# STRINGS AND SU(3) LATTICE GAUGE THEORY ☆

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Recent SU(3) lattice Monte Carlo data ($24^4$ and $16^4$) are analyzed in terms of a simple string model. Good agreement is found. Observed similarities between SU(3) and three-dimensional SU(2) indicate that the string theory is indeed an effective one.

One of the major results from numerical studies of lattice gauge theories has been the establishment of a nonvanishing string tension that exhibits approximate scaling. However, this fact does not necessarily mean that the mathematical idealization of an effective string theory is correct in describing Monte Carlo data. In ref. [1] this question was investigated by confronting the predictions of a simple string model with large $R \times T$ Wilson loops from three-dimensional SU(2). The agreement was remarkably good. Even the adjoint Wilson loops were successfully described by the corresponding adjoint ("gluon") string theory [2]. In this paper we pursue these ideas using recent four-dimensional SU(3) Monte Carlo data [3–7].

*The string picture.* A few years ago it was demonstrated [8] that chromoelectric flux tubes roughen when stretched out. The low-frequency fluctuations responsible for this delocalization also modify the interquark potential (or Wilson loops). In refs. [8,1] these modifications were studied by considering a general class of string models described by an effective action

$$S = c \iint \mathrm{d}r\,\mathrm{d}t\,[\nabla x_\perp \nabla x_\perp + \text{terms containing higher derivatives}]\,. \quad (1)$$

Performing the $R \times T$ integral over the wiggling transverse coordinates $x_\perp(r,t)$ one obtains for the Wilson loop (neglecting the higher derivatives)

$$-\ln W(R,T) = \sigma(\beta)RT + d(\beta)(R+T) + c(\beta) - (d-2)\left(\tfrac{1}{24}\pi T/R + \tfrac{1}{4}\ln R - \frac{1}{2}\sum_{n=1}^{\infty} \ln[1-\exp(-2\pi n T/R)]\right)\,. \quad (2)$$

Eq. (2) has two salient features. The vibrational terms (the bracket) have no parameters depending on the underlying local field theory. It only depends on the number of transverse dimensions, $d-2$, in which the string lives. Also, it is $R \leftrightarrow T$ symmetric despite its asymmetric appearance. The terms in the large parentheses strongly mimic a $(T/R + R/T)$-behaviour in the $R \times T$ region considered here (albeit with a coefficient differing from $(d-2)\frac{1}{24}\pi$). From eq. (2) one obtains the potential

$$V(R) = -\lim_{T\to\infty} \frac{\ln W(R,T)}{T} = \sigma(\beta)R + d(\beta) - (d-2)\tfrac{1}{24}\pi R\,. \quad (3)$$

The $1/R$-term in eq. (3) is universal and of long-distance origin in contrast to the ordinary Coulomb potential $\alpha_s(\Lambda)/R$. A similar result was obtained in ref. [9], where in the leading order of a $1/d$ expansion, the potential

$$V(R) = \sigma R(1 - R_c^2/R^2)^{1/2} \quad (4)$$

was obtained. This result has later been shown to be

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valid for arbitrary $d$ [10]. Here $R_c^2 = \pi(d-2)/12\sigma$ is the critical distance, below which the effective string picture breaks down. In four dimensions this amounts to $R_c \approx 0.7/\sqrt{\sigma}$ whereas in three dimensions one has $R_c \approx 0.5/\sqrt{\sigma}$. Expanding eq. (4) at large $R$ yields

$$V(R) = \sigma R - \tfrac{1}{24}\pi(d-2)/R - (\pi^2/288\sigma)(d-2)^2/R^3 \ldots . \quad (5)$$

Here we recognize the universal $1/R$-term of eq. (3) and we also observe that the next term is theory-dependent through $\sigma$. It is related to the terms containing higher derivatives (dim = 4) in eq. (1); their contributions to finite $T$ Wilson loops have not yet been calculated. It is presumably reasonable to assume a "brute force" symmetrized form

$$-\ln W_{(4)}(R,T) \sim [(d-2)^2/\sigma](T/R^3 + R/T^3). \quad (6)$$

In ref. [1] eq. (2) was successfully confronted with three-dimensional Monte Carlo data. Three reasons motivated this choice of theory:

(i) The fewer degrees of freedom allow the use of rather large loops (6 × 12) with reasonable statistics.

(ii) The "universal" $1/R$-term in eq. (2) differs from the real Coulomb term, which is logarithmic in three dimensions. They are hence easier to distinguish.

(iii) The critical distance $R_c$ is lower, in terms of lattice spacings, for three-dimensional SU(2) than for four-dimensional SU(2) and SU(3) in respective scaling region.

