PHYSICAL REVIEW D VOLUME 34, NUMBER 5 1 SEPTEMBER 1986

Do results from lattice gauge theories distinguish between different fragmentation models?

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(Received 30 December 1985)

The adjoint (gluon) string tension in SU(2) lattice gauge theory has been found to be $\frac{8}{3}$ times the corresponding fundamental (quark) string tension. This fact strongly favors the Lund-fragmentation-model prescription of treating hard gluons as kinks in the quark string.

The predictive power of quantum chromodynamics is presently limited to two sectors. One is perturbation theory which is used to compute matrix elements for dynamical processes at short distances. The other one is the lattice Monte Carlo approach, which has turned out to be very successful for computing static quantities. In particular, one now has in the latter case convincing evidence for confinement and string formation.[1]

When interpreting short-distance collision processes in terms of perturbative QCD one often encounters the problem of relating the experimentally observed jets of hadrons to the quark and gluon quanta they originate from. Fragmentation models based on the string picture,[2] where the QCD quanta hadronize in a cascading way, are phenomenologically very successful in this respect. For two-jet situations, as in $e^+e^- \to q\bar{q}$, the implementations of these models are unambiguous. However, when it comes to merging higher-order QCD matrix elements with the fragmentation schemes, it is not *a priori* clear how to proceed. As a well-known example of this situation, consider

$$e^+e^- \to q\bar{q}g \ . \quad (1)$$

Two prescriptions have been proposed to treat this process. One is independent fragmentation,[3] where the QCD quanta emerging according to the perturbative matrix element subsequently fragment independently [see Fig. 1(a)]. Different models may be used for the less known gluon fragmentation, but the main feature here is the existence of a gluonic jet. The other one is the string (or Lund) model[4] recipe. Here the hard gluon is treated as a kink on the $q\bar{q}$ string and does not appear as a gluonic jet. Rather, as this kink on the string develops in time, it gives rise to two $q\bar{q}$ systems [see Fig. 1(b)]. This scheme leads to distinct experimental signatures in terms of multiplicity asymmetries (the so-called string effect), which have been repeatedly verified experimentally.[5] Also, schemes based on perturbative QCD-shower calculations with angular-ordering constraints[6] reproduce the observed asymmetries. A central question is, therefore, to what extent this string phenomenon can be justified from nonperturbative studies in QCD. As will be discussed below it turns out that recent lattice Monte Carlo results for the adjoint or gluon string in SU(2) hint at a definite answer to this question.

FIG. 1. The $q\bar{q}g$ final state in e^+e^- annihilation: (a) independent fragmentation, (b) Lund-model fragmentation.

THE ADJOINT (GLUON) STRING IN LATTICE MONTE CARLO CALCULATIONS

One of the major successes of lattice Monte Carlo (MC) calculations has been the establishment of confinement with a linear force law (a nonvanishing string tension) at large distances for SU(2) and SU(3) (Ref. 7). The coexistence of a string tension with asymptotic freedom ensures that the calculations are relevant for the continuum limit. With a few exceptions all calculations have been performed within the so-called quenched approximation, i.e., the effects of quark loops are neglected. This limitation has no significance for our considerations below.

When determining the potential in lattice MC calculations one measures the Wilson-loop expectation value $W(R,T)$ for a heavy-quark loop in pure gauge configurations. The static interquark potential $V(R)$ is then given by

$$V(R) = -\lim_{T \gg R} \frac{\ln W(R,T)}{T} \ . \quad (2)$$

For large interquark separations R one expects from string models a linear force law plus a universal $1/R$ correction:[8,9]

$$V(R) = \sigma R - \frac{\pi}{12}\frac{1}{R} \ . \quad (3)$$

The second term in Eq. (3) is due to the vibrational degrees of freedom associated with the quantum-mechanical transverse fluctuations of the string and should not be confused with the short-distance Coulomb term. Analysis of presently available MC data for SU(3) in the large-R region is consistent with the presence of this $\pi/12R$ term.[1] This fact is the best circumstantial evidence so far for string formation in QCD.

Similar numerical calculations can be made with the adjoint (gluon) Wilson loop $W_{\text{gluon}}(R,T)$.[10–12] So far this has only been done for SU(2). Again one finds a nonvanishing string tension that exhibits approximate scaling with a value for $\sigma_{\rm gluon}$:[11,12]

$$\sigma_{\rm gluon} \approx \tfrac{8}{3}\sigma_{\rm quark} . \qquad (4)$$

This is what is expected from dimensional reduction.[11] At large distances the (3+1)-dimensional theory reduces to its (1+1)-dimensional counterpart. The latter is easily solvable in perturbation theory. This is the origin of the perturbative Casimir expectation value relating the gluonic and quarkic string tensions in Eq. (4). One should stress that the appearance of this perturbative Casimir factor in the (3+1)-dimensional theory therefore does not originate from the perturbative sector of this theory, but is rather an effect of the long-distance properties of the theory; at large distances the theory reduces to two dimensions. This phenomenon is in accordance with large-N expectations and is also expected in vacuum models, where the latter consists of random magnetic fluxes.[13]

