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THREE-DIMENSIONAL LATTICE GAUGE THEORY AND STRINGS

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We consider SU(2) lattice gauge theory in three dimensions. The Wilson loops are found to be well described by a simple string model in the approximate scaling region.

1. Introduction

Monte Carlo calculations have provided much insight into the nonperturbative structure of lattice gauge theories. In the four-dimensional case there is usually the difficulty that the lattice sizes considered are too small and, due to the exponential decrease of the string tension, the "scaling-window" is rather small. In this paper we shall therefore study SU(2) lattice gauge theory in 3 euclidean dimensions. The advantage is that (with limited computer time) the lattice size can be increased relative to the 4-dimensional case, and that the string tension decreases rather slowly like const/β^2. We use a $16^2 \times 32$ lattice which enables us to measure relatively large Wilson loops (up to 6×12) with reasonable statistics.

Previously SU(2) lattice gauge theories in three euclidean dimensions have been studied in ref. [1, 2]. The advantage of our work relative to the previous ones is that we use a larger lattice ($10^3 \to 16^2 \times 32$) and have larger statistics.

In a previous paper [3] it was pointed out that the 3-dimensional case is well suited for a comparison between QCD and (some) string models. This is in part due to the better statistics. However, it is also due to the fact that in 3 dimensions the "universal" Coulomb potential [4] $-\frac{1}{24}\pi/R$ is different from the "true" Coulomb potential which varies like ln R. Hence in 3 dimensions there is a better chance of distinguishing these 2 contributions to the potential.

The main results of this paper are that we find that the string tension scales to a good approximation and that the $R \times T$ Wilson average $W(R, T)$ is well represented by a simple string model with the action

$$S = c \int \mathrm{d}r \, \mathrm{d}t \, \nabla x_\mu \nabla x_\mu , \tag{1.1}$$

where $\mu = 1, 2, 3$ and $0 \leq r \leq R$, $0 \leq t \leq T$.

This paper is organized as follows: in sect. 2 we discuss the numerical methods. In sect. 3 we give results for the string tension and in sect. 4 we compare our numerical results with the simple string model (1.1). Sect. 5 contains some conclusions.

2. Numerical methods

For β between 3.0 and 6.5 our Monte Carlo data were generated by using the discrete 120 element subgroup of SU(2) [4]. The discrete group freezes for β around 7.5, and the advantage of using it disappears around 6.5 due to the low acceptance rate in the Metropolis algorithm. This low acceptance rate is forced upon us at high β since the change in the action, ΔS, when updating a link, cannot be chosen arbitrarily close to zero.

The number of sweeps used for thermalization of the lattice is given in table 1, and the number of measurments at each β is given in table 2. Two subsequent measurements were separated by 5 sweeps. One measurement included measuring all $R \times T$ loops, the maximum (R, T) being $(4, 6)$ at $\beta = 3.0$ and $(6, 12)$ at $\beta = 6.5$. The size of the lattice was $16 \times 16 \times 32$ and the average of the $R \times T$ loop was taken over all $2 \times 16 \times 16 \times 32$ $R \times T$ loops that could be placed in the lattice with the T-direction along the long lattice direction.

The statistical errors of the Wilson loop averages or of other observables like the Creutz ratios, were estimated by grouping the N measurements together in bunches of n. For these new N/n "measurements" we used the standard formula for the

TABLE 1

Number of sweeps used for thermalization

β	3.0	3.5	4.0	4.5
	cold start 600	start: 3.0 600	cold start 2800	start: 4,0 1400
β	5.0	5.5	6.0	6.5
	cold start 5700	start: 6.0 at 1000 6000	start: 5.0 at 3000 4000	start: 6.0 at 1000 6000

TABLE 2
Number of measurements

β	No. of measurements
3.0	400
3.5	400
4.0	400
4.5	400
5.0	800
5.5	800
6.0	800
6.5	800

statistical error $\sigma_n(0)$:

$$\sigma_n(0) = \sqrt{\frac{\sum_{i=1}^{N/n} (O_i - \langle O \rangle_n)^2}{N/n(N/n-1)}}\,. \tag{2.1}$$

We observed an increase in σ_n of the order of 20% when n increased from 1 to 20. For $n \geqslant 20$ $\sigma_n(0)$ stayed constant.

3. The string tension

The 3-dimensional SU(2) lattice gauge theory is defined by the standard Wilson action:

$$Z = \int \prod \mathrm{d}U_{r,\mu}\, \mathrm{e}^{+\beta S}\,, \tag{3.1}$$

$$S = \tfrac{1}{2} \sum_{r,\square} \mathrm{Tr}\, U_\square\,, \tag{3.2}$$

where the link variables $U_{r,\mu} \in \mathrm{SU}(2)$ and $\square$ stands for the plaquettes at r. The action reduces to the usual continuum action $S = \frac{1}{4}\int \mathrm{d}^3x\, F_{\mu\nu}^2$ if we write

