OBSERVATION OF A STRING IN THREE-DIMENSIONAL SU(2) LATTICE GAUGE THEORY

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SU(2) lattice gauge theory in three dimensions is investigated on a $16^2 \times 32$ lattice. We find that $R \times T$ Wilson loops are well described by a simple string theory for β in the (approximate) scaling region.

The existence of strings has so far been supported by lattice gauge theory in the sense that a string tension which scales approximately has been found. A more precise confrontation between strings and Monte Carlo calculations is, however, still lacking. In this letter we shall therefore make a comparison between SU(2) lattice gauge theory and a simple string model.

The main a priori difficulty of such a comparison is that in practice the lattices used in Monte Carlo calculations are rather small, and hence the Wilson loops that one can measure are also small. We have therefore decided to concentrate primarily on three-dimensional SU(2) lattice gauge theory with a $16^2 \times 32$ lattice. We are thus able to measure relatively large Wilson loops (up to 6×12) with reasonable statistics. Another advantage is that the "universal Coulomb term" [1] differs from the form of the usual perturbative Coulomb term in three dimensions. Hence in three dimensions there is a better chance of distinguishing the two types of terms than in four dimensions.

Another difficulty is that strings are expected to be mathematical idealizations. At smaller scales they should be replaced by flux tubes. Thus one can only hope to see strings at distances which are large relative to the effective width of the flux tube. According to a widespread folklore the flux tube becomes very narrow in the limit $N \to \infty$. However, since we calculate in SU(2) one would expect a width of the order of the inverse (lightest) glueball mass. Of course the truth is that nobody has shown how flux tubes emerge analytically in QCD, and hence one does not know if this folklore is right. We shall assume that even in SU(2) the flux tube is rather thin, and our results strongly indicate that this assumption is right.

Over the years several string models have been constructed. Our philosophy (or in the minds of some people, lack of philosophy) is to take a very simple model and compute the analytic expression for the quark–antiquark potential at finite values of the distance R and the time T. Since the values obtained for a Wilson loop $W(R,T)$ are in practice only obtainable for R and T relatively small, we clearly need an analytic expression for the potential which is sufficiently accurate at distances which are not asymptotically large.

As our string action we take

$$S = c\int dr\, dt\, \nabla \boldsymbol{x} \nabla \boldsymbol{x}, \quad 0 \leqslant r \leqslant R, \; 0 \leqslant t \leqslant T. \qquad (1)$$

One then expands $x_\mu = x_\mu^{cl} + \delta x_\mu$, where x_μ^{cl} corresponds to the "classical" minimal surface. The first term in S then gives the area behaviour. The transverse fluctuations give rise to a correction

$$\delta S = c\int \mathrm{d}r\, \mathrm{d}t\ \nabla \delta \boldsymbol{x}_\perp \nabla \delta \boldsymbol{x}_\perp . \qquad (2)$$

If we are in 3 dimensions $\delta x_\perp = \delta x_\perp(r,t)$ has only one component. In d dimensions it has d-2 components. It has been argued [1] that eq. (2) is a universal effective action for large T,R. However, in Polyakov's string [2] there are other variables than $x_\perp$, and hence universality is broken. In a saddle point approximation Polyakov's string does e.g. not give [3] the "universal" Coulomb term if the parameter in front of the Liouville term is nonvanishing. Hence eq. (2) should be regarded as a simple model.

The next step is to perform the functional integral over the wiggling $\delta x_\perp$, using the usual boundary condition

$$\delta x_\perp = 0 \quad \text{on } \partial D, \qquad (3)$$

where ∂D is the boundary of the Wilson loop. This functional integration can easily be done using the ζ-function regularization. With a few modifications due to (3) one obtains by the method used by Minami [4]

$$-\ln W(R,T) = \sigma RT - \tfrac{1}{24}\pi T/R$$
$$-\tfrac{1}{4}\ln(R\mu/2) + \tfrac{1}{2}\sum_{n=1}^{\infty} \ln(1 - \mathrm{e}^{-2\pi n T/R}). \qquad (4)$$

The sum of logarithms ensures that (4) is symmetric in R and T. It plays a minor role for $T > R$, falling off exponentially, but for $T < R$ is rapidly approaches $-\frac{1}{24}\pi(1/x - x) - \frac{1}{4}\log x, x = T/R$. The first term is the area term, the second is the (not quite) "universal" Coulomb term, whereas the third and fourth terms are dependent on the model [‡1]. In d dimensions the second, third and fourth terms should be multiplied by the factor (d-2).

