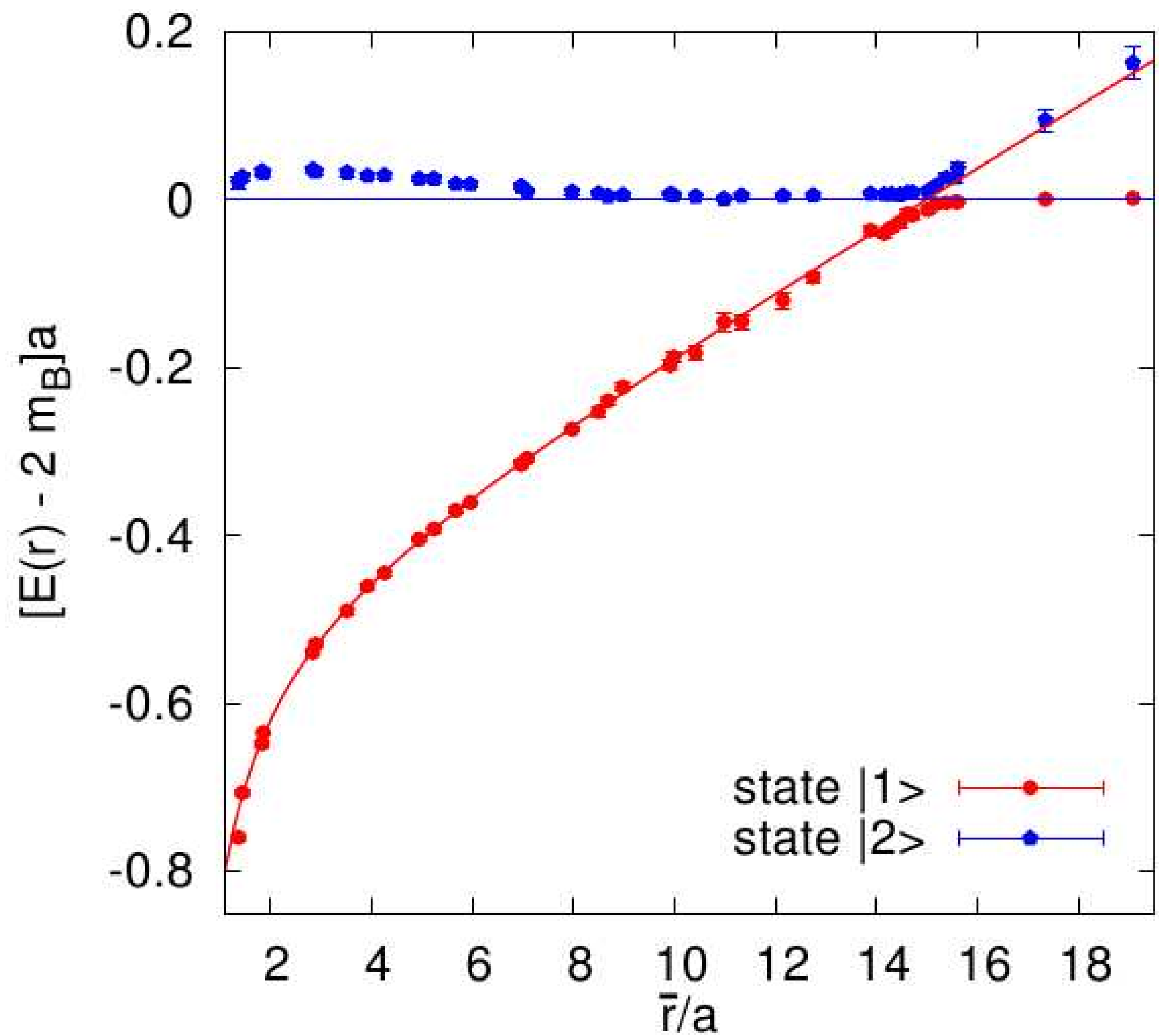
- Global quantity
- Light masses: slowing down of algorithms
- Larger hadrons: need bigger lattice
- typical size:
  - a: 0.15–0.04 fm
  - L: $\mathbf{(16{-}32)^3} \times \mathbf{(32{-}64)}$
  - sides: 1–3 fm
  - Importance sampling: hundreds of “points” (configurations)
  - Studying systematics more important than getting more points $\Longrightarrow$ vary $\boldsymbol{a}$, sizes, quark masses,. . .

Lattice QCD: many successes

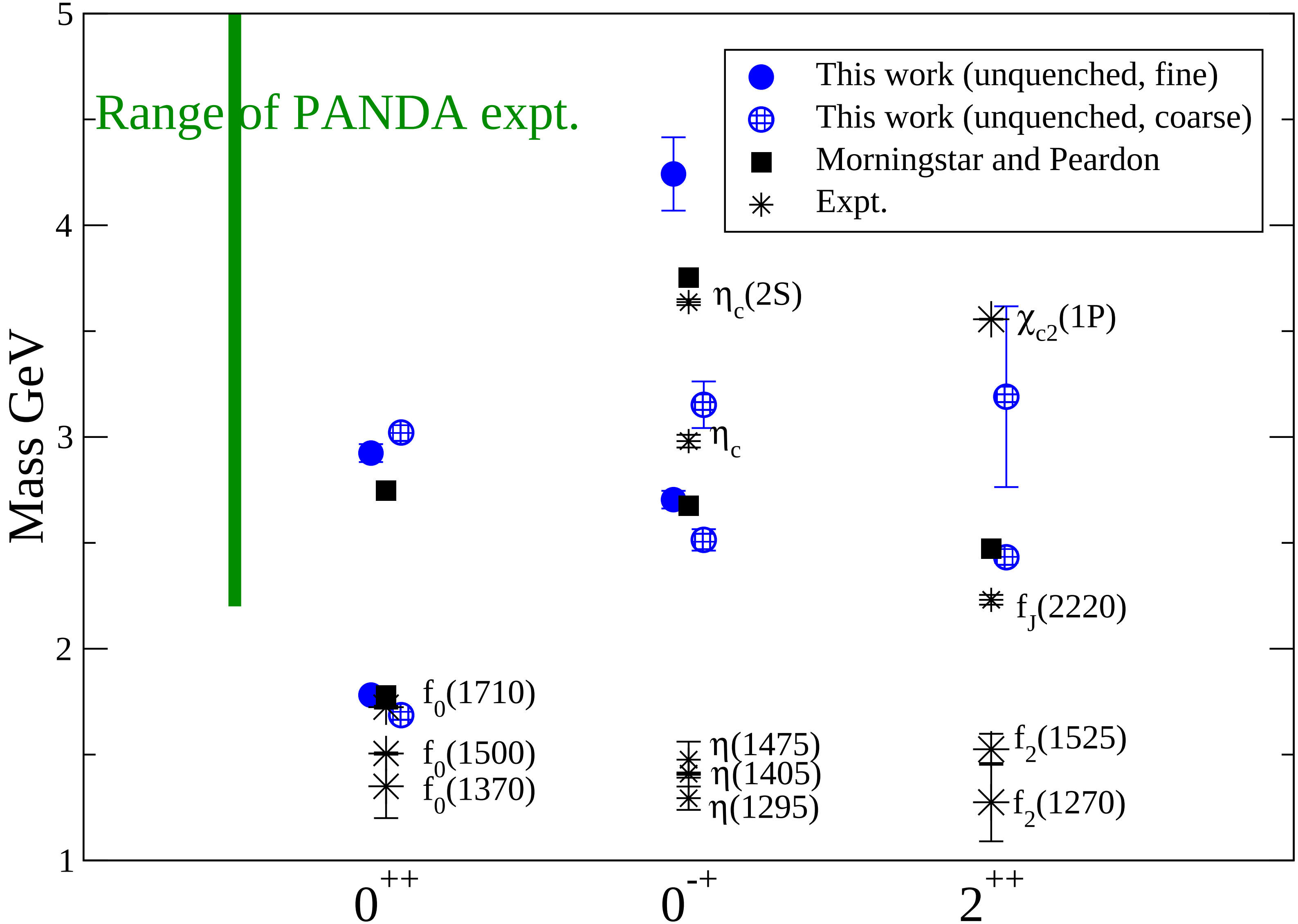


Lattice QCD: string breaking



String breaking = avoided level crossing

Lattice QCD: glueball masses



UKQCD
arXiv:
1005.2473

Wikipedia

http://en.wikipedia.org/wiki/
Effective_field_theory

In physics, an effective field theory is an approximate theory (usually a quantum field theory) that contains the appropriate degrees of freedom to describe physical phenomena occurring at a chosen length scale, but ignores the substructure and the degrees of freedom at shorter distances (or, equivalently, higher energies).

