Eta decays and other isospin breaking effects in meson physics

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Overview

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- Chiral Perturbation Theory
- Some Two Loop Results
  - Two Loop: General
  - Two Loop: Three Flavours
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    - $\pi\pi$, πK and scalar form factors
  - Isopin Violation at Two Loops
- Eta decays beyond p^4
- Isospin Violation in $K \to 3\pi$
- Two Loop: PQChPT and lattice QCD
- Conclusions

Introduction

Earlier talks:

electromagnetic effects at one-loop, chiral perturbation theory, isospin breaking at one loop.

This talk:

what is known (and what remains to be done) beyond one-loop

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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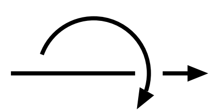
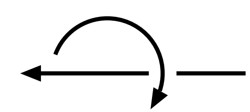
So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via



$v < c,\ m_q \neq 0 \Longrightarrow$

$v = c,\ m_q = 0 \not\Longrightarrow$

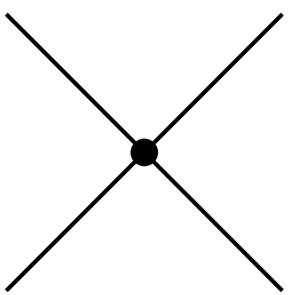


Chiral Perturbation Theory

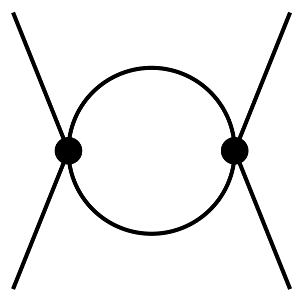
$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

Power counting in momenta:

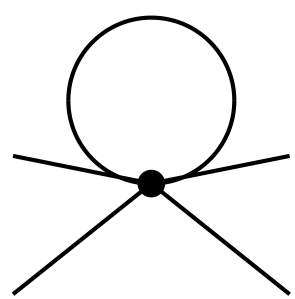


p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$



$(p^2)\,(1/p^2)\,p^4 = p^4$

$\int d^4p$ $\quad p^4$

Two Loop: General

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

Note
- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour is a special case of 3+3 PQ:

  $\hat{L}_i^r, K_i^r \to L_i^r, C_i^r$

Three Flavours at Two Loop

$\Pi_{VV\pi}$, $\Pi_{VV\eta}$, Π_{VVK}	Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera	
$\Pi_{VV\rho\omega}$	Maltman	
$\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η	Kambor, Golowich; Amorós, JB, Talavera	
Π_{SS}	Moussallam	L_4^r, L_6^r
Π_{VVK}, Π_{AAK}, F_K, m_K	Amorós, JB, Talavera	
$K_{\ell 4}$, $\langle \overline{q} q \rangle$	Amorós, JB, Talavera	L_1^r, L_2^r, L_3^r
F_M, m_M, $\langle \overline{q} q \rangle$ $(m_u \neq m_d)$	Amorós, JB, Talavera	$L_{5,7,8}^r, m_u/m_d$
$F_{V\pi}$, F_{VK^+}, F_{VK^0}	Post, Schilcher; JB, Talavera	L_9^r
$K_{\ell 3}$	Post, Schilcher; JB, Talavera	V_{us}
$F_{S\pi}$, F_{SK} (includes σ-terms)	JB, Dhonte	L_4^r, L_6^r
$K, \pi \to \ell \nu \gamma$	Geng, Ho, Wu	L_{10}^r
$\pi\pi$	JB,Dhonte,Talavera	
πK	JB,Dhonte,Talavera	

General Strategy and some comments

- Find enough inputs from experiment
- C_i^r:
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not σ)
  - quark masses: which scalars? unrealistically large estimates
- in p^6 physical or lowest order masses: thresholds in right place requires physical

General Strategy and some comments

Inputs:

$K_{\ell 4}$: $F(0)$, $G(0)$, λ — E865 BNL

$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$ — em with Dashen violation

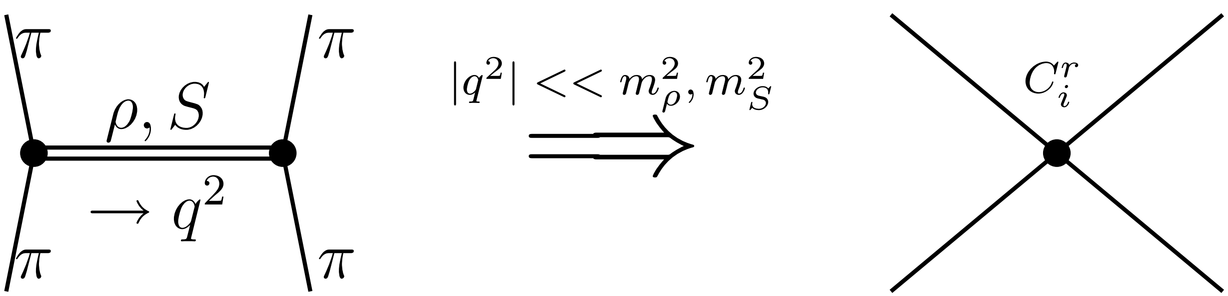
F_{π^+}

F_{K^+}/F_{π^+}

$m_s/\hat{m}$ — 24 (26) — $\hat{m} = (m_u + m_d)/2$

L^r_4, L^r_6

C^r_i from single resonance approximation



General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

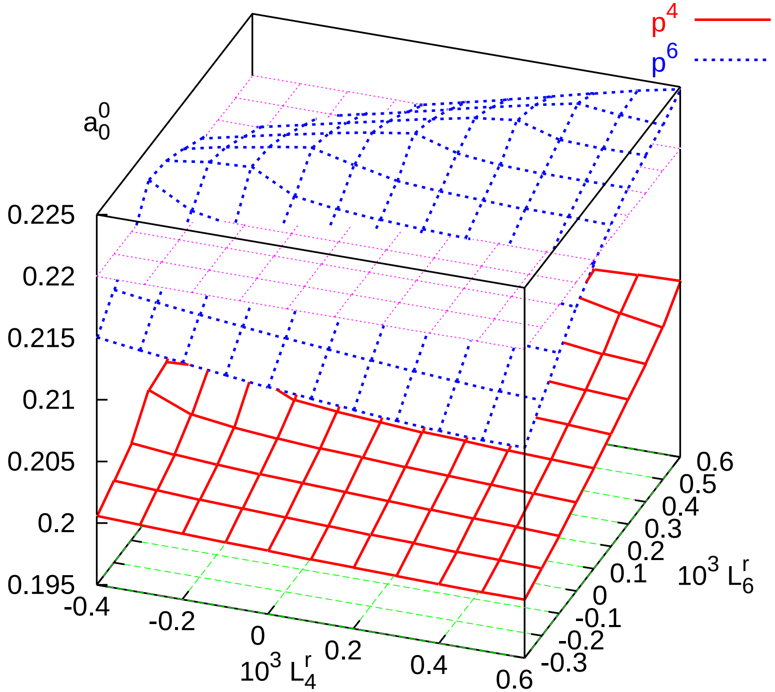
- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying C_i^r factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and πK thresholds

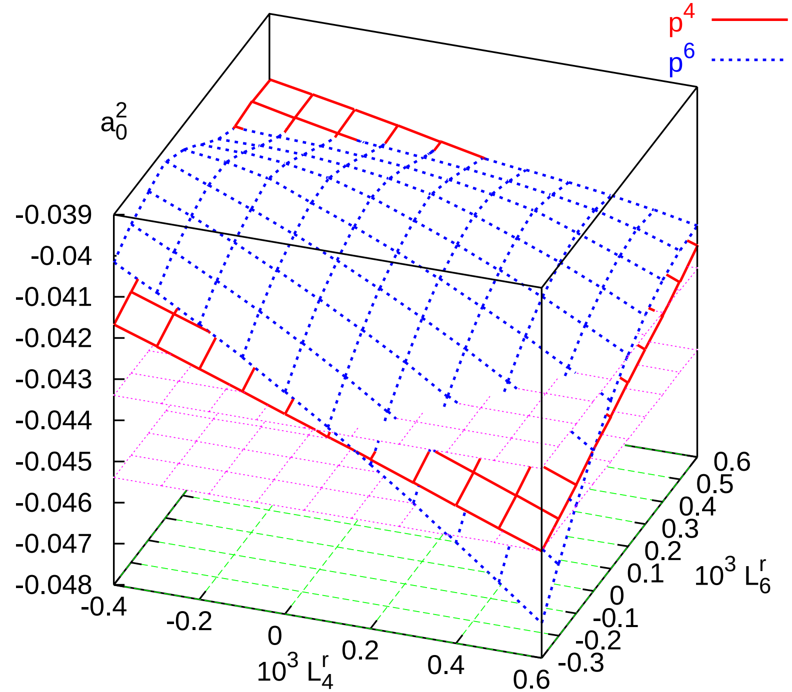
General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	0.009,$\equiv 0$	$-$0.138,0.009	$-$0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	0.075,$\equiv 0$	$-$0.149,0.094	$-$0.096,0.201
m_η^2: p^4, p^6	$-$0.052,0.318	0.013,$\equiv 0$	$-$0.197,0.073	$-$0.151,0.197
m_u/m_d	0.45$\pm$0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}$: p^4, p^6	0.169,0.051	0.22,$\equiv 0$	0.153,0.067	0.159,0.061

- $m_u = 0$ always very far from the fits
- F_0: pion decay constant in the chiral limit