$$U_{r,\mu} = \mathrm{e}^{igA_\mu^a(r)T^a} \tag{3.3}$$

$$\beta = 4/g_{\mathrm{L}}^2 a\,, \tag{3.4}$$

$$\mathrm{Tr}\, T^a T^b = \tfrac{1}{2}\delta^{ab}\,. \tag{3.5}$$

In three dimensions the bare coupling constant g_{L}^2 has the dimension of mass. The theory is super renormalizable and if we assume that g_{L} is the only mass scale which enters into the theory in the continuum limit, we can write for the string tension

$$\sigma = \frac{1}{a^2} f(g_{\mathrm{L}}^2 a)\,, \tag{3.6}$$

or (assuming that the string tension survives in the continuum limit $a \to 0$):

$$\sigma = C g_L^4 \quad (a \to 0)\,, \tag{3.7}$$

$$\sigma a^2 = c/\beta^2 \quad (a \to 0)\,. \tag{3.8}$$

These equations are of course based on the assumption that no additional mass parameter, like an infrared cutoff, is needed in order to define the theory.

In order to verify (3.8) we have measured the Ceutz ratios

$$\chi(R, T) = -\ln \frac{W(R, T)\, W(R-1, T-1)}{W(R, T-1)\, W(R-1, T)}\,, \tag{3.9}$$

where $W(R, T)$ is the expectation value of an $R \times T$ Wilson loop. $\chi(R, T)$ measures σa^2 if the exponent of $W(R, T)$ contains an area term, a parameter term and a constant.

In ref. [2] the authors have estimated that the crossover region occurs for $\beta \approx 2$–3. Since our data sampling starts at $\beta = 3$ we have thus passed the crossover region. Also, ref. [2] contains an estimate of the perturbative terms, which are of order $1/\beta$ with $1/\beta^2$ corrections. From [2] it is quite clear that these corrections cannot account for the data when $R > 1$ and $T > 1$. We refer the reader to [2] for a detailed discussion of these points.

In fig. 1 and table 3 we present our data for the Creutz ratios $\chi(R, R)$ together with the lowest-order perturbative results. These results are more accurate than those obtained in ref. [2], but within the error bars there is agreement, showing that the discrete group is reliable up to $\beta = 6.5$.

From fig. 1 it is seen that we are far from the lowest-order perturbative results for $\chi(R, R)$. It is also seen that for $\beta \gtrsim 5.0$ one has a relation

$$\chi(R, R) = \frac{c}{\beta^2} + C_2(R)\,, \tag{3.10}$$

where $C_2(R)$ is small and decreasing with R.

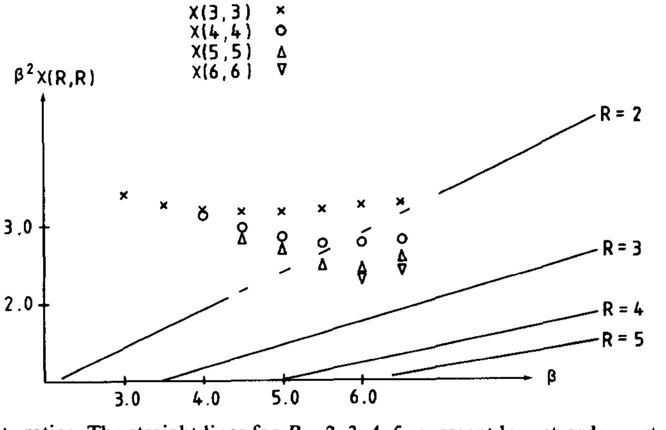

Fig. 1. The Creutz ratios. The straight lines for $R = 2, 3, 4, 5$ represent lowest-order perturbation theory.

TABLE 3
The Creutz ratios

β	$\chi(3,3)$	$\chi(4,4)$	$\chi(5,5)$	$\chi(6,6)$
3.0	0.380 (10)			
3.5	0.268 (5)	0.30 (3)		
4.0	0.200 (2)	0.195 (10)		
4.5	0.157 (2)	0.148 (6)	0.141 (27)	
5.0	0.1275 (20)	0.1135 (22)	0.108 (11)	
5.5	0.1060 (15)	0.0906 (22)	0.082 (8)	0.086 (20)
6.0	0.0908 (10)	0.0772 (20)	0.067 (3)	0.064 (10)
6.5	0.0778 (8)	0.0662 (14)	0.062 (3)	0.057 (5)

How can we understand a relation like (3.10)? A simple explanation seems to be possible. Some time ago Lüscher suggested that a string should have a universal Coulomb correction of the form $-(d-2)\pi/24R$, d being the number of space–time dimensions [5]. Since the correction is independent of β it would change (3.8) into a relation like (3.10). One can check that it has the right order of magnitude to account for the difference between (3.8) and (3.10) seen in the data of fig. 1. In the next section we will therefore investigate this possibility in detail.