Effective Field Theory (EFT)

Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples:

- Solid state physics: conductors: neglect the empty bands above the partially filled one
- Atomic physics: Blue sky: neglect atomic structure

EFT: Power Counting

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$\Longrightarrow$ Need some ordering principle: power counting
Higher orders suppressed by powers of $1/\Lambda$

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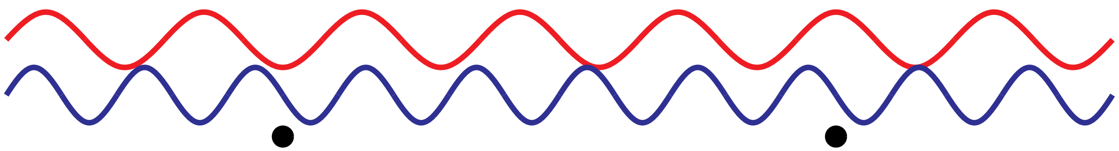
- Taylor series expansion does not work (convergence radius is zero when massless modes are present)
- Continuum of excitation states need to be taken into account

Example: Why is the sky blue ?

System: Photons of visible light and neutral atoms

Length scales: a few 1000 Å versus 1 Å

Atomic excitations suppressed by $\approx 10^{-3}$



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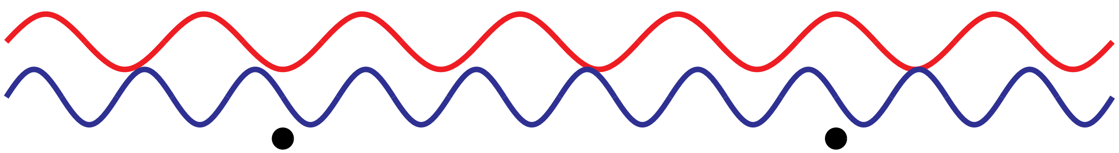

$$\mathcal{L}_A = \Phi_v^\dagger \partial_t \Phi_v + \ldots \qquad \mathcal{L}_{\gamma A} = G F_{\mu\nu}^2 \Phi_v^\dagger \Phi_v + \ldots$$

Units with $\not{h} = c = 1$: G energy dimension -3:

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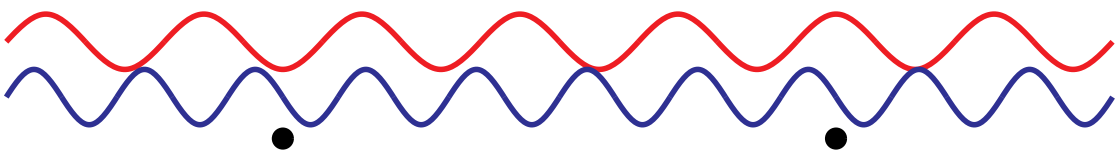

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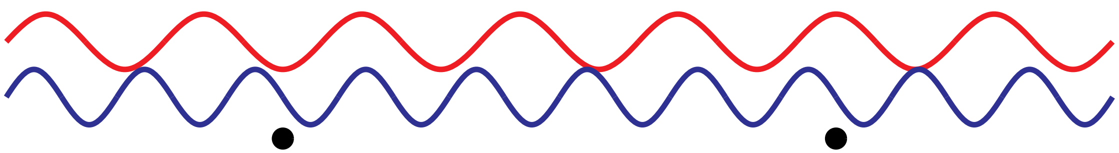

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blue light scatters a lot more than red $\left\{ \begin{array}{l} \Longrightarrow \text{red sunsets} \\ \Longrightarrow \text{blue sky} \end{array} \right.$

Higher orders suppressed by 1 Å$/\lambda_\gamma$.

EFT: Why Field Theory ?

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<u>Drawbacks</u>

- Many parameters (but finite number at any order)
  any model has few parameters but model-space is large
- expansion: it might not converge or only badly

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Advantages

- Calculations are (relatively) simple
- It is general: model-independent
- Theory $\Longrightarrow$ errors can be estimated
- Systematic: ALL effects at a given order can be included
- Even if no convergence: classification of models often useful

Examples of EFT

- Fermi theory of the weak interaction
- Chiral Perturbation Theory: hadronic physics
- NRQCD
- SCET
- General relativity as an EFT
- 2,3,4 nucleon systems from EFT point of view
- Magnons and spin waves

References

- A. Manohar, Effective Field Theories (Schladming lectures), hep-ph/9606222
- I. Rothstein, Lectures on Effective Field Theories (TASI lectures), hep-ph/0308266
- G. Ecker, Effective field theories, Encyclopedia of Mathematical Physics, hep-ph/0507056
- D.B. Kaplan, Five lectures on effective field theory, nucl-th/0510023
- A. Pich, Les Houches Lectures, hep-ph/9806303
- S. Scherer, Introduction to ChPT, hep-ph/0210398
- J. Donoghue, Introduction to the Effective Field Theory Description of Gravity, gr-qc/9512024

NRQCD

- Effective field theories for nonrelativistic bound states

  very much based on a talk by A. Vairo in Vienna EFT workshop November 2009

- The relevant scales of the non-relativistic bound state dynamics are:

  - $E \sim \dfrac{\mathbf{p}^2}{2m} \sim V \sim mv^2$,
  - $p \sim 1/r \sim mv$;

- a crucial observation:

  if v(elocity)$\ll 1$, then $m \gg mv \gg mv^2$.