$\pi\pi$



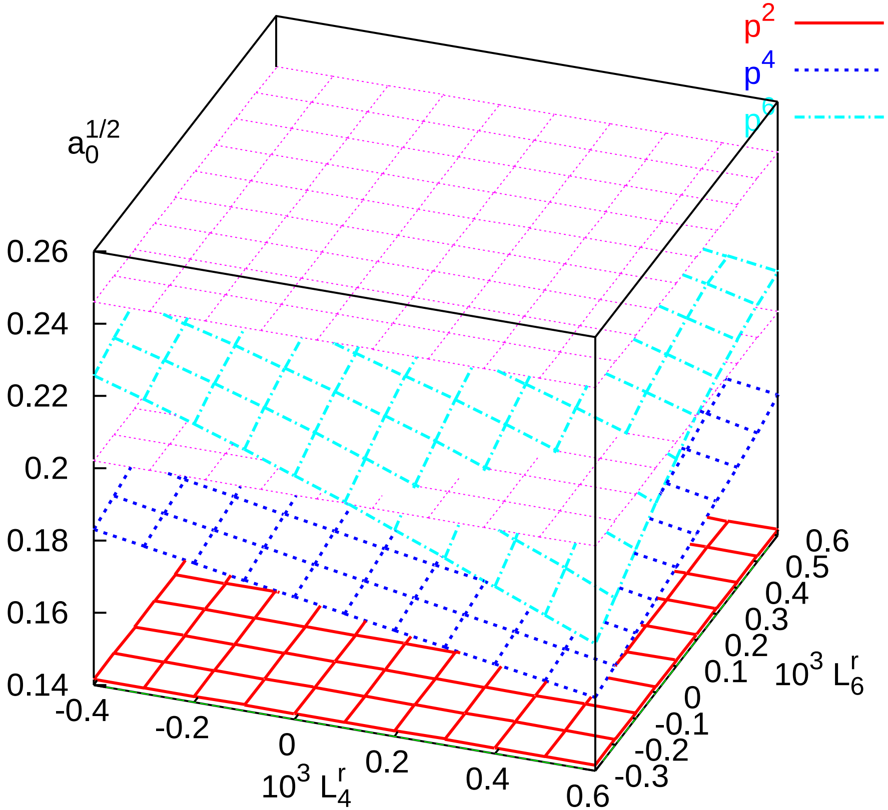


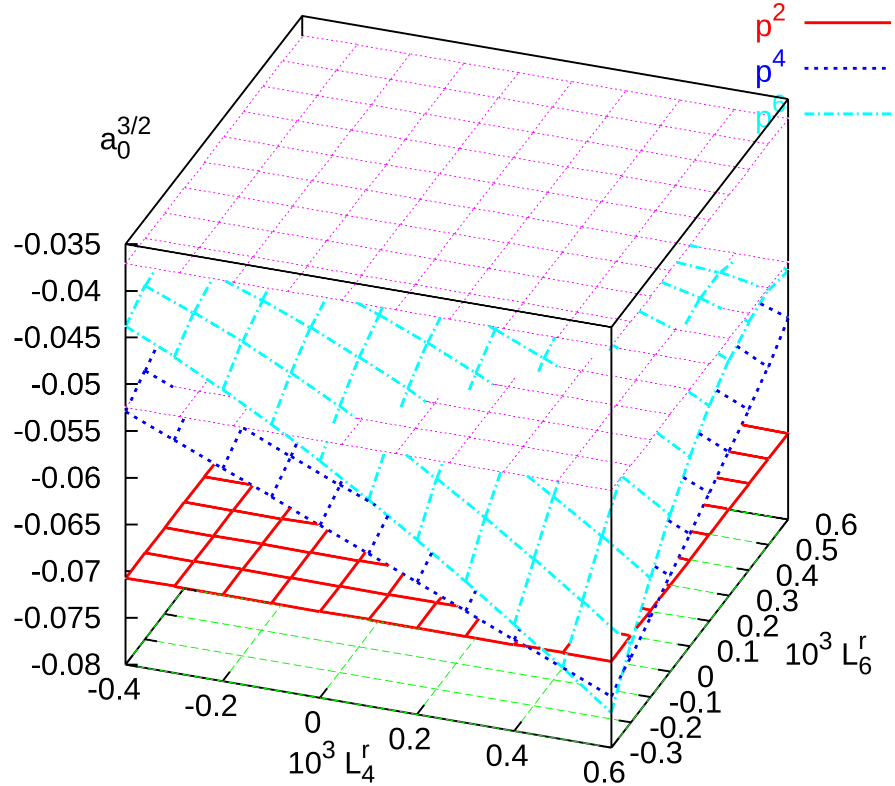
$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159$ $a_0^2 = -0.0454$ at order p^2

πK



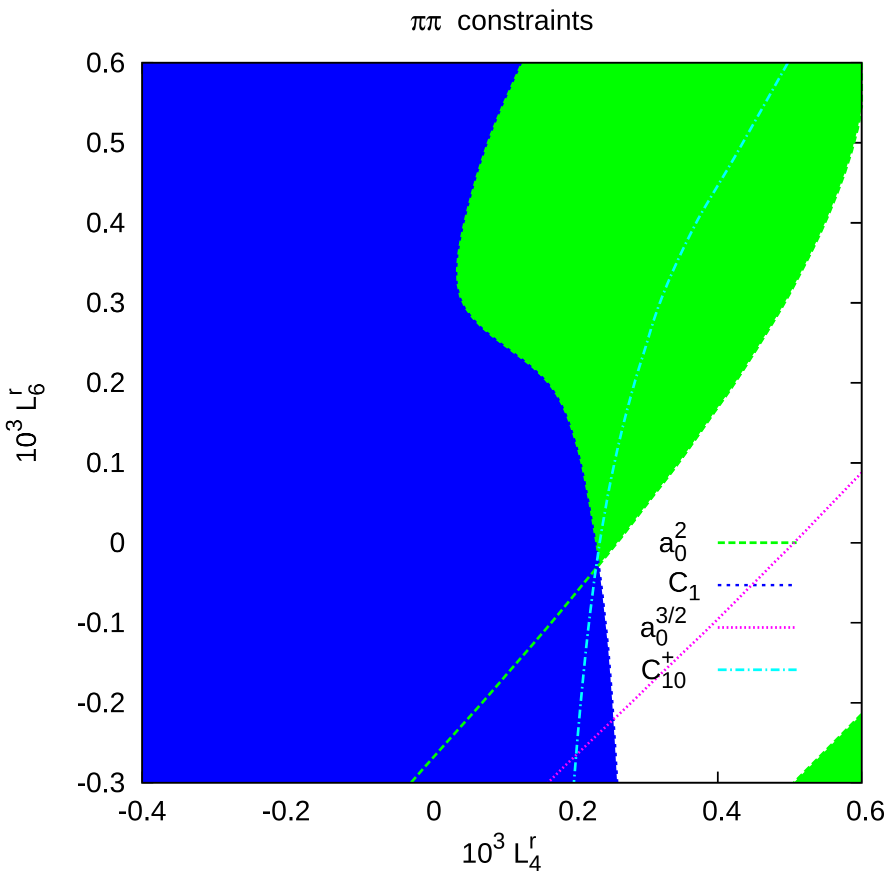


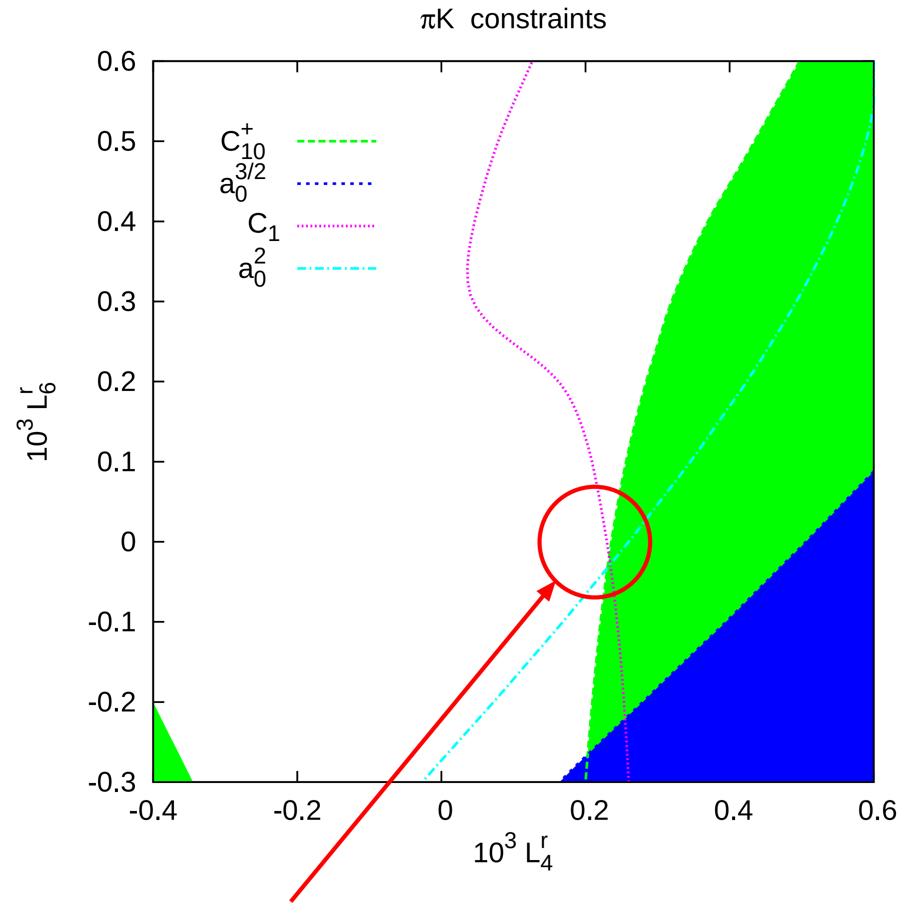
$a_0^{1/2} = 0.224 \pm 0.022$, $a^2_{3/2} = -0.0448 \pm 0.0077$

Büttiker, Descotes-Genon, Moussallam

$a_0^{1/2} = 0.142\ a_0^2 = -0.0708$ at order p^2

$\pi\pi$ and πK





preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

Isospin Violation at Two Loops

Known: $\Pi^{\rho\omega}_{VV}$ Maltman

masses, decay constants, $\langle\bar{q}q\rangle$ Amorós, JB, Talavera

$\epsilon_d = 1 - \dfrac{\langle\overline{d}d\rangle}{\langle\overline{u}u\rangle}$ in 10^{-2} with $H_2^r = 0$