Encouraged by this success we will now proceed to confront four-dimensional SU(3) Monte Carlo data with the simple string model (eq. (1)) despite the absence of the virtues mentioned above.

*SU(3) Monte Carlo data and its limitations.* The interquark potential can be extracted from Monte Carlo data either by measuring Wilson lines or Wilson loops. With each method there is an associated penalty; Wilson lines on a large lattice build up dominating parameter contributions, whereas Wilson loops are exponentially suppressed at large $R \times T$. At reasonably large $R$ and for $\beta \geqslant 6.0$ there are essentially five sets of Wilson-loop data at our disposal (see table 1).

Which loop sizes are relevant for a nonperturbative string model comparison at various $\beta$? The answer is given by the Alvarez bound (eq. (4)), $R_c = 0.7/\sqrt{\sigma(\beta)}$. In fig. 1 we show the loop sizes surviving this constraint at $\beta = 6.0$.

Table 1
Presently available high-statistic Wilson-loop data for $\beta \geqslant 6.0$, used in our fits.

| $\beta$ | Lattice | Max $R \cdot T$ | Authors [ref.] |
|---|---|---|---|
| 6.0 | $16^4$ | 8•8 | de Forcrand [6] |
| | $16^4$ | 8•8 | Bowler et al. [5] |
| | $16^4$ | 6•8 (8•8) | Barkai et al. [4] |
| | $12^3 \cdot 16$ | 6•8 | Brooks et al. [3] |
| 6.1 | $12^3 \cdot 16$ | 6•8 | Brooks et al. [3] |
| 6.2 | $16^4$ | 6•8 (8•8) | Barkai et al. [4] |
| | $12^3 \cdot 16$ | 6•8 | Brooks et al. [3] |
| 6.3 | $24^4$ | 12•12 | de Forcrand [7] |
| | $16^4$ | 8•8 | Bowler et al. [5] |
| | $12^3 \cdot 16$ | 6•8 | Brooks et al. [3] |
| 6.4 | $16^4$ | 6•8 | Barkai et al. [4] |
| | $12^3 \cdot 16$ | 6•8 | Brooks et al. [3] |

The pure perturbative region of $R$ and $T$ is on the other hand approximately limited by the value of $R$ where [11,12]

$$V(R) = \tfrac{16}{33}\pi\,\{R \ln[1/(\Lambda_p R)^2]\}^{-1} \times (1 - \tfrac{102}{121}\ln\{\ln[1/(\Lambda_p R)^2]\}/\ln[1/(\Lambda_p R)^2]) \quad (7)$$

exhibits a maximum. Using $\sigma/\Lambda_0^2 \sim 1.1 \times 10^4$ [3,4] and $\Lambda_p = 82.07\,\Lambda_0$ one finds $R_{max} \approx 0.4/\sqrt{\sigma(\beta)}$.

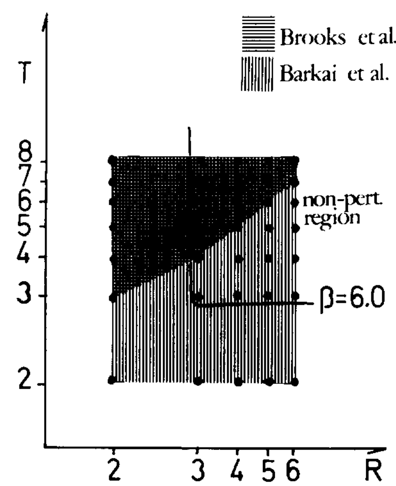


Fig. 1. Loop sizes included in the analyses of refs. [3,4]. The scales are chosen such that areas are approximately proportional to the corresponding sum of statistical weights. The drawn line limits the nonperturbative region $T, R > R_c \approx 0.7/\sqrt{\sigma}$ at $\beta = 6.0$.

*Previous analyses of the SU(3) potential.* The two data sets of Barkai et al. [4] and Brooks et al. [3] have been analyzed in terms of a string-potential by the respective authors. In both cases $R = 2$ loops are included in all fits involving a $1/R$-term. Since the total statistical weight of $\ln W(R,T)$ with $T \geqslant R \geqslant 2$ in these data sets exceeds that of the remaining points $T > R > 2$ by a factor 3 (see fig. 1), it is not unconceivable that the $1/R$-coefficient obtained in this way represents a mixture of the perturbative one in eq. (7) and $(d-2)\pi/24$ in eq. (3). Secondly the $T/R$-contribution to $\ln W(R,T)$ is an asymptotic form valid for $T \gg R$, and must therefore be accompanied by the remaining term in the large parentheses of eq. (2) for $T \approx R$. In ref. [4] Creutz ratios are considered including the domain $T < R$ where

$$\frac{1}{2}\sum_{n=1}^{\infty} \ln[1-\exp(-2\pi n T/R)] \qquad (8)$$

in eq. (2) gives significant contributions. In ref. [3] the constraint $T \geqslant R+1$ is used and nonleading terms, including $\log R$, are neglected. Below we will confront all SU(3) Monte Carlo data, referred to in table 1, with the full expression of eq. (2).