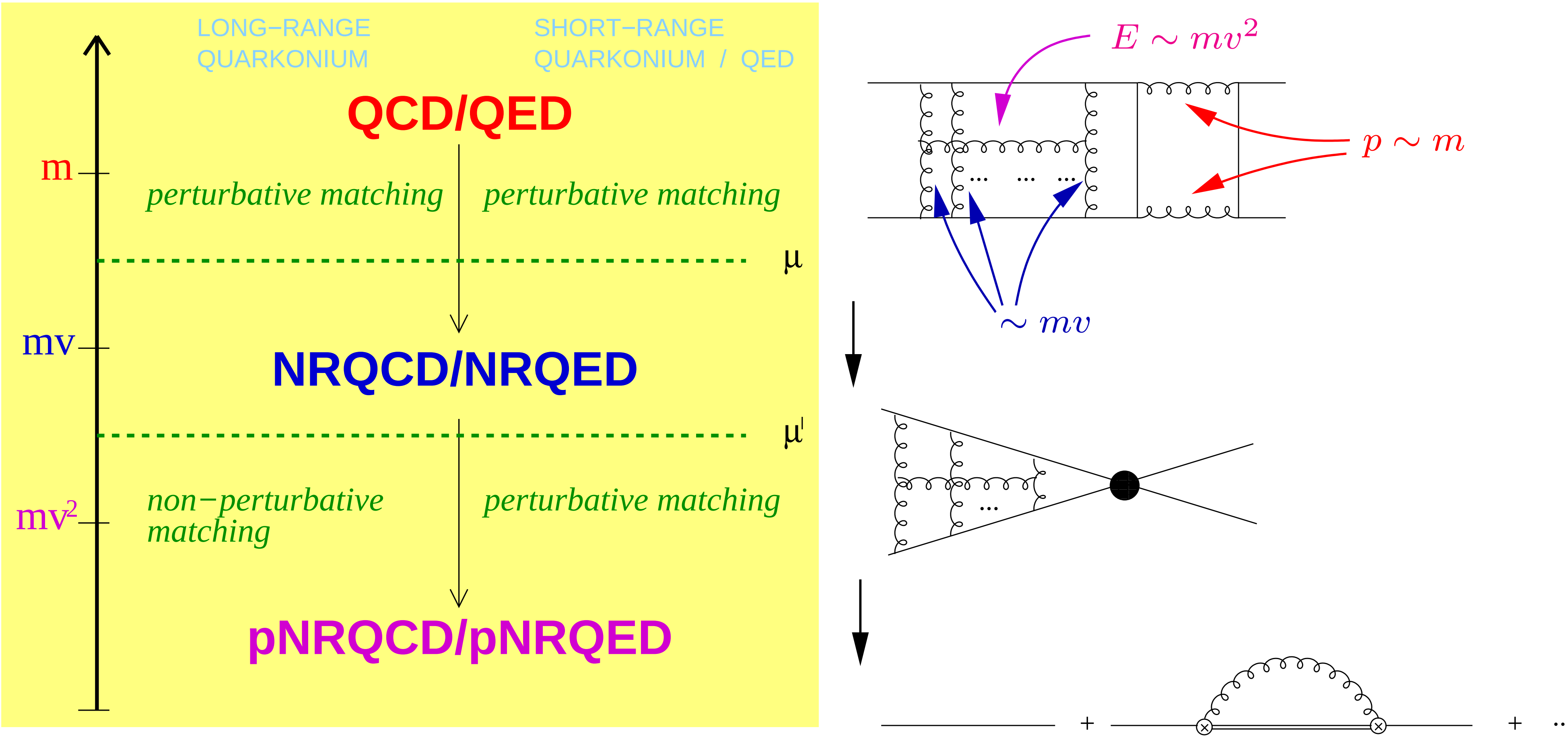
- Multiscale physics
- Bethe-Salpeter equation mixes all these scales: difficult

EFT

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(\Lambda/\mu)\frac{O_n(\mu,\lambda)}{\Lambda^n}$$

- Since at $\mu \sim \lambda$, $\langle O_n \rangle \sim \lambda^n$, the EFT is organized as an expansion in λ/Λ.
- The EFT is renormalizable order by order in λ/Λ.
- The matching coefficients $c_n(\Lambda/\mu)$ encode the non-analytic behaviour in Λ. They are calculated by imposing that $\mathcal{L}_{\mathrm{EFT}}$ and $\mathcal{L}$ describe the same physics at any finite order in the expansion: matching procedure.
- In QCD, if $\Lambda \gg \Lambda_{QCD}$ then $c_n(\Lambda/\mu)$ may be calculated in perturbation theory.

EFTs for two heavy quarks/fermions



- They exploit the expansion in v/ factorization of low and high energy contributions.
- They are renormalizable order by order in v.
- In perturbation theory, RG techniques provide resummation of large logs.

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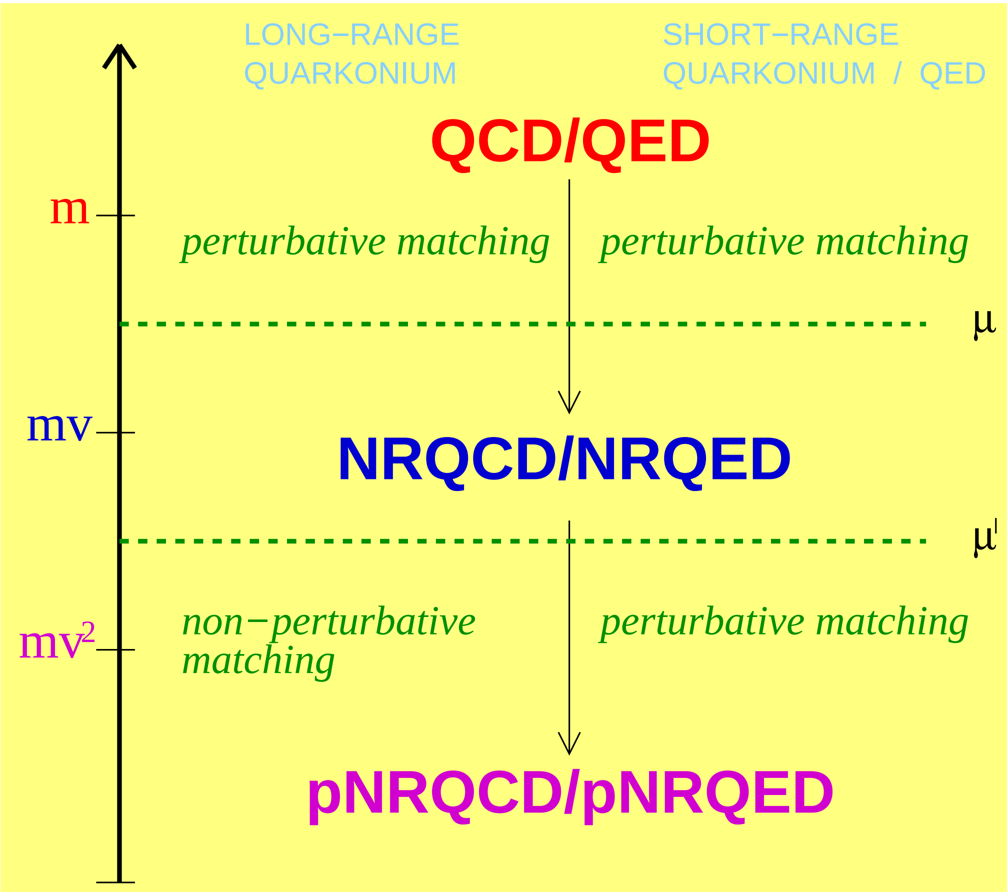

- Caswell Lepage PLB 167(86)437
- Lepage Thacker NP PS 4(88)199
- Bodwin et al PRD 51(95)1125, ...

- Pineda Soto NP PS 64(98)428
- Brambilla et al PRD 60(99)091502
- Brambilla et al NPB 566(00)275
- Kniehl et al NPB 563(99)200
- Luke Manohar PRD 55(97)4129
- Luke Savage PRD 57(98)413
- Grinstein Rothstein PRD 57(98)78
- Labelle PRD 58(98)093013
- Griesshammer NPB 579(00)313
- Luke et al PRD 61(00)074025
- Hoang Stewart PRD 67(03)114020, ...