Our Monte Carlo data were generated using the discrete 120 element subgroup of SU(2) for β between 3.0 and 6.5. The discrete group freezes at β around 7.5 and the advantage of using it disappears around 6.5. From the envelope of Creutz ratios we estimated scaling to begin at $\beta \approx 4.5 - 5.0$. At $\beta = 6.5$ we used 6000 sweeps for thermalizing and performed 800 measurements, each measurement 5 sweeps apart. One measurement included measuring all $T \times R$ loops up to 12×6, the average of the $T \times R$ loop being taken over the $2 \times 32 \times 16^2$ $T \times R$ loops that can be placed in the lattice with T-direction corresponding to the long lattice direction.

We compare with the prediction (4) from the string. Our fit has the form ($T \geqslant R$)

$$-\ln W(R,T) = \theta_1 RT + \theta_2(R+T) + \theta_3 T/R$$
$$+ \theta_4 \ln R + \theta_5 . \qquad (5)$$

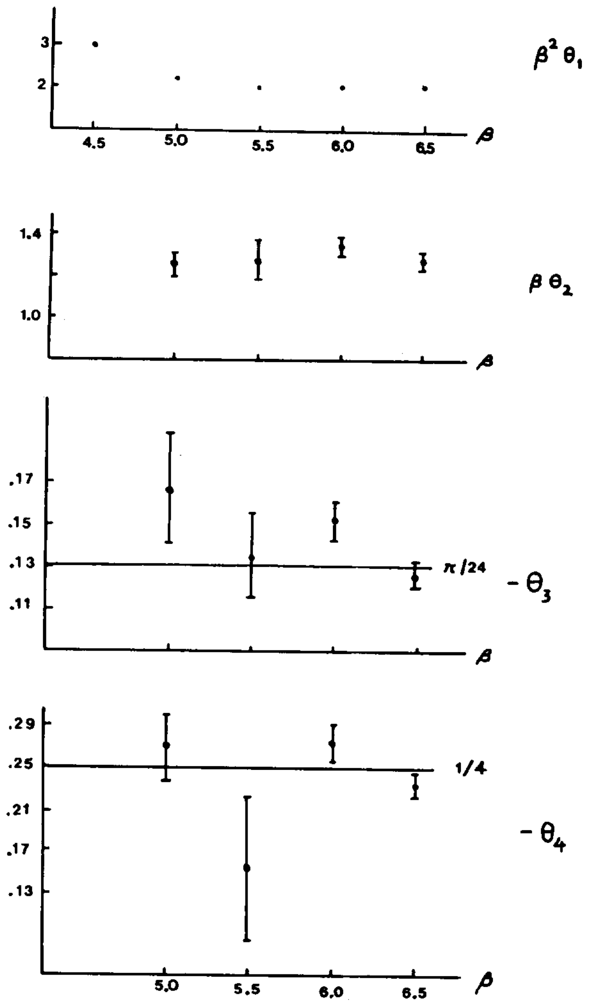

Fig. 1. The parameters $\theta_1, \theta_2, \theta_3, \theta_4$ defined by eq. (5). For θ_1 we see (approximate) scaling, $\theta_1 \approx$ const./β^2. The highest value of R is $R = 4$, and $R \leqslant T \leqslant 10$.

‡1 The third term also appears in the saddle point discussed in ref. [3], except that the coefficient is different from −1/4.

Table 1
Comparison of measured and fitted values for $W(R,T)$ for β = 6.0 (χ^2 = 0.33). The numbers in parantheses give the statistical errors in the last digits. The deviation is defined as $W(R,T)_{\text{measured}} - W(R,T)_{\text{fit}}$ divided by the standard deviation in the measured W. This deviation is calculated keeping one more digit in the W's. The θ's corresponding to the fit above are given by: θ_1 = 0.059 ± 0.002, θ_2 = 0.22 ± 0.02, θ_3 = −0.15 ± 0.03, θ_4 = −0.26 ± 0.07 and θ_5 = −0.14 ± 0.04.