4. Comparison with a string picture

A "universal" type of correction to the string tension was suggested some time ago by Lüscher [5]. He showed that the action (1.1) is to be considered the leading term in a wide class of string models. When integrating S in eq. (1.1) over x_μ in the path integral for $T \gg R$ he obtained

$$\int \mathrm{d}x_\mu(r,t) \exp(-S) = \exp -\left[\sigma RT + \delta(R+T) - \tfrac{1}{24}(d-2)\pi\frac{T}{R} + \mathrm{O}(T/R^2)\right], \qquad (4.1)$$

in d space–time dimensions. The third term is the "universal" Coulomb correction. It is not quite universal, however. As a counter-example one can mention the Polyakov string [6]. This string carries new degrees of freedom (the metric field) which are not governed by eq. (1.1). If the coefficient in front of the Liouville term in Polyakov's string model is not zero, the third term in (4.1) is vanishing [7].

In our Monte Carlo data we never really have $T \gg R$ satisfied, so we need to calculate (1.1) for all T, R. It is not clear that (1.1) is a good measure for the correction to the pure string when $T \approx R$. Indeed, corrections of the order $\mathrm{O}(T/R^2)$ might become large. We will assume that the action (1.1) is a reasonable approximation (even when $T \approx R$) and turn to the calculation of the Wilson average from eq. (1.1).

One expands $x_\mu = x_\mu^{\mathrm{cl}} + \delta x_\mu$, where x_μ^{cl} corresponds to the "classical" minimal surface of the Wilson loop. The first term in S in (1.1) then gives the area behaviour.

The transverse fluctuations give rise to a correction

$$\delta S = c \int \mathrm{d}r\,\mathrm{d}t\,(\nabla \delta \boldsymbol{x}_\perp)^2 . \tag{4.2}$$

If we are in 3 dimensions $\delta x_\perp = \delta x_\perp(r, t)$ has only one component. In d dimension it has $d-2$ components. The next step is to perform the functional integral over the wiggling $\delta x_\perp$, using the usual boundary condition

$$\delta x_\perp = 0 \qquad \text{on} \qquad \partial D , \tag{4.3}$$

where D is the $R \times T$ Wilson loop and ∂D its boundary. The contribution to the partition function Z from δS in (4.2) is

$$\int \mathscr{D}\delta x_\perp \exp - \left[c \int \mathrm{d}r\,\mathrm{d}t\,(\nabla \delta \boldsymbol{x}_\perp)^2 \right] = \det(-\nabla^2)^{1/2} . \tag{4.4}$$

If ω_α^2 are the eigenvalues of the laplacian $-\nabla^2$ on D with Dirichlet boundary conditions (4.3) we have

$$\omega_{n,m}^2 = \left(\frac{\pi n}{T}\right)^2 + \left(\frac{\pi m}{R}\right)^2 , \qquad n, m > 0 , \tag{4.5}$$

and the determinant in (4.4) can be written as

$$\begin{aligned} \det(-\nabla^2)^{1/2} &= \prod_q (\omega_q^2/\mu^2)^{-1/2} \\ &= \exp - \left[\tfrac{1}{2} \sum_q \ln(\omega_q^2/\mu^2) \right] = \exp \left\{ \frac{1}{2} \left[\frac{\partial}{\partial k} \sum_q (\omega_q^2/\mu^2)^{-k} \right] \right\} \Bigg|_{k=0} . \end{aligned} \tag{4.6}$$

The sum in the exponent can be written as

$$\sum_{n,m \geq 1} (\omega_{n,m}/\mu^2)^{-k} = \frac{1}{4} \left[Z_2\left(\frac{\pi}{\mu T}, \frac{\pi}{\mu R}; k\right) - Z_1\left(\frac{\pi}{\mu T}; k\right) - Z_1\left(\frac{\pi}{\mu R}; k\right) \right] , \tag{4.7}$$

where $Z_l(a_1, \dots a_l; k)$ is Epstein's zeta function defined by

$$\begin{aligned} Z_l(a_1, \dots a_l; k) &= \sum_{-\infty}^{+\infty} [(a_1 n_1)^2 + \cdots (a_l n_l)^2]^{-k} , \\ (n_2, \dots, n_l) &\neq (0, \dots, 0) . \end{aligned} \tag{4.8}$$

Using the known results for Z_l as given in ref. [8] it is not hard to evaluate (4.4), and one gets

$$\det(-\nabla^2)^{1/2} = \exp - \left[\tfrac{1}{24} \pi \frac{T}{R} - \tfrac{1}{2} \sum_{n=1}^{\infty} \ln(1 - \mathrm{e}^{-2\pi n T/R}) + \tfrac{1}{4} \ln(R\mu) \right] , \tag{4.9}$$

where μ is a constant. The expression is symmetric in T and R. Had we done the calculation not in 3 dimensions but in d dimensions, the exponent in (4.9) should be multiplied by a factor $(d-2)$.

If the Wilson loops $W(R, T)$ are well represented by a string we would now expect them to behave as

$$-\ln W(R,T) \equiv V(R,T) = \sigma(\beta)RT + d(\beta)(R+T) + \left[-\tfrac{1}{24}\pi\frac{T}{R} - \tfrac{1}{4}\ln R + \tfrac{1}{2}\sum_{n=0}^{\infty}\ln\left(1-e^{-2\pi nT/R}\right)\right]. \quad (4.10)$$

The perimeter term $d(\beta)(R+T)$ is needed when one uses other regularization methods than the zeta function regularization. (In our Monte Carlo calculations we are obviously using the lattice regularization.)