*Our analysis.* We focus on the $\beta \geqslant 6.0$ region. First we subtract from the measured $-\ln W(R,T)$ the term in eq. (8), which amount to a very small correction unless $T < R-1$. For each pair of $R$ and $\beta$, with $R \geqslant 3$, we fit to a straight line in $T$. As a result we obtain a slope and an intercept for each $R$ and $\beta$:

$$-\ln W(R,T) - (d-2)\frac{1}{2}\sum_{n=1}^{\infty} \ln[1-\exp(-2\pi n T/R)]$$
$$= [\Theta_1(\beta)R + \Theta_2(\beta) + \Theta_3(\beta)/R]\,T$$
$$+ [\Theta_4(\beta)\ln R + \Theta_2(\beta)R + \Theta_5(\beta)]\,. \qquad (9)$$

The perimeter term $\Theta_2$, appearing both in the slope and the intercept, is by far the dominating term. With a total of 53 pairs of $R$ and $\beta$ at our disposal, we thus perform 53 two-parameter fits, each to 6–8 different $T$-values. The statistical weights are $W(R,T)/\sigma(R,T)$, where $\sigma(R,T)$ denotes respective standard deviation. The bulk of the $\chi^2$/DOF values lies in the range 1–4 for these fits.

For the largest set of data, ref. [7], we have also

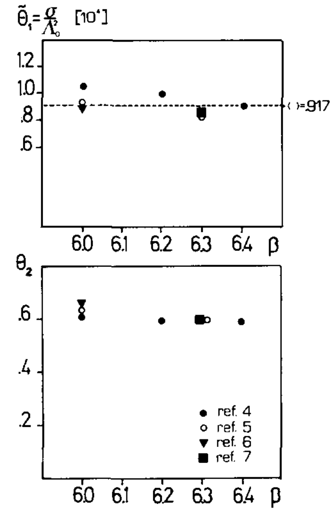


Fig. 2. The parameters $\Theta_1$ and $\Theta_2$ as defined by eq. (9). Instead of $\Theta_1$ we have plotted the rescaled variable $\tilde{\Theta}_1$ defined by $\tilde{\Theta}_1 = \sigma/\Lambda_0^2 = \Theta_1(8\pi^2\beta/33)^{-102/121}\exp(8\pi^2\beta/33)$. The $\beta$-dependence of $\Theta_2$ is negligible in the interval considered here. Statistical errors are (1–3)% (see text). The plotted values for $\Theta_2$ are the average of the ones extracted from the slope and the intercept of eq. (9).

tried to raise the minimum size of the loops to 4 and even 5 units. This increase does not change the values of the parameters significantly.

The $\Theta_i(\beta)$ resulting from a fit to these 53 points are shown in figs. 2 and 3. First we conclude that the string model predictions for $\Theta_3(\beta)$ and $\Theta_4(\beta)$ are very well described by the data. For the string tension we

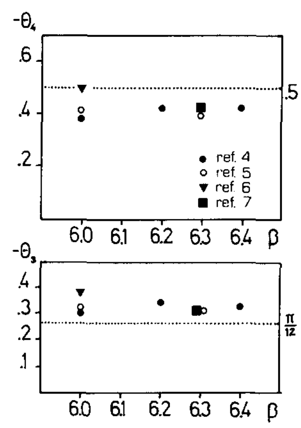


Fig. 3. The parameters $\Theta_3$ and $\Theta_4$ as defined in eq. (9). The dotted lines are the universal string model predictions of eq. (2). Statistical errors are (1–3)% (see text).

obtain the average

$$\sigma/\Lambda_0^2 = 0.917 \times 10^4, \qquad (10)$$

which is somewhat lower than in refs. [3,4]. Finally the perimeter term $\Theta_2(\beta)$ shows almost no $\beta$-behaviour in the interval we consider. This is as expected since this term originates from self-interactions and hence scales like $1/\beta$ because

$$\langle W \rangle \sim \exp\left(-\tfrac{1}{2}g^2 \oint_c \oint_c \Delta(x-y)\,dx_\mu\,dy_\mu\right), \qquad (11)$$

which with $\dim \Delta(x-y) = 2$ gives $\ln W \sim (R+T)/\beta$. Note that the perimeter term can be extracted from both the slope and the intercept in eq. (9).

