STATUS OF STRONG CHIRAL PERTURBATION THEORY

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Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- 50, 40, 35, 30, 25, 20 and 15 years ago
- Chiral Perturbation Theory (ChPT, CHPT, χPT)
- Expand in which quantities
- Two-flavour ChPT at NNLO: one mass
  - Calculations
  - LECs and Quark-mass dependence of m_π^2, F_π
- Three-flavour ChPT at NNLO: 3-5 masses
  - Calculations
  - Fits to data; some quark mass dependences
  - What about p^6 LECs; $\eta \to 3\pi$
- Even more flavours at NNLO (Partially Quenched)
- Renormalization group

Jubileum Papers: 50 years

The start:

- M. Goldberger and S. Treiman, *Decay of the pi meson*. Phys. Rev. 110:1178-1184,1958. (330 citations)
- Y. Nambu, *Axial Vector Current Conservation in Weak Interactions*, Phys. Rev. Lett. 4 (1960) 380 (530 citations)
- M. Gell-Mann and M. Lévy, *The axial vector current in beta decay*. Nuovo Cim. 16 (1960) 705 (1229 citations)

Jubileum Papers: 40 and 35 years

Tree level and beginning of loops:

- M. Gell-Mann, R.J. Oakes and B. Renner, *Behavior of current divergences under* $SU(3) \times SU(3)$, Phys. Rev. 175 (1968) 2195 (1264 citations)
- S. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. 1.*, Phys. Rev. 177 (1969) 2239 (1091 citations)
- C. Callan, S. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. 2.*, Phys. Rev. 177 (1969) 2247 (932 citations)
- P. Langacker and H. Pagels, *Applications of Chiral Perturbation Theory: Mass Formulas and the Decay* $\eta \to 3\pi$ Phys.Rev.D10:2904,1974.

Jubileum Papers: 30 and 25 years

The restart:

- Steven Weinberg, *Phenomenological Lagrangians*, Physica A96 (1979) 327 (1884 citations)
- J. Gasser and A. Zepeda, *Approaching The Chiral Limit In QCD*, Nucl. Phys. B174 (1980) 445 (preprint in 1979)
- Juerg Gasser and Heiri Leutwyler, *Chiral Perturbation Theory to One Loop*, Annals Phys. 158 (1984) 142 (2407 citations)
- Juerg Gasser and Heiri Leutwyler, *Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark* Nucl. Phys. B250 (1985) 465 (2431 citations)
- J. Bijnens, H. Sonoda and M. Wise, *On the Validity of Chiral Perturbation Theory for* K^0-$\overline{K^0}$ *Mixing* , Phys. Rev. Lett. 53 (1984) 2367 Here is where I started

Jubileum Papers: 20 and 15 years

LECs from elsewhere and first full two-loop:

- G. Ecker, J. Gasser, A. Pich and E. de Rafael, *The Role of Resonances in Chiral Perturbation Theory*, Nucl. Phys. B321 (1989) 311 (826 citations)
- G. Ecker,J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, *Chiral Lagrangians for Massive Spin 1 Fields*, Phys. Lett. B223 (1989) 425 (462 citations)
- S. Bellucci, J. Gasser and M.E. Sainio, *Low-energy photon-photon collisions to two loop order*, Nucl. Phys. B423 (1994) 80
- H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory*, Ann. Phys. 235 (1994) 165

Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

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Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory*, Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

For lectures, review articles: see
http://www.thep.lu.se/∼bijnens/chpt.html

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting in momenta/masses

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

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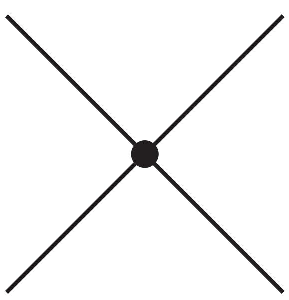
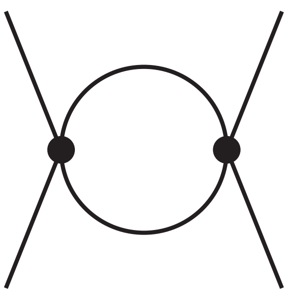
Power counting in momenta: Meson loops



p^2

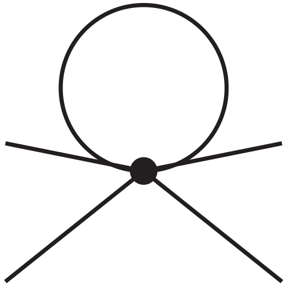


$(p^2)^2 \, (1/p^2)^2 \, p^4 = p^4$

$1/p^2$

$\int d^4p$

p^4



$(p^2) \, (1/p^2) \, p^4 = p^4$

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
- . . .
- $\Longrightarrow$ shortage of letters for the constants in the Lagrangians (LECs)

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
  - Two or Three (or even more) Flavours
  - Strong interaction and couplings to external currents/densities
  - Including (internal) electromagnetism
  - Including weak nonleptonic interactions
  - Treating kaon as heavy

Lagrangians

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta_8}{\sqrt{6}} \end{pmatrix} .$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}\,,$

$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu\,,$
left and right external currents: $r(l)_\mu = v_\mu + (-) a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$
quark masses via scalar density: $s = \mathcal{M} + \cdots$

$\langle A \rangle = Tr_F\,(A)$

Lagrangians

$$\mathcal{L}_4 = L_1\langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2\langle D_\mu U^\dagger D_\nu U\rangle\langle D^\mu U^\dagger D^\nu U\rangle$$
$$+L_3\langle D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U\rangle + L_4\langle D^\mu U^\dagger D_\mu U\rangle\langle\chi^\dagger U + \chi U^\dagger\rangle$$
$$+L_5\langle D^\mu U^\dagger D_\mu U(\chi^\dagger U + U^\dagger\chi)\rangle + L_6\langle\chi^\dagger U + \chi U^\dagger\rangle^2$$
$$+L_7\langle\chi^\dagger U - \chi U^\dagger\rangle^2 + L_8\langle\chi^\dagger U\chi^\dagger U + \chi U^\dagger\chi U^\dagger\rangle$$
$$-iL_9\langle F^R_{\mu\nu} D^\mu U D^\nu U^\dagger + F^L_{\mu\nu} D^\mu U^\dagger D^\nu U\rangle$$
$$+L_{10}\langle U^\dagger F^R_{\mu\nu} U F^{L\mu\nu}\rangle + H_1\langle F^R_{\mu\nu} F^{R\mu\nu} + F^L_{\mu\nu} F^{L\mu\nu}\rangle + H_2\langle\chi^\dagger\chi\rangle$$

L_i: Low-energy-constants (LECs)
H_i: Values depend on definition of currents/densities

These absorb the divergences of loop diagrams: $L_i \to L_i^r$
Renormalization: order by order in the powercounting

Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour special case of 3+3 PQ: $\hat{L}_i^r, K_i^r \rightarrow L_i^r, C_i^r$
- 53 $\rightarrow$ 52 arXiv:0705.0576 [hep-ph]

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms
- includes Isospin and the eightfold way ($SU(3)_V$)

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2}\log\frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu)\right] + \cdots$$

$M^2 = 2B\hat{m}$

$B \neq B_0$, $F \neq F_0$ (two versus three-flavour)

LECs and μ

$l_3^r(\mu)$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i}\, l_i^r(\mu) - \log \frac{M_\pi^2}{\mu^2}\,.$$

Independent of the scale μ.

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

μ :

- m_π, m_K: chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$ large N_c arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exists
- Express orders in terms of physical masses and quantities (F_π, F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering

I prefer physical masses

- Thresholds correct
- Chiral logs are from physical particles propagating

An example

$$m_\pi = \frac{m_0}{1 + a\frac{m0}{f0}} \qquad f_\pi = \frac{f_0}{1 + b\frac{m0}{f0}}$$

An example

$$m_\pi = \frac{m_0}{1 + a\frac{m0}{f0}} \qquad f_\pi = \frac{f_0}{1 + b\frac{m0}{f0}}$$

$$m_\pi = m_0 - a\frac{m_0^2}{f_0} + a^2\frac{m_0^3}{f_0^2} + \cdots$$

$$f_\pi = f_0\left(1 - b\frac{m_0}{f_0} + b^2\frac{m_0^2}{f_0^2} + \cdots\right)$$