Inspired by (4.10) we try a five-parameter fit to our data of the form

$$-\ln W(R,T) = \theta_1(\beta)RT + \theta_2(\beta)(R+T) - \theta_3(\beta)T/R - \theta_4(\beta)\ln R + \theta_5(\beta) \quad (4.11)$$

for $T \geq R$. The sum over the logarithmic terms are small for $T \geq R$ and we neglect it.

In fig. 2 we show the results of a least squares fit to the data for R between 2 and 4. The results can also be found in table 4. First of all it can be seen that the string tension $\theta_1(\beta)$ scales well for $\beta \geq 5.0$. We also note that θ_3 and θ_4 agree well with the values predicted from the string model (4.10). The error bars should be

TABLE 4a

Comparison of measured and fitted values for $W(R, T)$ with $\beta = 5.0$

R	T	$W(R, T)$ measured	$W(R, T)$ fitted	Deviation
2	2	0.4379 (4)	0.4360	4.63
2	3	0.3072 (6)	0.3079	−1.36
2	4	0.2167 (7)	0.2175	−1.27
2	5	0.1533 (6)	0.1536	−0.50
2	6	0.1086 (6)	0.1085	0.11
2	7	0.0768 (6)	0.0766	0.33
3	3	0.1898 (7)	0.1891	1.11
3	4	0.1186 (7)	0.1189	−0.52
3	5	0.0747 (6)	0.0748	−0.13
3	6	0.0471 (6)	0.0471	0.10
3	7	0.0296 (5)	0.0296	0.02
4	4	0.0662 (6)	0.0662	0.01
4	5	0.0374 (6)	0.0375	−0.25
4	6	0.0213 (5)	0.0212	0.16
4	7	0.0120 (5)	0.0120	−0.02

The numbers in brackets give the statistical errors in the last digits. The deviation is given by $W_{\text{measured}} - W_{\text{fit}}$ divided by the standard deviation in the measured W. This deviation is calculated keeping more digits in the W's. The θ's corresponding to the above fit are given by: $\theta_1 = 0.095 \pm 0.006$, $\theta_2 = 0.22 \pm 0.02$, $\theta_3 = 0.12 \pm 0.02$, $\theta_4 = 0.19 \pm 0.04$, $\theta_5 = -0.17 \pm 0.02$.

TABLE 4b

Comparison of measured and fitted values for $W(R, T)$ with $\beta = 5.5$

R	T	$W(R, T)$ measured	$W(R, T)$ fitted	Deviation
2	2	0.4842 (5)	0.4824	3.90
2	3	0.3564 (7)	0.3573	−1.25
2	4	0.2638 (8)	0.2646	−1.01
2	5	0.1955 (8)	0.1960	−0.58
2	6	0.1449 (8)	0.1451	−0.29
2	7	0.1075 (8)	0.1075	0.01
2	8	0.0797 (7)	0.0796	0.10
2	9	0.0590 (6)	0.0590	0.03
3	3	0.2360 (9)	0.2336	2.68
3	4	0.1581 (10)	0.1576	0.42
3	5	0.1063 (10)	0.1064	−0.15
3	6	0.0715 (9)	0.0718	−0.34
3	7	0.0483 (8)	0.0485	−0.23
3	8	0.0327 (7)	0.0327	−0.02
3	9	0.0221 (6)	0.221	0.03
3	10	0.0150 (5)	0.0149	0.12
4	4	0.0968 (9)	0.0958	0.99
4	5	0.0594 (9)	0.0595	−0.13
4	6	0.0366 (8)	0.0370	−0.46
4	7	0.0226 (6)	0.0230	−0.64
4	8	0.0143 (5)	0.0143	−0.02
4	9	0.0090 (5)	0.0089	0.19
4	10	0.0056 (4)	0.0055	0.08

The numbers in brackets give the statistical errors in the last digits. The deviation is given by $W_{\text{measured}} - W_{\text{fit}}$ divided by the standard deviation in the measured W. This deviation is calculated keeping one more digit in the W's. The θ's corresponding to the above fit are given by: $\theta_1 = 0.073 \pm 0.006$, $\theta_2 = 0.21 \pm 0.02$, $\theta_3 = 0.12 \pm 0.02$, $\theta_4 = 0.16 \pm 0.04$, $\theta_5 = -0.18 \pm 0.03$.

multiplied by a factor of 2 when we include the variations in $\theta_i(\beta)$ which come from making all kind of cuts in all data available for $2 \leq R \leq 6$; $T \leq R$. From table 4 it is apparent that a χ^2-fit using the 3-parameter ansatz

$$-\ln W(R, T) = \theta_1(\beta)RT + \theta_2(R+T) + \theta_5(\beta) - \left[\tfrac{1}{24}\pi\frac{T}{R} + \tfrac{1}{4}\ln R\right] \tag{4.12}$$

for $T \geq R$ will be quite good, as is indeed the case.

