Nuclear Physics B240 [FS12] (1984) 533-542

STOCHASTIC CONFINEMENT AND DIMENSIONAL REDUCTION (II). Three-dimensional SU(2) lattice gauge theory

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Received 3 May 1984

We study 3-dimensional SU(2) lattice gauge theory with respect to dimensional reduction By Monte Carlo calculations we find that this reduction is valid to a good approximation (within ≈10%) The adjoint string tension is found to scale approximately We also compare the adjoint string tension with a string theory

1. Introduction

In a previous paper [1] we have investigated the phenomenon of dimensional reduction in four-dimensional SU(2) lattice gauge theory It turns out that to a good approximation (≈10%) the four-dimensional theory reduces to the corresponding two-dimensional theory at large distances.

In the present paper we investigate whether a similar phenomenon holds in three-dimensional SU(2) lattice gauge theory Thus we want to investigate whether at large distances the three-dimensional theory reduces to the corresponding two-dimensional theory

Dimensional reduction has been shown explicitly to occur in solid state physics [2] for systems described by random fields If certain assumptions are satisfied a d-dimensional system in a random field has a large distance behaviour which is equivalent to the same $(d-2)$-dimensional system without the external random field If the QCD ground state can be characterized by a random colour magnetic field, it was then suggested by two of the authors [3] that a phenomenon similar to the solid state case could occur in QCD However, the reduction $d \to (d-2)$ is based on some simplifying assumptions, and in general more complicated reductions are expected even in solid state physics

The main motivation for the work reported in the present paper is therefore to study if the $d \to (d-2)$ reduction is valid in QCD. If it is valid, a three-dimensional system would not reduce approximately to a two-dimensional system at large distances

Our result is that to a good approximation a three-dimensional SU(2) lattice gauge theory does reduce to the same two-dimensional theory at large distances Thus we conclude that the $d \to (d-2)$ reduction is not operative in the QCD case.

The plan of this paper is the following: in sect. 2 we remind the reader of a few features relevant for dimensional reduction, and in sect 3 we present our results In sect 4 we compare with a string theory. Sect 5 contains a discussion of the confinement mechanism, and sect 6 contains some conclusions

2. Dimensional reduction

The phenomenon of dimensional reduction has been discussed in much detail in our previous paper [1] We shall therefore not repeat this discussion However, we would like to point out a few features which are of relevance for the three-dimensional case discussed in the present paper

In the solid state example discussed in ref. [2] the quenched free energy has the form

$$e^{-F[\boldsymbol{h}]} = \int d\boldsymbol{\sigma} \exp\left[-\int d^d x\{\tfrac{1}{2}(\nabla\boldsymbol{\sigma})^2 + \tfrac{1}{2}r|\boldsymbol{\sigma}|^2 + u|\boldsymbol{\sigma}|^4 + \boldsymbol{h}\boldsymbol{\sigma}\}\right], \qquad (2\ 1)$$

where $\boldsymbol{\sigma}$ is the order parameter The main point is thus that the $d \to (d-2)$ reduction is based on the simple linear coupling between the magnetic field $\boldsymbol{h}$ and the order parameter $\boldsymbol{\sigma}$ However, one can think of other possible couplings (e g a minimal coupling) and in general the reduction $d \to (d-2)$ is therefore not expected to be valid [4]

In the case of QCD the situation is indeed that we have a very complicated dynamical system which by its own self interactions is supposed to produce the disordered ground state Hence it is really not surprising that the situation in QCD is different from that of eq (2 1) In this connection it should also be remembered that in lattice QCD there is dimensional reduction in the strong coupling limit in the form any dim $\to$ 2 dim at large distance [1].

3. Results on dimensional reduction in the (approximate) scaling region

We shall now present our results on dimensional reduction in the approximate scaling region. In another paper [5] we have discussed the Monte Carlo method used to obtain the numerical data Let us only mention here that the lattice size was 32×16^2 and that we used the discrete subgroup of SU(2) to generate the configurations. The β range was $3.0 \leq \beta \leq 6.5$ and for the larger values of β we used $\approx$6000 sweeps for thermalization and subsequently performed 800 measurements. It was pointed out in ref [5] that for $\beta \geq 5\ 0$ one has approximate scaling We refer the reader to ref [5] for further discussion.

