RECENT RESULTS IN CHIRAL MESON PHYSICS

Johan Bijnens

Lund University

bijnens@thep.lu.se
http://thep.lu.se/~bijnens
http://thep.lu.se/~bijnens/chpt/

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Overview

1. Chiral Perturbation Theory
2. Determination of LECs in the continuum
3. Charged Pion Polarizabilities
4. Finite volume
5. Beyond QCD
6. A mesonic ChPT program framework
7. Leading logarithms
8. Conclusions
Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

Derivation from QCD:
H. Leutwyler,
*On The Foundations Of Chiral Perturbation Theory*,

For references to lectures see:
http://www.thep.lu.se/~bijnens/chpt.html
Chiral Perturbation Theory

A general Effective Field Theory:
- Relevant degrees of freedom
- A powercounting principle (predictivity)
- Has a certain range of validity

Chiral Perturbation Theory:
- Degrees of freedom: Goldstone Bosons from spontaneous breaking of chiral symmetry
- Powercounting: Dimensional counting in momenta/masses
- Breakdown scale: Resonances, so about $M_\rho$. 
Chiral Perturbation Theory

A general Effective Field Theory:
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Chiral Symmetry

QCD: $N_f$ light quarks: equal mass: interchange: $SU(N_f)_V$

But
\[
\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[ i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R) \right]
\]

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Spontaneous breakdown

- $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
- $SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$
- 8 generators broken $\implies$ 8 massless degrees of freedom and interaction vanishes at zero momentum
Goldstone Bosons

Power counting in momenta: Meson loops, Weinberg power counting

rules

\[ \int d^4 p \]

\[ (p^2)^2 \left( \frac{1}{p^2} \right)^2 p^4 = p^4 \]

one loop example

\[ \frac{1}{p^2} \]

\[ p^4 \]
Chiral Perturbation Theories

- Which chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$, for $N_f = 2, 3, \ldots$ and extensions to (partially) quenched
- Or beyond QCD
- Space-time symmetry: Continuum or broken on the lattice: Wilson, staggered, mixed action
- Volume: Infinite, finite in space, finite $T$
- Which interactions to include beyond the strong one
- Which particles included as non Goldstone Bosons
- My general belief: if it involves soft pions (or soft $K, \eta$) some version of ChPT exists
Lagrangians: Lowest order

\[ U(\phi) = \exp(i\sqrt{2}\Phi/F_0) \] parametrizes Goldstone Bosons

\[ \Phi(\chi) = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\
K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} 
\end{pmatrix}. \]

LO Lagrangian: \[ \mathcal{L}_2 = \frac{F_0^2}{4} \{ \langle D_\mu U^\dagger D^\mu U \rangle + \langle \chi^\dagger U + \chi U^\dagger \rangle \}, \]

\[ D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu, \]
left and right external currents: \( r(l)_\mu = v_\mu + (-)a_\mu \)

Scalar and pseudoscalar external densities: \( \chi = 2B_0(s + ip) \) quark masses via scalar density: \( s = M + \cdots \)

\[ \langle A \rangle = Tr_F (A) \]
### Lagrangians: Lagrangian structure

<table>
<thead>
<tr>
<th></th>
<th>2 flavour</th>
<th>3 flavour</th>
<th>PQChPT/N(_f) flavour</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p^2)</td>
<td>(F, B) 2</td>
<td>(F_0, B_0) 2</td>
<td>(F_0, B_0) 2</td>
</tr>
<tr>
<td>(p^4)</td>
<td>(l_i^r, h_i^r) 7+3</td>
<td>(L_i^r, H_i^r) 10+2</td>
<td>(\hat{L}_i^r, \hat{H}_i^r) 11+2</td>
</tr>
<tr>
<td>(p^6)</td>
<td>(c_i^r) 52+4</td>
<td>(C_i^r) 90+4</td>
<td>(K_i^r) 112+3</td>
</tr>
</tbody>
</table>

\(p^2\): Weinberg 1966
\(p^4\): Gasser, Leutwyler 84,85
\(p^6\): JB, Colangelo, Ecker 99,00

- \(L_i\): LEC = Low Energy Constants = ChPT parameters
- \(H_i\): contact terms: value depends on definition of currents/densities
- Finite volume: no new LECs
- Other effects: (many) new LECs
Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms
- includes Isospin and the eightfold way ($SU(3)_V$)
- Unitarity included perturbatively

\[
m^2 = 2B\hat{m} + \left( \frac{2B\hat{m}}{F} \right)^2 \left[ \frac{1}{32\pi^2} \log \frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu) \right] + \cdots
\]

\[
M^2 = 2B\hat{m}
\]
Overview

Let’s go over to the next point: dealing with the parameters

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(Partial) History/References


- $p^6$ 2 flavour: several papers (see later)


- Review article two-loops:

- Update of fits + new input:


- Lattice: FLAG reports:
Two flavour LECs

- \( \bar{l}_1 \) to \( \bar{l}_4 \): ChPT at order \( p^6 \) and the Roy equation analysis in \( \pi\pi \) and \( F_S \). Colangelo, Gasser and Leutwyler, *Nucl. Phys. B* 603 (2001) 125 [hep-ph/0103088] Compatible with Rios, Nebraska, Pelaez
- \( \bar{l}_5 \) and \( \bar{l}_6 \): from \( F_V \) and \( \pi \to l\nu\gamma \). JB,(Colangelo,)Talavera and from \( \Pi_V - \Pi_A \). González-Alonso, Pich, Prades

\[
\begin{align*}
\bar{l}_1 &= -0.4 \pm 0.6, \\
\bar{l}_2 &= 4.3 \pm 0.1, \\
\bar{l}_3 &= 2.9 \pm 2.4, \\
\bar{l}_4 &= 4.4 \pm 0.2, \\
\bar{l}_5 &= 12.24 \pm 0.21, \\
\bar{l}_6 &= 16.0 \pm 0.5 \pm 0.7.
\end{align*}
\]

- \( l_7 \sim 5 \cdot 10^{-3} \) from \( \pi^0-\eta \) mixing. Gasser, Leutwyler 1984
- Guesstimate including lattice: \( \bar{l}_3 = 3.0 \pm 0.8 \bar{l}_4 = 4.3 \pm 0.3 \)
Three flavour LECs: uncertainties

- $m_K^2, m_\eta^2 \gg m_\pi^2$
- Contributions from $\rho^6$ Lagrangian are larger
- Reliance on estimates of the $C_i$ much larger
- Typically: $C'_i$: (terms with)
  - kinematical dependence $\equiv$ measurable
  - quark mass dependence $\equiv$ impossible (without lattice)
    - 100% correlated with $L'_i$
- How suppressed are the $1/N_c$-suppressed terms?
- Are we really testing ChPT or just doing a phenomenological fit?
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Testing if ChPT works: relations

Systematic search for relations between observables that do not depend on the $C'_i$.
Included:
- $m^2_M$ and $F_M$ for $\pi, K, \eta$.
- 11 $\pi\pi$ threshold parameters.
- 14 $\pi K$ threshold parameters.
- 6 $\eta \rightarrow 3\pi$ decay parameters.
- 10 observables in $K_{\ell 4}$.
- 18 in the scalar formfactors.
- 11 in the vector formfactors.
- Total: 76
We found 35 relations.
Relations at NNLO: summary

- We did numerics for $\pi\pi$ (7), $\pi K$ (5) and $K_{\ell 4}$ (1)
- 13 relations
- $\pi\pi$: similar quality in two and three flavour ChPT
  The two involving $a_3^-$ significantly did not work well
- $\pi K$: relation involving $a_3^-$ not OK
  one more has very large NNLO corrections
- The relation with $K_{\ell 4}$ also did not work: related to that
  ChPT has trouble with curvature in $K_{\ell 4}$
- Conclusion: Three flavour ChPT “sort of” works
Fits: inputs


- $M_{\pi}, M_K, M_{\eta}, F_{\pi}, F_K/F_{\pi}$
- $\langle r^2 \rangle_{\pi}, c_S^\pi$ slope and curvature of $F_S$
- $\pi\pi$ and $\pi K$ scattering lengths $a_0^0, a_0^2, a_0^{1/2}$ and $a_0^{3/2}$.
- Value and slope of $F$ and $G$ in $K_{\ell 4}$
- $\frac{m_s}{\hat{m}} = 27.5$ (lattice)
- $\bar{l}_1, \ldots, \bar{l}_4$
- more variation with $C_i^r$, a penalty for a large $p^6$ contribution to the masses
- 17+3 inputs and 8 $L_i^r + 34$ $C_i^r$ to fit
## Main fit