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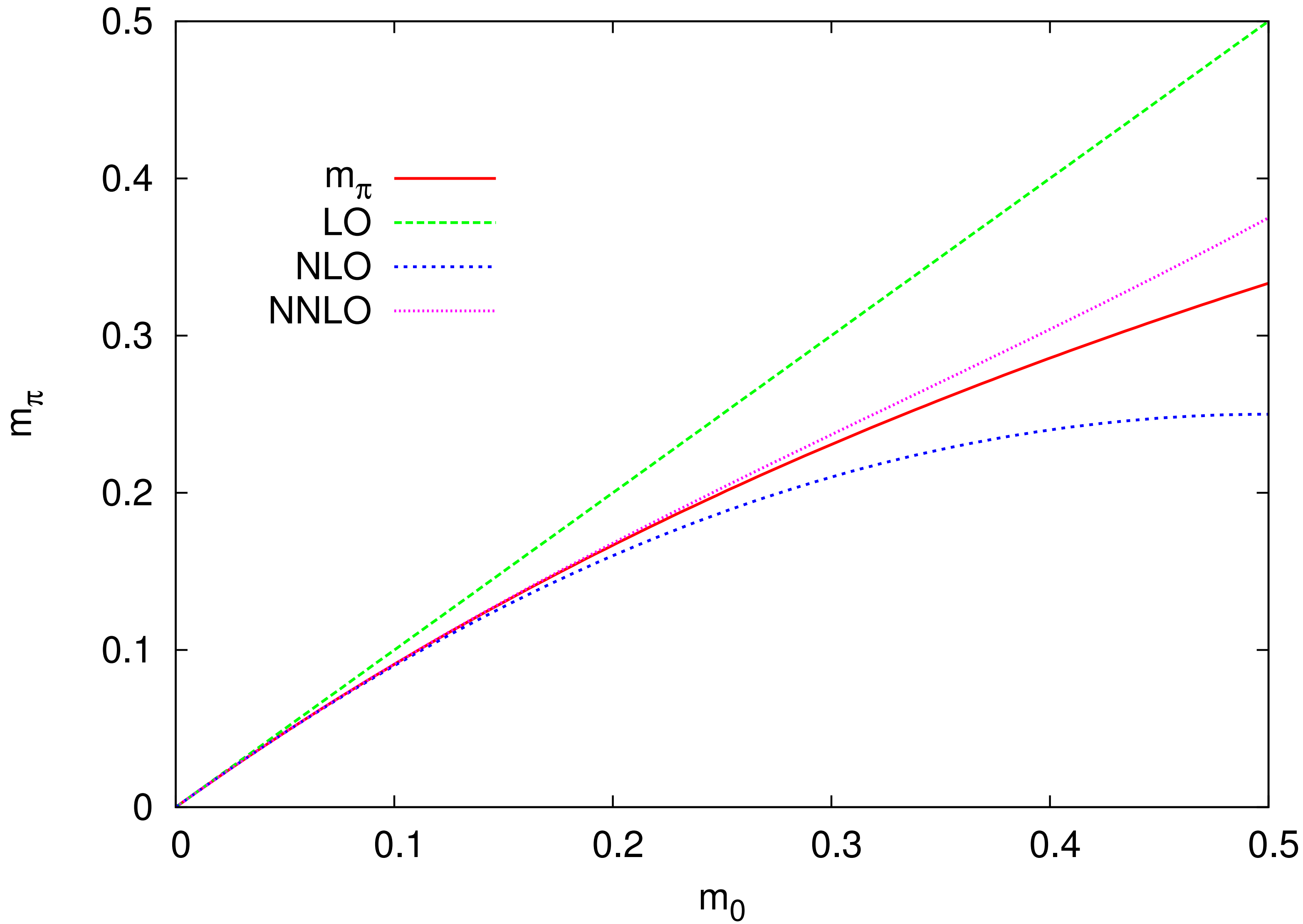
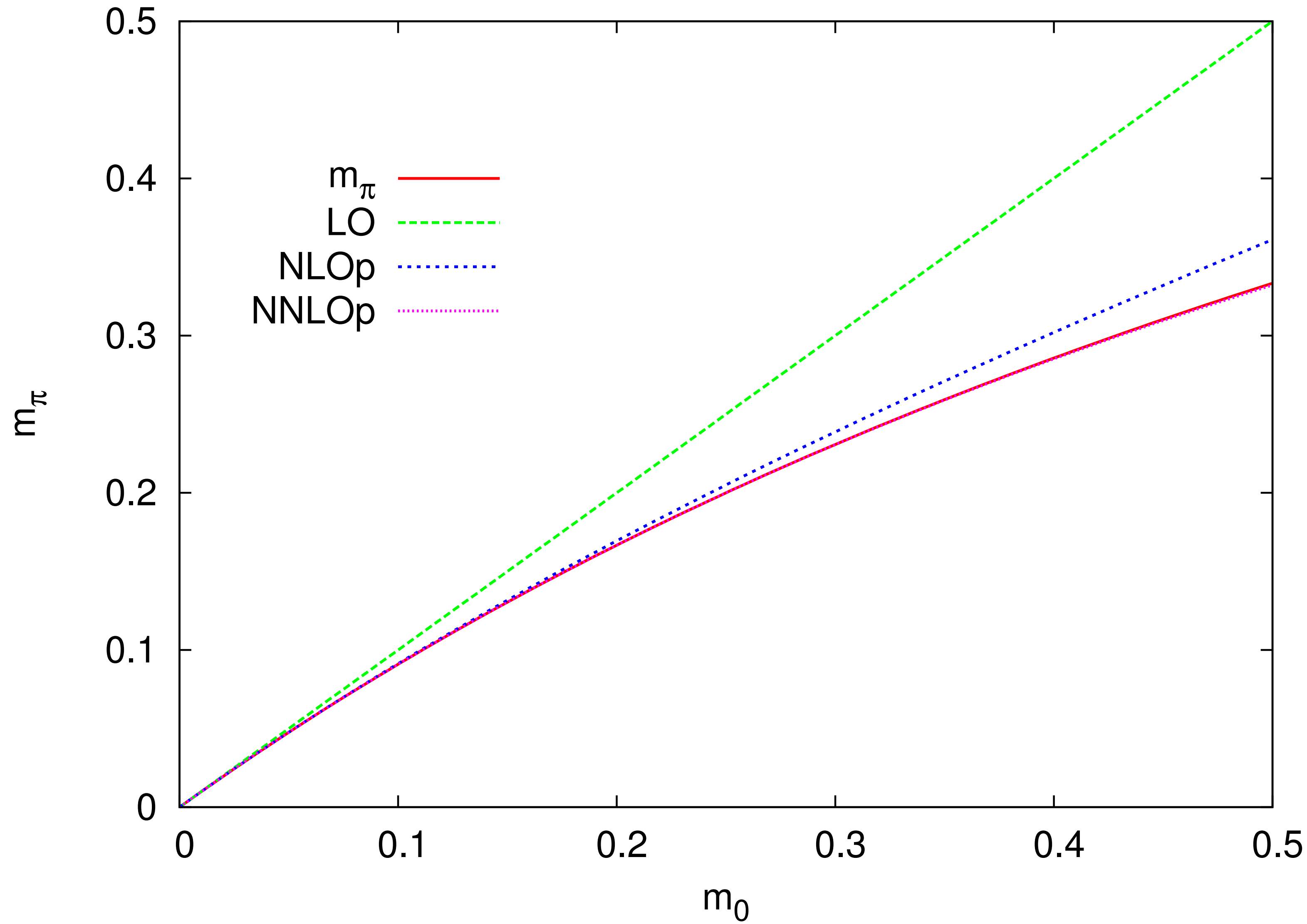
$a = 1 \quad b = 0.5 \quad f_0 = 1$

An example: m_0/f_0



An example: m_π / f_π



Two-loop Two-flavour

Review paper on Two-Loops: JB, hep-ph/0604043 Prog. Part. Nucl. Phys. 58 (2007) 521

Dispersive Calculation of the nonpolynomial part in q^2, s, t, u

- Gasser-Meißner: F_V, F_S: 1991 numerical
- Knecht-Moussallam-Stern-Fuchs: $\pi\pi$: 1995 analytical
- Colangelo-Finkemeier-Urech: F_V, F_S: 1996 analytical

Two-Loop Two-flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \to \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \to \pi^+\pi^-$, F_π, m_π: 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi$, F_π, m_π: 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t)$, $F_{S\pi}$: 1998
- JB-Talavera: $\pi \to \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \to \pi^0\pi^0$, $\gamma\gamma \to \pi^+\pi^-$: 2005-2006
- m_π, F_π, F_V, F_S, $\pi\pi$: simple analytical forms
- Colangelo-(Dürr-)Haefeli: Finite volume F_π, m_π 2005-2006
- Kampf-Moussallam: $\pi^0 \to \gamma\gamma$ 2009

LECs

$\bar{l}_1$ to $\bar{l}_4$: ChPT at order p^6 and the Roy equation analysis in $\pi\pi$ and F_S Colangelo, Gasser and Leutwyler, *Nucl. Phys.* B 603 (2001) 125 [hep-ph/0103088]

$\bar{l}_5$ and $\bar{l}_6$: from F_V and $\pi \to \ell\nu\gamma$ JB,(Colangelo,)Talavera

$$
\begin{aligned}
\bar{l}_1 &= -0.4 \pm 0.6\,,\\
\bar{l}_2 &= 4.3 \pm 0.1\,,\\
\bar{l}_3 &= 2.9 \pm 2.4\,,\\
\bar{l}_4 &= 4.4 \pm 0.2\,,\\
\bar{l}_6 - \bar{l}_5 &= 3.0 \pm 0.3\,,\\
\bar{l}_6 &= 16.0 \pm 0.5 \pm 0.7\,.
\end{aligned}
$$

$l_7 \sim 5 \cdot 10^{-3}$ from π^0-η mixing Gasser, Leutwyler 1984

LECs

Some combinations of order p^6 LECs are known as well: curvature of the scalar and vector formfactor, two more combinations from $\pi\pi$ scattering (implicit in b_5 and b_6),

