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The effective string and $SU(2)$ lattice MC data

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Abstract. We present high statistics MC calculations of the static potential in three-dimensional $SU(2)$ for a wide range of β values on a 24^3 lattice. The deviations from area law are unambiguously demonstrated by use of 2nd lattice R derivative. After a clear crossover at $\beta=4.5$ the data show signs of an effective string roughening up to $\beta=6.5$, while scaling is not strictly obeyed in this interval. Pure fermionic strings do not provide better fits. The effect of regularization prescription on the effective string model up to two-loop correction is discussed and is found to be small. We also make a comparative study of existing data on $Z(2)$ and $SU(3)$ together with new data on four-dimensional $SU(2)$ presented here. It is pointed out that standard variance reduction methods as applied especially to Wilson lines are plagued by severe long range auto-correlations, whereas larger Wilson loops are less affected.

1 Introduction

With the emergence of accurate lattice Monte Carlo data on the static interquark potential, efforts have recently been devoted to analyze this potential in terms of effective string theories [1–4]. Such an analysis is motivated for two reasons. First of all it is of course very interesting to get an understanding of the non-abelian long distance dynamics. Secondly, and of more urgent importance, such studies are crucial for the extraction of the string tension σ from the data. Since this extraction is model dependent the presence or absence or even choice of an effective string theory strongly affects the value for σ and also its scaling properties.

An effective string theory is defined by a general class of partition functions of the type (for the bosonic case)

$$Z(R,T)=e^{-\sigma RT}\int Dx_\perp \exp\left\{-\frac{\sigma}{2}\int_0^R\int_0^T drdt(\nabla x_\perp\cdot\nabla x_\perp) + \text{higher derivative terms}\right\}. \quad (1)$$

In the case of the Nambu string the quartic part of the last term amounts to

$$S_4=-\frac{\sigma}{2}\int_0^R\int_0^T drdt(-\tfrac{1}{4}[(\nabla x_\perp)^2]^2+(\partial_t x_\perp\wedge\partial_r x_\perp)^2) \quad (2)$$

where the fields $x_\perp$ are defined on a surface with appropriate boundary conditions on a $R\times T$ boundary loop. Fermionic fields can of course also be introduced. As is wellknown, such an effective theory, if indeed realized in MC data, will manifest itself with its zero energy roughening terms [5], which modifies the linear interquark potential. For a bosonic string one gets, keeping the gaussian term

$$V(r)=\sigma r-\frac{\pi}{24}(d-2)\frac{1}{r} \quad (3)$$

where $(d-2)$ is the number of transverse dimensions in which the string lives. When analyzing MC data for finite $R\times T$ Wilson one has for the bosonic string

$$-\log W(R,T)=\sigma RT+P(R+T) + c+(d-2)q_B(R,T) \quad (4)$$

where

$$q_B(R,T)=-\frac{\pi T}{24R}-\tfrac{1}{4}\log R + \tfrac{1}{2}\sum_{n=1}^{\infty}\log(1-e^{-2\pi n(T/R)}) \quad (5)$$

which is related to (3) through

$$V(R)=\lim_{T\to\infty} -\frac{1}{T}\log W(R,T). \quad (6)$$

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Corresponding expressions can be derived for the Polyakov counterparts [2,4]. When confronting string model predictions with MC data it is preferable to use (3) where also the finite T behaviour enters the analysis.

Encouraging results for the bosonic string exists for $SU(2)$ in 3 dimensions ($SU(2)_3$) [1] and $SU(3)$ [2,4]. The predictions of (1) and (2) are universal in the sense that they are first order prediction in loop expansions of the effective string theory and hold for a whole class of bosonic string theories [5]. Summing up all the terms [6,7], predictions for the position of the deconfining phase transition can be obtained [8] at least for the Nambu string. When comparing the values for $T_c/\sqrt{\sigma}$ for $SU(2)$ in 3 and 4 dimensions very good agreement is found for the Nambu string, whereas for $SU(3)$ and $SU(N\to\infty)$ another string model seems to be present [4].