We consider the Creutz ratios

$$\chi_j(R, T) = -\ln \frac{W_j(R, T)\, W_j(R-1, T-1)}{W_j(R, T-1)\, W_j(R-1, T)} \qquad (3\ 1)$$

Here $j = \frac{1}{2}, 1, \frac{3}{2},$ is the isospin quantum number and $W_j(R, T)$ is the Wilson average for quarks in the jth representation Thus $W_{1/2}$ is the usual Wilson average for quarks in the fundamental representation, whereas W_1 is the adjoint Wilson average As discussed in [1], dimensional reduction leads to the prediction

$$\frac{\chi_j(R, T)}{\chi_{1/2}(R, T)} \approx \frac{C_j}{C_{1/2}} = \frac{j(j+1)}{\frac{3}{4}}, \qquad (3\ 2)$$

provided β is large enough (This is the case for the data used in this paper) In eq (3 2) C_j in the Casimir operator for quarks with isospin j and in two dimensions (3 2) follows because the string tension there can be calculated from lowest-order perturbation theory the two-dimensional Coulomb potential is linear

In fig 1 we show our results for the fundamental string tension $\chi_{1/2}(R, T)$ If scaling holds one has $\chi_{1/2} \approx C/\beta^2$ This follows from the superrenormalizability of QCD in three dimensions as well as from the conjecture that the infrared cut-off disappears in gauge invariant quantities [6] For $\beta \geq 5\ 0$ it is seen that one can draw an envelope around $\beta^2\chi \approx 2$ In fact, a more detailed analysis allowing for the fact that the string between the heavy quarks can vibrate, shows that the string tension scales very well for $\beta \geq 5\ 0$ The χ ratios measure the string tension *and* the universal Coulomb potential [7] which comes from roughening This potential is independent

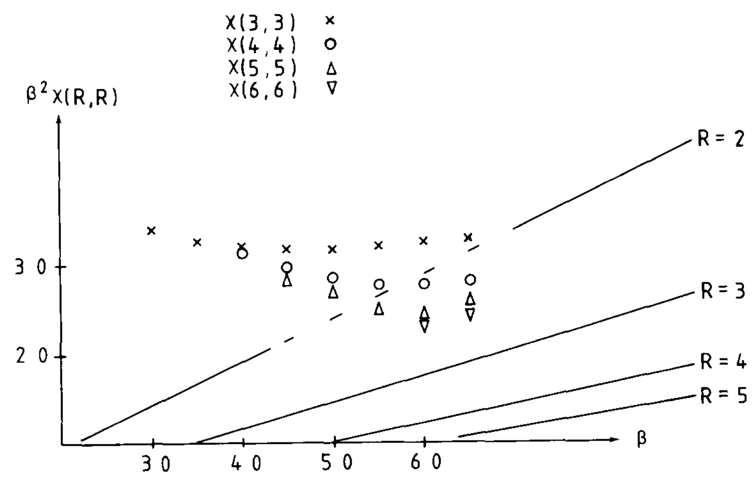

Fig 1 The Creutz ratios for the fundamental string tension The straight lines represent the perturbative (one-gluon exchange) predictions

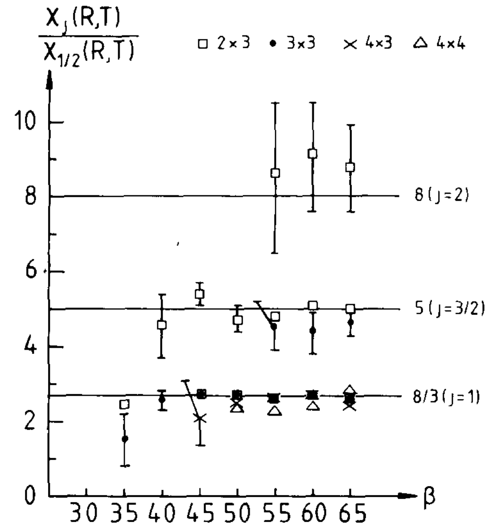

Fig 2 Comparison of the prediction (3 2) with our Monte Carlo data

of β and therefore the χ ratios do not have exact scaling (for details we refer to ref [5]) In sect 4 we are going to do a similar analysis for the "adjoint string"