Note: c_i^r for m_π, f_π, $\pi\pi$: small effect

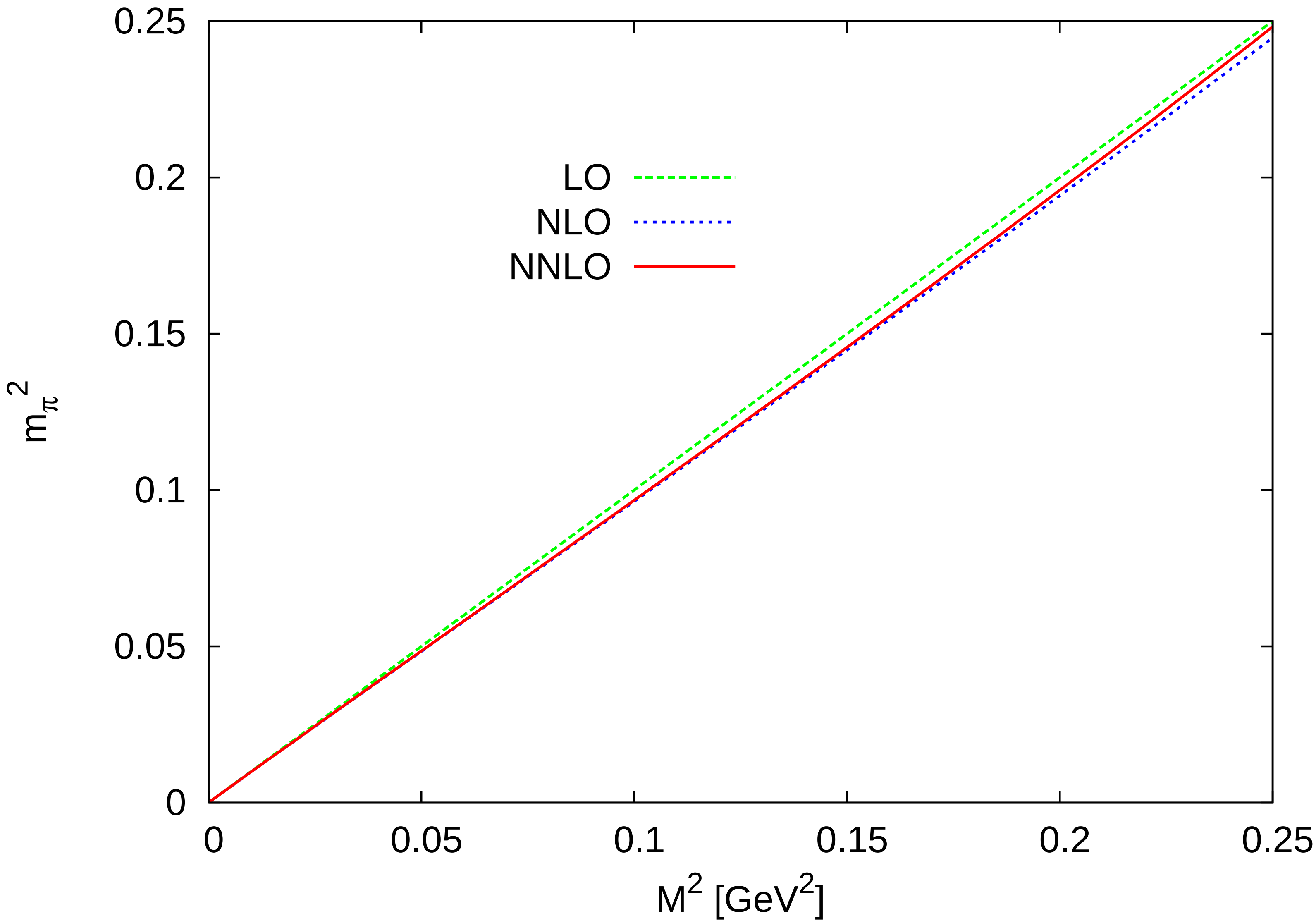
$c_i^r(770 MeV) = 0$ for plots shown

expansion in m_π^2/F_π^2 shown

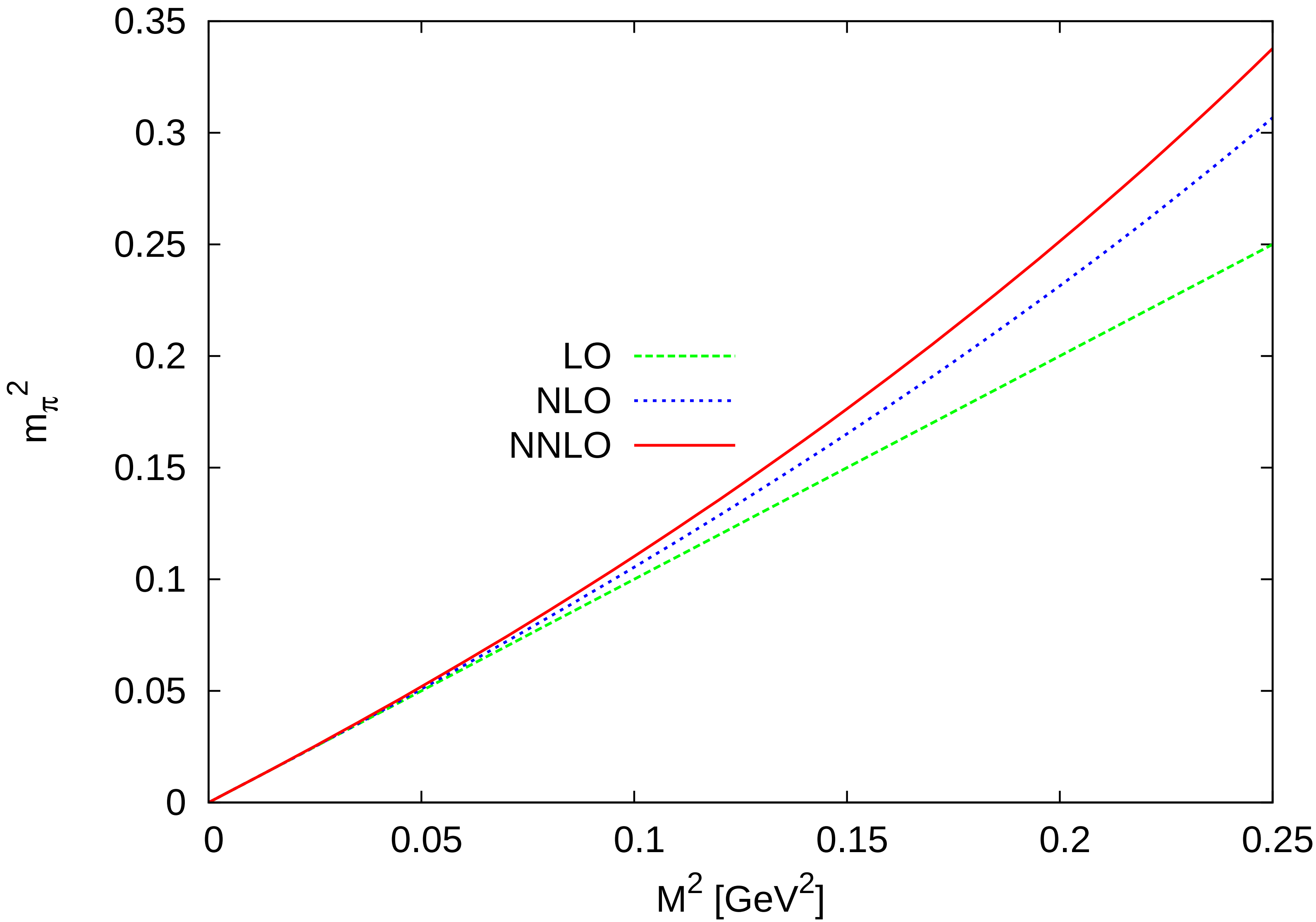
General observation:

- Obtainable from kinematical dependences: known
- Only via quark-mass dependence: poorely known

m_π^2

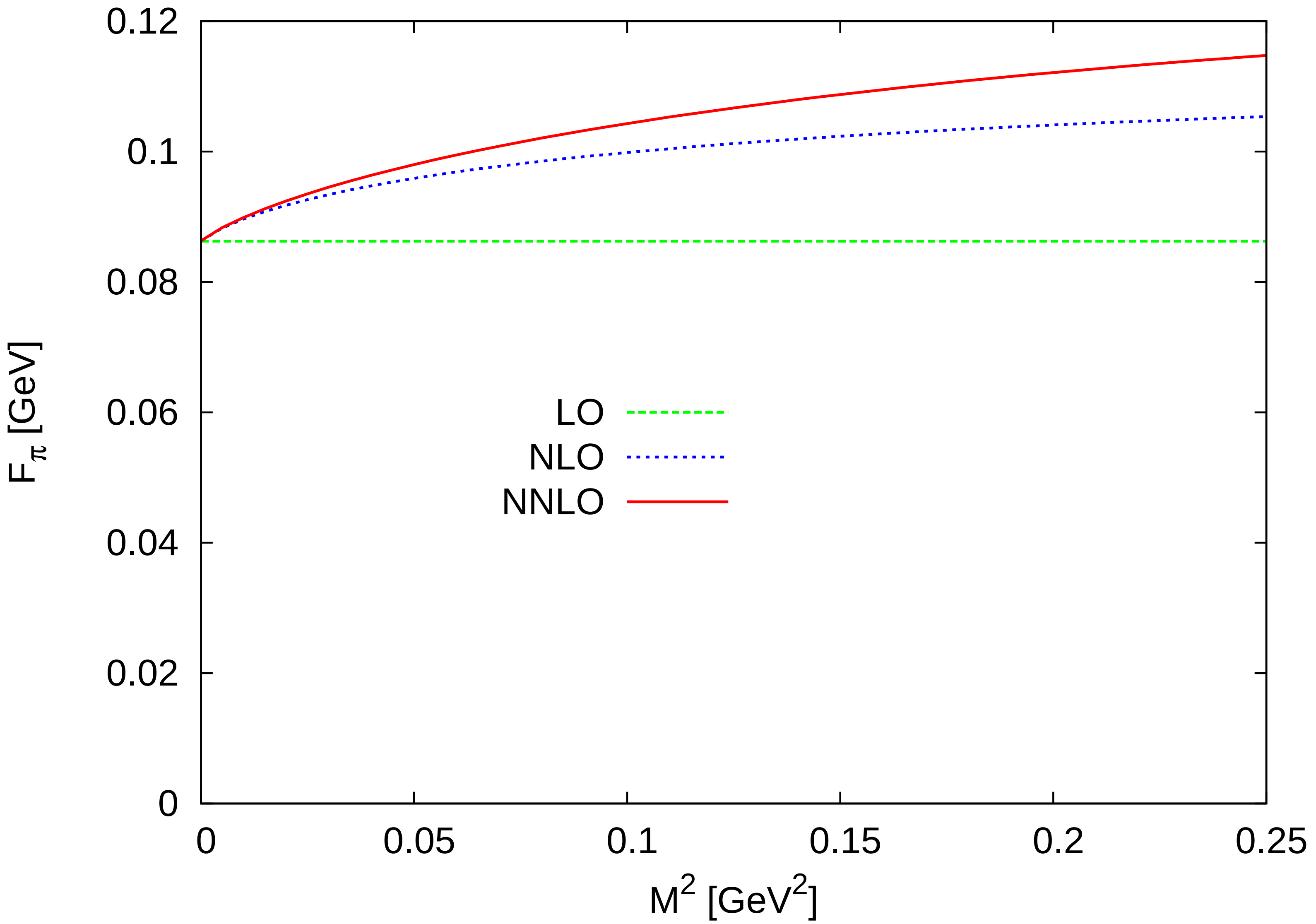


m_π^2 ($\bar{l}_3 = 0$)



0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
m_π^2
M^2 [GeV^2]
LO
NLO
NNLO

F_π


0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.05
0.1
0.15
0.2
0.25
F_π [GeV]
M^2 [GeV^2]
LO
NLO
NNLO

Two-loop Three-flavour, $\leq$2001

- $\Pi_{VV\pi}$, $\Pi_{VV\eta}$, Π_{VVK} Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
- $\Pi_{VV\rho\omega}$ Maltman
- $\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η Kambor, Golowich; Amorós, JB, Talavera
- Π_{SS} Moussallam $\boxed{L_4^r, L_6^r}$
- Π_{VVK}, Π_{AAK}, F_K, m_K Amorós, JB, Talavera
- $K_{\ell 4}$, $\langle \overline{q}q \rangle$ Amorós, JB, Talavera $\boxed{L_1^r, L_2^r, L_3^r}$
- F_M, m_M, $\langle \overline{q}q \rangle$ $(m_u \neq m_d)$ Amorós, JB, Talavera $\boxed{L_{5,7,8}^r, m_u/m_d}$

Two-loop Three-flavour, $\geq$2001

- $F_{V\pi}$, F_{VK^+}, F_{VK^0} Post, Schilcher; JB, Talavera $\boxed{L_9^r}$
- $K_{\ell 3}$ Post, Schilcher; JB, Talavera $\boxed{V_{us}}$
- $F_{S\pi}$, F_{SK} (includes σ-terms) JB, Dhonte $\boxed{L_4^r, L_6^r}$
- $K, \pi \to \ell\nu\gamma$ Geng, Ho, Wu $\boxed{L_{10}^r}$
- $\pi\pi$ JB,Dhonte,Talavera
- πK JB,Dhonte,Talavera
- relation l_i^r and L_i^r, C_i^r Gasser,Haefeli,Ivanov,Schmid
- Finite volume $\langle \overline{q}q \rangle$ JB,Ghorbani
- $\eta \to 3\pi$: JB,Ghorbani
- $K_{\ell 3}$ isospin breaking JB,Ghorbani

Two-loop Three-flavour

Known to be in progress

- Finite Volume: sunsetintegrals — JB,Lähde
- relation c_i^r and L_i^r, C_i^r — Gasser,Haefeli,Ivanov,Schmid
- More analytical work on $K_{\ell 3}$ — Greynat et al.