<table>
<thead>
<tr>
<th>$10^3 L^r_i$</th>
<th>ABT01</th>
<th>BJ12</th>
<th>$L^r_4$ free</th>
<th>BE14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3 L^r_1$</td>
<td>0.39(12)</td>
<td>0.88(09)</td>
<td>0.64(06)</td>
<td>0.53(06)</td>
</tr>
<tr>
<td>$10^3 L^r_2$</td>
<td>0.73(12)</td>
<td>0.61(20)</td>
<td>0.59(04)</td>
<td>0.81(04)</td>
</tr>
<tr>
<td>$10^3 L^r_3$</td>
<td>$-2.34(37)$</td>
<td>$-3.04(43)$</td>
<td>$-2.80(20)$</td>
<td>$-3.07(20)$</td>
</tr>
<tr>
<td>$10^3 L^r_4$</td>
<td>$\equiv 0$</td>
<td>0.75(75)</td>
<td>0.76(18)</td>
<td>$\equiv 0.3$</td>
</tr>
<tr>
<td>$10^3 L^r_5$</td>
<td>0.97(11)</td>
<td>0.58(13)</td>
<td>0.50(07)</td>
<td>1.01(06)</td>
</tr>
<tr>
<td>$10^3 L^r_6$</td>
<td>$\equiv 0$</td>
<td>0.29(8)</td>
<td>0.49(25)</td>
<td>0.14(05)</td>
</tr>
<tr>
<td>$10^3 L^r_7$</td>
<td>$-0.30(15)$</td>
<td>$-0.11(15)$</td>
<td>$-0.19(08)$</td>
<td>$-0.34(09)$</td>
</tr>
<tr>
<td>$10^3 L^r_8$</td>
<td>0.60(20)</td>
<td>0.18(18)</td>
<td>0.17(11)</td>
<td>0.47(10)</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>0.26</td>
<td>1.28</td>
<td>0.48</td>
<td>1.04</td>
</tr>
<tr>
<td>dof</td>
<td>1</td>
<td>4</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$F_0$ [MeV]</td>
<td>87</td>
<td>65</td>
<td>64</td>
<td>71</td>
</tr>
</tbody>
</table>

$\equiv (17 + 3) - (8 + 34)$
Main fit: Comments

- All values of the $C_i^r$ we settled on are “reasonable”
- Leaving $L_4^r$ free ends up with $L_4^r \approx 0.76$
- Keeping $L_4^r$ small: also $L_6^r$ and $2L_1^r - L_2^r$ small (large $N_c$ relations)
- Compatible with lattice determinations
- Not too bad with resonance saturation both for $L_i^r$ and $C_i^r$
- Decent convergence (but enforced for masses)
- Many prejudices went in: large $N_c$, resonance model, quark model estimates, ...
Some results of this fit

Mass:
\[
\begin{align*}
\frac{m_\pi^2}{m_{\pi,\text{phys}}^2} &= 1.055(p^2) - 0.005(p^4) - 0.050(p^6), \\
\frac{m_K^2}{m_{K,\text{phys}}^2} &= 1.112(p^2) - 0.069(p^4) - 0.043(p^6), \\
\frac{m_\eta^2}{m_{\eta,\text{phys}}^2} &= 1.197(p^2) - 0.214(p^4) + 0.017(p^6),
\end{align*}
\]

Decay constants:
\[
\begin{align*}
\frac{F_\pi}{F_0} &= 1.000(p^2) + 0.208(p^4) + 0.088(p^6), \\
\frac{F_K}{F_\pi} &= 1.000(p^2) + 0.176(p^4) + 0.023(p^6).
\end{align*}
\]

Scattering:
\[
\begin{align*}
a_0^0 &= 0.160(p^2) + 0.044(p^4) + 0.012(p^6), \\
a_0^{1/2} &= 0.142(p^2) + 0.031(p^4) + 0.051(p^6).
\end{align*}
\]
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An example where ChPT triumphed

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Charged pion polarizabilities: experiment


- Expand $\gamma\pi^\pm \to \gamma\pi^\pm$ near threshold: $(z_\pm = 1 \pm \cos \theta_{\text{cm}})$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}^{\text{Born}} - \frac{\alpha m^3_\pi ((s - m^2_\pi)^2)}{4s^2(sz_+ + m^2_\pi z_-)} \left( z_+^2 (\alpha - \beta) + \frac{s^2}{m^4_\pi} z_-^2 (\alpha + \beta) \right)$$

- Three ways to measure: (all assume $\alpha + \beta = 0$)
  - $\pi\gamma \to \pi\gamma$ (Primakoff, high energy pion beam)
    - Dubna (1985) $\alpha = (6.8 \pm 1.4) \times 10^{-4}$ fm$^3$
    - Compass (CERN, 2015) $\alpha = (2.0 \pm 0.6 \pm 0.7) \times 10^{-4}$ fm$^3$
  - $\gamma\pi \to \pi\gamma$ (via one-pion exchange)
    - Lebedev (1986) $\alpha = (20 \pm 12) \times 10^{-4}$ fm$^3$
    - Mainz (2005) $\alpha = (5.8 \pm 0.75 \pm 1.5 \pm 0.25) \times 10^{-4}$ fm$^3$
  - $\gamma\gamma \to \pi\pi$ (in $e^+e^- \to e^+e^-\pi^+\pi^-$)
    - MarkII data analyzed (1992) $\alpha = (2.2 \pm 1.1) \times 10^{-4}$ fm$^3$

- Extrapolation and subtraction: difficult experiments
Polarizabilities: extrapolations needed

\[ \gamma + \gamma \rightarrow \pi^+ + \pi^- \]

\[ \sigma_{\text{tot}}(|\cos(\theta_{\pi\pi})| < 0.6) \text{ (nb)} \]