$\Delta_\pi = \left(m^2_{\pi^+} - m^2_{\pi^0}\right)_{QCD}$ in 10^{-5} GeV^2

$\Delta_K = \left(m^2_{K^0} - m^2_{K^+}\right)_{QCD}$ in 10^{-3} GeV^2

	fit 10	fit B	fit D
ϵ_d	$0 - 0.70 - 0.10$	$0 - 0.70 - 0.19$	$0 - 0.69 - 0.15$
Δ_π	$4.04 + 1.49 + 1.93$	$4.27 + 1.01 - 2.63$	$4.00 + 1.21 - 1.93$
Δ_K	$5.11 - 0.56 + 1.71$	$6.51 - 0.91 + 0.67$	$5.80 - 0.86 + 1.33$

Numbers are: $p^2 + p^4 + p^6$

Isospin Violation at Two Loops

$$\epsilon_{0+} = \frac{F_{\pi^0 3} - F_{\pi^+}}{F_{\pi^+}}$$ in 10^{-4}

$$\epsilon_{\pi} = \frac{F_{\pi^0 8}}{F_{\pi^0 3}}$$ in 10^{-2}

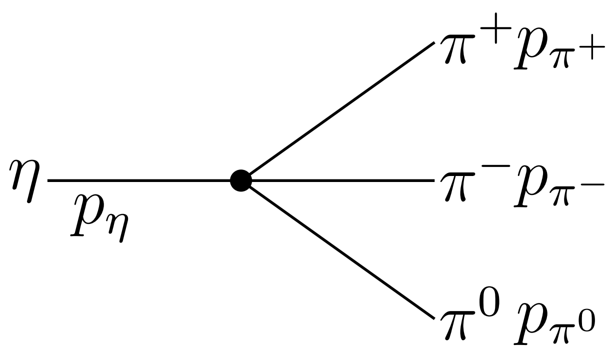
$$\epsilon_{\eta} = \frac{F_{\eta 3}}{F_{\eta 8}}$$ in 10^{-2}

	fit 10	fit B	fit D
ϵ_{0+}	$-1.02 + 0.21 + 0.65$	$-0.71 + 0.15 - 0.05$	$-0.78 + 0.17 - 0.03$
ϵ_{π}	$1.43 + 0.39 - 0.22$	$1.19 + 0.42 - 0.17$	$1.25 + 0.42 - 0.16$
ϵ_{η}	$-1.43 + 0.11 + 0.26$	$-1.19 - 0.15 + 0.49$	$-1.25 - 0.05 + 0.39$

Numbers are: $p^2 + p^4 + p^6$

Eta Decays beyond p^4: Basic

Review: JB, Gasser, Phys.Scripta T99(2002)34 [hep-ph/0202242]



$$\begin{aligned} s &= (p_{\pi^+} + p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2 \\ t &= (p_{\pi^-} + p_{\pi^0})^2 = (p_\eta - p_{\pi^+})^2 \\ u &= (p_{\pi^+} + p_{\pi^0})^2 = (p_\eta - p_{\pi^-})^2 \end{aligned}$$

$$s + t + u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0 \,.$$

$$\langle \pi^0\pi^+\pi^- \text{out}|\eta\rangle = i\,(2\pi)^4\ \delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right)\ A(s,t,u)\,.$$

$$\langle \pi^0\pi^0\pi^0 \text{out}|\eta\rangle = i\,(2\pi)^4\ \delta^4\left(p_\eta - p_1 - p_2 - p_3\right)\ \overline{A}(s_1,s_2,s_3)$$

$$\overline{A}(s_1,s_2,s_3) = A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2)\,,$$

Eta Decays beyond p^4: Lowest order

Pions are in $I=1$ state $\Longrightarrow$ $A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \to \pi^+\pi^-\pi^0\gamma$ needs to be included directly

ChPT:Cronin 67: $A(s,t,u) = \dfrac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \dfrac{3(s-s_0)}{m_\eta^2 - m_\pi^2}\right\}$

or with $Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$, $\quad \hat{m} = \frac{1}{2}(m_u + m_d)$

$$A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\,\frac{M(s,t,u)}{3\sqrt{3}F_\pi^2}\,,$$

with at lowest order $M(s,t,u) = \dfrac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}\,.$

Eta Decays beyond p^4: p^2 and p^4

$\Gamma\left(\eta \to 3\pi\right) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

Other Source from $m^2_{K^+} - m^2_{K^0} \Longrightarrow Q^{-2}$

$$Q = 20.0 \pm 1.5$$

Lowest order prediction $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 140$ eV .

$Q \approx 24$ gives lowest order $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 66$ eV .

At order p^4: $\dfrac{\int dLIPS|A_2 + A_4|^2}{\int dLIPS|A_2|^2} = 2.4\,,$ ($LIPS$=Lorentz invariant phase-space)

Major source: large S-wave final state rescattering

Eta Decays beyond p^4: Dispersive

Try to resum the S-wave rescattering:

Anisovich-Leutwyler (AL), Kambor,Wiesendanger,Wyler (KWW)

Different method but similar approximations

Here: simplified version of AL

Up to p^8: No absorptive parts from $\ell \geq 2$

$\Longrightarrow M(s,t,u) =$

$$M_0(s) + (s-u)M_1(t) + (s-t)M_1(t) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

M_I: “roughly” contributions with isospin 0,1,2

Eta Decays beyond p^4: Dispersive

3 body dispersive: difficult: keep only 2 body cuts

start from $\pi\eta \to \pi\pi$ ($m_\eta^2 < 3m_\pi^2$) standard dispersive analysis

analytically continue to physical m_η^2.

$$M_I(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \, \frac{\mathrm{Im} M_I(s')}{s' - s - i\varepsilon}$$

$$\mathrm{Im} M_I(s') \longrightarrow \mathrm{disc} M_I(s) = \frac{1}{2i}\left(M_I(s+i\varepsilon) - M_I(s-i\varepsilon)\right)$$

$$\begin{aligned}
M_0(s) &= a_0 + b_0 s + c_0 s^2 + \frac{s^3}{\pi}\int \frac{ds'}{s'^3}\frac{\mathrm{disc} M_0(s')}{s'-s-i\varepsilon}\,,\\
M_1(s) &= a_1 + b_1 s + \frac{s^2}{\pi}\int \frac{ds'}{s'^2}\frac{\mathrm{disc} M_1(s')}{s'-s-i\varepsilon}\,,\\
M_2(s) &= a_2 + b_2 s + c_2 s^2 + \frac{s^3}{\pi}\int \frac{ds'}{s'^3}\frac{\mathrm{disc} M_2(s')}{s'-s-i\varepsilon}\,.
\end{aligned}$$

Eta Decays beyond p^4

- Technical complications in solving
- Only 4 relevant constants:

$$M(s,t,u) = a + bs + cs^2 - d(s^2 + tu)$$

$$M_0(s) + \frac{4}{3}M_2(s)\;\; sM_1(s) + M_2(s) + s^2\frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)}$$

converge better

$$\begin{aligned} c &= c_0 + \frac{4}{3}c_2 = \frac{1}{\pi}\int\frac{ds'}{s'^3}\left\{\mathrm{disc}M_0(s') + \frac{4}{3}\mathrm{disc}M_2(s')\right\}, \\ d &= -\frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)} + \frac{1}{\pi}\int\frac{ds'}{s'^3}\left\{s'\mathrm{disc}M_1(s') + \mathrm{disc}M_2(s')\right\} \end{aligned}$$

Fix a, b by matching to tree level or p^4 amplitude

Eta beyond p^4: technical trouble

$$M_I(s) = \frac{1}{\pi}\int \frac{ds'}{s'-s-i\varepsilon}\sin\delta_I(s)e^{-i\delta_I(s)}\left\{M_I(s)+\hat{M}_I(s)\right\}$$

$\hat{M}_I(s) = \sum_{n,I'}\int_{-1}^{1} d\cos\theta\ \cos^n\theta c_{nII'}M_{I'}(t)$: t,u channel

need all s in dispersion $\Rightarrow$ t,u outside physical domain, cuts in plane $\Rightarrow$ choose path carefully

$$M_I(s) = \frac{1}{\pi}\int \frac{ds'}{s'-s-i\varepsilon}\sin\delta_I(s)e^{-i\delta_I(s)}\left\{M_I(s)+\hat{M}_I(s)\right\}$$

has nontrivial solutions $\Rightarrow$ need to pick the physically correct one

Disperse in $m_I = M_I/\Omega_I$ $\quad \Omega_I(s) = \exp\left\{\frac{s}{\pi}\int\frac{ds'}{s'}\frac{\delta_I(s')}{s'-s-i\varepsilon}\right\}$

Eta Decays beyond p^4

$$\frac{M_0(s)}{\Omega_0(s)} = \alpha_0 + \beta_0 s + \gamma_0 s^2 + \frac{s^2}{\pi}\int ds' \frac{\sin\delta_0(s')\,\hat{M}_0(s')}{|\Omega_0(s')|s'^2(s'-s-i\varepsilon)}\,,$$
$$\frac{M_1(s)}{\Omega_1(s)} = \beta_1 s + \frac{s}{\pi}\int ds' \frac{\sin\delta_1(s')\,\hat{M}_1(s')}{|\Omega_1(s')|s'(s'-s-i\varepsilon)}\,,$$
$$\frac{M_2(s)}{\Omega_2(s)} = \frac{s^2}{\pi}\int ds' \frac{\sin\delta_2(s')\,\hat{M}_2(s')}{|\Omega_2(s')|s'^2(s'-s-i\varepsilon)}\,. \qquad \text{(-3)}$$

find a $\delta_{0,1,2}(s) \Longrightarrow$ solve for M_1, M_2, M_3.
fix $\alpha_0, \beta_0, \gamma_0, \beta_1$

$$\gamma_0 \approx 0 \qquad \beta_1 \approx -\frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)}$$

α_0, β_0 values depend on where in s, t, u plane matching is done

Eta Decays beyond p^4

AL: Lowest order is $M(s,t,u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$

zero at $s_4/3\,m_\pi^2$: remains in the neighbourhood:

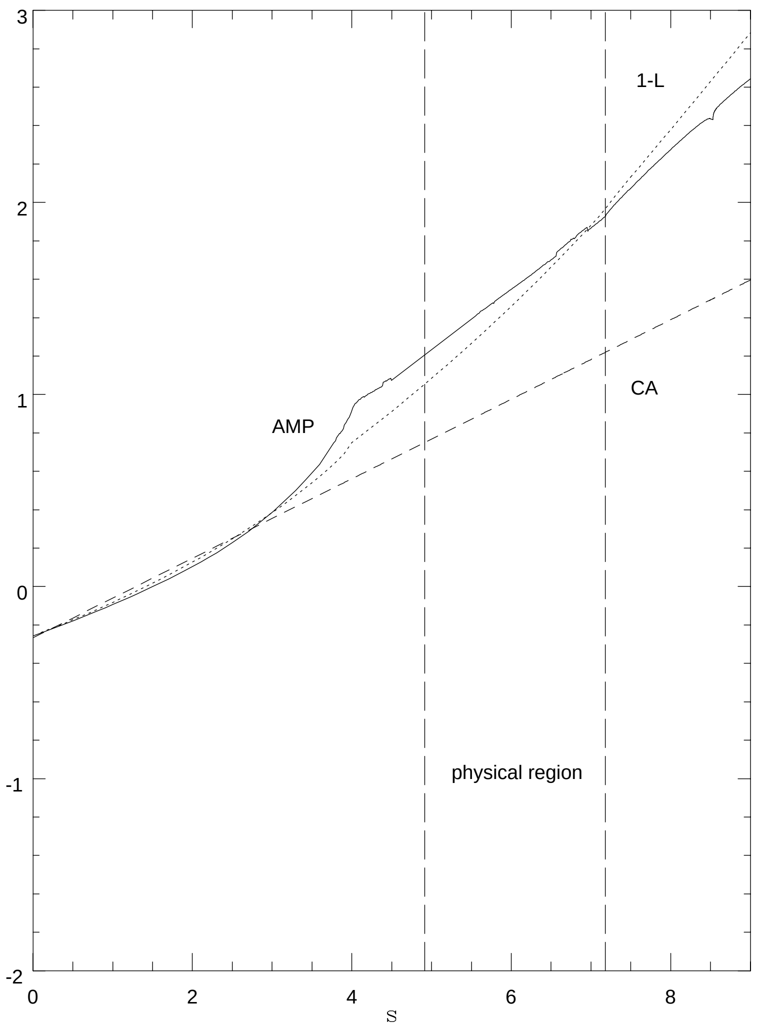
match position of s_A and slope of Adler zero.

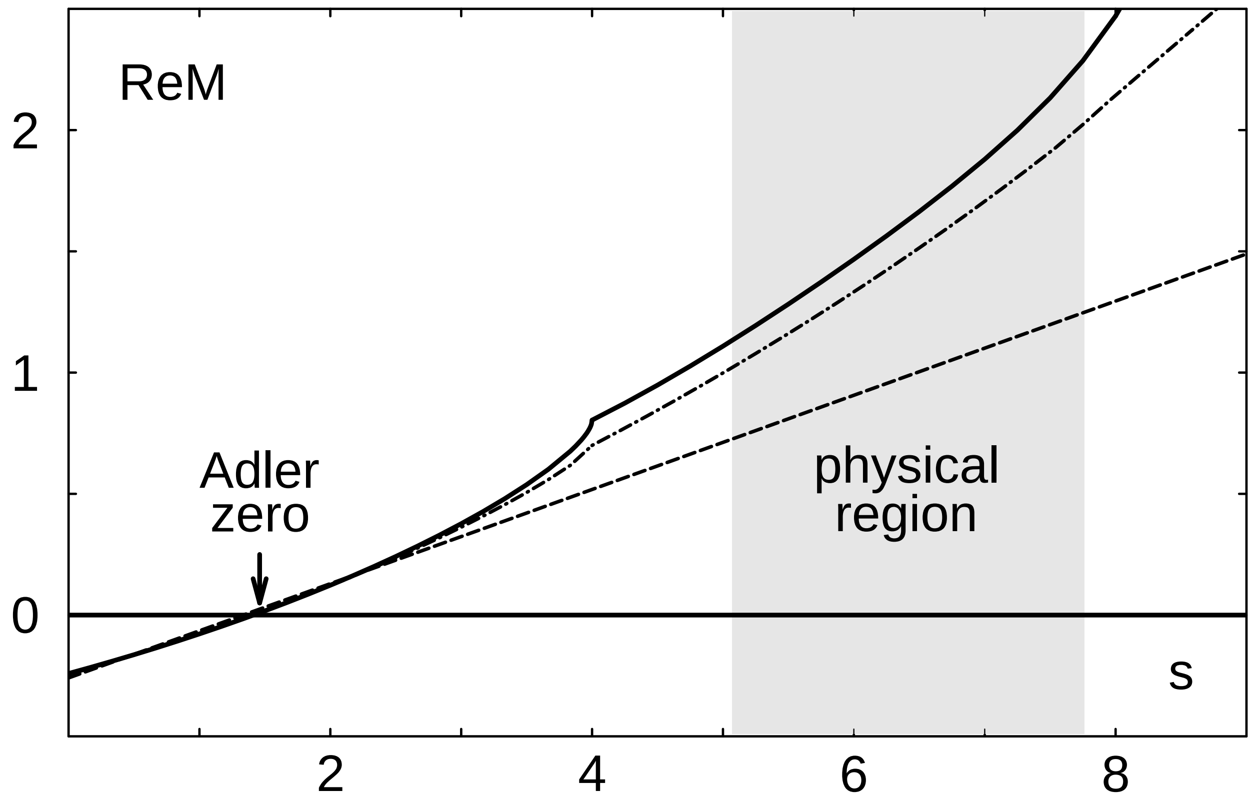
KWW: fix amplitude at some place(s) in s,t,u plane to be equal to p^4

$s = u$ of AL $\Longleftrightarrow$ KWW and simplified with $\hat{M}_I = 0$

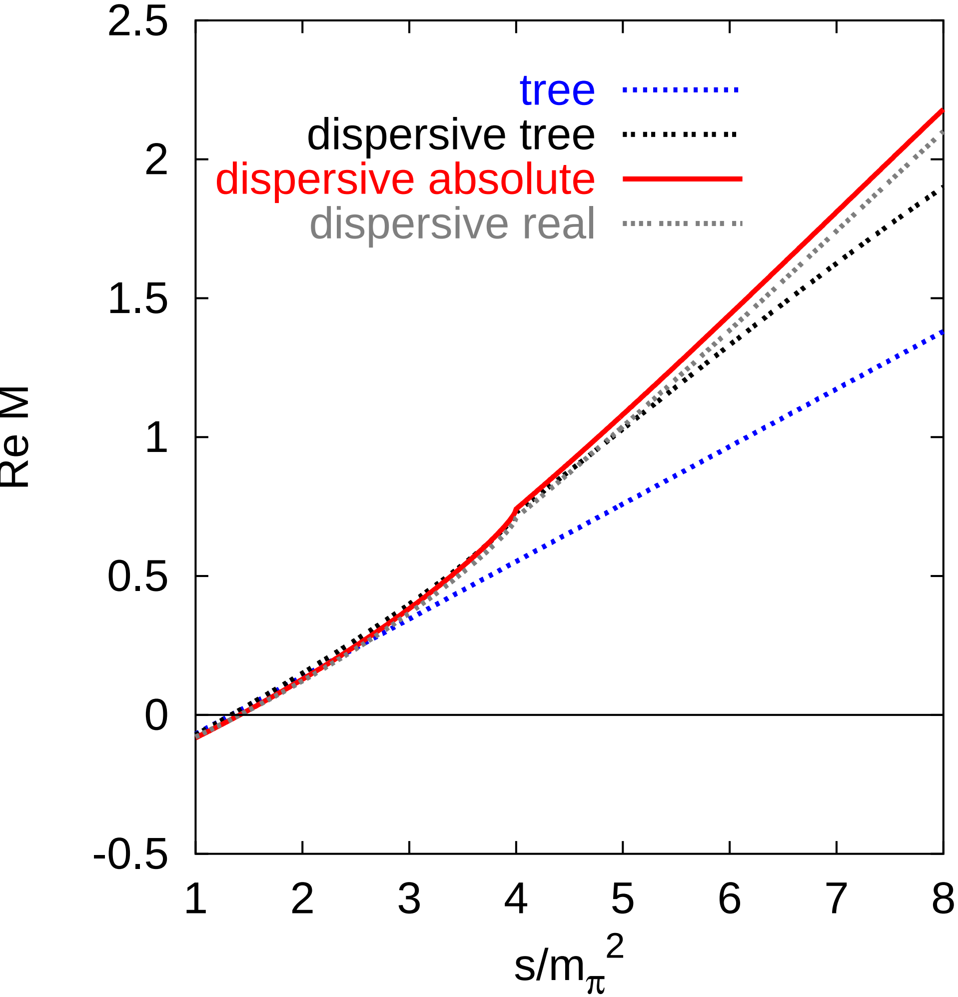
Dalitzplot distributions provide a check

Eta Decays beyond p^4





Eta Decays beyond p^4



Eta Decays beyond p^4

$$\begin{cases} x &= \sqrt{3}\,\frac{T_+ - T_-}{Q_\eta} = \frac{\sqrt{3}}{2M_\eta Q_\eta}(u-t)\,, \\ y &= \frac{3T_0}{Q_\eta} - 1 = \frac{3}{2m_\eta Q_\eta}\left\{(m_\eta - m_{\pi^0})^2 - s\right\} - 1\,, \\ Q_\eta &= m_\eta - 2m_\pi^+ - m_\pi^0 \end{cases}$$

$$r \equiv \Gamma(\eta \to \pi^0\pi^0\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0) = 1.44 \pm 0.04(PDG2004)\,,$$

charged decay: $1 + ay + by^2 + cx^2$, normalized at $x = y = 0$

Neutral decay: $1 + g(x^2 + y^2)$

	a	b	c	g
tree	-1.00	0.25	0.00	0.000
one-loop	-1.33	0.42	0.08	0.03[a]
dispersive (KWW)	-1.16	0.26	0.10	-0.014 — -0.028
tree dispersive	-1.10	0.31	0.001	-0.013
absolute dispersive	-1.21	0.33	0.04	-0.014

Eta decays beyond p^4

Experimental results for the charged decay.

	a	b	c
Layter	-1.08 ± 0.14	0.034 ± 0.027	0.046 ± 0.031
Gormley	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04
Crystal Barrel	-0.94 ± 0.15	0.11 ± 0.27	
Crystal Barrel	-1.22 ± 0.07	0.22 ± 0.11	0.06 fixed
KLOE	-1.072 ± 0.009	0.117 ± 0.008	0.047 ± 0.008

Experimental results for the the neutral decay.