The quoted errors for the parameters $\Theta_i(\beta)$ are quite small ((1–3)%). One should keep in mind, though, that we have a rather small set of data at our disposal and the error definition is therefore somewhat arbitrary, due to correlations. We have omitted the $12^3 \times 16$ data of ref. [3] in figs. 2 and 3, since these points show a somewhat larger spread although their average are well consistent with remaining higher statistics data. The spread of data points in figs. 2 and 3 might either indicate a realistic error estimate or the presence of systematic errors in the Monte Carlo data.

We conclude from this analysis that the simple string model (eq. (1)) nicely describes available SU(3) data for $R > R_c$.

*Hints of higher order effects.* The many fits performed allow us to notice a small but systematic deviation from the linearity in $T$ expected from eq. (2). This small discrepancy appears at all values of $\beta$ and $R$ (see fig. 4a). These deviations are of course also present and larger when fitting any other linear form of $T$ to $\ln W(R,T)$. The deviations are not finite lattice size effects since they are present for all sets of data. Neither do they seem to depend on the particular choice of theory, since they also appear in three-dimensional SU(2) as shown in fig. 5. Do these deviations result from the higher derivative term in eq. (5)? In fig. 4a we have indicated the functional form of a $(T/R^3 + R/T^3)$-term (eq. (6)). It certainly works in the right direction. When performing a three-parameter $\chi^2$-fit with this functional form, the deviations no longer show any systematics (see fig. 4b). Furthermore the $T/R^3$-coefficient comes out with the right sign. (Needless to say, the quality of the data is insufficient for precise fit.) Finally, when comparing four-dimensional SU(3) with three-dimensional SU(2) at compatible $\beta$ (say $\beta = 6.2$ with $\sigma = 0.04$ and $\beta = 5.5$ with $\sigma = 0.073$ respectively) we find that the ratio of the deviations approximately equals 8, which is what

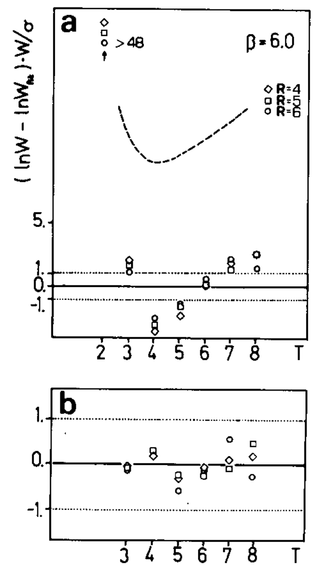


Fig. 4. (a) Deviations of eq. (9) from a straight line in $T$ (after subtracting eq. (8)) for $R$ = 4, 5 and 6 at $\beta$ = 6.0 using the data of ref. [6]. The dashed line shows the expected functional form of the higher order term (cf. eq. (6)). (b) The same deviations as in (a) after including the term of eq. (6) in the fit.

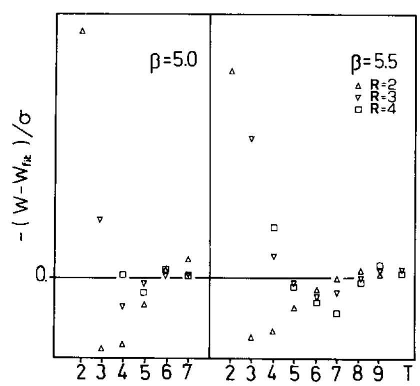


Fig. 5. Deviations of eq. (9) from a straight line in $T$ (after subtracting eq. (8)) for three-dimensional SU(2) [1] for $R$ = 2, 3 and 4 at $\beta$ = 5.0 and 5,5 respectively.

one expects from the $(d-2)^2/\sigma$-factor in eqs. (5) and (6). We take this fact as a further evidence that the string model really works as an effective long-distance theory for non-abelian lattice gauge theories in the continuum region.

In summary we have found that existing SU(3) Monte Carlo data are in good agreement with the leading "universal" term (eq. (2)) from the simple string theory of eq. (1). For the string tension we find the average $\sigma/\Lambda_0^2 = 0.917 \times 10^4$. A minor deviation from eq. (2) is observed, which hints at the presence of the correction term in eq. (6). The magnitude of this deviation when comparing SU(3) with three-dimensional SU(2) is as expected from eq. (6). This fact is an additional support for the string model indeed being the effective theory for non-abelian theories at large distances.

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