From Pasquini et al. 2008
Recent results in chiral meson physics

Johan Bijnens

Chiral Perturbation Theory

Determination of LECs in the continuum

Charged Pion Polarizabilities

Finite volume

Beyond QCD

A mesonic ChPT program framework

Leading logarithms

Conclusions

Charged pion polarizabilities: theory

- **ChPT:**
    \[ \alpha + \beta = 0, \quad \alpha = (2.8 \pm 0.2) \times 10^{-4} \, \text{fm}^3 \]
    
    Input \( \pi \rightarrow e\nu\gamma \) (error only from this)
  - Two-loop: Bürgi, 1996, Gasser, Ivanov, Sainio 2006
    \[ \alpha + \beta = 0.16 \times 10^{-4} \, \text{fm}^3, \quad \alpha = (2.8 \pm 0.5) \times 10^{-4} \, \text{fm}^3 \]

- Dispersive analysis from \( \gamma\gamma \rightarrow \pi\pi \):
  - Fil’kov-Kashevarov, 2005
    \[ (\alpha_1 - \beta_1) = (13.0^{+2.6}_{-1.9}) \times 10^{-4} \, \text{fm}^3 \]
  - Critized by Pasquini-Drechsel-Scherer, 2008
    “Large model dependence in their extraction”
    “Our calculations... are in reasonable agreement with ChPT for charged pions”
    \[ (\alpha_1 - \beta_1) = (5.7) \times 10^{-4} \, \text{fm}^3 \] perfectly possible
Overview

An example of other effects:

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Finite volume

- Lattice QCD calculates at different quark masses, volumes boundary conditions, ... 
- A general result by Lüscher: relate finite volume effects to scattering (1986)
- Chiral Perturbation Theory is also useful for this
  - $M_\pi, F_\pi, \langle \bar{q}q \rangle$ one-loop equal mass case
- I will stay with ChPT and the $p$ regime ($M_\pi L \gg 1$)
- $1/m_\pi = 1.4$ fm 
  - may need to go beyond leading $e^{-m_\pi L}$ terms
- Convergence of ChPT is given by $1/m_\rho \approx 0.25$ fm
Finite volume: selection of ChPT results

- masses and decay constants for $\pi, K, \eta$ one-loop

- $M_\pi$ at 2-loops (2-flavour)

- $\langle \bar{q}q \rangle$ at 2 loops (3-flavour)

- Twisted mass at one-loop

- Twisted boundary conditions

- This talk:
  - Twisted boundary conditions and some funny effects
  - Some results on masses 3-flavours at two loop order
Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i / L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions: $q(x^i + L) = e^{i\theta^i_q} q(x^i)$
- Then momenta are $p^i = \theta^i / L + 2\pi n^i / L$. Allows to map out momentum space on the lattice much better Bedaque, . . .
- But:
  - Box: Rotation invariance $\rightarrow$ cubic invariance
  - Twisting: reduces symmetry further

Consequences:
- $m^2(\vec{p}^2) = E^2 - \vec{p}^2$ is not constant
- There are typically more form-factors
- In general: quantities depend on more (all) components of the momenta
- Charge conjugation involves a change in momentum
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Twisted boundary conditions: volume correction masses

$\mathbf{m}_\pi \mathbf{L} = 2$, $\mathbf{\tilde{\theta}}_u = (\theta, 0, 0)$, $\mathbf{\tilde{\theta}}_d = \mathbf{\tilde{\theta}}_s = 0$

$\mathbf{\Delta}^V \mathbf{X} = \mathbf{X}^V - \mathbf{X}^\infty$ (dip is going through zero)
Volume correction decay constants: $F_{\pi^+}$


- \[ \langle 0 | A_\mu^M | M(p) \rangle = i \sqrt{2} F_M p_\mu + i \sqrt{2} F_V^M \]

- Extra terms are needed for Ward identities

\[ \theta = 0 \]
\[ \theta = \pi/8 \]
\[ \theta = \pi/4 \]
\[ \theta = \pi/2 \]

Relative for $F_{\pi}$

Extra for $\mu = x$
Volume correction electromagnetic formfactor

- earlier two-flavour work:

\[ \langle M'(p')|j_\mu|M(p)\rangle = f_\mu = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu \]

- Extra terms are again needed for Ward identities

- Note that masses have finite volume corrections
  - \( q^2 \) for fixed \( \vec{p} \) and \( \vec{p}' \) has corrections
  - small effect
  - This also affects the ward identities, e.g.
  \[ q^\mu f_\mu = (p^2 - p'^2) f_+ + q^2 f_- + q^\mu h_\mu = 0 \]
  - is satisfied but all effects should be considered
Volume correction electromagnetic formfactor