- *for a review* Brambilla Pineda Soto Vairo RMP 77(04)1423

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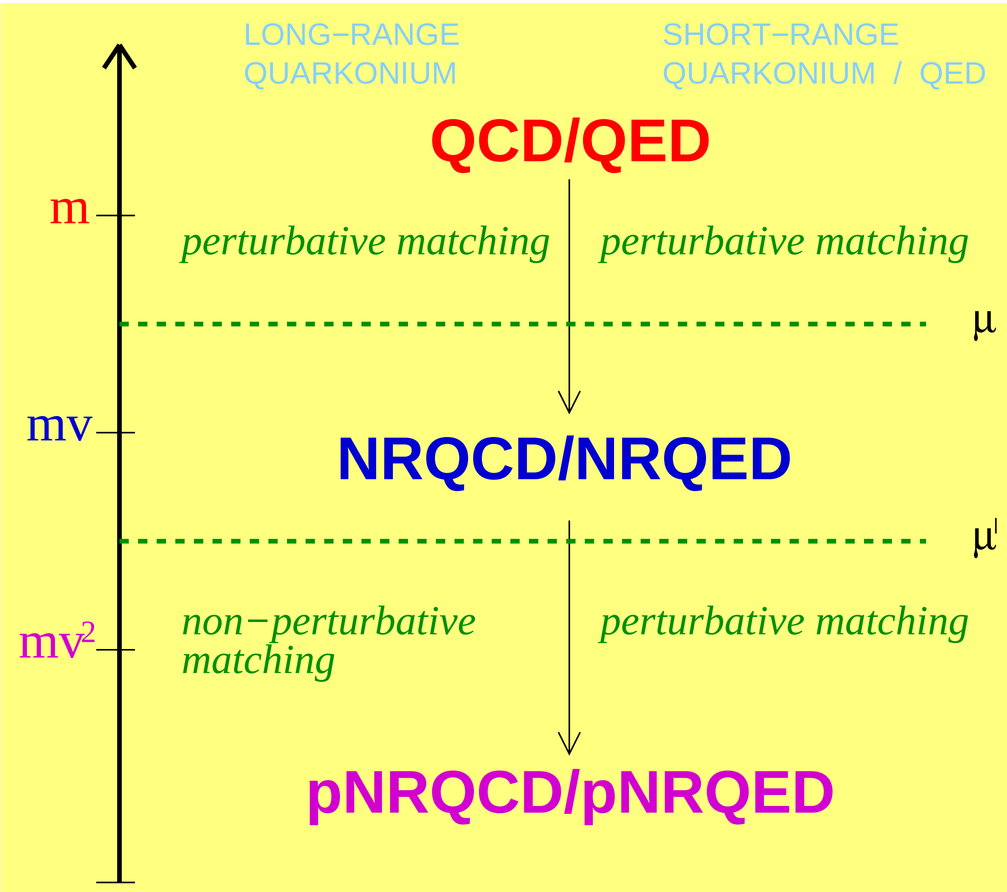
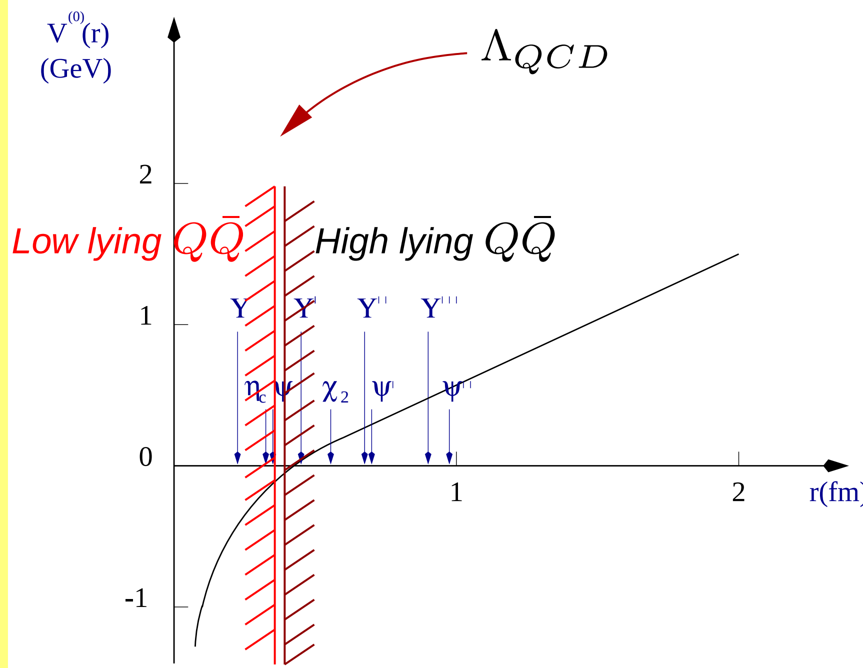




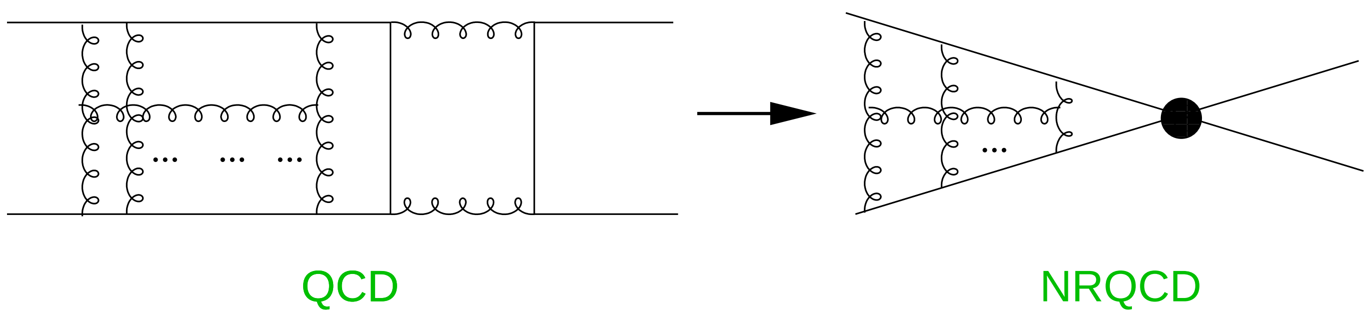
○ Godfrey Isgur PRD 32(85)189

A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{QCD}$

NRQCD

NRQCD is obtained by integrating out modes associated with the scale m



$\times c(\mu/m)$

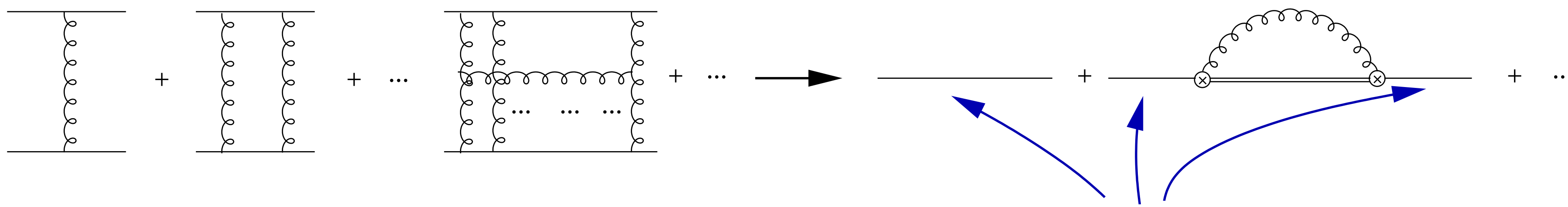
- The matching is perturbative.
- The Lagrangian is organized as an expansion in $1/m$ and $\alpha_S(m)$:

$$\mathcal{L}_{\mathrm{NRQCD}} = \sum_n c(\alpha_S(m/\mu)) \times O_n(\mu,\lambda)/m^n$$

Suitable to describe annihilation and production of quarkonium.

pNRQCD

pNRQCD is obtained by integrating out modes associated with the scale $\frac{1}{r} \sim mv$



$$\frac{1}{E - p^2/m - V(r)}$$

NRQCD　　　　pNRQCD

- The Lagrangian is organized as an expansion in $1/m$, r, and $\alpha_S(m)$:

$$\mathcal{L}_{\mathrm{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} \times c_k(\alpha_S(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

Case 1: pNRQCD for $mv \gg \Lambda_{QCD}$

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \mathrm{Tr}\left\{\mathrm{S}^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s\right)\mathrm{S} \right.$$
$$\left. + \ \mathrm{O}^\dagger\left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o\right)\mathrm{O}\right\}$$

LO in r

The equation of motion of the singlet,

$$\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s\right)\mathrm{S} = 0,$$

is the Schrödinger equation!