Inputs

Fit: Amoros, JB Talavera 2001

$K_{\ell 4}$: $F(0)$, $G(0)$, λ E865 BNL

$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$ em with Dashen violation

F_{π^+}

F_{K^+}/F_{π^+}

$m_s/\hat{m}$ 24 (26) $\hat{m} = (m_u + m_d)/2$

L^r_4, L^r_6

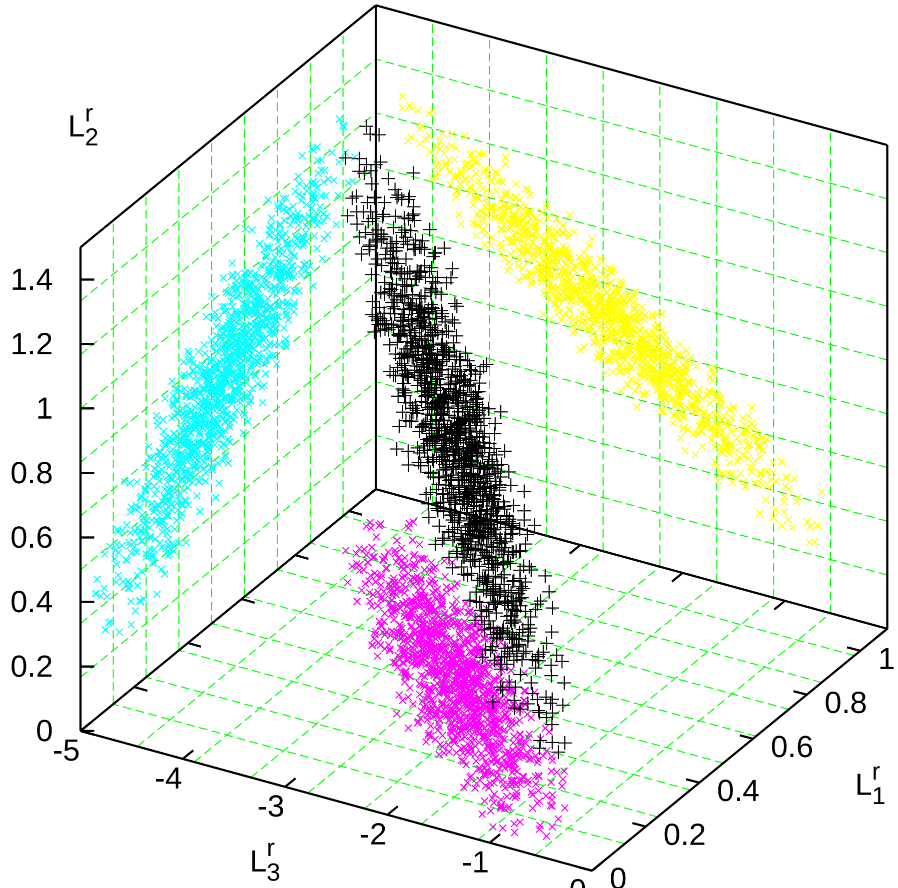
Comprehensive new fit in progress

Outputs: I

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.35 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying C_i^r factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and πK thresholds

Correlations



(older fit)

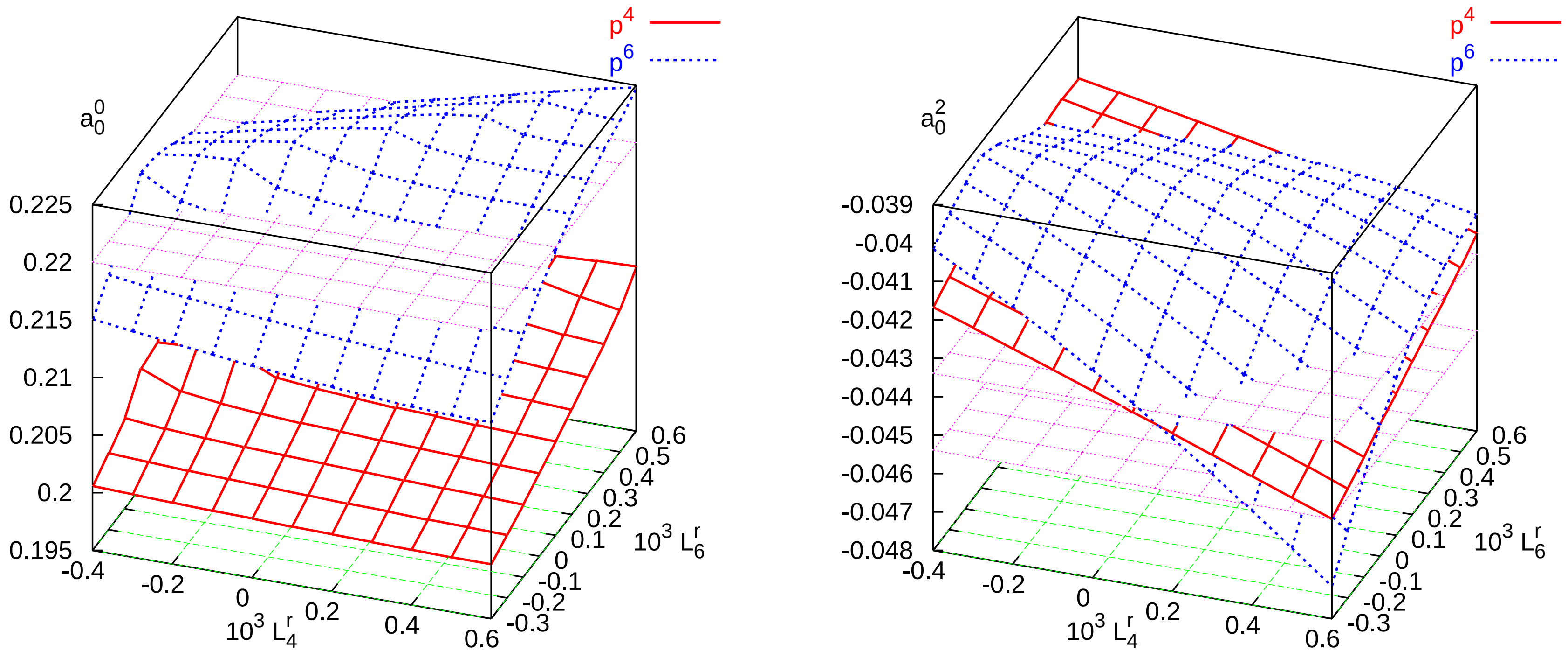
$$10^3\, L_1^r = 0.52 \pm 0.23$$
$$10^3\, L_2^r = 0.72 \pm 0.24$$
$$10^3\, L_3^r = -2.70 \pm 0.99$$

Outputs: II

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	0.009,$\equiv 0$	$-$0.138,0.009	$-$0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	0.075,$\equiv 0$	$-$0.149,0.094	$-$0.096,0.201
m_η^2: p^4, p^6	$-$0.052,0.318	0.013,$\equiv 0$	$-$0.197,0.073	$-$0.151,0.197
m_u/m_d	0.45$\pm$0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}$: p^4, p^6	0.169,0.051	0.22,$\equiv 0$	0.153,0.067	0.159,0.061

- $m_u = 0$ always very far from the fits
- F_0: pion decay constant in the chiral limit

$\pi\pi$

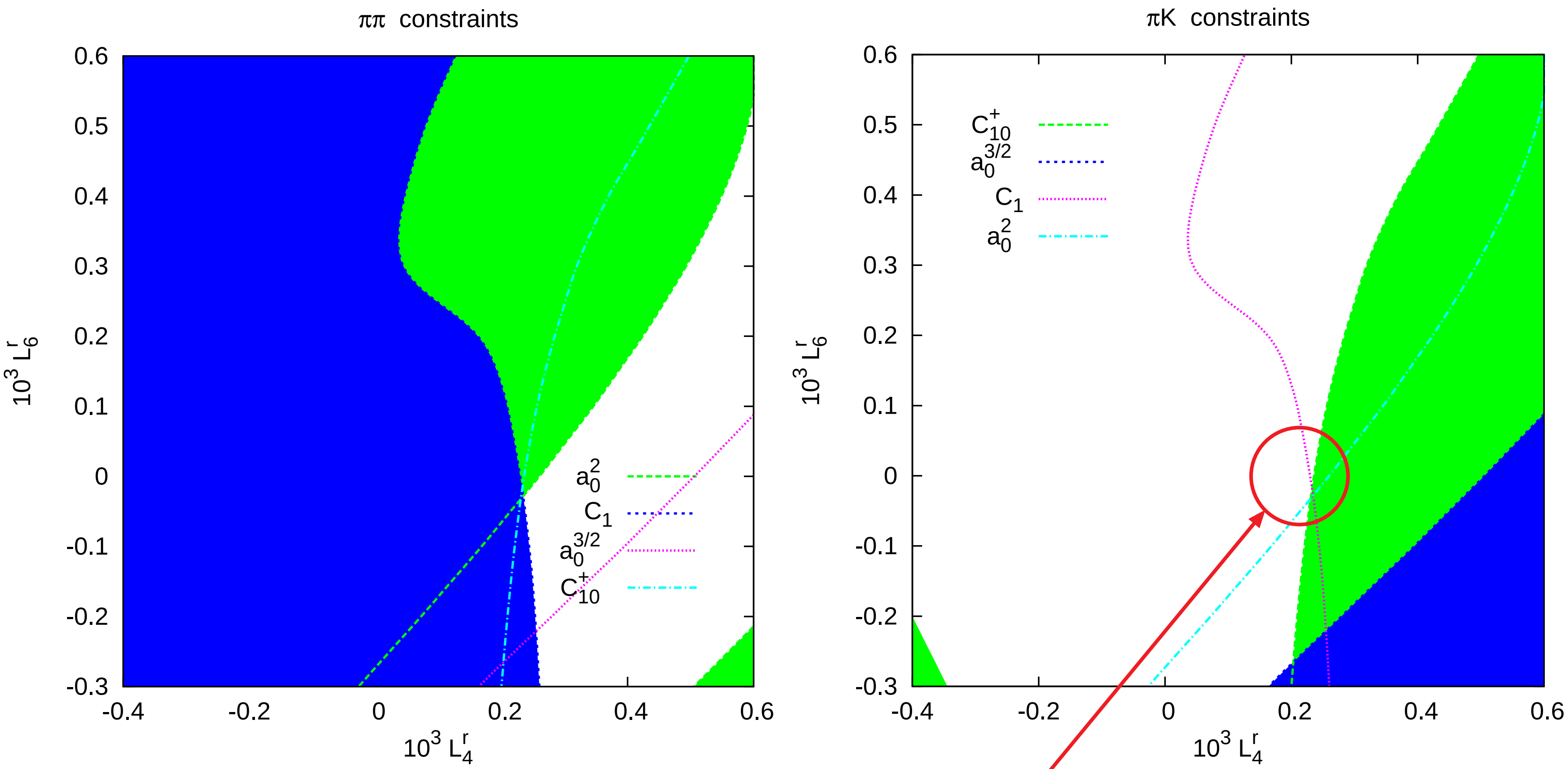


$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159$ $a_0^2 = -0.0454$ at order p^2