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Volume correction electromagnetic formfactor

\[ f_\mu = -\frac{1}{\sqrt{2}} \langle \pi^0(p') | \bar{d} \gamma_\mu u | \pi^+(p) \rangle \]
\[ = (1 + f_+^\infty + \Delta^V f_+) (p + p')_\mu + \Delta^V f_- q_\mu + \Delta^V h_\mu \]

- Pure loop plotted: \( f_+^\infty (p + p'), \Delta^V f_+ (p + p') \) and \( \Delta^V f_\mu \)

Finite volume corrections large, different for different \( \mu \)
Masses at two-loop order

- Sunset integrals at finite volume done
  

- Loop calculations:
  

\begin{itemize}
  \item Agreement for $N_f = 2, 3$ for pion
  \item $K$ has no pion loop at LO
\end{itemize}
Decay constants at two-loop order

- Sunset integrals at finite volume done
  

- Loop calculations:


![Graph showing decay constants at two-loop order](image)

- Agreement for $N_f = 2, 3$ for pion
- $K$ now has a pion loop at LO
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ChPT for other theories:

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QCDlike and/or technicolor theories

• One can also have different symmetry breaking patterns from underlying fermions
• Three generic cases
  • $SU(N) \times SU(N)/SU(N)$
  • $SU(2N)/SO(2N)$
  • $SU(2N)/Sp(2N)$

• Many one-loop results existed especially for the first case (several discovered only after we published our work)
• Equal mass case pushed to two loops **JB, Lu, 2009-11**
$N_F$ fermions in a representation of the gauge group

- **complex (QCD):**
  - $q^T = (q_1 \ q_2 \ldots \ q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R$
    - $q_L \rightarrow g_Lq_L$ and $g_R \rightarrow g_Rq_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \bar{q}_j q_i \rangle \propto \delta_{ij}$
  - $g_L = g_R$ then $\Sigma_{ij} \rightarrow \Sigma_{ij} \implies$ conserved $H = SU(N_F)_V$:

- **Real (e.g. adjoint):** $\hat{q}^T = (q_{R1} \ldots q_{RN_F} \ \tilde{q}_{R1} \ldots \tilde{q}_{RN_F})$
  - $\tilde{q}_{Ri} \equiv C\tilde{q}_{Li}^T$ goes under gauge group as $q_{Ri}$
  - some Goldstone bosons have baryonnumber
  - Global $G = SU(2N_F)$ and $\hat{q} \rightarrow g\hat{q}$
  - $\langle \bar{q}_j q_i \rangle$ is really $\langle (\hat{q}_j)^T C\hat{q}_i \rangle \propto J_{Sij}$ $J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
  - Conserved if $gJ_Sg^T = J_S \implies H = SO(2N_F)$
$N_F$ fermions in a representation of the gauge group

- complex (QCD):
  - $q^T = (q_1 \ q_2 \ldots q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R$
  - $q_L \rightarrow g_L q_L$ and $g_R \rightarrow g_R q_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \bar{q}_j q_i \rangle \propto \delta_{ij}$
  - $g_L = g_R$ then $\Sigma_{ij} \rightarrow \Sigma_{ij} \implies$ conserved $H = SU(N_F)_V$:

- Real (e.g. adjoint): $\hat{q}^T = (q_{R1} \ldots q_{RN_F} \ \tilde{q}_{R1} \ldots \tilde{q}_{RN_F})$
  - $\tilde{q}_{Ri} \equiv C \tilde{q}_{Li}^T$ goes under gauge group as $q_{Ri}$
  - some Goldstone bosons have baryon number
  - Global $G = SU(2N_F)$ and $\hat{q} \rightarrow g \hat{q}$
  - $\langle \bar{q}_j q_i \rangle$ is really $\langle (\hat{q}_j)^T C \hat{q}_i \rangle \propto J_{Sij}$ $J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
  - Conserved if $gJ_S g^T = J_S \implies H = SO(2N_F)$. 

Recent results in chiral meson physics

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Charged Pion Polarizabilities

Finite volume

Beyond QCD

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Leading logarithms

Conclusions
$N_F$ fermions in a representation of the gauge group

- Complex (QCD): $q^T = (q_1 \, q_2 \ldots \, q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R$ $q_L \to g_L q_L$ and $g_R \to g_R q_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \bar{q}_j q_i \rangle \propto \delta_{ij}$
  - Conserved $H = SU(N_F)_V$: $g_L = g_R$ then $\Sigma_{ij} \to \Sigma_{ij}$