If we include $R = 1$ we do not obtain good fits. We therefore conclude that the string picture seems to work from distances $R \geq 2a$, or, introducing the string tension σ: $\sqrt{\sigma}R \geq 0.5$. It is interesting that this lower limit almost exactly coincides with the critical distance $R_c = \sqrt{\pi/12\sigma}$ found by Alvarez [9]. For $R < R_c$ one expects the string picture to break down (see ref. [9]).

TABLE 4c

Comparison of measured and fitted values for $W(R, T)$ with $\beta = 6.0$

R	T	$W(R, T)$ measured	$W(R, T)$ fitted	Deviation
2	3	0.4012 (7)	0.4010	0.37
2	4	0.3084 (9)	0.3085	−0.08
2	5	0.2373 (9)	0.2374	−0.08
2	6	0.1826 (9)	0.1826	−0.02
2	7	0.1405 (9)	0.1405	0.03
2	8	0.1081 (9)	0.1081	−0.01
2	9	0.0831 (8)	0.0832	−0.05
2	10	0.0640 (8)	0.0640	0.06
3	3	0.2800 (9)	0.2798	0.24
3	4	0.1979 (11)	0.1982	−0.24
3	5	0.1402 (11)	0.1404	−0.14
3	6	0.0994 (10)	0.0994	−0.01
3	7	0.0706 (9)	0.0704	0.16
3	8	0.0499 (8)	0.0499	0.06
3	9	0.0353 (7)	0.0353	−0.03
3	10	0.0250 (6)	0.0250	0.00
4	4	0.1293 (12)	0.1293	0.01
4	5	0.0850 (11)	0.0852	−0.13
4	6	0.0562 (10)	0.0561	0.02
4	7	0.0371 (9)	0.0370	0.10
4	8	0.0244 (8)	0.0244	0.08
4	9	0.0160 (6)	0.0161	−0.04
4	10	0.0106 (5)	0.0106	−0.02

The numbers in brackets give the statistical errors in the last digits. The deviation is given by $W_{\text{measured}} - W_{\text{fit}}$ divided by the standard deviation in the measured W. This deviation is calculated keeping one more digit in the W's. The θ's corresponding to the above fit are given by: $\theta_1 = 0.062 \pm 0.004$, $\theta_2 = 0.20 \pm 0.01$, $\theta_3 = 0.12 \pm 0.01$, $\theta_4 = 0.22 \pm 0.01$, $\theta_5 = -0.12 \pm 0.01$.

Another check of eq. (4.10) consists of taking the second derivative of $V(R, T)$ with respect to R:

$$-\frac{\partial^2}{\partial R^2} V(R, T) = -\frac{\partial^2}{\partial R^2}\left[-\tfrac{1}{24}\pi\frac{T}{R} - \tfrac{1}{4}\ln R + \sum_{n=1}^{\infty} \ln\left(1 - e^{-2\pi n T/R}\right)\right]. \quad (4.13)$$

In fig. 3 we have plotted the discrete version of

$$-\frac{R^3}{2T}\frac{\partial^2}{\partial R^2} V(R, T) = \tfrac{1}{24}\pi - \frac{1}{8}\frac{R}{T} + \frac{R^3}{2T}\frac{\partial^2}{\partial R^2}\left[\sum_{n=1}^{\infty} \ln\left(1 - e^{-2\pi n T/R}\right)\right], \quad (4.14)$$

constructed from $R = 2, 3, 4$. The data agree very well with (4.14).

As a final consistency check we have shown in fig. 4 the measured Creutz ratios $\chi(R, T)$ and the "corrected" string tension $\chi_{\text{string}}(R, T)$ obtained by subtracting

TABLE 4d

Comparison of measured and fitted values for $W(R, T)$ with $\beta=6.5$

R	T	$W(R, T)$ measured	$W(R, T)$ fitted	Deviation
2	2	0.5614 (4)	0.5589	5.56
2	3	0.4424 (6)	0.4431	−1.10
2	4	0.3504 (7)	0.3512	−1.23
2	5	0.2781 (8)	0.2784	−0.48
2	6	0.2206 (8)	0.2207	−0.14
2	7	0.1750 (8)	0.1750	0.02
2	8	0.1388 (7)	0.1387	0.19
2	9	0.1102 (6)	0.1100	0.33
2	10	0.0873 (6)	0.0872	0.19
2	11	0.0690 (6)	0.0691	−0.11
2	12	0.0547 (5)	0.0548	−0.08
3	3	0.3224 (8)	0.3227	−0.38
3	4	0.2374 (9)	0.2382	−0.89
3	5	0.1756 (10)	0.1758	−0.24
3	6	0.1298 (10)	0.1298	0.05
3	7	0.0959 (9)	0.0958	0.17
3	8	0.0708 (8)	0.0707	0.13
3	9	0.0523 (7)	0.0522	0.14
3	10	0.0385 (7)	0.0385	0.05
3	11	0.0284 (6)	0.0284	−0.10
3	12	0.0210 (6)	0.0210	−0.04
4	4	0.1637 (11)	0.1640	−0.30
4	5	0.1138 (11)	0.1139	−0.17
4	6	0.0791 (10)	0.0791	−0.04
4	7	0.0551 (9)	0.0550	0.20
4	8	0.0382 (8)	0.0382	0.08
4	9	0.0267 (7)	0.0265	0.25
4	10	0.0185 (6)	0.0184	0.10
4	11	0.0127 (6)	0.0128	−0.13
4	12	0.0089 (4)	0.0089	−0.07