$\pi\pi$ and πK



preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

General fitting needs more work and systematic studies

Quark mass dependences

Updates of plots in

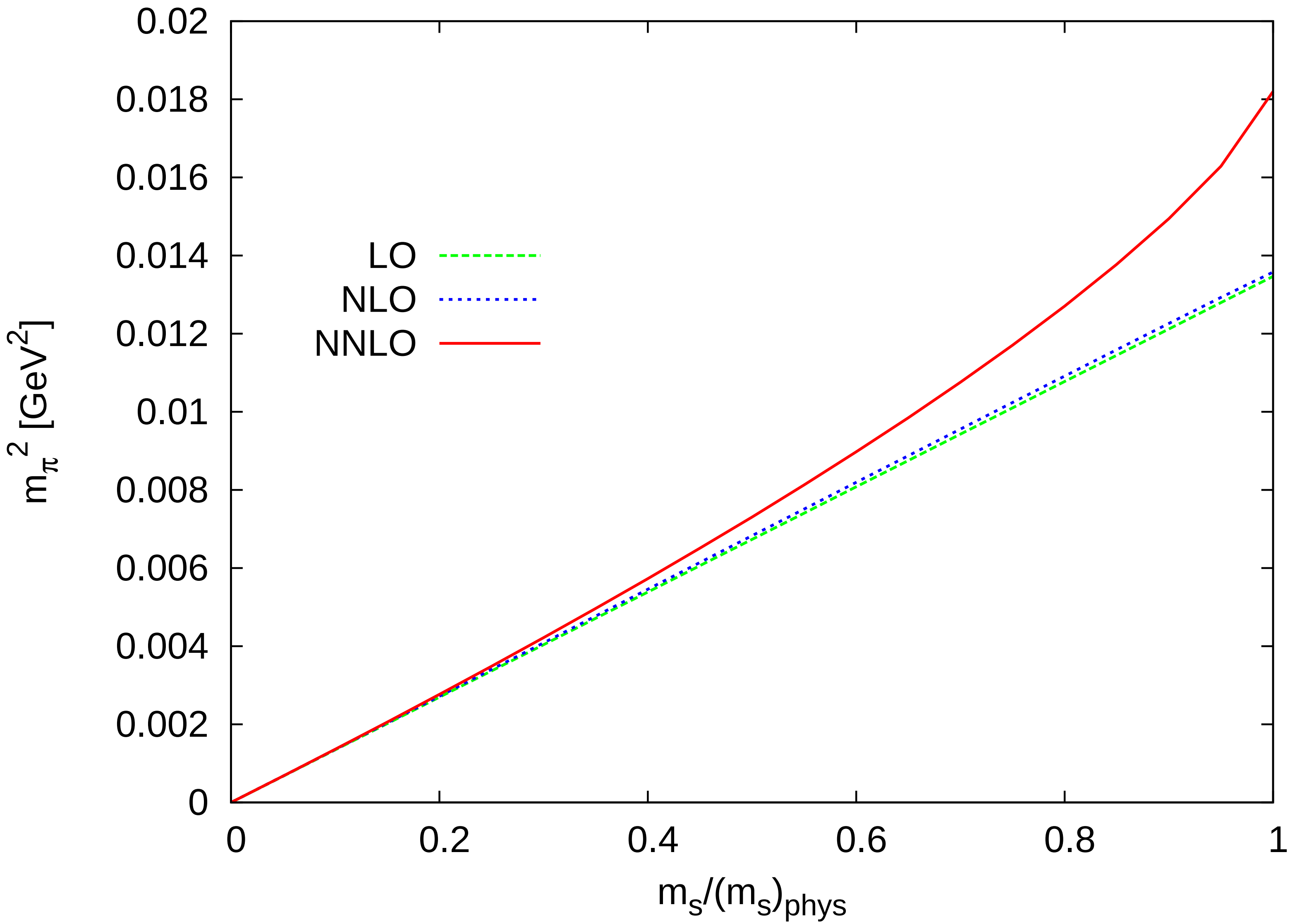
Amorós, JB and Talavera, hep-ph/0003258, Nucl. Phys. B585 (2000) 293

Some new ones

Procedure: calculate a consistent set of $m_\pi, m_K, m_\eta, f_\pi$ with the given input values (done iteratively)

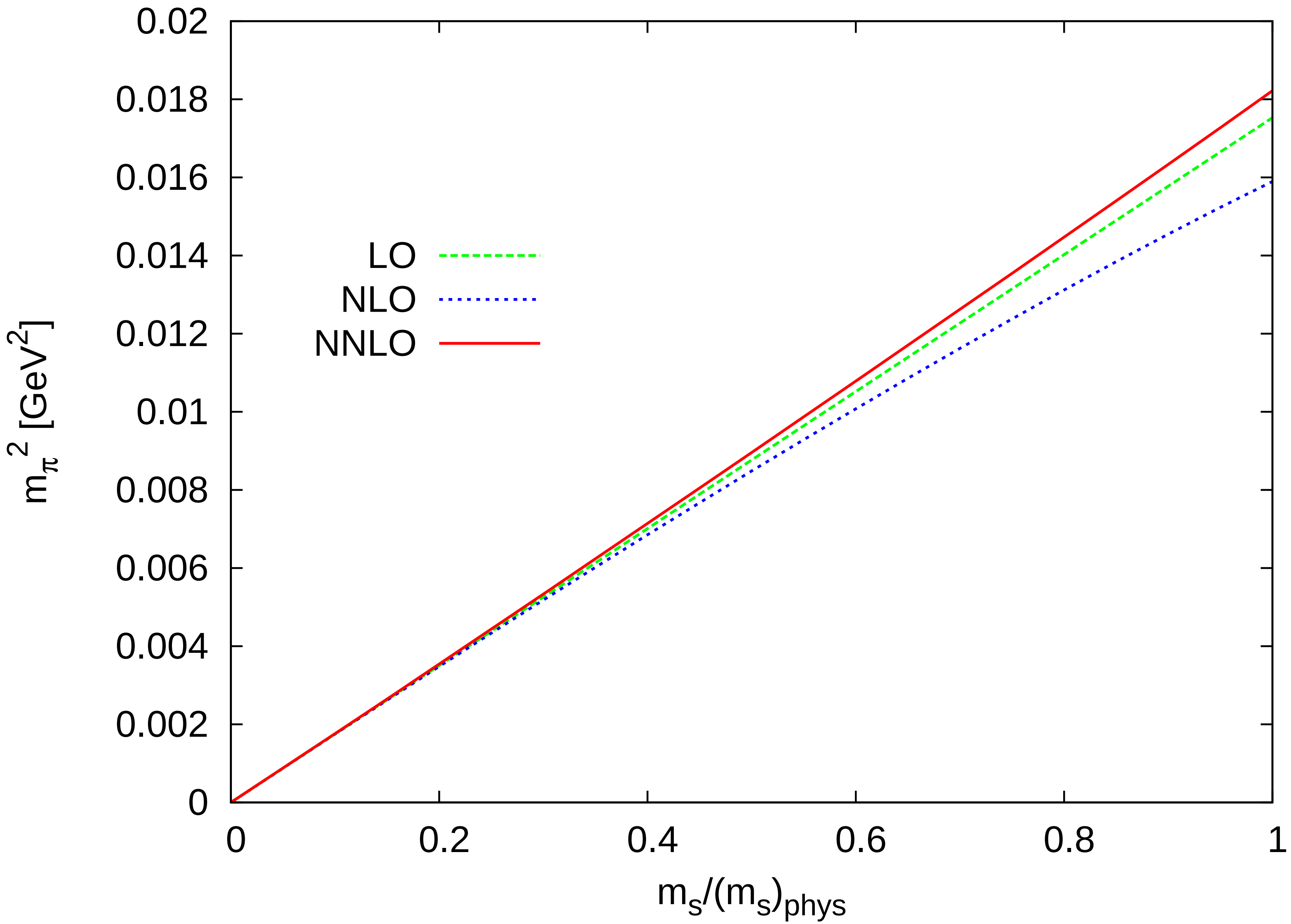
- vary $m_s/(m_s)_{phys}$, keep $m_s/\hat{m} = 24$
  m_π^2, F_K/F_π

m_π^2 fit 10



0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.2
0.4
0.6
0.8
1
LO
NLO
NNLO
m_π^2 [GeV^2]
$m_s/(m_s)_{phys}$

m_π^2 fit D



0.02
0.018
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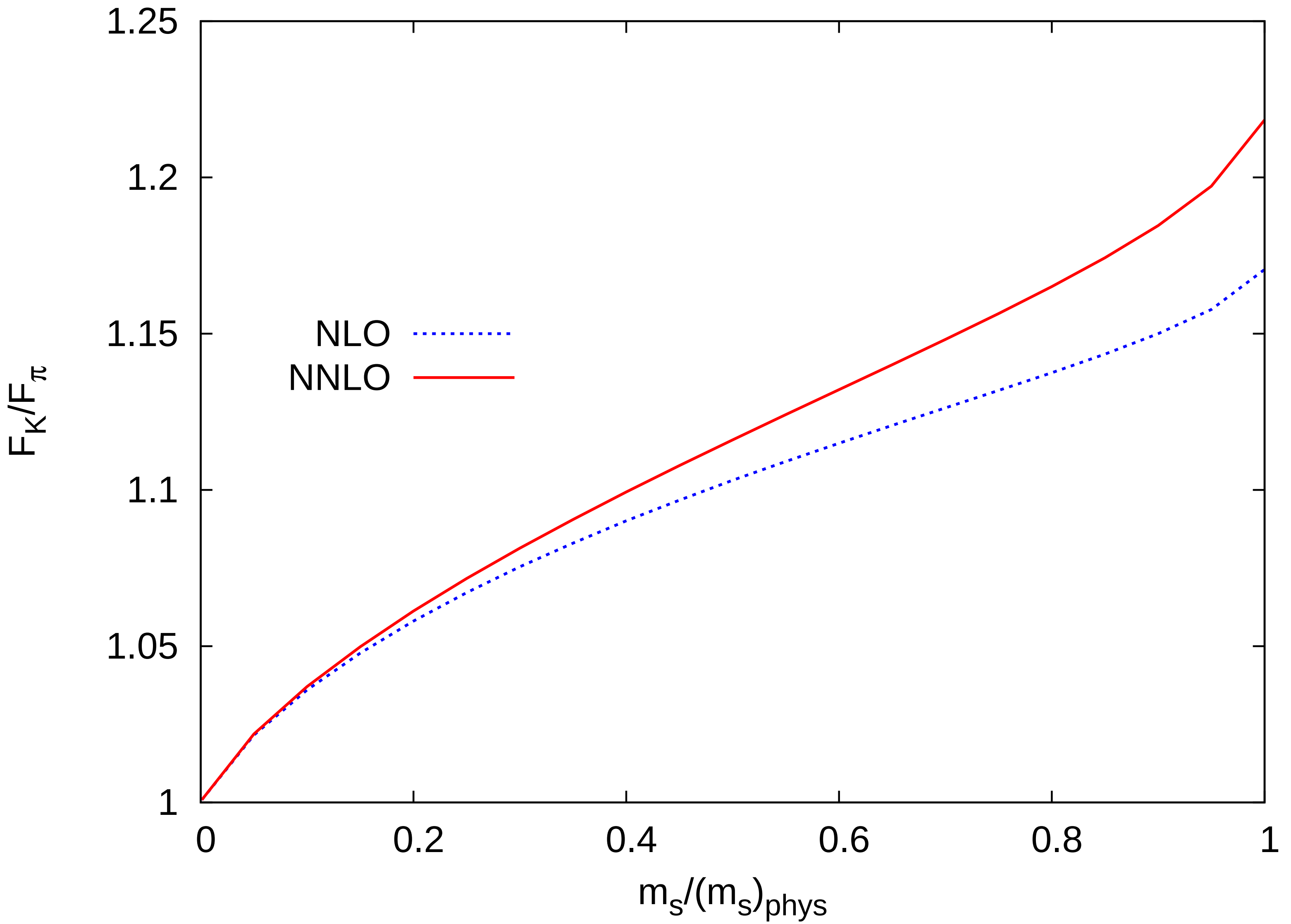
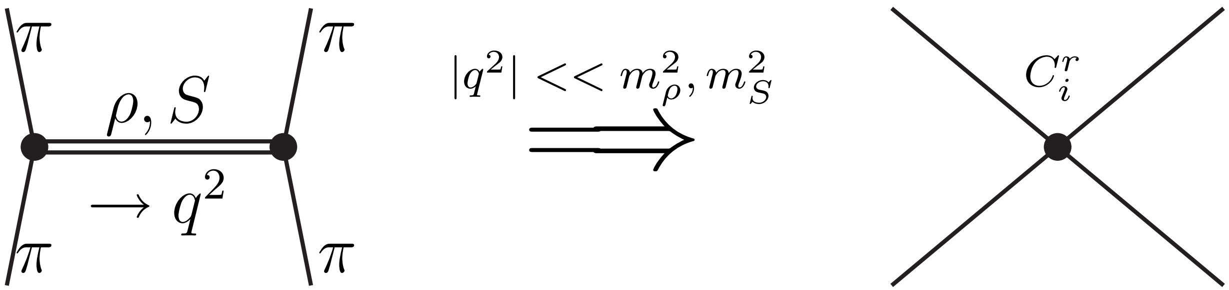
0.2
0.4
0.6
0.8
1
LO
NLO
NNLO
m_π^2 [GeV^2]
$m_s/(m_s)_{phys}$