Recently also fermionic string theories have been proposed for describing the long distance properties of gauge theories. Predictions for the Neveau-Schwarz model are discussed in [9] and a pure Dirac string is treated in [10]. At first sight fermionic strings might seem to be odd creatures for pure gauge theories (all data analyzed so far are in the quenched approximation). However, there are indications from strong coupling analysis of $Z(2)_3$ gauge theory that the string formation is restricted to paths in a way that resembles constraints from Fermi-Dirac statistics [11,12]. In fact, the theory can be formulated in terms of a 2 dimensional theory of quantized surfaces with only fermionic degrees of freedom. It is therefore not totally unreasonable that the resulting theory, quartic in fermion fields, could show up in an effective fermionic string. If so, the effective string might also apply to the strong coupling region of $SU(2)_3$. The authors of ref. [10] have analyzed MC data on $Z(2)_3$ gauge theory with a pure fermionic (Dirac) string theory and claim encouraging results.

It is therefore worthwhile to reexamine the situation in the non-abelian cases of $SU(2)_3$, $SU(2)$ and $SU(3)$. In this paper we present new high statistics data on $SU(2)_3$ and $SU(2)$ and also reanalyze available data on $Z(2)_3$, $SU(2)$ and $SU(3)$. With our new data on $SU(2)_3$ we are able to probe distances out to 4–5 correlation lengths and are thus safely out of the perturbative region [4].

We will consider the second lattice R derivative of the logarithm of the W-loop expectation values. This operation disentangles the roughening contribution from the area law [1]. The linearity in T compensate the increase of error as T increases. The deviations from area law are unambiguosly extracted without use of any assumed functional form or fitting procedure. The onset of finite size effects can be seen for large loops.

We have also supplemented our analysis of the data with a simpleminded test of how lattice artifacts might influence the potential. That is, to what extent are (3) and (5) valid when only a few lattice spacings are involved? One should remember that (3) and (5) are continuum formulae that contain all roughening modes contributing when the sources are at a distance R apart. On the lattice, at least in the strong coupling sense, it might be difficult for all modes to contribute for small or moderate R. By explicitly summing up the modes with a lattice cutoff, some feeling of the size of corrections arising from a change of the regularization prescription, or to some extent of lattice artifacts, may be obtained. We have performed this up to two-loop corrections.

Finally, we will discuss the purely numerical matter of long-range autocorrelations that we have found when implementing standard methods of variance reduction [13] into the MC algorithm. This should serve as a warning that some earlier MC data may suffer from underestimated errors.

2 String theory predictions

We will shortly review the results within a continuum formulation of the effective string where the necessary regularization is performed using Riemann ζ-function methods. For a bosonic string we will also comment on the effect upon those expressions when we impose some other prescription, exemplified by a lattice cutoff. For a more complete review of the former case, see ref. [4].

A. Bosonic string

For the bosonic string we have already given the modification of the parametrization of the Wilson loop in (4) and (5). We note that the expression, (5), spite its appearance, is symmetric in $R\leftrightarrow T$, while here put in a form to ease the investigations as $R\leqslant T$. While the T/R coefficient is universal even when higher order derivative terms are included, this is not necessarily true for the whole expression. Actually, also the Polyakov smooth string has been shown to possess this T/R behaviour in the limit of large d and R [14].

The two-loop contribution of the Nambu string is obtained as the expectation value of S_4 in (2) with respect to the gaussian part of the action. It can be expressed in terms modular functions [4,15] and has a $1/\sigma$ dependence.

In evaluating the determinant appearing from the gaussian part of the action, as well as the infinite sums occuring in the two-loop expressions, some specific regularization prescription must be used when summing up zero-modes. While our physical results of course should not depend on the scheme, the string as realized in the finite distance MC data may well be sensitive to the cutoff. After all we are examining continuum strings down to distances of a few lattice spacings. In order to test the sensitivity of our results to the procedure chosen, we have calculated the lattice regulated version of the bosonic one-loop contribution in (5). Using the ordinary lattice laplacian one gets

$$-\log W(R,T)=\frac{d-2}{2}\{(R-1)(T-1)\log\frac{\sigma}{2\pi}+\sum_p E_p\} \quad (7)$$

where

$$E_p=4-2\cos p_1-2\cos p_2$$

$$p_1=\frac{\pi}{T},\frac{2\pi}{T},\ldots,\frac{(T-1)\pi}{T}$$

$$p_2=\frac{\pi}{R},\frac{2\pi}{R},\ldots,\frac{(R-1)\pi}{R}.$$

Subtracting those parts which renormalize σ, P and c, we define the one-loop contribution as