In the more realistic case of SU(3), dimensional reduction prescribes

$$\sigma_{\rm gluon} = \tfrac{9}{4}\sigma_{\rm quark} , \qquad (5)$$

and for large N_c it is easily demonstrated that

$$\sigma_{\rm gluon} \rightarrow 2\sigma_{\rm quark} , \qquad (6)$$

since the adjoint loop factorizes at large N_c. Thus it is very likely that for all non-Abelian theories one has

$$\sigma_{\rm gluon} > 2\sigma_{\rm quark} , \qquad (7)$$

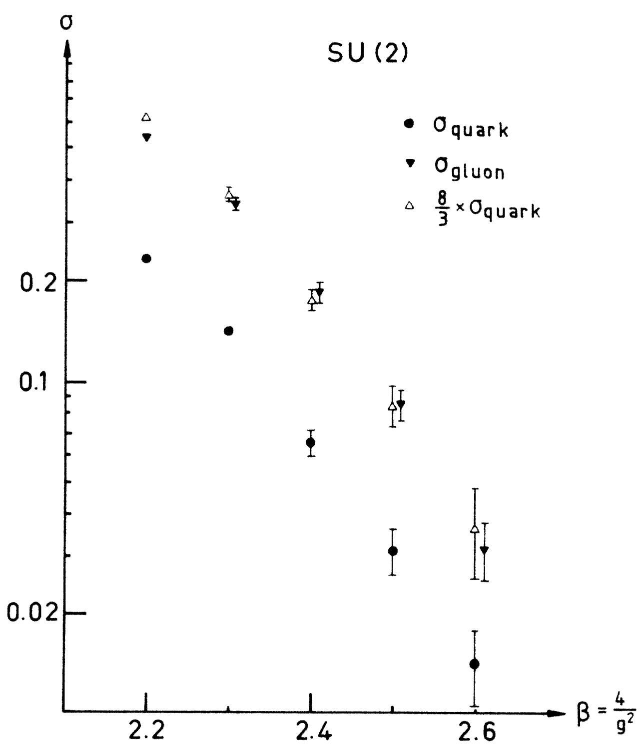

FIG. 2. Results from lattice Monte Carlo calculations on SU(2) (Ref. 11) for quark and gluon string tensions as functions of the inverse coupling squared.

regardless of the underlying gauge group. As mentioned above *we have at present only numerical evidence for this feature in the* SU(2) *case.* In Fig. 2 we show $\sigma_{\rm gluon}$ and $\sigma_{\rm quark}$ as functions of the inverse coupling squared ($\beta = 4/g^2$). It is clear from Fig. 2 that there is profound numerical support for Eq. (4). A corresponding investigation of the SU(3) gluon string tension has not yet been performed. (The gluonic string tensions are hard to measure in general just because of Eqs. (4)–(7) and the area-law suppression.) Even if it is done the result will be less conclusive for our purposes since $\frac{8}{3}$ is more distinctively different from 2 than the case for $\frac{9}{4}$. *In what follows we will assume that Eq. (5) is valid for* SU(3).

IMPLICATIONS FOR THE TREATMENT OF HARD GLUONS IN FRAGMENTATION PROCESSES

Let us now consider the final state in Eq. (1) assuming that what has been demonstrated for SU(2) also holds for SU(3) [cf. Eq. (5)]. It is then energetically favorable for the outgoing gluon not to form a gluonic jet. (This connection between the value of the gluonic string tension and fragmentation structure was first pointed out in Ref. 14.) Rather it will form a kink on the $q\bar{q}$ string and subsequently split up into two quark strings [see Fig. 1(b)]. The emerging hadrons will then stem from two bent strings with the observed asymmetries as results. A similar situation occurs in

$$\Upsilon \rightarrow ggg , \qquad (8)$$

where the three gluons produced split up into a triangle-shaped quark-line structure for the same reason [see Fig. 3(b)]. (Other diagrams, where more than one quark line connect two sources, will be suppressed since they are energetically unfavorable.) In this case there are no asymmetries associated with this treatment of the hard gluons. However, if the gluons fragmented independently with spectra different from those of the quarks one would expect different particle ratios, etc., on the Υ resonance than in the continuum region. No significant differences have been observed,[15] which again supports the Lund-model treatment of the hard gluons.

One should also mention that expectations from the MIT bag model are in conflict with Eqs. (4)–(7). For the gluonic string tension one obtains[16]

$$\sigma_{\rm gluon} = (\tfrac{9}{4})^{1/2}\sigma_{\rm quark} , \qquad (9)$$

for SU(3). This value, which is less than 2, would rather favor the independent fragmentation scheme. As dis-

FIG. 3. The ggg final state in Υ decay: (a) independent fragmentation, (b) Lund-model fragmentation.

cussed above the latter is excluded by the data.

In summary, we restate our main conclusion. The ratio $\sigma_{gluon}/\sigma_{quark}$ as obtained from recent lattice MC calculations in SU(2) strongly favors the Lund-model prescription for treating hard gluons (kinks). It is amusing that similar results can be obtained from almost pure perturbative QCD considerations with angular constraints.[6] To our knowledge this is the first occasion when pure perturbative and nonperturbative analyses match.

Note added. After the completion of this work, data on the SU(3) adjoint potential were published.[17] The results of Ref. 17 are consistent with Eq. (5). This fact provides further support of the central theme in this paper.

This work was supported by the Swedish Natural Science Council under Contract No. NFR F 70717-109.

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