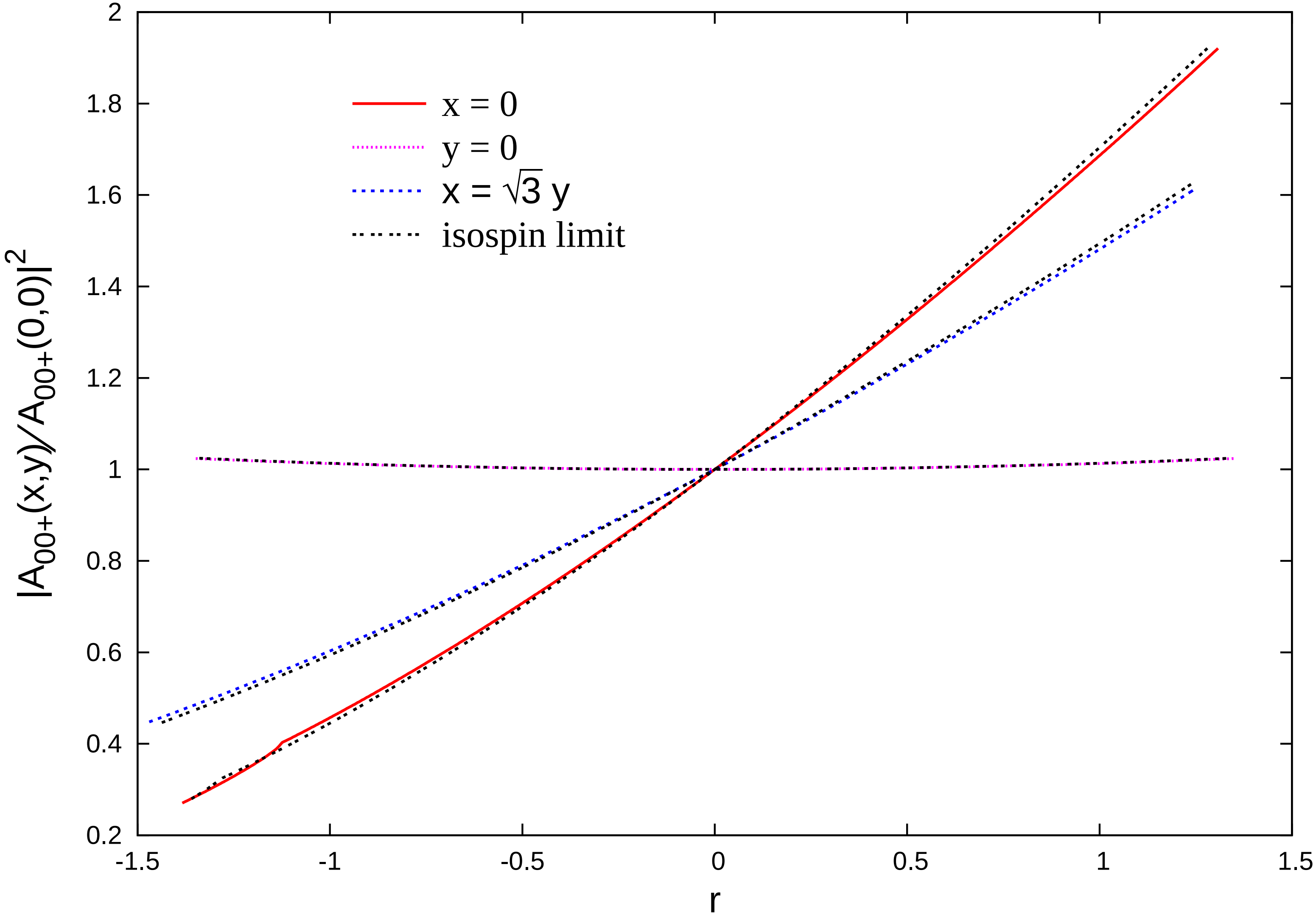
	g
Alde	-0.044 ± 0.046
Crystal Barrel	-0.104 ± 0.039
Crystal Ball	-0.062 ± 0.008
SND	-0.020 ± 0.023

Isospin Breaking in $K \to 3\pi$

- $K \to 3\pi$ Decays in Chiral Perturbation Theory, J. Bijnens, P. Dhonte and F. Persson, hep-ph/0205341, Nucl. Phys. B648 (2003) 317-344.
- Isospin Breaking in $K \to 3\pi$ Decays I: Strong Isospin Breaking, J. Bijnens and F. Borg, hep-ph/0405025, Nuclear Physics B697 (2004) 319-342.
  e^2p^2 also by Gamiz, Prades, Scimemi, JHEP 0310 (2003) 042
- Isospin Breaking in $K \to 3\pi$ Decays II: Radiative Corrections, J. Bijnens and F. Borg, hep-ph/0410333, Eur. Phys. J. C39 (2005) 347
- Isospin Breaking in $K \to 3\pi$ Decays III: Bremsstrahlung and Fit to Experiment, J. Bijnens and F. Borg, hep-ph/0501163, Eur. Phys. J. C40 (2005) 383

Note: Fredrik Borg = Fredrik Persson, obtained PhD 28/1/2005

$K^+ \to \pi^0\pi^0\pi^+$



Note: Disagree numerically with Nehme, hep-ph/0406209

Experimental Inputs

- Decay Rates
- Dalitz Plot Parameters

$$\left|\frac{A(s_1,s_2,s_3)}{A(s_0,s_0,s_0)}\right|^2 = 1 + gy + hy^2 + kx^2$$

g, h, k for $K_L \to \pi^+\pi^-\pi^0$,
$K^+ \to \pi^0\pi^0\pi^+$,
$K^+ \to \pi^+\pi^+\pi^-$

h for $K_L \to \pi^0\pi^0\pi^0$
($g = 0$ and $k = h/3$)

$A^S_{+-0} = \gamma_S x - \xi_S xy$ for $K_S \to \pi^+\pi^-\pi^0$

- All input taken from PDG2004

Results

$K \to \pi\pi$ only: essentially same as Ecker et al.

Tree level: $G_8 = 10.36 \quad G_{27} = 0.550$

Full: $G_8 = 5.39 \quad G_{27} = 0.359$

Results

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Full: $G_8 = 5.39$ $G_{27} = 0.359$

Full fit:

μ	0.77 GeV	1.0 GeV	0.6 GeV	0.77 GeV
G_8	5.39(1)	4.60(1)	6.43(1)	5.39(1)
G_{27}	0.359(2)	0.301(1)	0.438(2)	0.359(2)
$\delta_2 - \delta_0$	$-57.9(1.5)^o$	$-57.3(1.4)^o$	$-58.9(1.4)^o$	$-57.9(1.4)^o$
$10^3 \tilde{K}_1/G_8$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$
$10^3 \tilde{K}_2/G_8$	48.5(2.4)	56.5(2.4)	41.2(1.9)	46.6(1.6)
$10^3 \tilde{K}_3/G_8$	2.6(1.2)	$-$1.7(1.1)	6.7(1.0)	3.5(0.8)
$10^3 \tilde{K}_4/G_{27}$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$
$10^3 \tilde{K}_5/G_{27}$	$-$41.2(16.9)	$-$52.0(17.7)	$-$31.1(12.0)	$-$27.0(8.3)
$10^3 \tilde{K}_6/G_{27}$	$-$102(105)	$-$114(105)	$-93(76)$	$\equiv 0$
$10^3 \tilde{K}_7/G_{27}$	78.6(33)	78.0(33.5)	79.6(22.7)	50.0(13.0)
χ^2/DOF	29.3/10	27.2/10	33.0/10	30.5/11

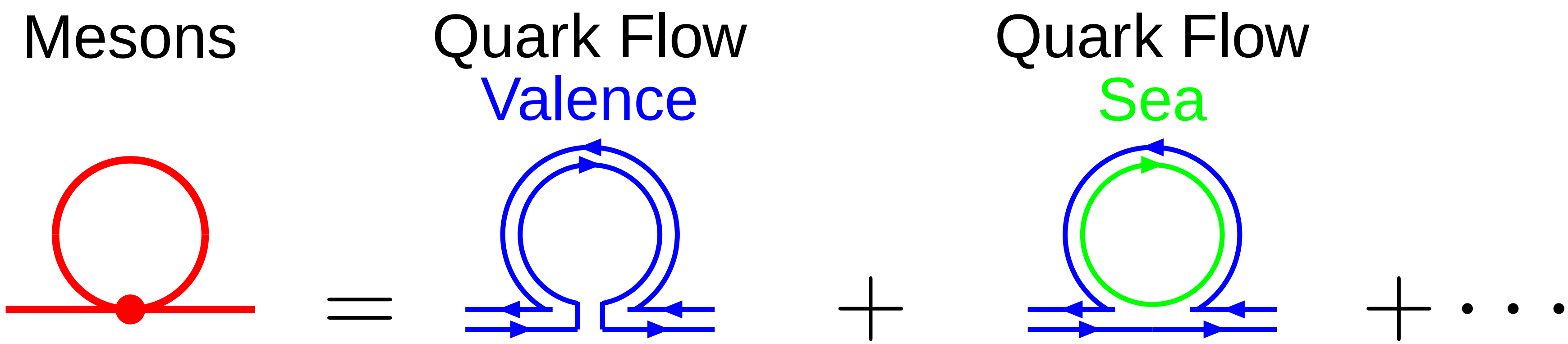
vary $\tilde{K}_1$, $\tilde{K}_4$ see first paper