In fig 2 we show our results for the quantities (3 2) It is seen that to a good approximation dimensional reduction holds The result (3 2) in three dimensions cannot be of perturbative nature From fig 1 it is obvious that even for the smallest Wilson loop considered, the $\chi_{1/2}$ ratios are far from their perturbative values Neither has (3 2) anything to do with strong coupling The strong coupling expansion predicts a ratio of $2j$ instead of $\frac{4}{3}\times j(j+1)$, simply because the leading term in the strong coupling expansion for isospin j quarks consists of $2j$ layers of plaquettes covering the minimal surface spanned by the Wilson loop

It should be emphasized that the string tension for $j=1$ (and, in particular, for $j=2, 3, \quad$) is "unconventional", whereas the $j=\frac{1}{2}, \frac{3}{2}, \quad$ string tensions are conventional The emergence of "unconventional" string tensions in 4 dimensions has been observed by Bernard [8] and by us [1] For very large loops (larger than present day lattices) one expects

$$W_{\mathrm{adj}}(R, T) = \frac{1}{N^2} e^{-k(R+T)} + c\, e^{-\sigma_{\mathrm{adj}} RT - b(R+T)} + \text{possibly other terms} \qquad (3\ 3)$$

Here the first term dominates for very large loops This is due to the fact that the adjoint string can be broken by screening of the adjoint quarks by means of a gluon pair created from the vacuum. However, our numerical results shown in fig 2 (as well as the four-dimensional results [1, 8]) indicate that σ_{adj} scales to the same extent that the fundamental string tension scales Thus we expect the second term in eq (3 3) to survive in the continuum limit, irrespective of the fact that it is subdominant

To put this in more physical terms the adjoint string can of course break, but before it breaks it has to be formed.

4. The "adjoint string"

In this section we shall investigate the adjoint string in more detail

As already mentioned, Luscher, Symanzik and Weisz [7] found that for quarks with isospin $j = \frac{1}{2}$, $-\ln W_{1/2}(R, T)$ contains a "Coulomb" term

$$-\tfrac{1}{24}\pi(d-2)\times T/R, \qquad T \gg R \tag{4 1}$$

In (4 1) d is the dimension of space–time Eq (4 1) is derived under the assumption that one has a string between the heavy quarks in the Wilson loop Physically (4 1) arises from roughening the string vibrates in transverse directions (hence the factor $d-2$ in (4 1)) As Luscher showed [7], the "transverse" effective action at large R, T should be

$$S_{\text{eff}} = \sigma \int d^2\xi \, \partial_\alpha x_\perp \, \partial_\alpha x_\perp + \text{subdominant terms} \tag{4 2}$$

This follows from locality and symmetry considerations In eq (4 2) $x_\perp$ is the transverse displacement of the string In our MC data we do not have $T \gg R$ When calculating (4 2) with $x_\perp = 0$ on the boundary, one gets (see ref [5])

$$-\ln W(R, T) = \sigma RT + d(R+T) - \left[\tfrac{1}{24}\pi T/R + \tfrac{1}{4}\ln(R\mu) + \tfrac{1}{2}\sum_{n=1}^{\infty} \ln\left(1 - e^{-2\pi n T/R}\right)\right] \tag{4 3}$$

The function in the last bracket is symmetric in R and T despite its asymmetric appearance

