NUMERICAL EVIDENCE FOR A MASS GAP IN THREE-DIMENSIONAL SU(2) ☆

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Numerical evidence for a nonvanishing 0^{++} glueball mass in three-dimensional SU(2) is presented By using a long-distance correlation Monte Carlo method we obtain $m_{0^{++}} = (4\,7 \pm 1\,2)\sqrt{\sigma}$ in the β-range 4 0–6 5 The measurements are consistent with $1/\beta$-scaling The relevance of this result for SU(2) in four dimensions at finite temperature is briefly discussed

1. Introduction. A non-vanishing lowest 0^{++} glueball mass is now well established in both (3 + 1)-dimensional SU(2) and SU(3) from lattice Monte Carlo calculations ‡1. The first generation of measurements employed the straightforward technique of computing the plaquette–plaquette correlation function

$$G(r) = \langle \square(r) \square(0) \rangle - \langle \square \rangle^2 . \quad (1)$$

The lowest glueball mass is then obtained by fitting eq. (1) with an exponential at large r. This method is plagued with a large noise-to-signal ratio for $r \geqslant 3$ at desirable β-values. Subsequently new techniques have been developed; source methods [2], the Langevin equation [3], fixed boundary conditions [4,5] and adjoint Polyakov loops [6]. These permit the probing of larger distances and hence give more reliable values for $m_{0^{++}}$. Typical results for both SU(2) and SU(3) are that $m_{0^{++}} \approx 2\sqrt{\sigma}$ and that one observes approximate scaling at least for SU(2) [1].

The target of this work is to measure $m_{0^{++}}$ in (2 + 1)-dimensional SU(2) [$SU(2)_3$]. The motivation for this is twofold. First of all it is of general interest to establish a mass gap for this theory. Gauge theories in three euclidean dimensions are very peculiar in one aspect. They are super-renormalizable and dimensional quantities are therefore believed to scale with the bare coupling. In ref. [7] a nonvanishing string tension was found for $SU(2)_3$, which exhibited this scaling behaviour. Another and more topical reason to study $SU(2)_3$ is its relevance for $SU(2)_4$ at finite temperature. There is now substantial evidence from Monte Carlo calculations ‡2 that $SU(2)_4$ and $SU(3)_4$ undergo deconfining phase transitions at critical temperatures T_c. However, it has been pointed out [9] that in the plasma phase above T_c there could be more degrees of freedom than just free gluons. Such a scenario is expected from dimensional reduction [10] where at large enough temperatures $SU(N)_4$ effectively reduces to $SU(N)_3$. A glueball in $SU(2)_3$ could then correspond to a finite T excitation (plasmon) of $SU(2)_4$ [9].

The $SU(2)_3$ theory. We define the $SU(2)_3$ lattice gauge theory by the Wilson action

$$Z = \int \prod_{r,\mu} \mathrm{d}U_{r,\mu} \exp(\beta S) , \quad S = \tfrac{1}{2} \sum_{r,\square} \mathrm{Tr}\, U_\square , \quad (2a,b)$$

where $U_{r,\mu}$ are the link variables and $U_\square$ the plaquettes In the continuum limit the action reduces to $S = \frac{1}{4} \int \mathrm{d}^3x\, F_{\mu\nu}^2$ with

$$\beta = 4/g_3^2 a , \quad (3)$$

where the coupling constant g_3^2 has the dimension of mass. Assuming g_3^2 to be the only mass scale entering in the continuum limit the theory is super-renormalizable and one has for the string tension and the mass gap

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‡1 For a review see ref. [1]

‡2 For a recent review see ref. [8].

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$$\sigma a^2 = c_1/\beta^2\,, \quad Ma = c_2/\beta\,, \tag{4a,b}$$

in the $a \to 0$ limit (c_1 and c_2 are constants).

The first numerical study of $SU(2)_3$ was performed by d'Hoker [11], who made rough estimates of the string tension σ from relatively small Creutz ratios and also attempted to establish a mass gap. A non-vanishing σ was later confirmed in ref. [7] where also good scaling was observed according to eq. (4a) for $\beta \geqslant 5.0$ with $c_1 \approx 2$.

The mass gap in $SU(2)_3$. It is very difficult to measure the mass gap in this theory even if one here has one less dimension. One reason is that continuum sets in at $\beta \approx 5.0$, where the values for $\langle\square\rangle$ are quite close to unity. Therefore correlation measurements like in eq. (1) are out of the question for large distances. Another difficulty is that at these high β-values the isocahedral subgroup of SU(2) freezes so one has to use the full SU(2) group, which is more time consuming. The effect of the subgroup approximation on the average plaquette is shown in fig. 1. Also, and most importantly, the mass gap in $SU(2)_3$ turns out to be larger than in the four-dimensional case, which implies smaller signals.