The numbers in brackets give the statistical error sin the last digits. The deviation is given by $W_{\text{measured}} - W_{\text{fit}}$ divided by the standard deviation to the meausred W's. The θ's corresponding to the above fit are given by: $\theta_1 = 0.050 \pm 0.003$, $\theta_2 = 0.196 \pm 0.004$, $\theta_3 = 0.128 \pm 0.005$, $\theta_4 = 0.23 \pm 0.01$, $\theta_5 = -0.11 \pm 0.02$.

from $\chi(R, T)$ the part coming from the string vibration. The correction has the form

$$\chi(R, T) = \tilde{\chi}(R, T) + \tfrac{1}{24}\pi \frac{1}{R(R-1)} + \Delta(R, T)\,, \tag{4.15}$$

where $\Delta(R, T)$ is a minor positive correction coming from the sum of logarithms in $V(R, T)$. It is only important for $R=T$, $R=T-1$ and $R=T-2$. From fig. 4 it follows that

$$\tilde{\chi}(R, T) = \sigma\,, \tag{4.16}$$

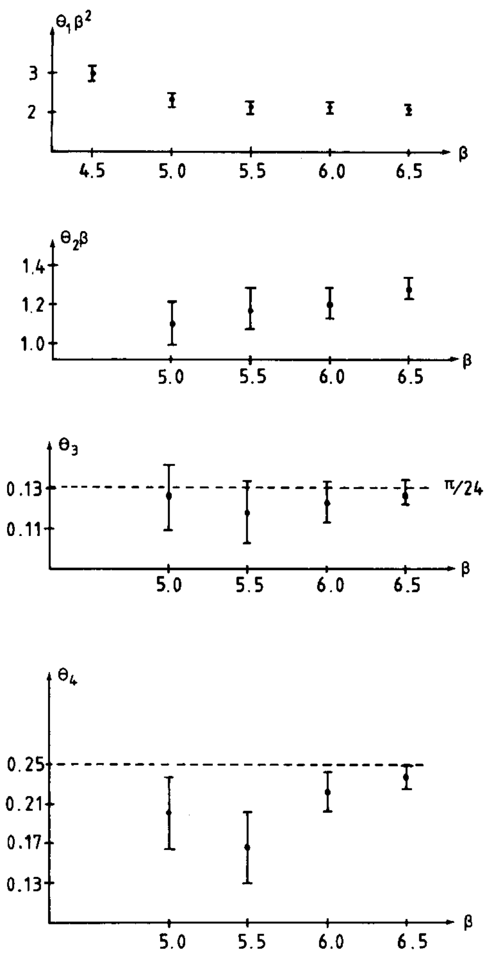

Fig. 2. The parameters θ_1, θ_2, θ_3 and θ_4 as defined by eq. (4.11).

where σ is independent of (R, T) as it should be if it were a measure for the string tension.

By now we would be tempted to conclude that by the same time scaling to the continuum limit begins, the Wilson loops $W(R, T)$ are very well represented by the simple string model with action (1.1). We should however check that the deviation from scaling that is seen in the measured Creutz ratios could not have an alternative perturbative explanation, although this seems somewhat unlikely when we consider the perturbative regions in fig. 1.

The lowest-order perturbative potential is easily calculated using the continuum propagators. These are excellent approximations to the lattice propagators when