R	T	$W(R,T)$ measured	$W(R,T)$ from fit	Deviation
2	2	0.5247 (05)	0.5241	1.15
2	3	0.4007 (08)	0.4022	−1.81
2	4	0.3076 (10)	0.3087	−1.08
2	5	0.2363 (11)	0.2369	−0.58
2	6	0.1816 (11)	0.1818	−0.23
2	7	0.1394 (10)	0.1396	−0.12
2	8	0.1070 (10)	0.1071	−0.13
2	9	0.0820 (09)	0.0822	−0.20
2	10	0.0631 (08)	0.0631	0.02
3	3	0.2793 (10)	0.2791	0.18
3	4	0.1968 (12)	0.1969	−0.09
3	5	0.1390 (12)	0.1390	0.04
3	6	0.0983 (11)	0.0981	0.14
3	7	0.0695 (10)	0.0692	0.23
3	8	0.0489 (09)	0.0489	0.08
3	9	0.0345 (08)	0.0345	0.08
3	10	0.0244 (07)	0.0243	0.08
4	4	0.1285 (12)	0.1280	0.40
4	5	0.0841 (12)	0.0841	−0.02
4	6	0.0553 (11)	0.0552	0.04
4	7	0.0363 (10)	0.0363	0.05
4	8	0.0241 (08)	0.0238	0.38
4	9	0.0159 (06)	0.0157	0.35
4	10	0.0104 (05)	0.0103	0.17
5	5	0.0521 (09)	0.0513	0.93
5	6	0.0319 (08)	0.0315	0.41
5	7	0.0196 (07)	0.0194	0.37
5	8	0.0117 (06)	0.0119	−0.41
5	9	0.0070 (06)	0.0073	−0.49
5	10	0.0044 (06)	0.0045	−0.23
6	6	0.0184 (07)	0.0180	0.50
6	7	0.0102 (06)	0.0104	−0.34
6	8	0.0056 (05)	0.0060	−0.81
6	9	0.00343 (39)	0.00345	−0.08
6	10	0.00217 (33)	0.00199	0.54

Including the logarithms from eq. (4) for $T \geqslant R$ only changes the θ's by a few percent and they are therefore left out for $T \geqslant R$. We include the perimeter term which is expected in QCD. Such a term also emerges for some string models if one does not use the ζ-function regularization [3].

In fig. 1 we show the values obtained for the parameters $\theta_1, \theta_2, \theta_3, \theta_4$ and θ_5. From the values of θ_1 we see that one has approximate scaling (i.e., $\theta_1 \approx c/\beta^2$). It should be noticed that we are far away from the strong coupling region. We see the remarkable feature that θ_3 is close to the string prediction $-\pi/24$. Similarl θ_4 is close to $-1/4$. In table 1 we show the equally rem able "goodness" of the fit for β = 6.0. The χ^2 of the fit is 0.33 and the least squares sum is 0.0005. It is seen that we have included R and T values for which $R \geqslant 2$ and $T \geqslant R$.

To see if the string picture is valid for R = 1 we ther included the loops 1 × 2, 1 × 3, 1 × 4, 1 × 5 and 1 × 6 in the fit. It turns out that the fit becomes much less spectacular: The χ^2 value is now 13.6. Thus we conclude that $T \times R$ Wilson loops with R = 1 cannot be described in a precise way by a string. The critical distance is therefore R = 2.

To check the sensitivity to the lowest values used for R, we have also made a cut in the data such that we start at $R \geqslant 3$ instead of $R \geqslant 2$. As can be seen fror table 1 we lose 9 data points (≈26% of the available data). It now becomes difficult for the data to distinguish the logarithmic term from the constant term in eq. (5). Thus θ_3 and θ_4 obtain large error bars. However, inside the error bars θ_3 and θ_4 are consistent with the values obtained from $R \geqslant 2$.

We have made a somewhat different check of the string picture. If one forms the quantity

$$Q(R,T) = \ln W(R,T) + \ln W(R-2,T) - 2\ln W(R-1,T), \tag{6}$$

then the area and perimeter terms cancel. From eq. (4 one obtains (keeping the last term for n = 1)

$$\tfrac{1}{2}T^{-1}R(R-1)(R-2)Q(R,T) = \tfrac{1}{24}\pi - R(R-2)/8T(l$$
$$+ \tfrac{1}{2}T^{-1}R(R-1)(R-2)$$
$$\times \left[\tfrac{1}{2}\left(e^{-2\pi T/R} + e^{-2\pi T/(R-2)}\right) - 2e^{-2\pi T/(R-1)}\right]. \tag{7}$$