If we now turn to the adjoint string one would naively expect it to be composed of two strings in the same sense as one can imagine an isospin-1 quark to be made of two isospin-$\frac{1}{2}$ quarks These strings can of course interact From fig 2 we have seen that the Creutz ratios $\chi_j(R, T)$ follow the prediction from dimensional reduction quite well, i e the adjoint χ's are $\frac{8}{3}$ times the fundamental χ's It is therefore natural to assume that the two strings forming the adjoint string interact in such a way as to give the quadratic Casimir operator This corresponds to vector addition

In analogy with (4 3) we therefore make a fit of the type

$$-\ln W_{j=1}(R, T) = \theta_1(\beta)RT + \theta_2(\beta)(R+T) + \theta_3(\beta) - \frac{8}{3}\left[\tfrac{1}{24}\pi T/R + \tfrac{1}{4}\ln(R\mu) - \tfrac{1}{2}\sum_{n=1}^{\infty} \ln\left(1 - e^{-2\pi n T/R}\right)\right] \tag{4 4}$$

In tables 1–4 we give the resulting χ^2 fits We see that the data for $\ln W_{j=1}(R, T)$ are indeed very well represented by a fit of the type (4 4) It should be mentioned

TABLE 1

Comparison of measured and fitted values for $W_{adj}(R, T)$ with $\beta = 5\ 0$

R	T	$W(R, T)$ measured	$W(R, T)$ from fit	Deviation
2	2	0 1129 (3)	0 1127	−0 5
2	3	0 0442 (3)	0 0447	1 6
2	4	0 0177 (2)	0 0177	0 1
2	5	0 0072 (2)	0 0704	−1 3
2	6	0 0029 (1)	0 0028	−0 5
2	7	0 0012 (2)	0 0011	−0 3
3	3	0 0126 (2)	0 0125	−0 7
3	4	0 0034 (1)	0 0036	1 2
3	5	0 00098 (13)	0 00101	0 3
4	4	0 00078 (14)	0 00074	−0 3

The numbers in brackets give the statistical errors in the last digits The deviation is given by $W_{fit} - W_{measured}$ divided by the standard deviation of the measured W The deviation is calculated keeping one more digit in the W's The θ's corresponding to the above fit are given by $\theta_1 = 0\ 27 \pm 0\ 01$, $\theta_2 = 0\ 55 \pm 0\ 03$, $\theta_3 = -0\ 30 \pm 0\ 06$ The χ^2 of the fit is 0 97

TABLE 2

Comparison of measured and fitted values for $W_{adj}(R, T)$ with $\beta = 5\ 5$

R	T	$W(R, T)$ measured	$W(R, T)$ from fit	Deviation
2	2	0 1466 (4)	0 1465	−0 2
2	3	0 0650 (4)	0 0653	0 7
2	4	0 0293 (3)	0 0292	−0 6
2	5	0 0132 (2)	0 0130	−0 7
2	6	0 0058 (2)	0 0058	−0 03
2	7	0 0026 (1)	0 0026	0 3
2	8	0 0010 (1)	0 0012	1 0
2	9	0 0005 (1)	0 0005	0 2
3	3	0 0219 (3)	0 0222	0 9
3	4	0 0076 (2)	0 0077	0 6
3	5	0 0026 (2)	0 0027	0 3
3	6	0 0010 (2)	0 0009	−0 6
3	7	0 00054 (10)	0 00032	−2 3
4	4	0 0022 (1)	0 0021	−0 5
4	5	0 00054 (12)	0 00058	0 3

The numbers in brackets give the statistical errors in the last digits The deviation is given by $W_{fit} - W_{measured}$ divided by the standard deviation of the measured W The deviation is calculated keeping one more digit in the W's The θ's corresponding to the above fit are $\theta_1 = 0\ 197 \pm 0\ 008$, $\theta_2 = 0\ 59 \pm 0\ 02$, $\theta_3 = -0\ 40 \pm 0\ 04$ The χ^2 of the fit is 1 07