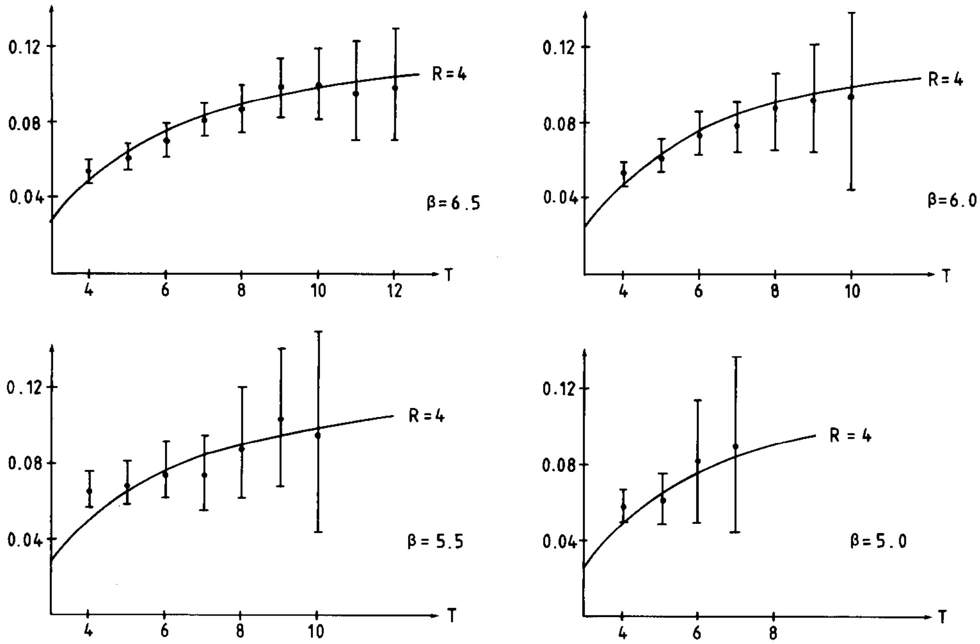

Fig. 3. Comparison of the discrete version of eq. (4.14) with the data.

the distance is two or more lattice spacings. One gets

$$V(R,T)_{\text{pert}} = C_2 \frac{4}{\beta}\frac{1}{2\pi}[T \ln R + R \ln T + T(\sqrt{1+(R/T)^2} - \ln(1+\sqrt{1+(R/T)^2})) + R(\sqrt{1+(T/R)^2} - \ln(1+\sqrt{1+(T/R)^2})) + C(R+T)], \tag{4.17}$$

where C_2 is the Casimir of the fundamental representation of SU(2): $C_2 = \frac{3}{4}$ and the constant C depends on the cutoff used. For $C = 0.32$ eq. (4.17) fits the exact values calculated in ref. [2] within a few percent for $R, T \geqslant 2$.

Calculating the correction to $\chi(R,R)$ from (4.17) one gets the analogue of eq. (4.15)

$$\chi(R,R) = \bar{\chi}(R,R) + \frac{3}{\pi\beta}\left[\ln\frac{R}{R-1} + \Delta(R)\right], \tag{4.18}$$

where $\Delta(R)$ is a small negative quantity of the order of 10% relative to the logarithmic term.

The result of applying (4.15) and (4.18) to square loops is shown in fig. 5. As already mentioned, the corrected $\bar{\chi}(R,R)$ gives a consistent interpretation as a string tension in the sense that it is independent of R. This is clearly not the case for

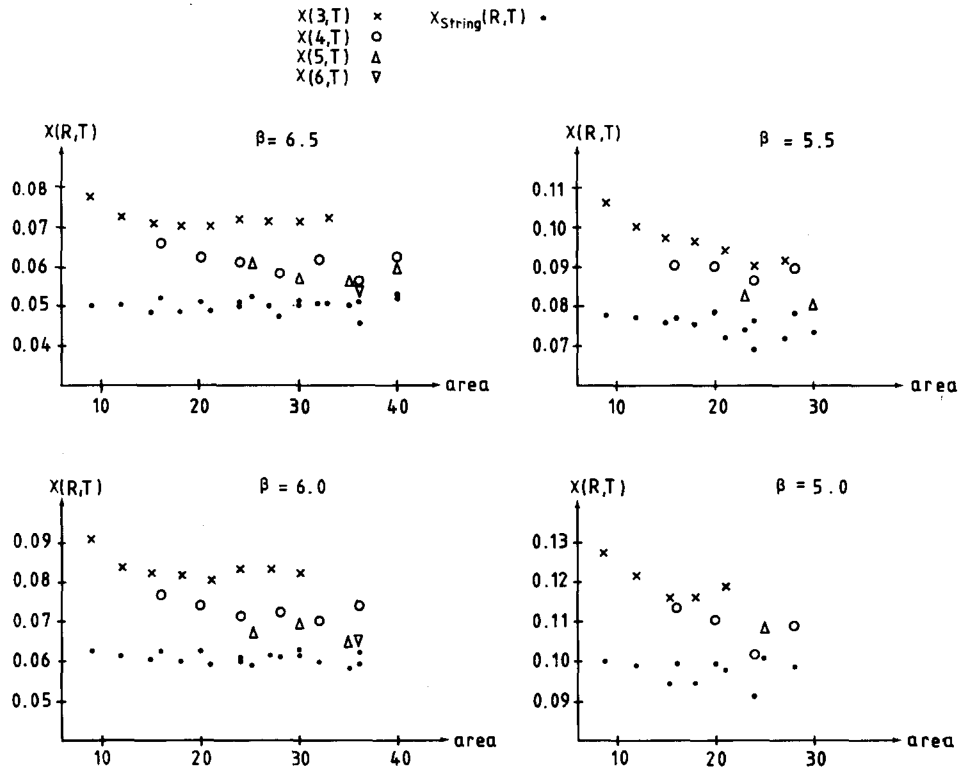

Fig. 4. The Creutz ratios for different areas and different β's. The black dots represent the Creutz ratios corrected for string vibrations (see eq. (4.15)).