- Pseudoreal (e.g. two-colours):
  $\hat{q}^T = (q_{R1} \ldots \, q_{RN_F} \, \tilde{q}_{R1} \ldots \, \tilde{q}_{RN_F})$
  - $\tilde{q}_{R\alpha i} \equiv \epsilon_{\alpha \beta} C \tilde{q}_L^{\beta i}$ goes under gauge group as $q_{R\alpha i}$
  - some Goldstone bosons have baryonnumber
  - Global $G = SU(2N_F)$ and $\hat{q} \to g \hat{q}$
  - $\langle \bar{q}_j q_i \rangle$ is really $\epsilon_{\alpha \beta} \langle (\hat{q}_{\alpha j})^T C \hat{q}_\beta i \rangle \propto J_{Aij}$ $J_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$
  - Conserved if $gJ_A g^T = J_A \implies H = Sp(2N_F)$
Lagrangians

JB, Lu, arXiv:0910.5424: 3 cases similar with \( u = \exp \left( \frac{i}{\sqrt{2F}} \phi^a X^a \right) \)

But the matrices \( X^a \) are:

- Complex or \( SU(N) \times SU(N)/SU(N) \): all \( SU(N) \) generators
- Real or \( SU(2N)/SO(2N) \): \( SU(2N) \) generators with \( X^a J_S = J_S X^{aT} \)
- Pseudoreal or \( SU(2N)/Sp(2N) \): \( SU(2N) \) generators with \( X^a J_A = J_A X^{aT} \)
- Note that the latter are not the usual ways of parametrizing \( SO(2N) \) and \( Sp(2N) \) matrices
The main useful formulae

Calculating for equal mass case goes through using:

Complex:
\[ \langle X^a A X^a B \rangle = \langle A \rangle \langle B \rangle - \frac{1}{N_F} \langle AB \rangle , \]
\[ \langle X^a A \rangle \langle X^a B \rangle = \langle AB \rangle - \frac{1}{N_F} \langle A \rangle \langle B \rangle . \]

Real:
\[ \langle X^a A X^a B \rangle = \frac{1}{2} \langle A \rangle \langle B \rangle + \frac{1}{2} \langle AJ_S B^T J_S \rangle - \frac{1}{2N_F} \langle AB \rangle , \]
\[ \langle X^a A \rangle \langle X^a B \rangle = \frac{1}{2} \langle AB \rangle + \frac{1}{2} \langle AJ_S B^T J_S \rangle - \frac{1}{2N_F} \langle A \rangle \langle B \rangle . \]

Pseudoreal:
\[ \langle X^a A X^a B \rangle = \frac{1}{2} \langle A \rangle \langle B \rangle + \frac{1}{2} \langle AJ_A B^T J_A \rangle - \frac{1}{2N_F} \langle AB \rangle , \]
\[ \langle X^a A \rangle \langle X^a B \rangle = \frac{1}{2} \langle AB \rangle - \frac{1}{2} \langle AJ_A B^T J_A \rangle - \frac{1}{2N_F} \langle A \rangle \langle B \rangle . \]

So can do the calculations for all cases
\( \phi \phi \rightarrow \phi \phi \)

- **\( \pi \pi \) scattering**
  - Amplitude in terms of \( A(s, t, u) \)
    \[
    M_{\pi\pi}(s, t, u) = \delta^{ab} \delta^{cd} A(s, t, u) + \delta^{ac} \delta^{bd} A(t, u, s) + \delta^{ad} \delta^{bc} A(u, s, t). 
    \]
  - Three intermediate states \( I = 0, 1, 2 \)

- **Our three cases**
  - Two amplitudes needed \( B(s, t, u) \) and \( C(s, t, u) \)
    \[
    M(s, t, u) = \begin{cases} 
    \left[ \langle X^a X^b X^c X^d \rangle + \langle X^a X^d X^c X^b \rangle \right] B(s, t, u) \\
    + \left[ \langle X^a X^c X^d X^b \rangle + \langle X^a X^b X^d X^c \rangle \right] B(t, u, s) \\
    + \left[ \langle X^a X^d X^b X^c \rangle + \langle X^a X^c X^b X^d \rangle \right] B(u, s, t) \\
    + \delta^{ab} \delta^{cd} C(s, t, u) + \delta^{ac} \delta^{bd} C(t, u, s) + \delta^{ad} \delta^{bc} C(u, s, t). 
    \end{cases}
    \]

  \[
  B(s, t, u) = B(u, t, s) \quad C(s, t, u) = C(s, u, t). 
  \]

  - 7, 6 and 6 possible intermediate states

- All formulas similar length to \( \pi\pi \) cases but there are so many of them
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\[ \phi \phi \rightarrow \phi \phi: a_0^l/n \]
Conclusions for “Beyond QCD”

Calculations done:

- $M_{\text{phys}}^2$
- $F_{\text{phys}}$
- Meson-meson scattering
- Equal mass case: allows to get fully analytical result just as for 2-flavour ChPT
- Two-point functions relevant for $S$-parameter

To remember:

- Different symmetry patterns can appear for different gaugegroups and fermion representations
- Nonperturbative: lattice needs extrapolation formulae
Overview

Making the programs more accessible for others to use:

1. Chiral Perturbation Theory
2. Determination of LECs in the continuum
3. Charged Pion Polarizabilities
4. Finite volume
5. Beyond QCD
6. A mesonic ChPT program framework
7. Leading logarithms
8. Conclusions
Program availability

- Two-loop results have very long expressions
- Many not published but available from http://www.thep.lu.se/~bijnens/chpt/
- Many programs available on request from the authors
- Idea: make a more general framework

CHIRON:

JB,

“CHIRON: a package for ChPT numerical results at two loops,”

http://www.thep.lu.se/~bijnens/chiron/
Program availability: CHIRON

- Present version: 0.52
- Classes to deal with $L_i$, $C_i$, $L_i^{(n)}$, $K_i$, standardized in/output, changing the scale,…
- Loop integrals: one-loop and sunset integrals
- Included so far (at two-loop order):
  - Masses, decay constants and $\langle \bar{q}q \rangle$ for the three flavour case
  - Masses and decay constants at finite volume in the three flavour case
  - Masses and decay constants in the partially quenched case for three sea quarks
- A large number of example programs is included
Can we calculate something of high loop orders?

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Leading Logarithms

- Take a quantity with a single scale: $F(M)$
- The dependence on the scale in field theory is typically logarithmic
- $L = \log (\mu / M)$
- $F = F_0 + F_1^1 L + F_0^1 + F_2^2 L^2 + F_1^2 L + F_0^2 + F_3^3 L^3 + \cdots$
- Leading Logarithms: The terms $F_m^m L^m$

The $F_m^m$ can be more easily calculated than the full result

- $\mu (dF / d\mu) \equiv 0$
- Ultraviolet divergences in Quantum Field Theory are always local
Weinberg’s argument

- Weinberg, Physica A96 (1979) 327
- Two-loop leading logarithms can be calculated using only one-loop: Weinberg consistency conditions
- Proof at all orders:
  - using $\beta$-functions: Büchler, Colangelo, hep-ph/0309049
- Proof relies on
  - $\mu$: dimensional regularization scale
  - $d = 4 - w$
  - at $n$-loop order ($\hbar^n$) must cancel:
    - $1/w^n, \log \mu/w^{n-1}, \ldots, \log^{n-1} \mu/w$
    - This allows for relations between diagrams
    - All needed for $\log^n \mu$ coefficient can be calculated from one-loop diagrams
Mass to $\hbar^2$

- $\hbar^1$: $\begin{array}{c} \hline 0 \\ \end{array} \rightarrow \begin{array}{c} \hline 1 \\ \end{array}$

- $\hbar^2$: $\begin{array}{c} \hline 1 \\ \end{array} \rightarrow \begin{array}{c} \hline 0 \\ \end{array} \rightarrow \begin{array}{c} \hline 2 \\ \end{array}$

- but also needs $\hbar^1$: $\begin{array}{c} \hline 0 \\ \end{array} \rightarrow \begin{array}{c} \hline 0 \\ \end{array} \rightarrow \begin{array}{c} \hline 1 \\ \end{array}$
General

- Calculate the divergence
- rewrite it in terms of a local Lagrangian
  - Luckily: symmetry kept: we know result will be symmetrical, hence do not need to explicitly rewrite the Lagrangians in a nice form
  - Luckily: we do not need to go to a minimal Lagrangian
  - So everything can be computerized
- We keep all terms to have all 1PI (one particle irreducible) diagrams finite
Massive $O(N)$ sigma model

- $O(N + 1)/O(N)$ nonlinear sigma model
- \[ \mathcal{L}_{n\sigma} = \frac{F^2}{2} \partial_\mu \Phi^T \partial^\mu \Phi + F^2 \chi^T \Phi. \]
  - $\Phi$ is a real $N + 1$ vector; $\Phi \rightarrow O\Phi$; $\Phi^T \Phi = 1$.
  - Vacuum expectation value $\langle \Phi^T \rangle = (1 \ 0 \ldots 0)$
  - Explicit symmetry breaking: $\chi^T = (M^2 \ 0 \ldots 0)$
  - Both spontaneous and explicit symmetry breaking
  - $N$-vector $\phi$

- $N$ (pseudo-)Nambu-Goldstone Bosons
- $N = 3$ is two-flavour Chiral Perturbation Theory
Results

\[ M_{\text{phys}}^2 = M^2(1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \ldots) \]

\[ L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{M^2} \]