F_K/F_π fit 10



C_i^r

Most analysis use:
C_i^r from (single) resonance approximation



Motivated by large N_c: large effort goes in this

Ananthanarayan, JB, Cirigliano, Donoghue, Ecker, Gamiz, Golterman, Kaiser, Kampf, Knecht, Moussallam, Peris, Pich, Prades, Portoles, de Rafael,. . .

C_i^r

$$
\begin{aligned}
\mathcal{L}_V &= -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}m_V^2\langle V_\mu V^\mu\rangle - \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle \\
&\quad -\frac{ig_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu,u^\nu]\rangle + f_\chi\langle V_\mu[u^\mu,\chi_-]\rangle \\
\mathcal{L}_A &= -\frac{1}{4}\langle A_{\mu\nu}A^{\mu\nu}\rangle + \frac{1}{2}m_A^2\langle A_\mu A^\mu\rangle - \frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle \\
\mathcal{L}_S &= \frac{1}{2}\langle\nabla^\mu S\nabla_\mu S - M_S^2 S^2\rangle + c_d\langle S u^\mu u_\mu\rangle + c_m\langle S\chi_+\rangle \\
\mathcal{L}_{\eta'} &= \frac{1}{2}\partial_\mu P_1\partial^\mu P_1 - \frac{1}{2}M_{\eta'}^2 P_1^2 + i\tilde{d}_m P_1\langle\chi_-\rangle\,.
\end{aligned}
$$

$f_V = 0.20$, $f_\chi = -0.025$, $g_V = 0.09$, $c_m = 42$ MeV, $c_d = 32$ MeV, $\tilde{d}_m = 20$ MeV,

$m_V = m_\rho = 0.77$ GeV, $m_A = m_{a_1} = 1.23$ GeV, $m_S = 0.98$ GeV, $m_{P_1} = 0.958$ GeV

f_V, g_V, f_χ, f_A: experiment

c_m and c_d from resonance saturation at $\mathcal{O}(p^4)$

C_i^r

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

C_i^r

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

What we did about it:

- Vary resonance estimate by factor of two
- Vary the scale μ at which it applies: 600-900 MeV
- Check the estimates for the measured ones
- Again: kinematic can be had, quark-mass dependence difficult

Comparisons of C_i^r

Kampf-Moussallam 2006 using our $\pi\pi$ and πK results

input	$C_1^r + 4C_3^r$	C_2^r	$C_4^r + 3C_3^r$	$C_1^r + 4C_3^r + 2C_2^r$
$\pi K : C_{30}^+, C_{11}^+, C_{20}^-$	20.7 ± 4.9	-9.2 ± 4.9	9.9 ± 2.5	2.3 ± 10.8
$\pi K : C_{30}^+, C_{11}^+, C_{01}^-$	28.1 ± 4.9	-7.4 ± 4.9	21.0 ± 2.5	13.4 ± 10.8
$\pi\pi$			23.5 ± 2.3	18.8 ± 7.2
Resonance model	7.2	-0.5	10.0	6.2

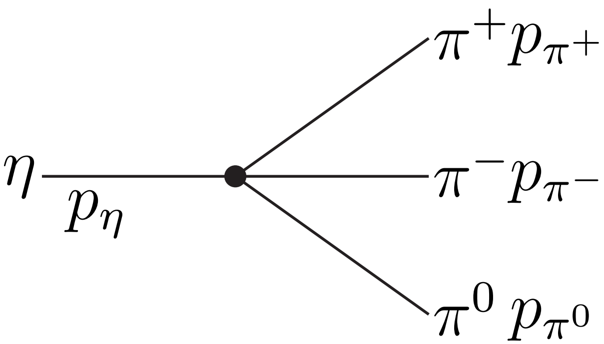
Correlated analysis in progress (slowly)

see talk by Ilaria Jemos

$\eta \to 3\pi$

Reviews: JB, Gasser, Phys.Scripta T99(2002)34 [hep-ph/0202242]

JB, Acta Phys. Slov. 56(2005)305 [hep-ph/0511076]



$$\begin{aligned} s &= (p_{\pi^+}+p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2 \\ t &= (p_{\pi^-}+p_{\pi^0})^2 = (p_\eta - p_{\pi^+})^2 \\ u &= (p_{\pi^+}+p_{\pi^0})^2 = (p_\eta - p_{\pi^-})^2 \end{aligned}$$

$$s+t+u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0\,.$$

$$\langle \pi^0\pi^+\pi^-\text{out}|\eta\rangle = i\,(2\pi)^4\;\delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right)\;A(s,t,u)\,.$$

$$\langle \pi^0\pi^0\pi^0\text{out}|\eta\rangle = i\,(2\pi)^4\;\delta^4\left(p_\eta - p_1 - p_2 - p_3\right)\,\overline{A}(s_1,s_2,s_3)$$

$$\overline{A}(s_1,s_2,s_3) = A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2)\,,$$

$\eta \to 3\pi$: Lowest order (LO)

Pions are in $I = 1$ state $\Longrightarrow A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \to \pi^+\pi^-\pi^0\gamma$ needs to be included directly

$\eta \to 3\pi$: Lowest order (LO)

Pions are in $I = 1$ state $\Longrightarrow A \sim (m_u - m_d)$ or α_{em}

ChPT:Cronin 67: $A(s,t,u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \frac{3(s - s_0)}{m_\eta^2 - m_\pi^2}\right\}$

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with $Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$ or $R \equiv \frac{m_s - \hat{m}}{m_d - m_u}$ $\quad\quad \hat{m} = \frac{1}{2}(m_u + m_d)$

$$A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\,\frac{\mathcal{M}(s,t,u)}{3\sqrt{3}F_\pi^2}\,,$$

$$A(s,t,u) = \frac{\sqrt{3}}{4R}\,M(s,t,u)$$

LO: $\mathcal{M}(s,t,u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$ $\quad\quad M(s,t,u) = \frac{1}{F_\pi^2}\left(\frac{4}{3}m_\pi^2 - s\right)$

$\eta \longrightarrow 3\pi$ beyond p^4: p^2 and p^4

$\Gamma\left(\eta \to 3\pi\right) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

$\boxed{Q \approx 24}$ gives lowest order $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 66$ eV .

Other Source from $m^2_{K^+} - m^2_{K^0} \sim Q^{-2} \Longrightarrow \boxed{Q = 20.0 \pm 1.5}$

Lowest order prediction $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 140$ eV .

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Lowest order prediction $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 140$ eV .

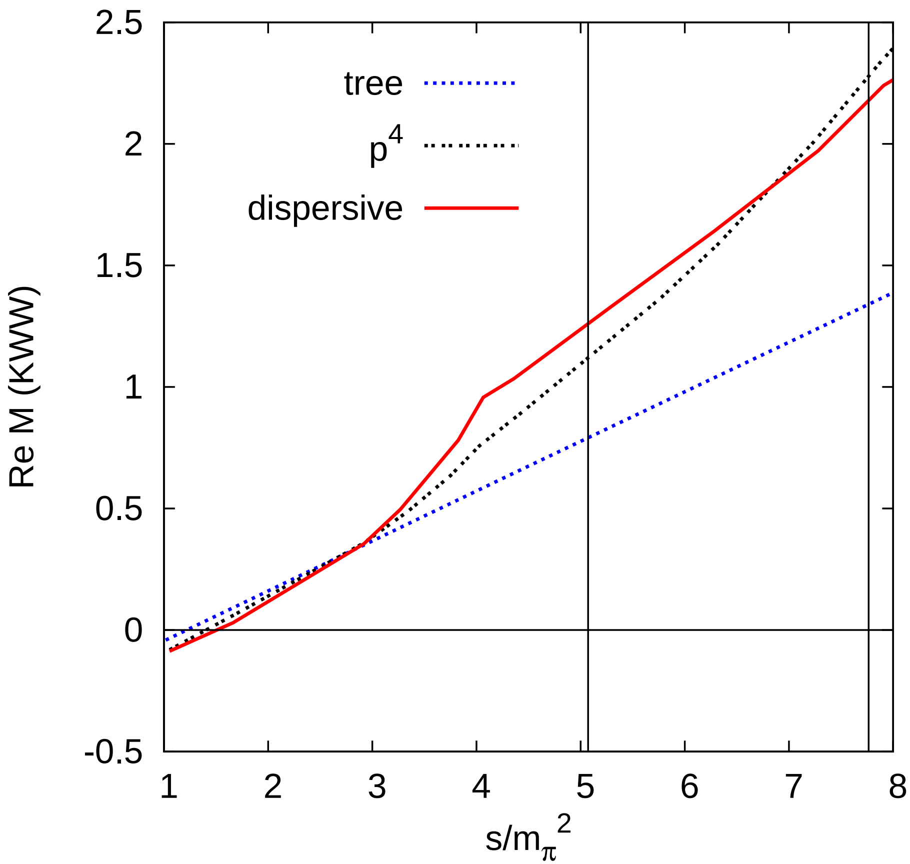
At order p^4 Gasser-Leutwyler 1985:
$$\frac{\int dLIPS|A_2 + A_4|^2}{\int dLIPS|A_2|^2} = 2.4\,,$$

($LIPS$=Lorentz invariant phase-space)