We have chosen to use the fixed boundary condi-

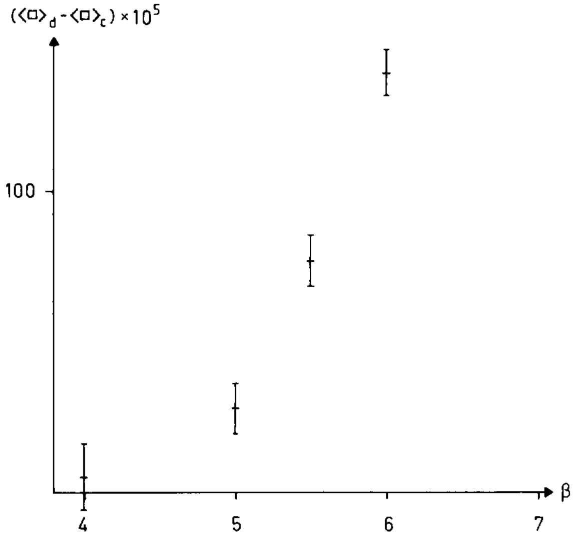

Fig 1. The difference between the average plaquette computed with the isocaheder ($\langle\square\rangle_d$) and full SU(2) group ($\langle\square_c\rangle$) respectively, as a function of β on a 12^3 lattice.

tion method of ref. [4], which seems to be most efficient as long as one is not interested in projecting out particular momentum and spin states [2]. Here one measures the response of plaquettes inside concentric cubes in a cubic N^3 lattice to the change of boundary conditions, fixed and periodic, respectively. We refer the reader to ref. [4] for details on this algorithm. The first step is to measure the differences $\Delta(d)$ of the average plaquettes inside the cube for the 2 different boundary conditions as functions of the distance d to the lattice boundary. The measured differences $\Delta(d)$ can than be expressed as a sum over all correlations between the plaquettes on the boundary (P) and those inside the cube (Q).

$$\Delta(d) \sim \sum_{P,Q} \rho(P,Q)\,. \tag{5}$$

In ref. [4] these correlations are assumed to be of the form

$$\rho(P,Q) \approx \exp[-r(P,Q)/\xi]\,, \tag{6}$$

where $\xi = 1/Ma$ is the correlation length and

$$r = \sum_{i=1}^{3} \sqrt{x_i^2} \tag{7}$$

is the euclidian distance between the plaquettes. We have performed straightforward plaquette–plaquette correlation measurements (eq. (1)) on a 12^3 lattice in order to explicitly check to what extent the euclidean distance r is the appropriate argument for ρ at distances of a few lattice units. The results for $\beta = 5.0$ are shown in fig. 2. Similar results were obtained for β = 4.0, 5.5, 6.0 and 6.5. It is clear from fig. 2 that the falloff of ρ exhibits a directional dependence. It is a function of

$$r' = \sum_{i=1}^{3} |x_i|\,, \tag{8}$$

rather than r at small distances. In other words the rotational symmetry is not restored for plaquette correlations at small distances. (This observation is in contrast to the case of Wilson line correlations [12], where the restoration of rotational symmetry holds down to 1 to 2 lattices spacings in the continuum region.) Although ρ should become a function of r as r becomes larger, it thus seems plausible that $\rho = \rho(r')$

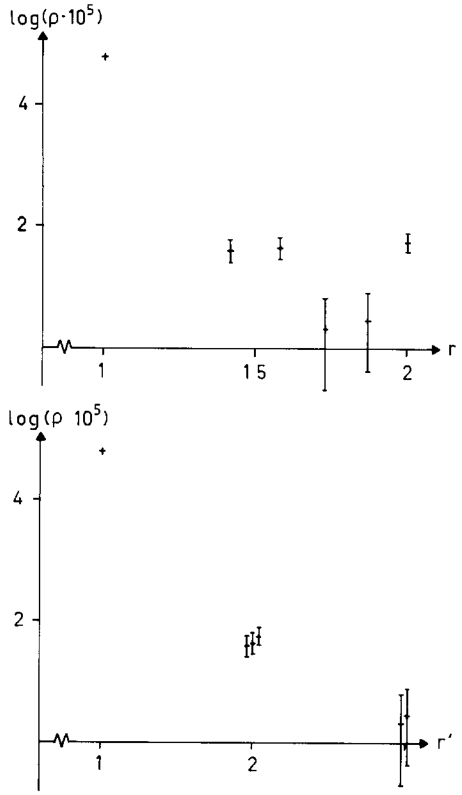

Fig. 2. Correlations between different pairs of not necessarily parallel plaquettes measured with eq. (1) on a 12^3 lattice The correlation function ρ is shown as a function of r and r', respectively