Data and Fits

Decay	Width [GeV]	ChPT [GeV]	Fact. [GeV]
$K^+ \to \pi^+\pi^0$	$(1.1231 \pm 0.0078) \cdot 10^{-17}$	$1.123 \cdot 10^{-17}$	$1.127 \cdot 10^{-17}$
$K_S \to \pi^0\pi^0$	$(2.2828 \pm 0.0104) \cdot 10^{-15}$	$2.282 \cdot 10^{-15}$	$2.283 \cdot 10^{-15}$
$K_S \to \pi^+\pi^-$	$(5.0691 \pm 0.0108) \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$
$K_L \to \pi^0\pi^0\pi^0$	$(2.6748 \pm 0.0358) \cdot 10^{-18}$	$2.618 \cdot 10^{-18}$	$2.698 \cdot 10^{-18}$
$K_L \to \pi^+\pi^-\pi^0$	$(1.5998 \pm 0.0271) \cdot 10^{-18}$	$1.658 \cdot 10^{-18}$	$1.711 \cdot 10^{-18}$
$K^+ \to \pi^0\pi^0\pi^+$	$(9.195 \pm 0.0213) \cdot 10^{-19}$	$8.934 \cdot 10^{-19}$	$8.816 \cdot 10^{-19}$
$K^+ \to \pi^+\pi^+\pi^-$	$(2.9737 \pm 0.0174) \cdot 10^{-18}$	$2.971 \cdot 10^{-18}$	$2.933 \cdot 10^{-18}$

Decay	Quantity	Experiment	ChPT	Fact.
$K_L \to \pi^0\pi^0\pi^0$	h	-0.0050 ± 0.0014	-0.0062	-0.0025
$K_L \to \pi^+\pi^-\pi^0$	g	0.678 ± 0.008	0.678	0.654
	h	0.076 ± 0.006	0.088	0.083
	k	0.0099 ± 0.0015	0.0057	0.0068
$K_S \to \pi^+\pi^-\pi^0$	γ_S	$(3.3 \pm 0.5) \cdot 10^{-8}$	$3.0 \cdot 10^{-8}$	$2.9 \cdot 10^{-8}$
$K^\pm \to \pi^0\pi^0\pi^\pm$	g	0.638 ± 0.020	0.636	0.648
	h	0.051 ± 0.013	0.077	0.080
	k	0.004 ± 0.007	0.0047	0.0069
$K^+ \to \pi^+\pi^+\pi^-$	g	-0.2154 ± 0.0035	-0.215	-0.226
	h	0.012 ± 0.008	0.012	0.019
	k	-0.0101 ± 0.0034	-0.0034	-0.0033

ChPT and Lattice QCD



Valence is easy to deal with in lattice QCD
Sea is very difficult

They can be treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

PQChPT at Two Loop

Subject started:
valence equal mass, 3 sea equal mass:
$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0406017
Other mass combinations:
$F^2_{\pi^+}$: JB,Lähde, hep-lat/0501014
$F^2_{\pi^+}$, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004
In progress: the other charged masses

Actual Calculations:
- heavy use of FORM Vermaseren
- use PQ without super Φ_0 in supersymmetric formalism
- Main problem: sheer size of the expressions

Iso breaking from lattice data: a and L extrapolations needed

PQChPT at Two Loop

Problem: Plotting with many input parameters

Plot masses as a function of lowest order mass squared

$$\chi_i = 2B_0 m_i = m_M^{2(0)}$$

Remember: $\chi_i \approx 0.3\ GeV^2 \approx (550\ MeV)^2 \sim$ border ChPT

1+1 case: Valence: $\chi_1 = \chi_2 = \chi_3$
Sea: $\chi_4 = \chi_5 = \chi_6$

PQChPT at Two Loop

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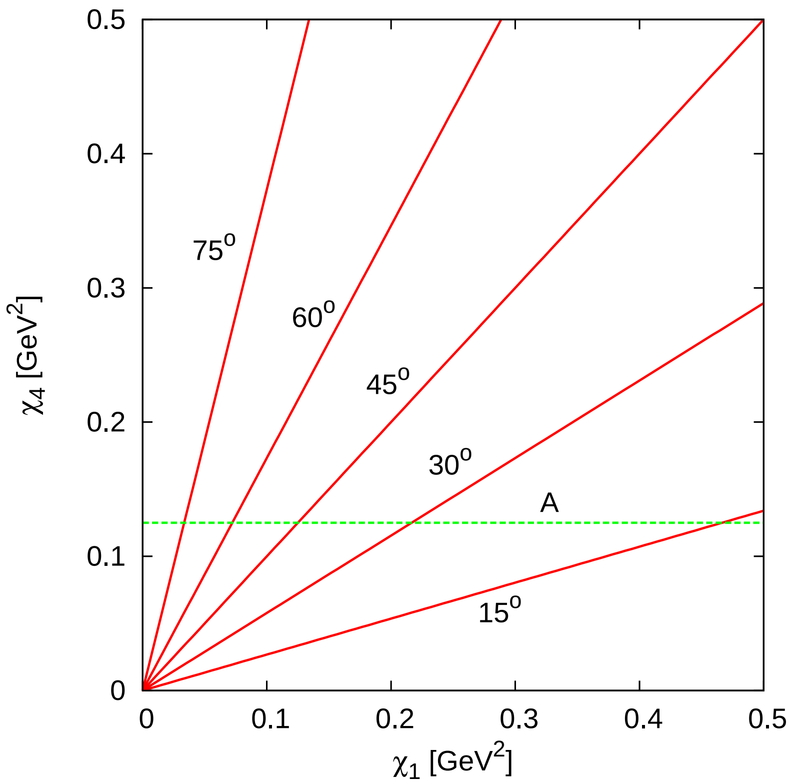
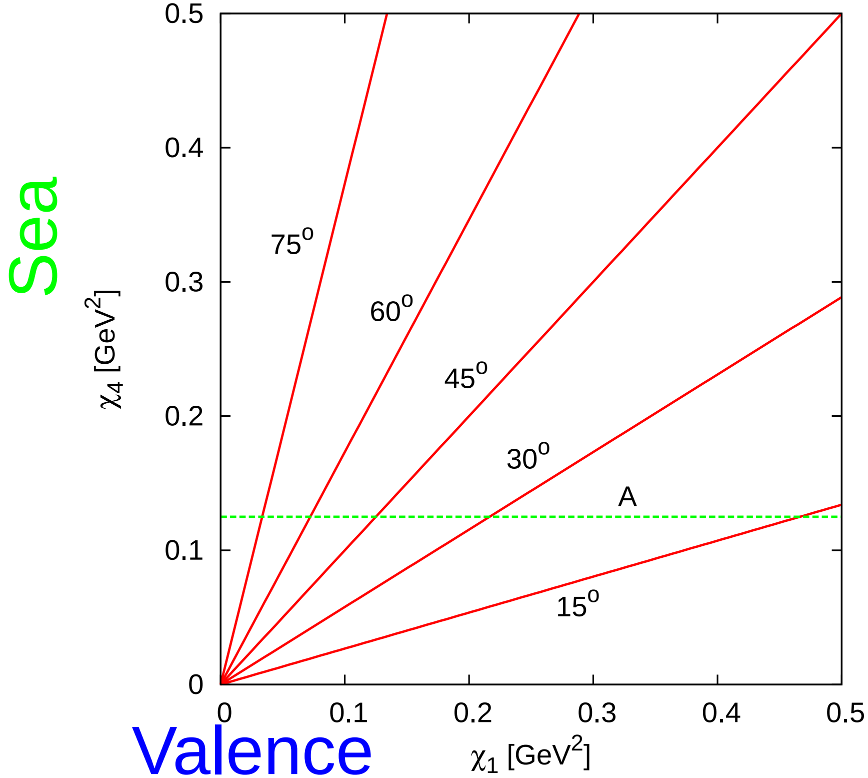
1+1 case: Valence: $\chi_1 = \chi_2 = \chi_3$
Sea: $\chi_4 = \chi_5 = \chi_6$

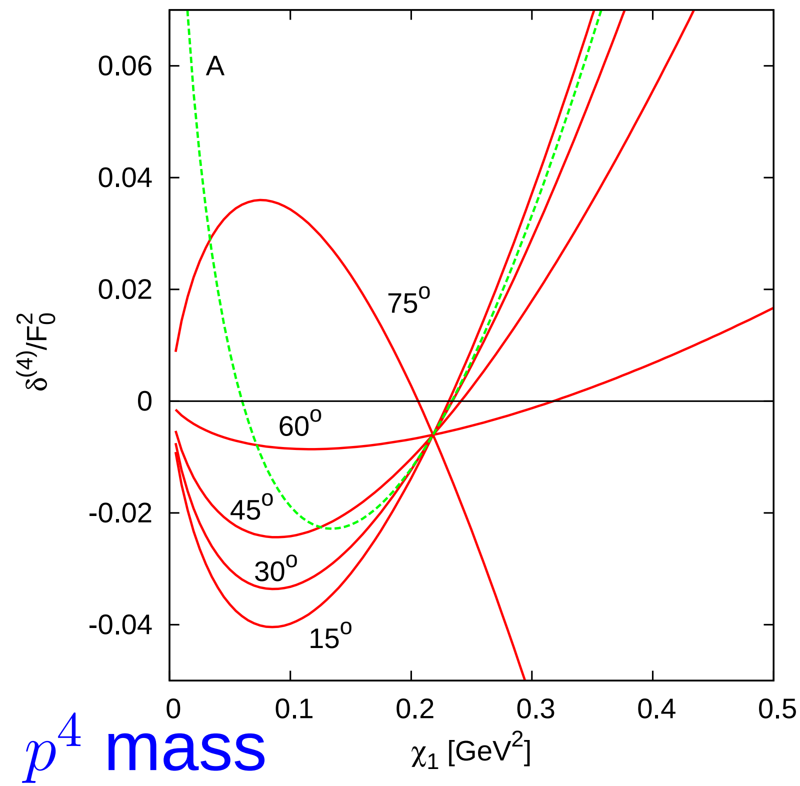
Plot along curves: $\chi_4 = \tan\theta\ \chi_1$ or χ_4 constant