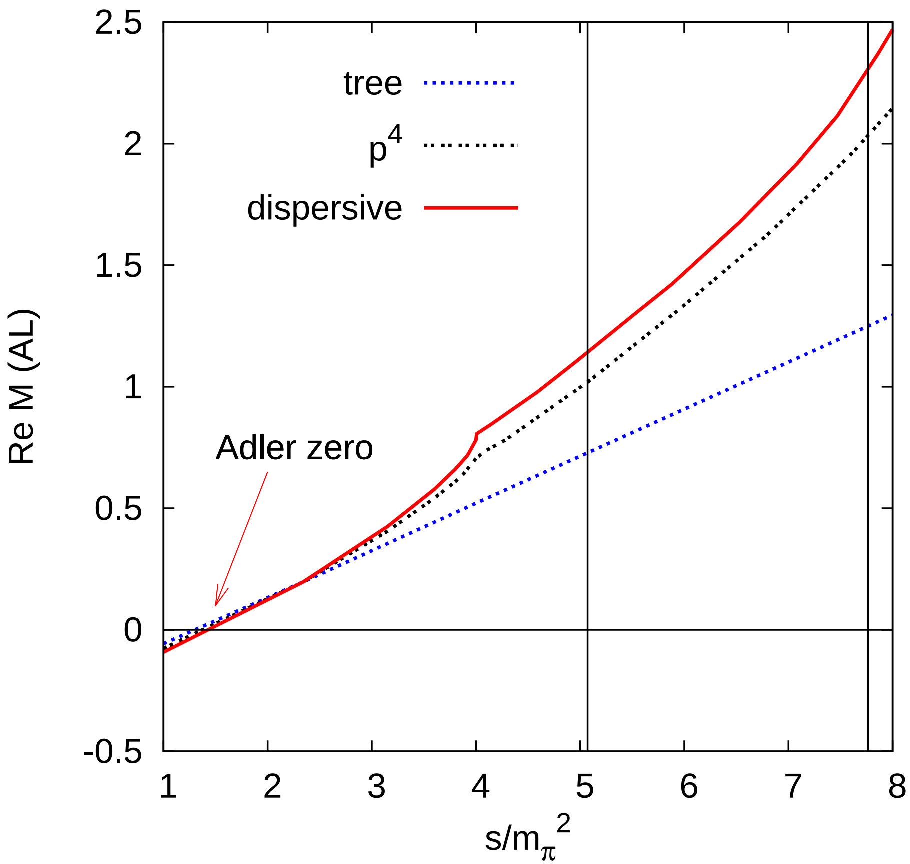
Major source: large S-wave final state rescattering

Experiment: 295 ± 17 eV (PDG 2006)

$\eta \to 3\pi$ beyond p^4: dispersive



Along $s = u$ KWW

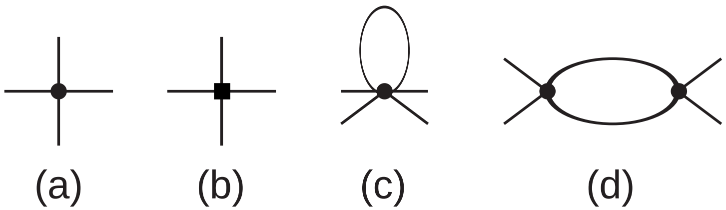


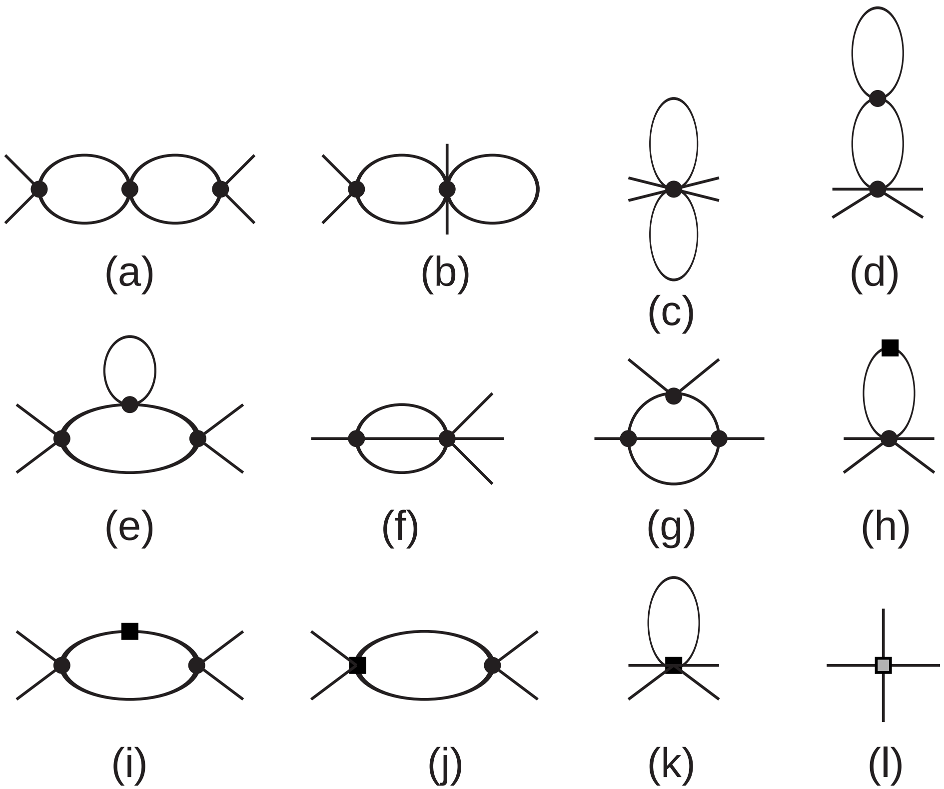
Along $s = u$ AL

Two Loop Calculation: why

- In $K_{\ell 4}$ dispersive gave about half of p^6 in amplitude
- Same order in ChPT as masses for consistency check on m_u/m_d
- Check size of 3 pion dispersive part
- At order p^4 unitarity about half of correction
- Technology exists:
  - Two-loops: Amorós,JB,Dhonte,Talavera,...
  - Dealing with the mixing π^0-η: Amorós,JB,Dhonte,Talavera 01
- JB, Ghorbani, arXiv:0709.0230 [hep-ph]
  - Dealing with the mixing π^0-η: extended to $\eta \to 3\pi$

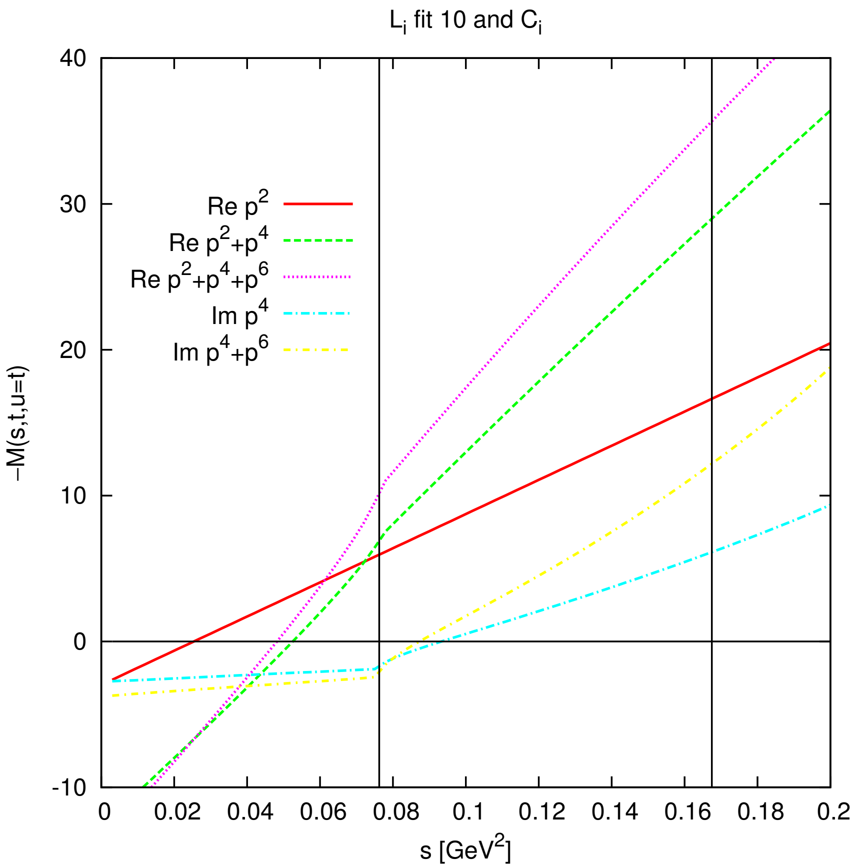
Diagrams



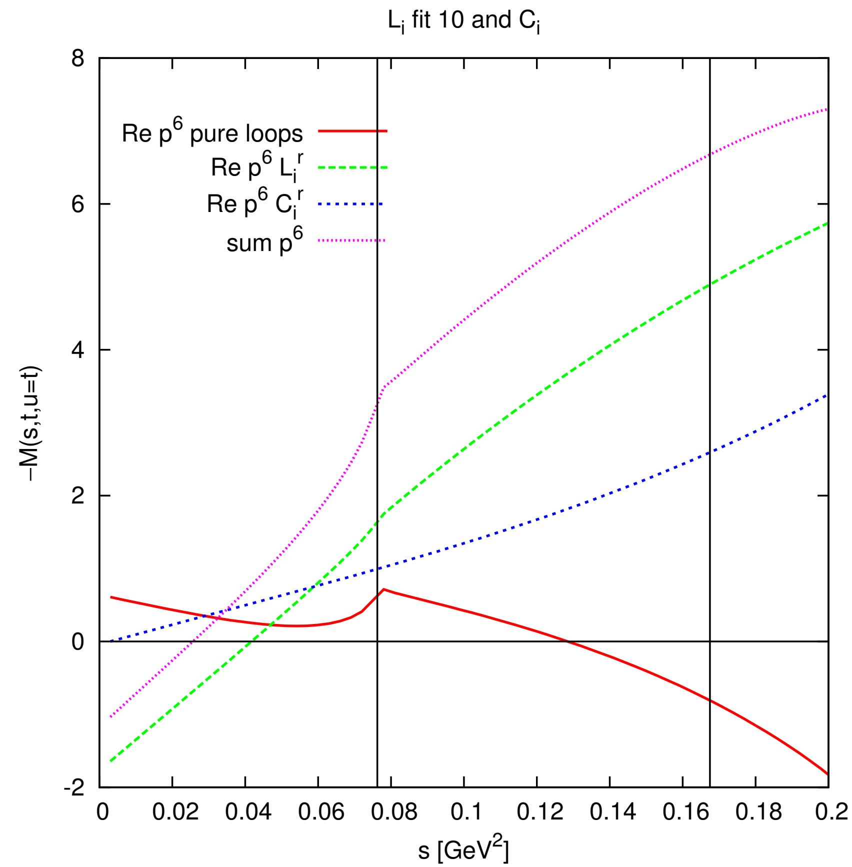


- Include mixing, renormalize, pull out factor $\frac{\sqrt{3}}{4R}$, ...
- Two independent calculations (comparison major amount of work)