TABLE 3

Comparison of measured and fitted values for $W_{adj}(R, T)$ with $\beta = 6\ 0$

R	T	$W(R, T)$ measured	$W(R, T)$ from fit	Deviation
2	2	0 1820 (4)	0 1818	−0 6
2	3	0 0893 (4)	0 0898	1 5
2	4	0 0447 (3)	0 0445	−0 7
2	5	0 0222 (2)	0 0220	−0 7
2	6	0 0108 (2)	0 0109	0 4
2	7	0 0055 (2)	0 0054	−0 5
2	8	0 0027 (2)	0 0027	−0 3
2	9	0 0014 (2)	0 0013	−0 5
2	10	0 00085 (15)	0 00066	−1 3
3	3	0 0345 (3)	0 0351	1 9
3	4	0 0139 (3)	0 0140	0 1
3	5	0 0057 (2)	0 0056	−0 8
3	6	0 0023 (2)	0 0022	−0 5
3	7	0 0008 (1)	0 0009	0 4
4	4	0 0046 (2)	0 0046	−0 2
4	5	0 0017 (1)	0 0015	−1 5
4	6	0 00049 (15)	0 00050	0 04

The numbers in brackets give the statistical errors in the last digits The deviation is given by $W_{fit} - W_{measured}$ divided by the standard deviation of the measured W The deviation is calculated keeping one more digit in the W's The θ's corresponding to the above fit are $\theta_1 = 0\ 159 \pm 0\ 005$, $\theta_2 = 0\ 56 \pm 0\ 01$, $\theta_3 = -0\ 35 \pm 0\ 03$ The χ^2 of the fit is 1 00

that if we had changed the factor $\frac{8}{3}$ in front of the last term in (4 4) to 1, the χ^2 values increase from ≈ 1 to ≈ 10 Further it should be mentioned that if we had used the lower limit $R \geqslant 1$ instead of $R \geqslant 2$, the resulting fits would have been bad This is in accordance with a remark in ref [5], where it was noticed that a critical distance $R_C = 2$ almost exactly coincides with the critical distance $R_C = \sqrt{\pi / 12\sigma}$ found by Alvarez [9]. It was discovered by Alvarez that for distances below R_C the string picture breaks down This is in complete agreement with our Monte Carlo results

In fig 3 we show the values for $\theta_1\beta^2$ obtained for the adjoint string and for the fundamental string [5] We compare the adjoint values for $\theta_1\beta^2$ with the prediction from dimensional reduction obtained by multiplying the fundamental $\theta_1\beta^2$ by $\frac{8}{3}$ It is seen that the agreement is excellent

5. On the confinement mechanism

The results obtained in this paper as well as in ref [1] indicate that dimensional reduction works well (within $\approx 10\%$) on lattice sizes which can be handled with present day computers From the observed approximate scaling (see e g fig 3) one

TABLE 4

Comparison of measured and fitted values for $W_{\mathrm{adj}}(R, T)$ with $\beta = 6\ 5$

R	T	$W(R, T)$ measured	$W(R, T)$ from fit	Deviation
2	2	0 2171 (4)	0 2166	−1 1
2	3	0 1154 (4)	0 1163	2 1
2	4	0 0625 (4)	0 0626	0 4
2	5	0 0339 (3)	0 0337	−0 9
2	6	0 0184 (2)	0 0179	2 3
2	7	0 0101 (3)	0 0098	−1 1
2	8	0 0055 (2)	0 0053	−1 1
2	9	0 0028 (1)	0 0028	0 6
2	10	0 0014 (1)	0 0015	1 5
2	11	0 00075 (15)	0 00082	0 5
3	3	0 04998 (42)	0 05072	1 8
3	4	0 0224 (3)	0 0225	0 3
3	5	0 0101 (2)	0 00998	−0 3
3	6	0 0045 (2)	0 0044	−0 6
3	7	0 0020 (2)	0 0020	0 1
3	8	0 00096 (15)	0 00087	−0 6
3	9	0 00041 (12)	0 00039	−0 2
4	4	0 0083 (2)	0 0084	0 5
4	5	0 0033 (2)	0 0032	−0 8
4	6	0 0013 (1)	0 0012	−1 3
4	7	0 00053 (14)	0 00045	−0 6
5	5	0 00087 (14)	0 00102	1 0