PQChPT: 1+1 case

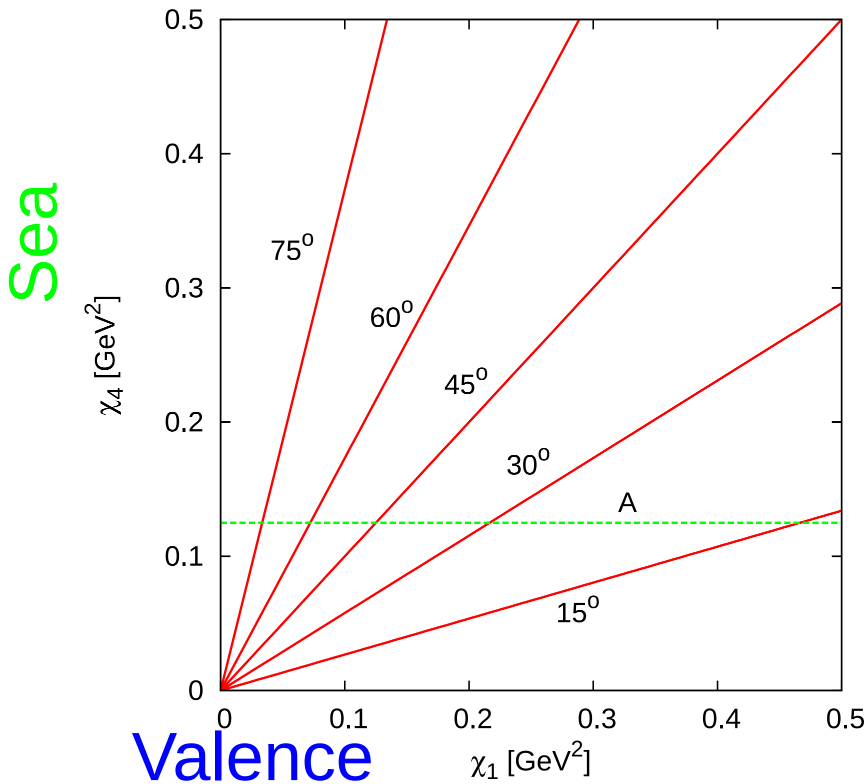


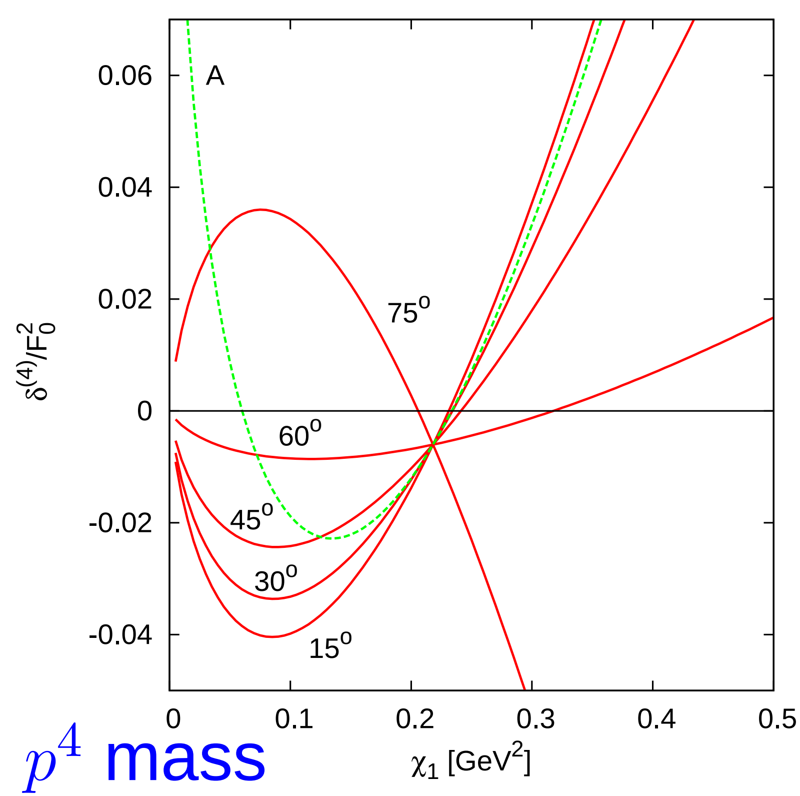
PQChPT: 1+1 case





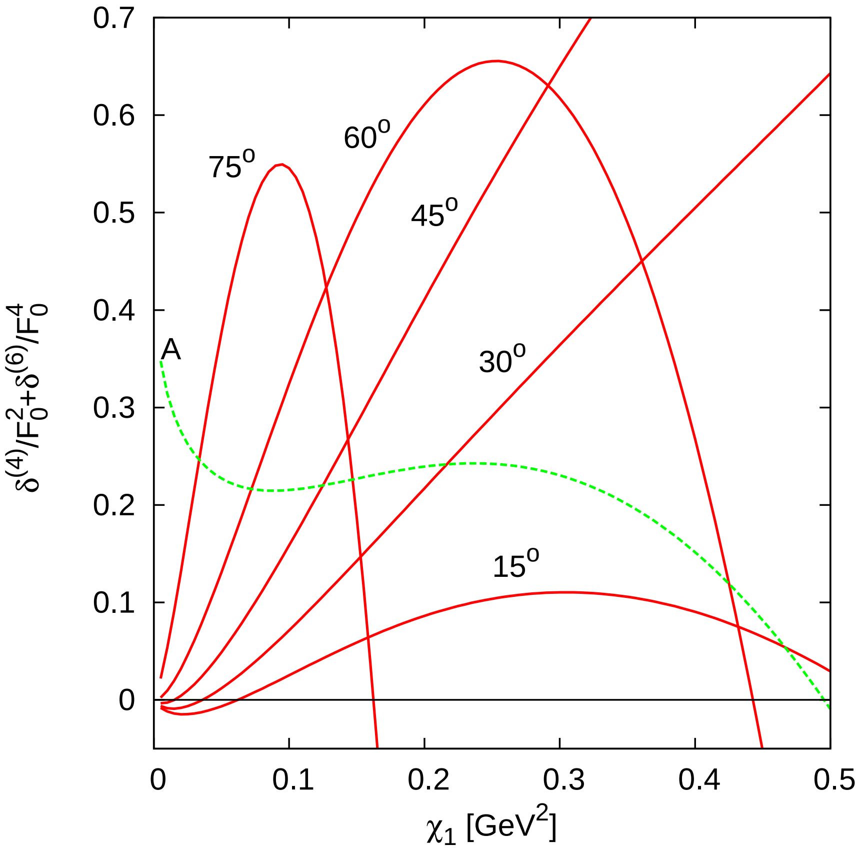
PQChPT: 1+1 case





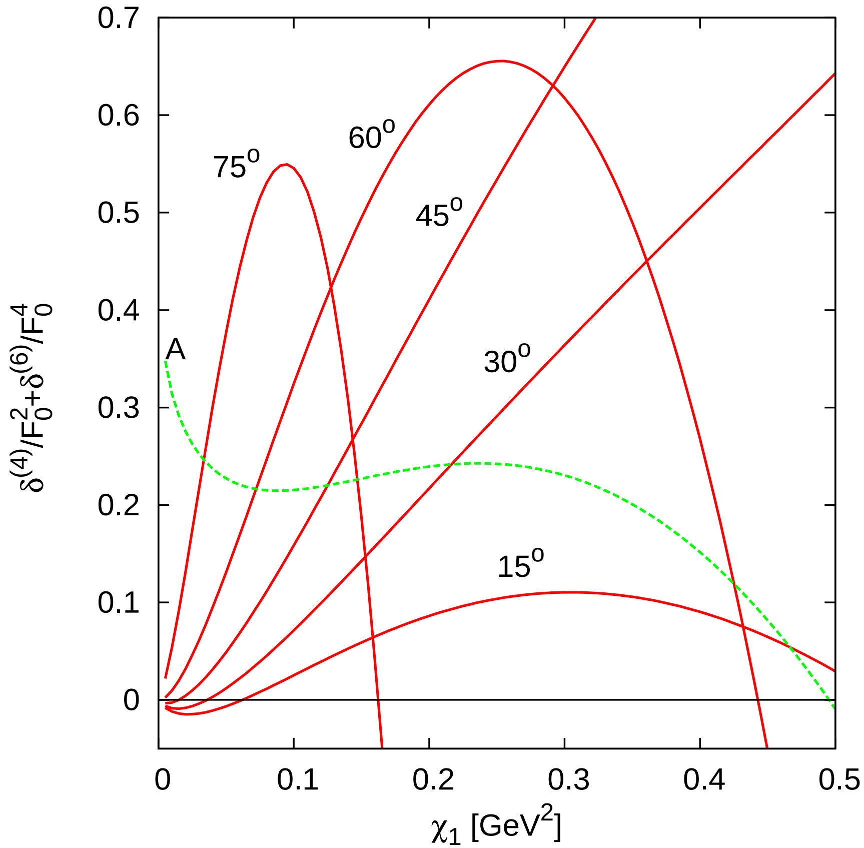
Notice the Quenched Chiral Logs: $\frac{m_\pi^2}{\chi_1} = 1 + \frac{a}{F^2}\chi_4 \log \chi_1 + \cdots$