$\eta \to 3\pi$: $M(s, t = u)$

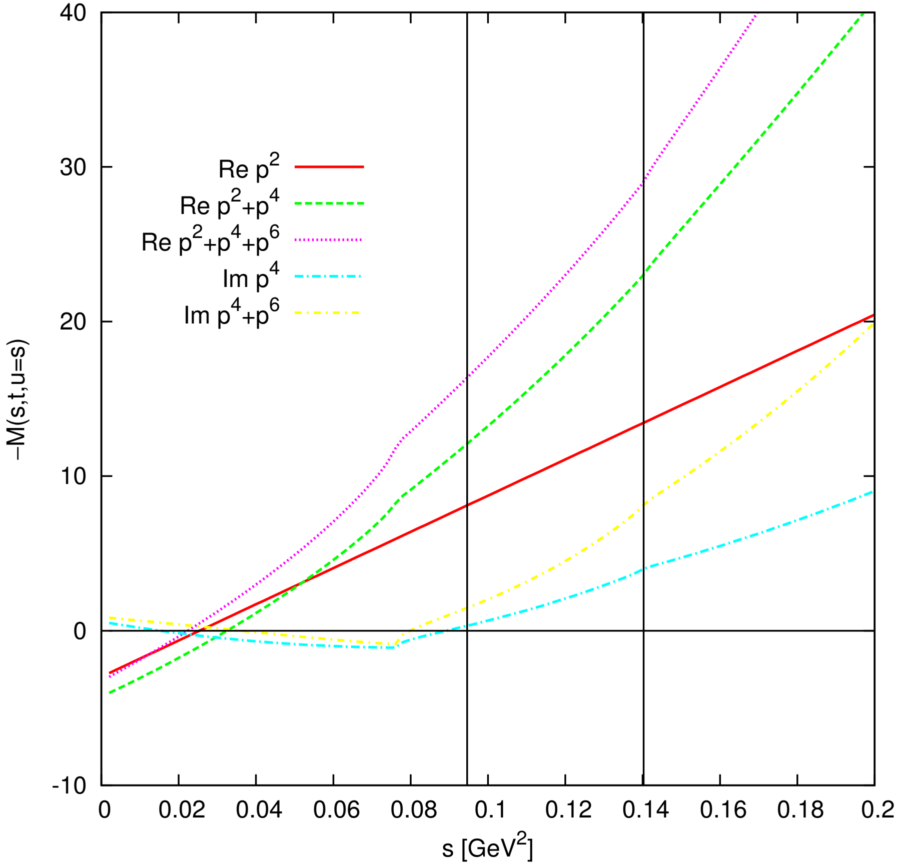


Along $t = u$



Along $t = u$ parts

$\eta \to 3\pi$: $M(s = u, t)$



Along $s = u$

Shape agrees with AL

Correction larger:
20-30% in amplitude

Dalitz plot

$$x = \sqrt{3}\frac{T_+ - T_-}{Q_\eta} = \frac{\sqrt{3}}{2m_\eta Q_\eta}(u-t)$$

$$y = \frac{3T_0}{Q_\eta} - 1 = \frac{3\left((m_\eta - m_{\pi^o})^2 - s\right)}{2m_\eta Q_\eta} - 1 \stackrel{\text{iso}}{=} \frac{3}{2m_\eta Q_\eta}(s_0 - s)$$

$$Q_\eta = m_\eta - 2m_{\pi^+} - m_{\pi^0}$$

T^i is the kinetic energy of pion π^i

$$z = \frac{2}{3}\sum_{i=1,3}\left(\frac{3E_i - m_\eta}{m_\eta - 3m_\pi^0}\right)^2$$

E_i is the energy of pion π^i

$$|M|^2 = A_0^2\left(1 + ay + by^2 + dx^2 + fy^3 + gx^2y + \cdots\right)$$

$$|\overline{M}|^2 = \overline{A}_0^2\left(1 + 2\alpha z + \cdots\right)$$

Experiment: charged

Exp.	a	b	d
KLOE	$-1.090 \pm 0.005^{+0.008}_{-0.019}$	$0.124 \pm 0.006 \pm 0.010$	$0.057 \pm 0.006^{+0.007}_{-0.016}$
Crystal Barrel	-1.22 ± 0.07	0.22 ± 0.11	0.06 ± 0.04 (input)
Layter et al.	-1.08 ± 0.014	0.034 ± 0.027	0.046 ± 0.031
Gormley et al.	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04

KLOE has: $f = 0.14 \pm 0.01 \pm 0.02$.

Crystal Barrel: d input, but a and b insensitive to d

Theory: charged

	A_0^2	a	b	d	f
LO	120	-1.039	0.270	0.000	0.000
NLO	314	-1.371	0.452	0.053	0.027
NLO ($L_i^r = 0$)	235	-1.263	0.407	0.050	0.015
NNLO	538	-1.271	0.394	0.055	0.025
NNLOp (y from T^0)	574	-1.229	0.366	0.052	0.023
NNLOq (incl $(x,y)^4$)	535	-1.257	0.397	0.076	0.004
NNLO ($\mu = 0.6$ GeV)	543	-1.300	0.415	0.055	0.024
NNLO ($\mu = 0.9$ GeV)	548	-1.241	0.374	0.054	0.025
NNLO ($C_i^r = 0$)	465	-1.297	0.404	0.058	0.032
NNLO ($L_i^r = C_i^r = 0$)	251	-1.241	0.424	0.050	0.007
dispersive (KWW)	—	-1.33	0.26	0.10	—
tree dispersive	—	-1.10	0.33	0.001	—
absolute dispersive	—	-1.21	0.33	0.04	—
error	18	0.075	0.102	0.057	0.160

NLO to NNLO: Little change

Error on $|M(s,t,u)|^2$:

$|M^{(6)} M(s,t,u)|$

Experiment: neutral

Exp.	α
KLOE 2007	$-0.027 \pm 0.004^{+0.004}_{-0.006}$
KLOE (prel)	$-0.014 \pm 0.005 \pm 0.004$
Crystal Ball	-0.031 ± 0.004
WASA/CELSIUS	$-0.026 \pm 0.010 \pm 0.010$
Crystal Barrel	$-0.052 \pm 0.017 \pm 0.010$
GAMS2000	-0.022 ± 0.023
SND	$-0.010 \pm 0.021 \pm 0.010$

	$\overline{A}_0^2$	α
LO	1090	0.000
NLO	2810	0.013
NLO ($L_i^r = 0$)	2100	0.016
NNLO	4790	0.013
NNLOq	4790	0.014
NNLO ($C_i^r = 0$)	4140	0.011
NNLO ($L_i^r = C_i^r = 0$)	2220	0.016
dispersive (KWW)	—	$-(0.007$—$0.014)$
tree dispersive	—	-0.0065
absolute dispersive	—	-0.007
Borasoy	—	-0.031
error	160	0.032

Note: NNLO ChPT gets a_0^0 in $\pi\pi$ correct

α is difficult

Expand amplitudes and isospin:

$$M(s,t,u) = A\left(1+\tilde{a}(s-s_0)+\tilde{b}(s-s_0)^2+\tilde{d}(u-t)^2+\cdots\right)$$

$$\overline{M}(s,t,u) = A\left(3+\left(\tilde{b}+3\tilde{d}\right)\left((s-s_0)^2+(t-s_0)^2+(u-s_0)^2\right)+$$

Gives relations ($R_\eta = (2m_\eta Q_\eta)/3$)

$$a = -2R_\eta\,\mathrm{Re}(\tilde{a}), \quad b = R_\eta^2\left(|\tilde{a}|^2+2\mathrm{Re}(\tilde{b})\right), \quad d = 6R_\eta^2\,\mathrm{Re}(\tilde{d}).$$

$$\alpha = \frac{1}{2}R_\eta^2\,\mathrm{Re}\left(\tilde{b}+3\tilde{d}\right) = \frac{1}{4}\left(d+b-R_\eta^2|\tilde{a}|^2\right) \le \frac{1}{4}\left(d+b-\frac{1}{4}a^2\right)$$

equality if Im$(\tilde{a})=0$

Large cancellation in α, overestimate of b likely the problem

r and decay rates

$$r \equiv \frac{\Gamma(\eta \to \pi^0\pi^0\pi^0)}{\Gamma(\eta \to \pi^+\pi^-\pi^0)}$$

$$\begin{aligned} r_{\mathrm{LO}} &= 1.54 \\ r_{\mathrm{NLO}} &= 1.46 \\ r_{\mathrm{NNLO}} &= 1.47 \\ r_{\mathrm{NNLO}\,C_i^r=0} &= 1.46 \end{aligned}$$

PDG 2006

$$\begin{aligned} r &= 1.49 \pm 0.06 \quad \text{our average}\,. \\ r &= 1.43 \pm 0.04 \quad \text{our fit}\,, \end{aligned}$$

Good agreement

R and Q

	LO	NLO	NNLO	NNLO $(C_i^r = 0)$
R (η)	19.1	31.8	42.2	38.7
R (Dashen)	44	44	37	—
R (Dashen-violation)	36	37	32	—
Q (η)	15.6	20.1	23.2	22.2
Q (Dashen)	24	24	22	—
Q (Dashen-violation)	22	22	20	—

LO from $R = \dfrac{m_{K^0}^2 + m_{K^+}^2 - 2m_{\pi^0}^2}{2\left(m_{K^0}^2 - m_{K^+}^2\right)}$ (QCD part only)

NLO and NNLO from masses: Amorós, JB, Talavera 2001

$$Q^2 = \frac{m_s + \hat{m}}{2\hat{m}} R = 12.7 R \qquad (m_s/\hat{m} = 24.4)$$

≥ 3-flavour: PQChPT

PQChPT: treat closed quark-loops differently from external quarks,

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT

$\Longrightarrow$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

E.g. $L_1^r = L_0^{r(3pq)}/2 + L_1^{r(3pq)}$

PQChPT

One-loop: Bernard, Golterman, Sharpe, Shoresh, Pallante,. . .

with electromagnetism: JB,Danielsson, hep-lat/0610127

Two loops:

$m^2_{\pi^+}$ simplest mass case: JB,Danielsson,Lähde, hep-lat/0406017

F_{π^+}: JB,Lähde, hep-lat/0501014

F_{π^+}, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004

$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0602003

Neutral masses: JB,Danielsson, hep-lat/0606017

Lattice data: a and L extrapolations needed

Programs available from me (Fortran)

Formulas: `http://www.thep.lu.se/∼bijnens/chpt.html`

Renormalization group

Weinberg 79: nonlocal diverences must cancel $\Longrightarrow$ consistency conditions between graphs with different numbers of loops (but same order in the power counting)

This allows to calculate the leading logarithms to any order from one-loop diagrams Buchler Colangelo 2003

- double logs in $\pi\pi$ Colangelo 95
- all double logs JB, Ecker, Colangelo 1998
- leading logs to five loops for (massless) Scalar two-point function Bissegger Fuhrer 2007
- three loops for generalized GPD Kivel Polyakov 2007
- Recursion relations in the massless $O(N+1)/O(N)$ sigma model for many quantities Kivel, Polyakov, Vladimirov 2008

Renormalization group

Underlying practical problem: the number of needed terms increases fast with order $\Longrightarrow$ need a good way to handle this.

KPV: write the 4-meson vertex using Legendre polynomials
could perform all loopintegrals
$\Longrightarrow$ algebraic recursion relations

It works in the chiral limit since tadpoles vanish: simplification: the number of external legs in the vertices needed does not go up.

Conclusions

- Modern ChPT is doing fine:
- Two flavour ChPT is in good shape: precision science in many ways
- Three flavour ChPT: corrections are larger
  there seem to be some problems, but many parameters (scalar sector) rather uncertain, errors very quantity dependent
- Partially quenched: useful for the lattice, please use them
- New application areas continue to be found