The numbers in brackets give the statistical errors in the last digits The deviation is given by $W_{\mathrm{fit}} - W_{\mathrm{measured}}$ divided by the standard deviation in the measured W The deviation is calculated keeping one more digit in the W's The θ's corresponding to the above fit are given by $\theta_1 = 0\ 134 \pm 0\ 003$, $\theta_2 = 0\ 53 \pm 0\ 01$, $\theta_3 = -0\ 29 \pm 0\ 02$ The χ^2 of the fit is 1 12

then expects that this phenomenon should occur also on larger lattices and in the continuum

As explained in [1], dimensional reduction leads to a disordered vacuum This can be seen in terms of the spectral density Thus, all our results can be understood by saying that the colour magnetic flux α, defined by the Wilson loop with respect to the path C

$$u(C) = \Omega \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix} \Omega^{+}, \tag{5 1}$$

is stochastic with a probability distribution $\rho_C(\alpha)$ satisfying ($C = C_1 + C_2$)

$$\rho_C(\alpha) = \frac{2}{\pi} \int_0^{\pi} d\beta \sin^2 \beta \rho_{C_1}(\beta) \rho_{C_2}(\alpha - \beta) \tag{5 2}$$

This also explains the emergence of an "adjoint string"

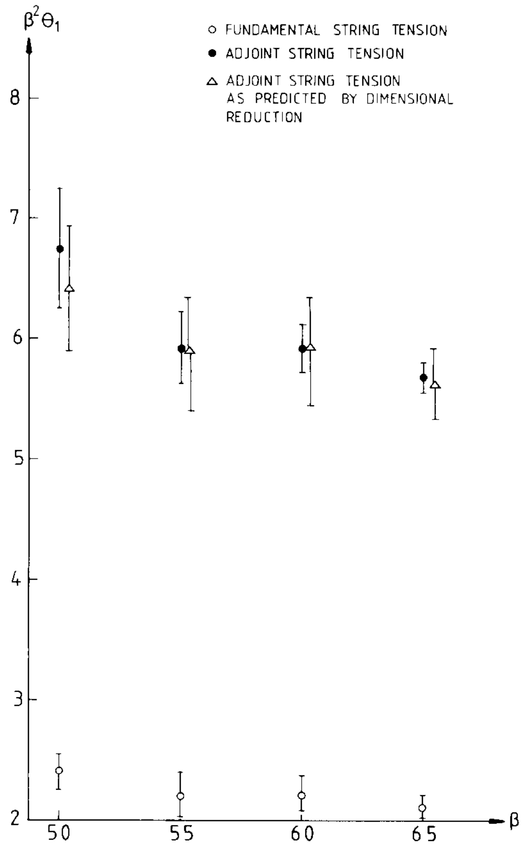

Fig 3 The parameter $\theta_1\beta^2$ for the fundamental and the adjoint strings, as well as a comparison with dimensional reduction

As already mentioned, the fact that the adjoint string can break (and hence lead to the perimeter behaviour) does not prevent one from asking why it is formed in the first place The situation is somewhat similar to the fundamental string in the presence of light quarks This string can also break, thus leading to a perimeter behaviour The main physical point is, however, the ability of the vacuum to form the string itself For further discussion we refer to [1]

6. Conclusions

The main conclusion of this paper is that dimensional reduction from $d=3$ to $d=2$ works well within $\approx 10\%$ for SU(2) lattice gauge theory Therefore the situation is not similar to the solid state example in ref [2] Another conclusion is that the adjoint string tension scales (approximately) to the same degree that the fundamental string tension scales This means that confinement schemes based on the centre (which the adjoint representation cannot see) seem to be ruled out In view of the similar results obtained for four dimensions [1] it thus appears that the confinement mechanism is entirely due to disorder of the colour magnetic flux (5.1).

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