Case 1: pNRQCD for $mv \gg \Lambda_{QCD}$

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \mathrm{Tr}\left\{\mathrm{S}^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s\right)\mathrm{S}\right.$$
$$\left. + \ \mathrm{O}^\dagger\left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o\right)\mathrm{O}\right\}$$
$$+V_A\,\mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{S} + \mathrm{S}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{O}\right\}$$
$$+\frac{V_B}{2}\mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{O} + \mathrm{O}^\dagger\mathrm{O}\mathbf{r}\cdot g\mathbf{E}\right\}$$
$$+\cdots$$

- At leading order in the multipole expansion, the equation of motion of the EFT is the Schrödinger equation. Higher-order terms correct this picture (these higher order terms are responsible, for instance, for the Lamb shift).
- The Schrödinger potential, V_s, emerges as a Wilson coefficient of the EFT. As such, it undergoes renormalization, develops scale dependence and satisfies renormalization group equations, which allow to resum large logarithms.

The static potential at N^4LO

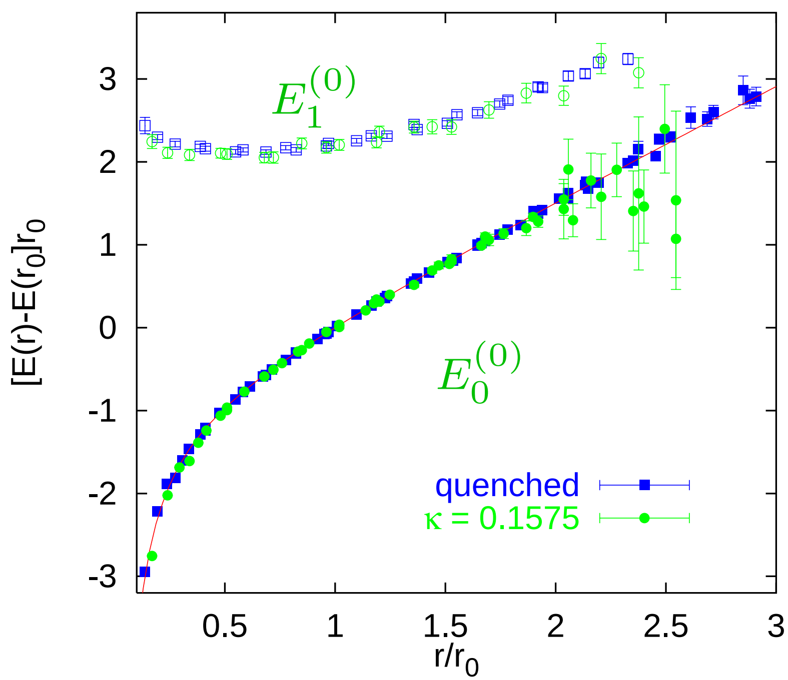
$$
\begin{aligned}
V_s(r,\mu) \quad = \quad & -C_F\,\frac{\alpha_S(1/r)}{r}\left[1 + a_1\,\frac{\alpha_S(1/r)}{4\pi} + a_2\left(\frac{\alpha_S(1/r)}{4\pi}\right)^2\right.\\
& + \left(\frac{16\,\pi^2}{3}C_A^3\,\ln r\mu + a_3\right)\left(\frac{\alpha_S(1/r)}{4\pi}\right)^3\\
& \left. + \left(a_4^{L2}\ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\,C_A^3\beta_0(-5+6\ln 2)\right)\ln r\mu + a_4\right)\left(\frac{\alpha_S(1/r)}{4\pi}\right)^4\right]
\end{aligned}
$$

$$
\begin{aligned}
a_4^{L2} \quad &= \quad -\frac{16\pi^2}{3}C_A^3\,\beta_0\\
a_4^{L} \quad &= \quad 16\pi^2 C_A^3\left[a_1 + 2\gamma_E\beta_0 + n_f\left(-\frac{20}{27}+\frac{4}{9}\ln 2\right)\right.\\
&\qquad\qquad \left. + C_A\left(\frac{149}{27} - \frac{22}{9}\ln 2 + \frac{4}{9}\pi^2\right)\right]
\end{aligned}
$$

- Brambilla et al PRD 60(99)091502, PLB 647(07)185

Case 2: pNRQCD for $mv \sim \Lambda_{QCD}$

- All scales above mv^2 are integrated out (including Λ_{QCD}).
- All gluonic excitations between heavy quarks are integrated out since they develop a gap of order Λ_{QCD} with the static $Q\bar{Q}$ energy.



○ Bali et al PRD 62(00)054503
($r_0 \simeq 0.5$ fm)

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- All scales above mv^2 are integrated out (including Λ_{QCD}).
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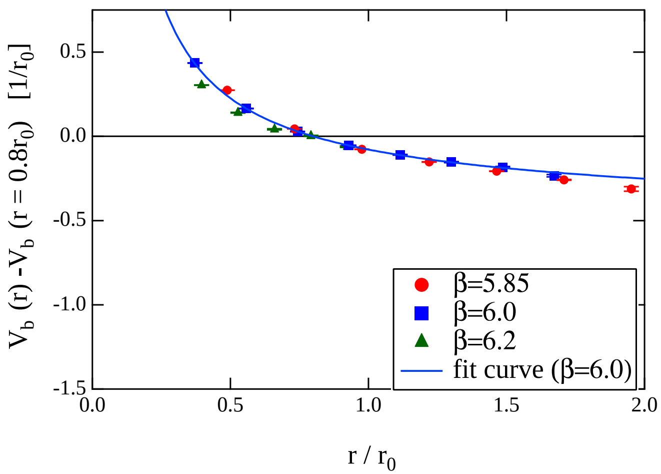
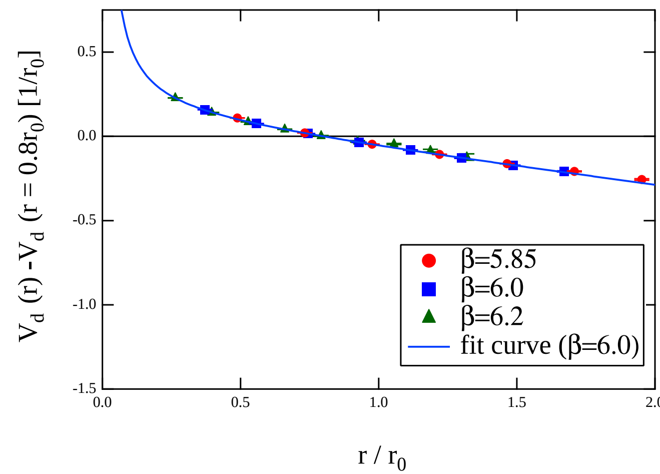
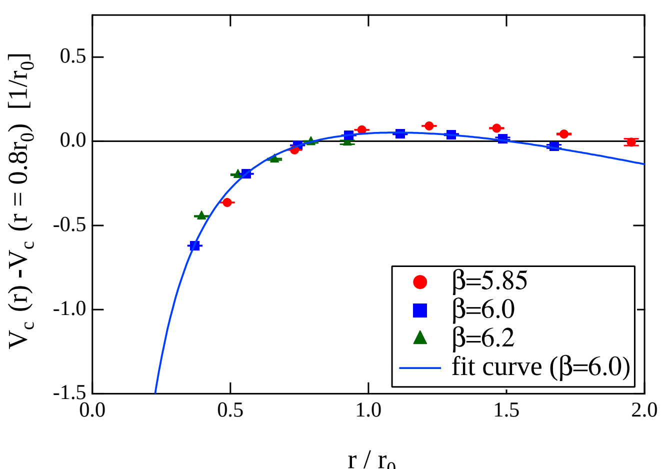
$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only degree of freedom of pNRQCD (up to ultrasoft hadrons, e.g. pions).