The quantity Q is approximately the second derivative

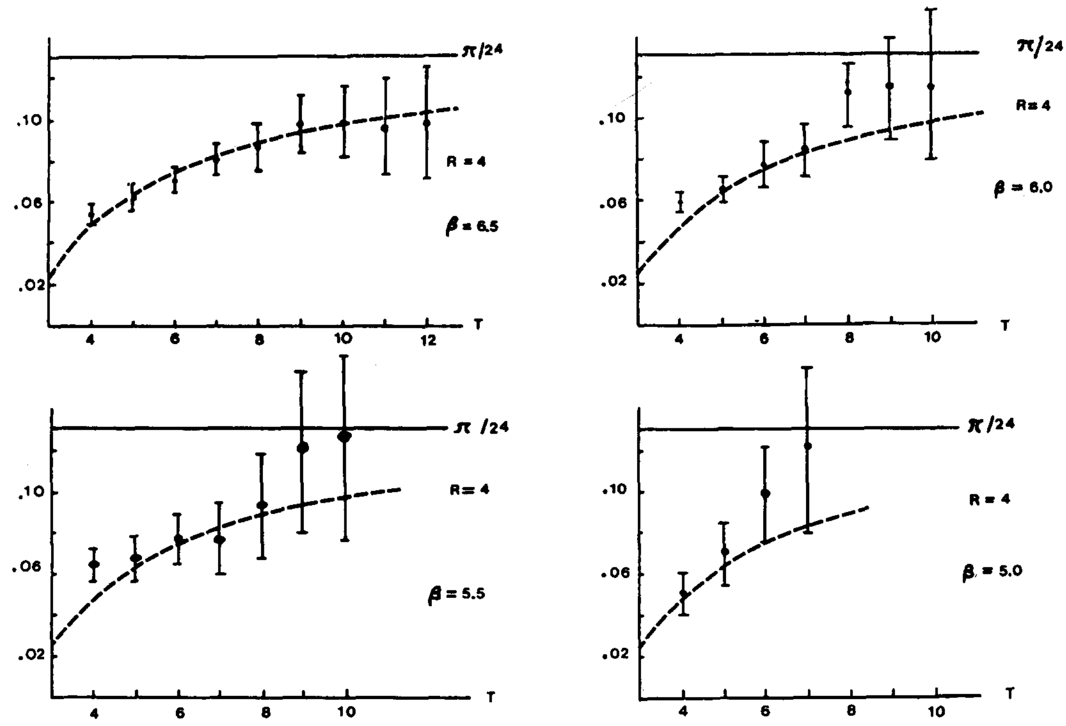

Fig. 2. The quantity $(R-1)(R^2-2R)Q(R,t)/2T$ plotted as a function of T. The theoretical curve is given by eq. (7).

of ln W with respect to R. Therefore the statistical errors are relatively large. In fig. 2 we have compared the measured values with the prediction [‡2] (7). We see good agreement. Also it is seen that the "universal" Coulomb term cannot represent the data alone. One also needs the $\ln(R\mu)$ contribution.

It should be mentioned that we have also made a fit to $W(R,T)$, using perturbative corrections (the one gluon exchange) instead of string corrections. We shall report these results in a forthcoming paper. However, we mention that although good results are obtained, the fit is physically rather meaningless: In order for the fit to succeed, it turns out to be necessary that the one gluon exchange dominates the potential, even if the test quarks are separated by more than one correlation length.

As a conclusion we have seen that an excellent representation of the data for $W(R,T)$ can be obtained from a simple string theory, provided $R \geqslant 2$ and $T \geqslant R$.

‡2 The first two terms on the right hand side of eq. (7) are quite dominant relative to the last term.

Thus, under these circumstances the string manages to wiggle in three dimensions. Therefore the width of the flux tube must be quite small even in SU(2). This is perhaps somewhat contrary to expectations, and may be an indication that the confinement mechanisms are the same in SU(2) and in SU(∞).

It should be stressed that although we cannot measure the non-leading terms of the type $\ln(1-e^{-2\pi nT/R})$ directly, there is nevertheless indirect evidence in our data for the existence of these terms within the assumed framework of the simple string model (2). The reason for this is that our data are invariant under $T \rightleftharpoons R$. Eq. (4) is also invariant due to the non-leading terms mentioned above.

It should be pointed out that the non-leading terms are in general dependent on the particular dual model. Thus, if one includes terms with higher derivatives in the string action, then this leads to terms of order T/R^n $(n \geqslant 2)$, as mentioned in ref. [1]. Such terms are not considered in this paper, since their concrete forms are not known.

Further results concerning scaling in three dimensions (and concerning dimensional reduction) will appear in a forthcoming paper [5].

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