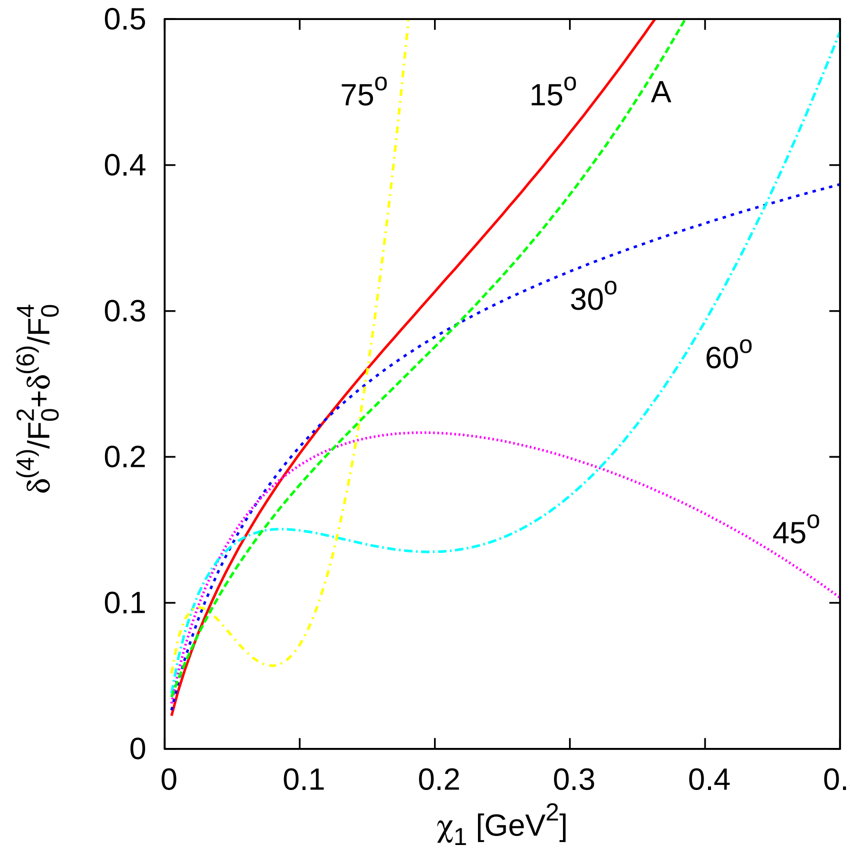
PQChPT at Two Loops



$p^4 + p^6$ relative correction mass

PQChPT at Two Loops



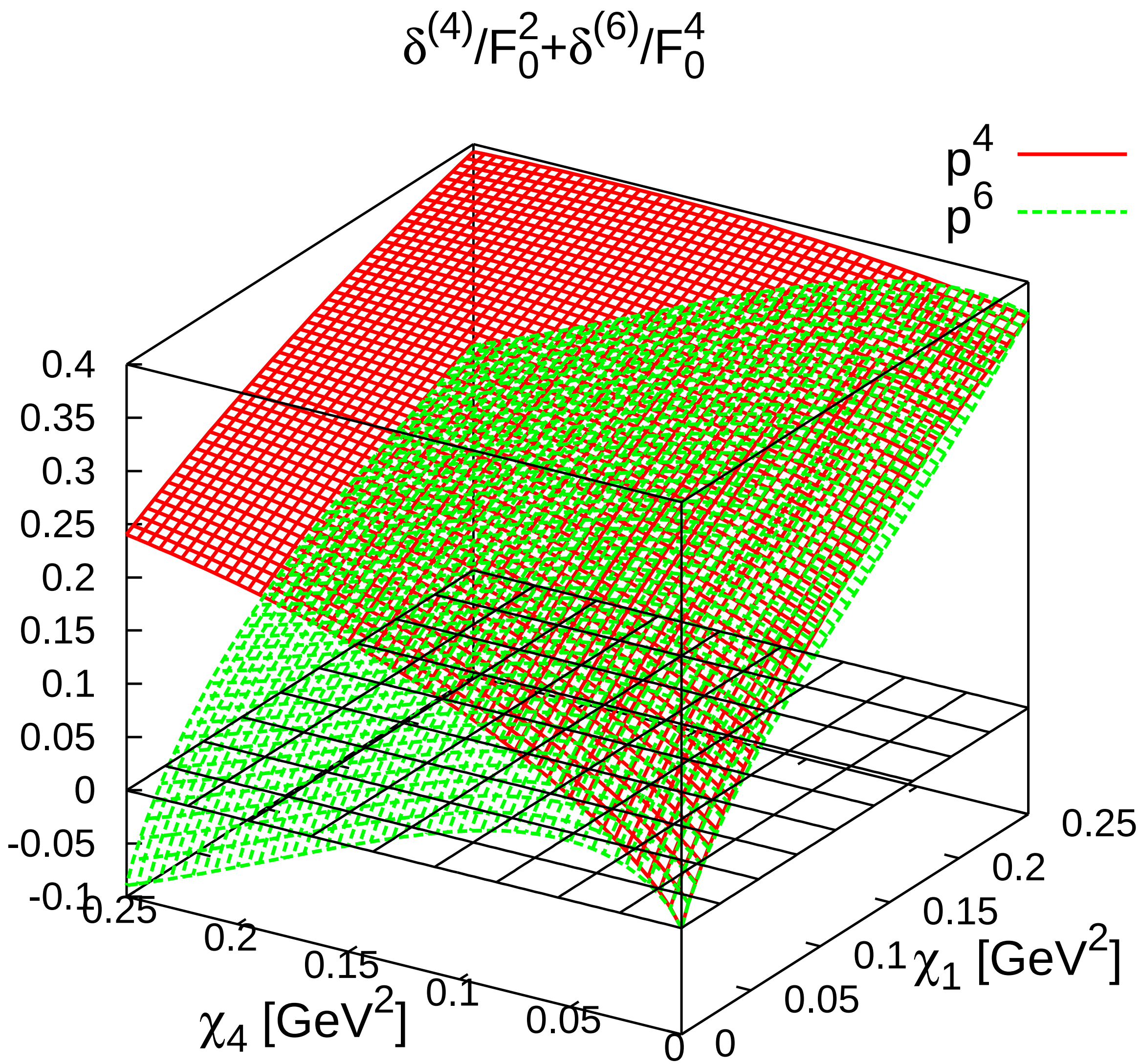


$p^4 + p^6$ relative correction mass

decay constant

(preliminary)

PQChPT at Two Loops



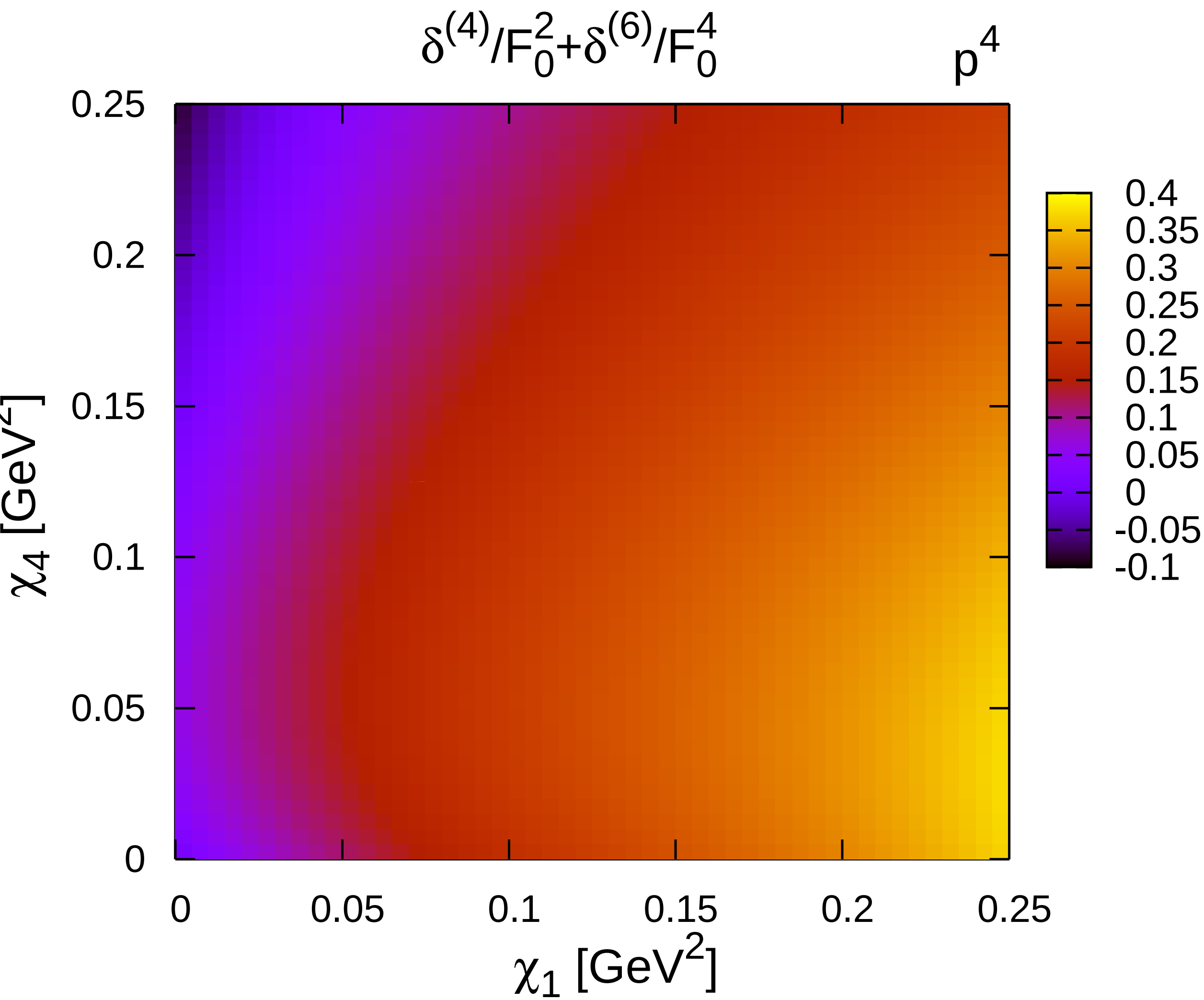
$p^4 + p^6$

correction

decay constant

fit 10

PQChPT at Two Loops



$p^4 + p^6$
correction
decay constant
fit 10

Conclusions

- 3 flavour ChPT at 2 loops
  - many calculations done
  - things seem to work but convergence is fairly slow
  - “kinematical” and “vector” C_i^r seem to be OK
  - L_4^r, L_6^r nonzero but reasonable for large N_c
  - $\eta \to 3\pi$, isobreaking in $K_{\ell 3}$: parts done
- PQChPT at 2 loops: subject just beginning
- $\eta \to 3\pi$: isospin breaking part needed to p^6, compare with dispersive (see $K_{\ell 4}$)
- $K \to 3\pi$: CP violating observables, higher orders for Cabibbo proposal