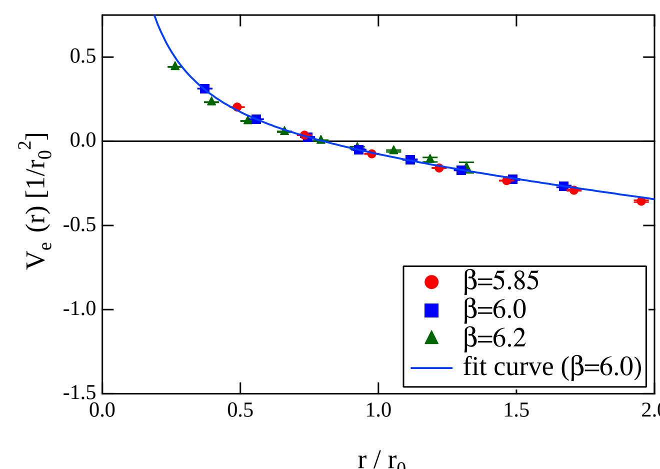
$\Rightarrow$ Higher order potentials are well defined and can be calculated from the lattice or QCD vacuum models.

The non-perturbative spin-independent p^2/m^2 potentials



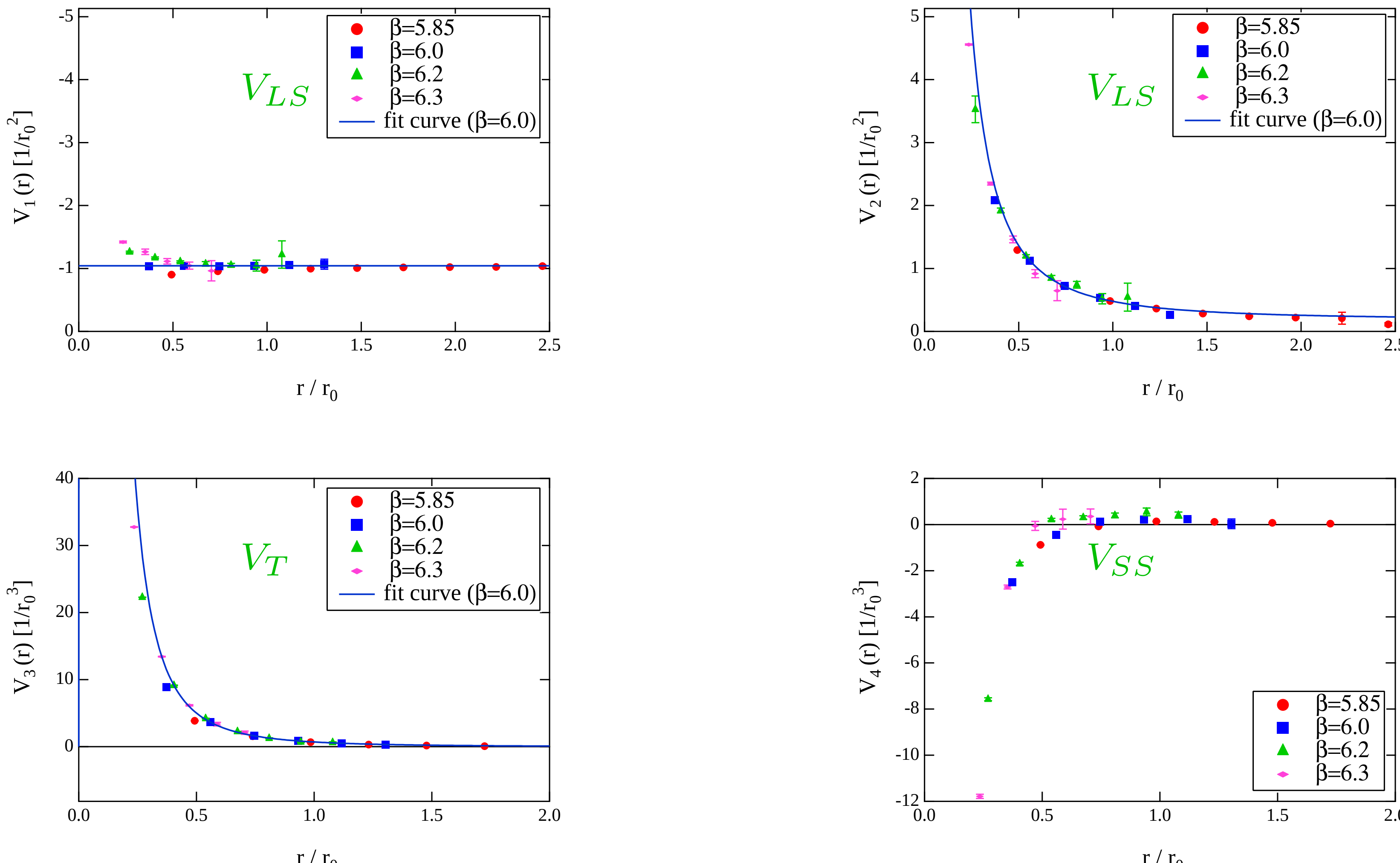






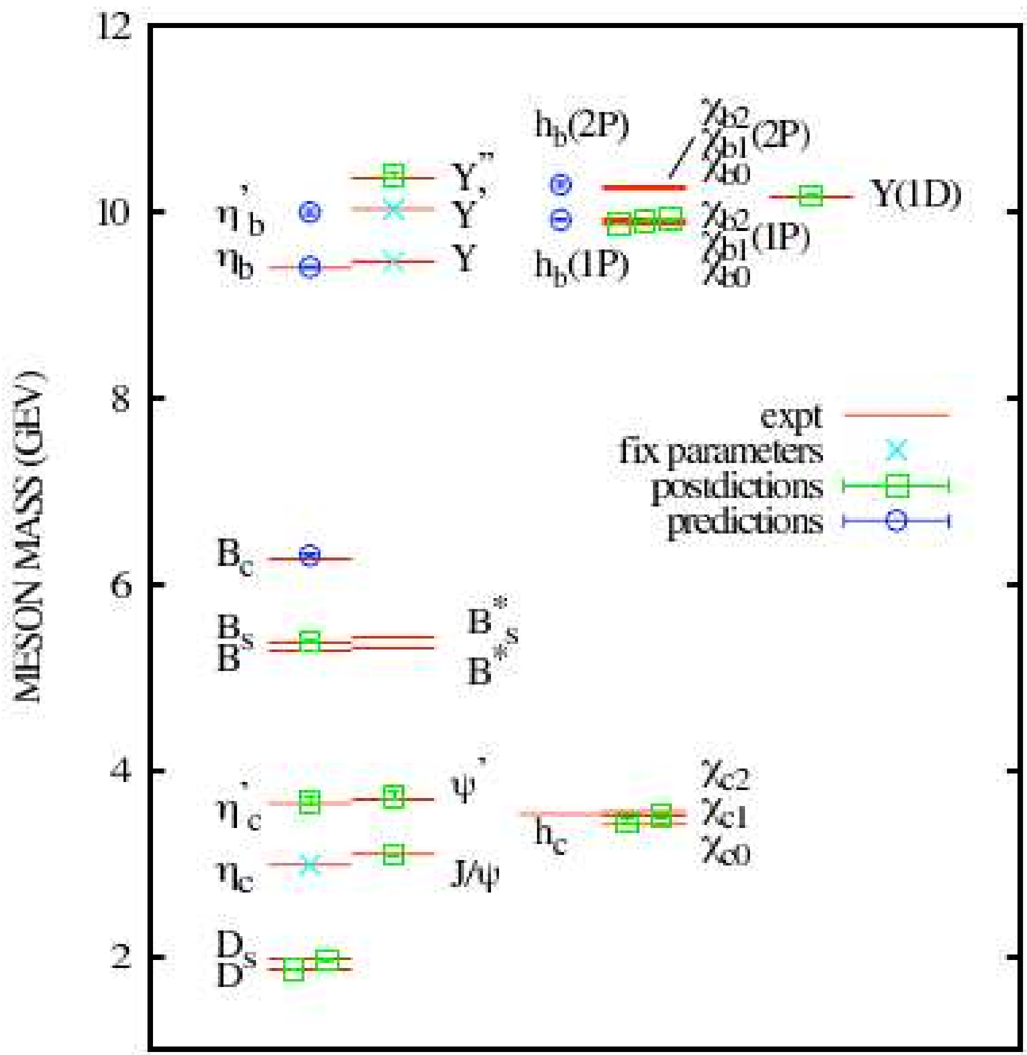
- Koma Koma Wittig PoS LAT2007(07)111, Koma Koma arXiv:0911.3204