is a good approximation in our measurements. Furthermore, the data in fig. 2 indicates that the exponential form of the correlation function is not valid down to 1 lattice spacing. Hence we have used the values of $\rho(1)$ $\rho(2)$ from our measurements of plaquette–plaquette correlation for $r' = 1$ and replaced $\rho(r' = 1) = \exp(-1/\xi)$ by $\rho(r' = 1) = c\rho(r' = 2)$, where c has been taken from our measurements of plaquette–plaquette correlations. Taking these two modifications into account, our ansatz reads

$$\Delta(d) \sim \sum_{\substack{P,Q \\ r'=1}} c \exp(-2/\xi) + \sum_{\substack{P,Q \\ r'>1}} \exp[-r'(p,Q)/\xi] , \quad (9)$$

rather than eqs. (5), (6), which were used in ref. [5].

We have worked on a 12^3 lattice with $\beta = 4.0$, 5.0, 5.5, 6.0 and 6.5, using the Metropolis algorithm for

Table 1
Details of the Monte Carlo runs with fixed boundary conditions. Approximately 1000 sweeps were used for thermalization for each β.

	β				
	4.0	5.0	5.5	6.0	6.5
no. of sweeps between two measurements	1	3	1	1	1
no. of configurations used for measurements	3600	2100	6500	6500	8000
distances d used	2	2.3	2 3	2 3	2.3

updating. In order to improve the statistics for large d the innermost cubes were updated more frequently than the outer ones. The relative frrequencies were chosen to be 1 : 1 : 5 . 25 for the 12^3, 10^3, 8^3 and 6^3 concentric cubes. Further details of the Monte Carlo runs are found in table 1. Only one updating per link was performed. Unfortunately, the relative number of surface plaquettes is smaller in three dimensions than in four, which makes the signals smaller. This fact together with the difficulties in three-dimensional SU(2) mentioned above limits us to measure $\Delta(d)$ for $d \geqslant 4$. One should keep in mind though that we are actually probing larger distances than d. The effective distances d, which are obtained from

$$\bar{d} = \left(\sum_{P,Q} r'(P,Q) \exp[-r'(P,Q)/\xi] \right) \times \left(\sum_{P,Q} \exp[-r'(P,Q)/\xi] \right)^{-1} \quad (10)$$

are shown in table 2 for typical values of ξ.

Table 2
Effective distances $\bar{d}$ (in lattice units) for different values of ξ.

d	ξ				
	0 5	0.7	0.9	1.1	1.3
2	2.93	3.53	4.05	4.51	4.91
3	3.92	4.51	5.03	5.49	5.88

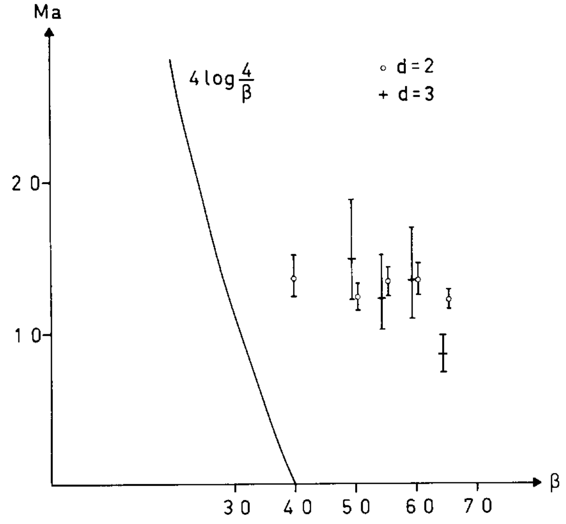

Fig. 3. The $SU(2)_3$ mass gap Ma as a function of β The solid line is the lowest-order strong coupling expansion.

Using eq. (9) we have extracted the mass gap Ma, which is shown in fig. 3 as a function of β. There is a clear evidence for a mass gap. By comparing the measurements with the lowest-order strong coupling expansion it is also obvious that the results are in the weak coupling region. The $d = 2$ and $d = 3$ data seem to approximately coincide at $\beta = 5.0$ and then depart slightly. In fig. 4 the same quantity multiplied with β is shown and one sees that the data is consistent with $1/\beta$ scaling.