$\bar{\chi}(R, R)$. It should also be mentioned that the order of magnitude of the "string tension" $\bar{\chi}(R, R)$ is such that the correlation length $1/\sqrt{\bar{\chi}(R, R)}$ is smaller than the size of the largest loops $(5\times5, 6\times6)$. This makes it hard to understand why one should add a perturbative part to the potential for distances larger than the correlation length, because then the propagation of massless gluons should be strongly damped and one-gluon perturbative corrections irrelevant. In three dimensions we are in the lucky situation that even for a fixed β we are able to distinguish clearly between the perturbative Coulomb potential (as in (4.18)) and the "universal" string Coulomb potential (as in (4.15)). This is in sharp contrast to the situation in four dimensions. The data definitely seem to favour the string model potential.

5. Conclusions

We have seen that an excellent representation of the data from $W(R, T)$ is obtained by the simple string model with the action of eq. (1.1).

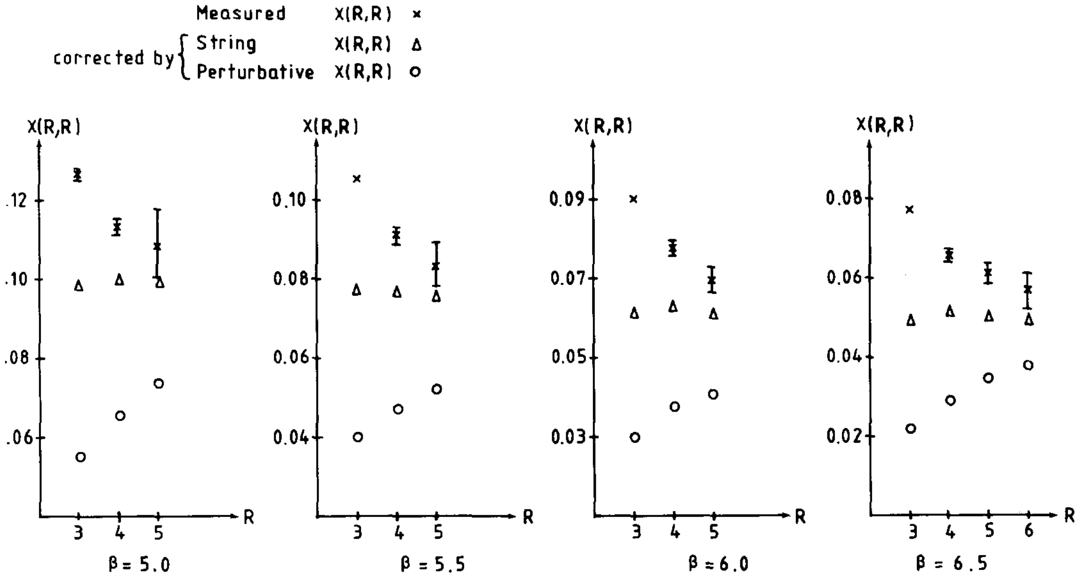

Fig. 5. The Creutz ratios for square loops. The string correction is given by eq. (4.15). The perturbatively corrected χ's are given by eq. (4.18). It is seen that the perturbative corrections lead to a string tension which depends on R, in contrast to the string corrections.

It is of importance to realize that we cannot see the short-distance behaviour in the transverse direction of the string (flux tube). Thus eq. (1.1) should be regarded as an effective action for distances much larger than the width of the flux tube.

It is well known that Lorentz invariance is broken for the simple string (1.1) (except for a critical dimension) [10]. QCD is of course expected to be Lorentz invariant and one may criticize our fit because of this. However, the lack of Lorentz invariance only occurs if one insists that the string picture is mathematically correct down to distances compatible with the transverse extension of the flux tube. Such an assumption is of course physically unreasonable. On the contrary, one expects the string to be a mathematical idealization, which is valid only for distances larger than the width of the flux tube. Therefore the breaking of Lorentz invariance is not an objection to the action (1.1) if one takes the point of view that it is an effective action, valid only for distances large compared to the transverse extension of the flux tube (the appearance of a tachyon in dual models is also an ultraviolet problem).

We have found that the string picture occurs already at relatively small distances between the test quarks. Therefore one would expect the lowest glueball mass to be rather large since it is expected to be of the inverse flux tube width. We are at the moment investigating this expectation by a direct calculation of the lowest glueball mass.

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References

[1] E. D'Hoker, Nucl. Phys. B180 [FS2] (1981) 341
[2] J. Ambjørn, A.J.G. Hey and S. Otto, Nucl. Phys. B210 [FS6] (1982) 347
[3] J. Ambjørn, P. Olesen and C. Peterson, Phys. Lett. B to be published
[4] D. Petcher and D.H. Weingarten, Phys. Rev. D22 (1980) 2465
[5] M. Lüscher, Nucl. Phys. B180 [FS2] (1981) 317
[6] A.M. Polyakov, Phys. Lett. 103B (1981) 207
[7] B. Duurhus, P. Olesen and J.L. Petersen, Nucl. Phys. B232 (1984) 291
[8] J. Ambjørn and S. Wolfram, Ann. of Phys. 147 (1983) 1
[9] O. Alvarez, Phys. Rev. D24 (1981) 440
[10] P. Goddard, J. Goldstone, C. Rebbi and C. Thorn, Nucl. Phys. B56 (1972) 109