The non-perturbative spin-dependent $1/m^2$ potentials



- Koma Koma NPB 769(07)79, Koma Koma arXiv:0911.3204

NRQCD: results



Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

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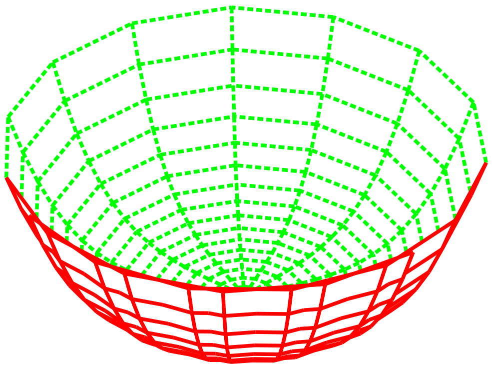
Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory,* Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

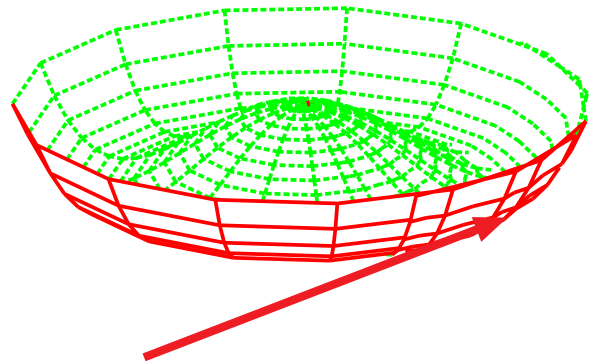
The mass gap: Goldstone Modes

UNBROKEN: $V(\phi)$



Only massive modes
around lowest energy
state (=vacuum)

BROKEN: $V(\phi)$



Need to pick a vacuum
$\langle\phi\rangle \neq 0$: Breaks symmetry
No parity doublets
Massless mode along bottom

For more complicated symmetries: need to describe the bottom mathematically: $G \to H \Longrightarrow G/H$

The power counting

Very important:

Low energy theorems: Goldstone bosons do not interact at zero momentum

Heuristic proof:

- Which vacuum does not matter, choices related by symmetry
- $\phi(x) \to \phi(x) + \alpha$ should not matter
- Each term in $\mathcal{L}$ must contain at least one $\partial_\mu \phi$

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
  - Two or Three (or even more) Flavours
  - Strong interaction and couplings to external currents/densities
  - Including electromagnetism
  - Including weak nonleptonic interactions
  - Treating kaon as heavy

Many similarities with strongly interacting Higgs

Hard pion ChPT?

- In Meson ChPT: the powercounting is from all lines in Feynman diagrams having soft momenta
- thus powercounting = (naive) dimensional counting

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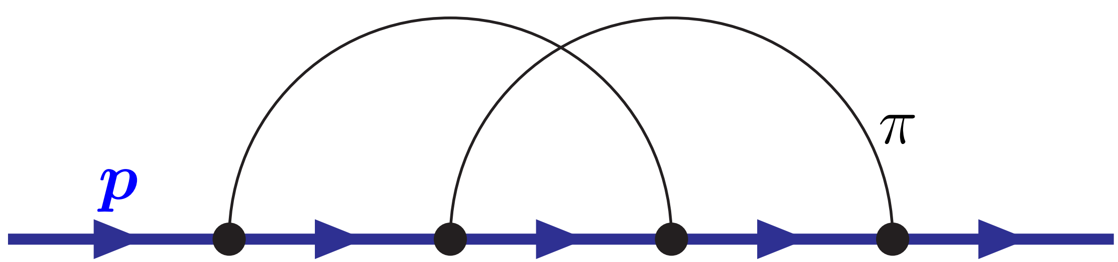
  - $p = M_B v + k$
  - Everything else soft
  - Works because baryon or b or c number conserved so the non soft line is continuous



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  - Works because baryon or b or c number conserved so the non soft line is continuous
  - Decay constant works: takes away all heavy momentum
  - General idea: M_p dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon always goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,. . .
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
  - Works like all the previous *heavy* ChPT

Hard pion ChPT?

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  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
- Flynn-Sachrajda also argued that $K_{\ell 3}$ could be done for q^2 away from q^2_{max}.
- JB-Celis, JB-Jemos Generalizes to other processes with hard/fast pions and applied to $K \to \pi\pi$ resp. $B \to \pi \ell \nu$
- General idea: heavy/fast dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle \pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

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$$\lim_{q\to 0}\langle \pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

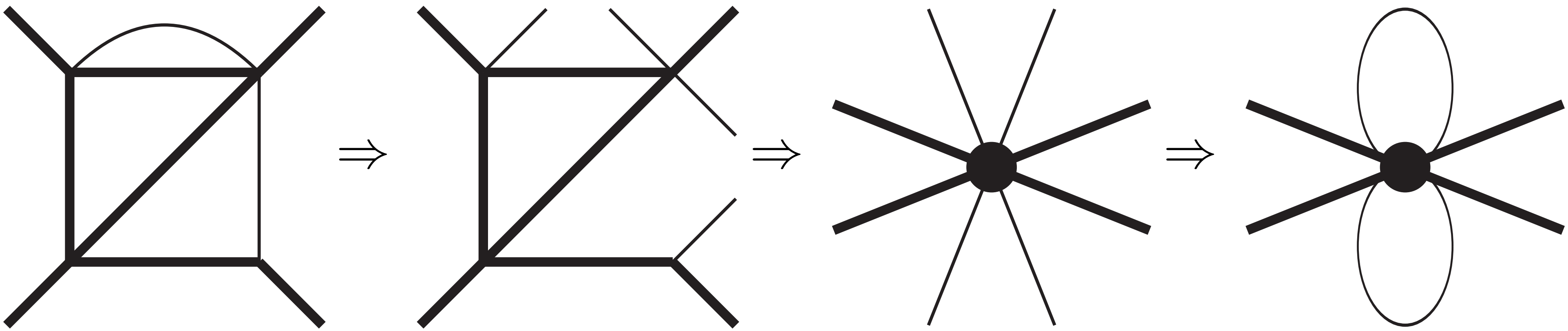
- Nothing prevents hard pions to be in the states α or β
- So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence

Hard pion ChPT?

Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta
- If symmetries present, Lagrangian should respect them
- Lagrangian should be complete in *neighbourhood*
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses
  Crucial part of the argument

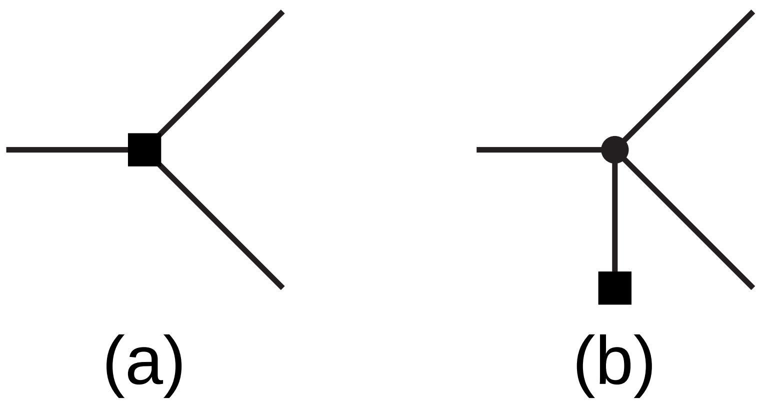
Hard pion ChPT?