<table>
<thead>
<tr>
<th>i</th>
<th>(a_i, N = 3)</th>
<th>(a_i) for general (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-\frac{1}{2})</td>
<td>(1 - \frac{N}{2})</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{17}{8})</td>
<td>(\frac{7}{4} - \frac{7N}{4} + \frac{5N^2}{8})</td>
</tr>
<tr>
<td>3</td>
<td>(-\frac{103}{24})</td>
<td>(\frac{37}{12} - \frac{113N}{24} + \frac{15N^2}{4} - N^3)</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{24367}{1152})</td>
<td>(\frac{839}{144} - \frac{1601N}{144} + \frac{695N^2}{48} - \frac{135N^3}{16} + \frac{231N^4}{128})</td>
</tr>
<tr>
<td>5</td>
<td>(-\frac{8821}{144})</td>
<td>(\frac{33661}{2400} - \frac{1151407N}{43200} + \frac{197587N^2}{4320} - \frac{12709N^3}{300} + \frac{6271N^4}{320} - \frac{7N^5}{2})</td>
</tr>
</tbody>
</table>

\(F_{\text{phys}}, \langle \bar{q}i\gamma_{\mu}q_i \rangle\) as well done

Anyone recognize any funny functions?

Many more and larger tables in the papers
Numerical results: masses

Left: \( \frac{M_{\text{phys}}^2}{M^2} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots \)

Right: \( \frac{M_{\text{phys}}^2}{M^2} = 1 + d_1 L_{M_{\text{phys}}} + d_2 L_{M_{\text{phys}}}^2 + d_3 L_{M_{\text{phys}}}^3 + \cdots \)

\( F = 90 \text{ MeV} \) (left), \( \mu = 0.77 \text{ GeV} \), \( F_\pi = 93 \text{ MeV} \) (right)
Numerical results: decay constants

Left: \( \frac{F_{\text{phys}}}{F} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots \)

Right: \( \frac{F_{\text{phys}}}{F} = 1 + d_1 L_{M_{\text{phys}}} + d_2 L_{M_{\text{phys}}}^2 + d_3 L_{M_{\text{phys}}}^3 + \cdots \)

\( F = 90 \text{ MeV} \) (left), \( \mu = 0.77 \text{ GeV} \), \( F_\pi = 93 \text{ MeV} \) (right)
Numerical results: $\pi\pi$ scattering

\[ a_0^0, a_2^2 \]

$$\mu = 0.77 \text{ GeV}, \quad F_\pi = 93 \text{ MeV} \quad \text{(right)}$$

Here the leading logs actually dominate.
Anomaly for $O(4)/O(3)$


\[ \mathcal{L}_{WZW} = -\frac{N_c}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \left\{ \epsilon^{abc} \left( \frac{1}{3} \Phi^0 \partial_\mu \Phi^a \partial_\nu \Phi^b \partial_\rho \Phi^c - \partial_\mu \Phi^0 \partial_\nu \Phi^a \partial_\rho \Phi^b \Phi^c \right) \nu_\sigma^0 
\]
\[ + (\partial_\mu \Phi^0 \Phi^a - \Phi^0 \partial_\mu \Phi^a) \nu_\nu^0 \partial_\rho \nu_\rho^0 + \frac{1}{2} \epsilon^{abc} \Phi^0 \Phi^a \nu_\mu^0 \nu_\nu^0 \nu_\sigma^0 \right\}. \]

\[ A(\pi^0 \to \gamma(k_1)\gamma(k_2)) = \]
\[ \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\mu}_1(k_1)\epsilon^{*\nu}_2(k_2) k_1^\alpha k_2^\beta F_{\pi\gamma\gamma}(k_1^2, k_2^2) \]

\[ F_{\pi\gamma\gamma}(k_1^2, k_2^2) = \frac{e^2}{4\pi^2 F_{\gamma}} \hat{F} F_{\gamma}(k_1^2) F_{\gamma}(k_2^2) F_{\gamma\gamma}(k_1^2, k_2^2) \]

\[ \hat{F} : \text{on-shell photon}; \quad F_{\gamma}(k^2) : \text{formfactor}; \quad F_{\gamma\gamma} \text{ nonfactorizable part} \]
Anomaly for $O(4)/O(3)$

- Done to six-loops
  \[ \hat{F} = 1 + 0 - 0.000372 + 0.000088 + 0.000036 + 0.000009 + 0.0000002 + \ldots \]
- Really good convergence
- $F_{\gamma\gamma}$ only starts at three-loop order (could have been two)
- $F_{\gamma\gamma}$ in the chiral limit only starts at four-loops.
- The leading logarithms thus predict this part to be fairly small.
- $F_{\gamma}(k^2)$: plot
Anomaly for $O(4)/O(3)$

Leading logs small, converge fast
Conclusions Leading Logs

- Many quantities in massive $O(N)$
- Had hoped: recognize the series also for other cases
- Limited essentially by CPU time and size of intermediate files
- Some studies on convergence etc.
- $\pi\pi, F_V$ and $F_S$ to four-loop order ($F_V$ higher)
- The technique can be generalized to other models/theories
- More details, nucleon and many more results: talk in parallel session
Conclusions

ChPT is a tool for many different areas of phenomenology. I talked about a few of them:

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