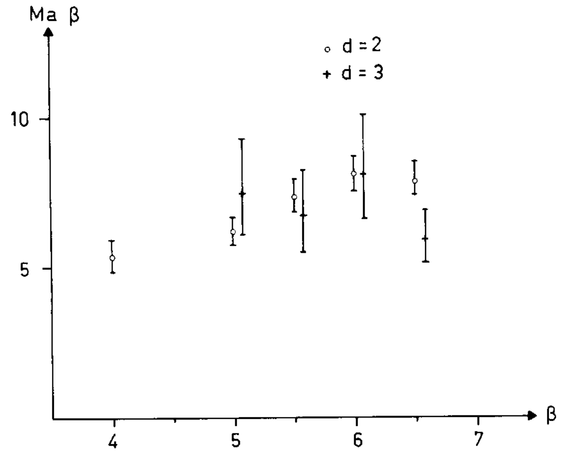

Fig. 4. The dimensionless quantity $Ma \cdot \beta$ as a function of β.

Averaging over the $d = 3$ data points and using the σ-values determined in ref. [7] we find for the glueball mass

$$m = (4.7 \pm 1.2)\sqrt{\sigma}\,, \tag{11}$$

which should be compared with $m \approx 2\sqrt{\sigma}$ as found in four-dimensional theories [1].

How can one understand this difference? It is here instructive to consult potential or bag model as a guide-line, the lowest 0^{++} models and see how the lowest 0^{++} mass state arises. Taking the bag state consists of 2 valence gluons in transverse electric modes ($J^P = 1^-$) which together form 0^{++} and 2^{++} states. With the interactions turned off these states are degenerate. The short-distance one-gluon exchange potential splits this degeneracy and one obtains [13]

$$m_{0^{++}} \approx 0.5 m_{0^{++}/2^{++}}\,, \quad m_{2^{++}} \approx 1.3\, m_{0^{++}/2^{++}}\,. \tag{12}$$

Normalizing to the lowest 0^{++} state to $2\sqrt{\sigma}$ one finds

$$m_{2^{++}} \approx 5\sqrt{\sigma}\,, \quad M_{0^{++}/2^{++}} \approx 4\sqrt{\sigma}\,, \quad m_{0^{++}} \approx 2\sqrt{\sigma}\,. \tag{13}$$

A similar pattern occurs in potential models.

In (2 + 1) dimensions the situation is different since one here has two E-field components and *one scalar B-field.* As a consequence the valence gluons have no magnetic moments and therefore do not induce one-gluon exchange spin–spin forces. Hence the *0^{++} and 2^{++} states* are expected to be *degenerate* with a relatively high mass gap as a result. It is thus very likely that the mass we have measured (eq. (11)) corresponds to a degenerate $0^{++}/2^{++}$ state. In this case one would then expect a higher $m/\sqrt{\sigma}$ value than in the (3 + 1)-dimensional case. (Cf. $m \approx 4\sqrt{\sigma}$ in eq. (13)).

As discussed in refs. [9,10] one expects $SU(2)_4$ to reduce to $SU(2)_3$ plus an adjoint Higgs field at finite temperatures above T_c. The corresponding three- and four-dimensional couplings β and β_4 are then related through

$$\beta_4 = \beta a T\,. \tag{14}$$

In ref. [14] the finite-temperature behaviour of the $SU(3)_4$ mass gap was investigated in a Monte Carlo calculation. Two important observations were made. The β_4-behaviour of Ma is consistent with eq. (14). Also a sharp rise of Ma was observed at T_c. The latter property is expected from the results of this work neglecting the effects from the Higgs field· If the four-dimensional reduces to its three-dimensional counterpart for $T > T_c$ $m_{0^{++}}$ should increase since one then gets the $0^{++}/2^{++}$ degeneracy discussed above.

The $SU(2)_4$ glueball mass revisited. In ref. [5] the $SU(2)_4$ glueball mass was computed using the same fixed boundary method [4] as in this work with one exception eq. (6) was used to extract ξ rather than eq. (9). For $\beta \geqslant 2.2$ and $d = 3$ one found

$$m_{0^{++}} = (2.1 \pm 0.21)\sqrt{\sigma}. \qquad (15)$$

No analysis of whether $\rho(\mathrm{P}, \mathrm{Q})$ in this case is a function of r at r' at small distance is available to us. We have nevertheless reanalyzed the data using $\rho(r')$ and find

$$m_{0^{++}} = (1.65 \pm 0.17)\sqrt{\sigma}. \qquad (16)$$

The choice of argument for ρ is thus crucial when determining the mass gap with this fixed boundary method.

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