This procedure works at one loop level, matching at tree level, nonanalytic dependence at one loop:

- Toy models and vector meson ChPT JB, Gosdzinsky, Talavera
- Recent work on relativistic meson ChPT Gegelia, Scherer et al.
- Extra terms kept in $K \to 2\pi$, $B \to \pi\ell\nu$: one-loop check
- Some preliminary two-loop checks

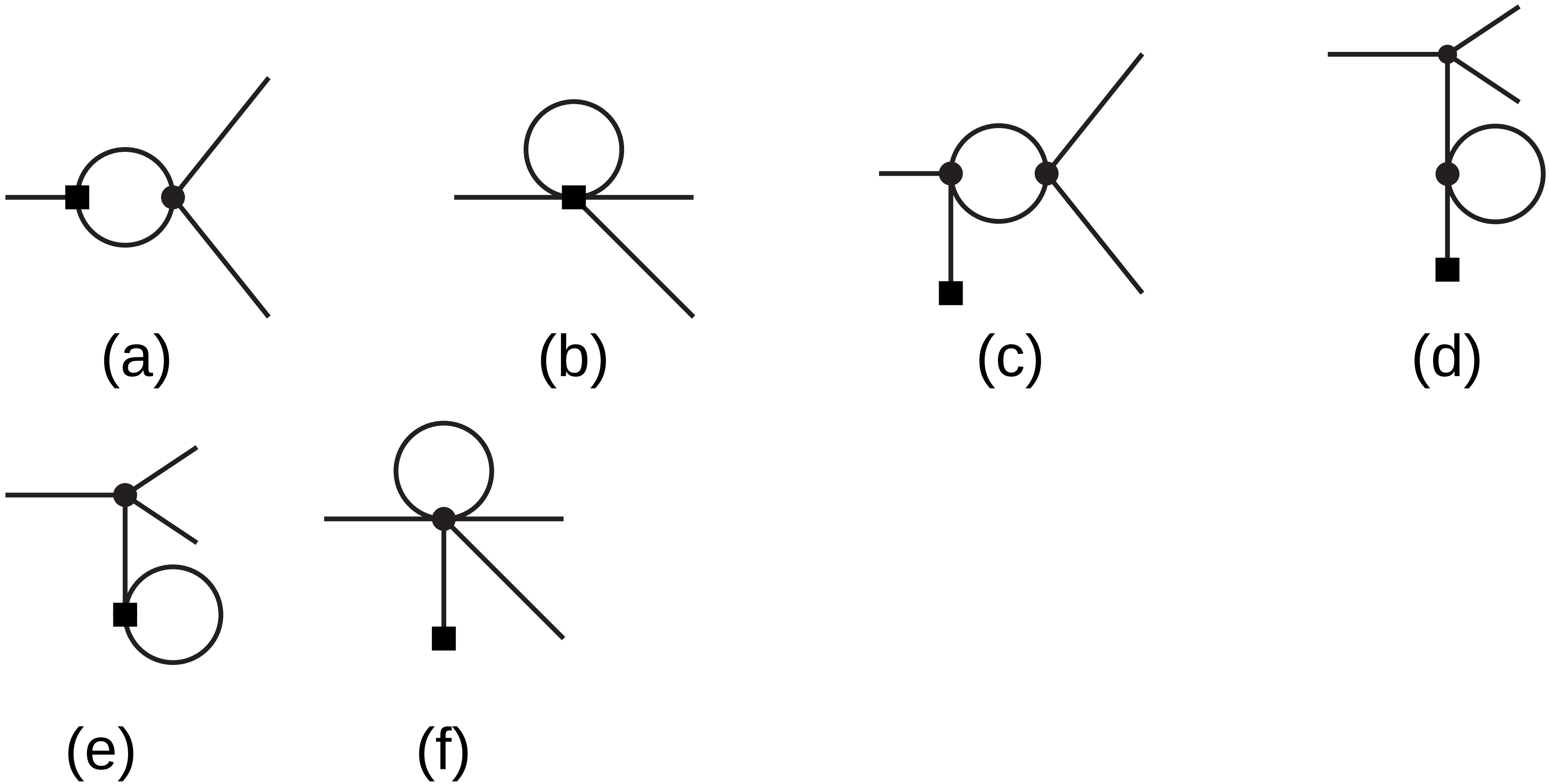
$K \to \pi\pi$: Tree level



$$A_0^{LO} = \frac{\sqrt{3}i}{2F^2}\left[-\frac{1}{2}E_1 + (E_2 - 4E_3)\,\overline{M}_K^2 + 2E_8\overline{M}_K^4 + A_1E_1\right]$$

$$A_2^{LO} = \sqrt{\frac{3}{2}}\frac{i}{F^2}\left[(-2D_1 + D_2)\,\overline{M}_K^2\right]$$

$K \to \pi\pi$: One loop



$K \to \pi\pi$: One-loop

$$
\begin{aligned}
A_0^{NLO} &= A_0^{LO}\left(1+\frac{3}{8F^2}\overline{A}(M^2)\right)+\lambda_0 M^2+\mathcal{O}(M^4)\,,\\
A_2^{NLO} &= A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathcal{O}(M^4)\,.
\end{aligned}
$$

$\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$

$B \to \pi\ell\nu$: One-loop

$$\left\langle P_f(p_f) \left| \overline{q}_i \gamma_\mu q_f \right| P_i(p_i) \right\rangle = (p_i + p_f)_\mu f_+(q^2) + (p_i - p_f)_\mu f_-(q^2)$$

$$= \left[(p_i + p_f)_\mu - q_\mu \frac{(m_i^2 - m_\pi^2)}{q^2} \right] f_+(q^2) + q_\mu \frac{(m_i^2 - m_f^2)}{q^2} f_0(q^2)$$

$$f_{0/+}(q^2) \quad = \quad f_{0/+}^{\text{Tree}}(q^2) \left[1 + \left(\frac{3}{4} + \frac{9}{4} g^2 \right) \overline{A}(m_\pi^2) \right],$$

$$\overline{A}(M^2) = -\frac{M^2}{16\pi^2} \log \frac{M^2}{\mu^2}$$

$$q^2 \neq q^2_{\max}$$

at $q^2_{\max}$ reproduce known results

Done in relativistic and heavy meson formalism

Hard Pion ChPT: A two-loop check

- Similar arguments to JB-Celis, Flynn-Sachrajda work for the pion vector and scalar formfactor
- Therefore at any t the chiral log correction must go like the one-loop calculation.
- But note the one-loop log chiral log is with $t >> m_\pi^2$
- Predicts
$$F_V(t, M^2) = F_V(t, 0)\left(1 - \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
$$F_S(t, M^2) = F_S(t, 0)\left(1 - \frac{5}{2}\frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
- Note that $F_{V,S}(t, 0)$ is now a coupling constant and can be complex

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- Note that $F_{V,S}(t,0)$ is now a coupling constant and can be complex
- Take the full two-loop ChPT calculation JB,Colangelo,Talavera and expand in $t >> m_\pi^2$.

A two-loop check

Full two-loop ChPT JB,Colangelo,Talavera, expand in $t >> m_\pi^2$:

$$F_V(t,M^2) = F_V(t,0)\left(1 - \frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
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with

$$F_V(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6}\ln\frac{t}{\mu^2}\right)$$
$$F_S(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(1 - 16\pi^2 l_4^r + i\pi - \ln\frac{t}{\mu^2}\right)$$

- The needed coupling constants are complex
- Both calculations have two-loop diagrams with overlapping divergences
- The chiral logs should be valid for any t where a pointlike interaction is a valid approximation

Hard Pion ChPT

Why is this useful:

- Lattice works actually around the strange quark mass
- need only extrapolate in m_u and m_d.
- Applicable in momentum regimes where usual ChPT might not work
- In progress: $B \to \pi, K$ semileptonic decays
- Thinking about PANDA possibilities here

Conclusions

A bit of an overview of three techniques for making predictions for PANDA physics

- Lattice QCD
- NRQCD and pNRQCD (and lattice)
- Hard pion ChPT