$$\begin{aligned}q_L(R,T)=&\sum_p\log(4-2\cos p_1-2\cos p_2)\\&-\left(\frac{2G}{\pi}-\log 2\right)RT\\&-\tfrac{1}{4}\log\frac{16}{3+2\sqrt{2}}(R+T)+\tfrac{3}{8}\log 2\end{aligned} \quad (8)$$

where $G\approx 0.916$ is Catalan's constant. The sum over p we calculate numerically.

Two-loop contributions can be treated in a similar way, but the expressions become more involved. A typical sum has the form (using a central lattice derivative)

$$\sum_p\frac{\sin^2 p_1/2\,\sin^2 p_2/2}{E_p^2}.$$

In order to compare with the continuum expression for the Nambu string we have calculated the full form for each R and T numerically. The contribution to the area term can be explicitly found and is $-RT(d-2)/2\sigma$.

The tachyonic content of the bosonic string sets a lower bound where the interquark potential becomes imaginary. Of course, an effective string theory must therefore be supplemented by a restriction of its validity to distances R larger than some critical R_C, below which the ultraviolet features of these theories appear. Alvarez [6] derived a closed expression for the potential in a $1/d$ expansion

$$V(R)=\sigma R\sqrt{(1-(R_C/R)^2)}$$

where

$$R_C=\sqrt{\pi(d-2)/12\sigma}. \quad (9)$$

This expression is actually valid for any d in the case of the Nambu string [7]. Our standpoint will be to exclude from the fits those points representing physical distances below R_C.

B. Fermionic string

A fermionic string will in general result in a T/R term with a coefficient different from the bosonic one. The value of the coefficient depends on the boundary conditions. For a Dirac spinor field with boundary phase $\phi=\pi/2$, for which the authors of [10] claim encouraging results, one obtains

$$q_D(R,T)=\frac{-\pi}{96}\frac{T}{R}-\frac{1}{4}\sum_{n=1}^{\infty}\log(1+e^{-2\pi(n-1/2)T/R}) \quad (10)$$

We note the disappearance of the $\log R$ term, as compared to the bosonic case of (5).

C. Neveu-Schwarz sector

Finally we give the roughening contribution for a Neveu-Schwarz string

$$\begin{aligned}q_{NS}(R,T)&=q_B(R,T)+q_M(R,T)\\&=\frac{-\pi}{16}\frac{T}{R}-\tfrac{1}{4}\log R-\frac{1}{2}\sum_{n=1}^{\infty}\log\frac{1+e^{-2\pi(n-1/2)T/R}}{1+e^{-2\pi nT/R}}\end{aligned} \quad (11)$$

where the contribution from the antiperiodic Majorana spinor is $q_M=2q_D$. We will only shortly comment on this possibility since the dominance of the bosonic contribution makes it somewhat difficult to distinguish it from the scalar string.

3 Calculations

A. Algorithm

For our measurements of Wilson loops in $SU(2)$ we have used the isocahedral subgroup. We have worked on a 24^3 lattice, using the Metropolis algorithm for updating. In the measuring procedure we have implemented the variance reduction techniques of [13]. This amounts to interchanging links to larger objects with equal mean but smaller variance. In practice we replace a measured link by its expectation value U'_μ with respect to its neighbours. Let X^+ denote the sum of the ordered products of the other three links in plaquettes $U_\square$ with U_μ at its border, so that

$$U_\mu X_\mu^+=\sum_{\square\supset\mu}U_\square, \quad (12)$$

In the case of $SU(2)$ the result of the integration can be written as

$$U'_\mu=X_\mu\frac{I_2(\beta\lambda)}{I_1(\beta\lambda)}\frac{1}{\lambda},\quad \lambda=\sqrt{\det X_\mu} \quad (13)$$

where I_1 and I_2 are modified Bessel functions. For the isocaheder subgroup the operation takes us outside the group and must therefore be performed after a translation to the continuous group. The new link values and its new norm are then found from a dense tabulation, and our measured objects are finally constructed using the continuous group. Measurements will thereby take considerably longer time than

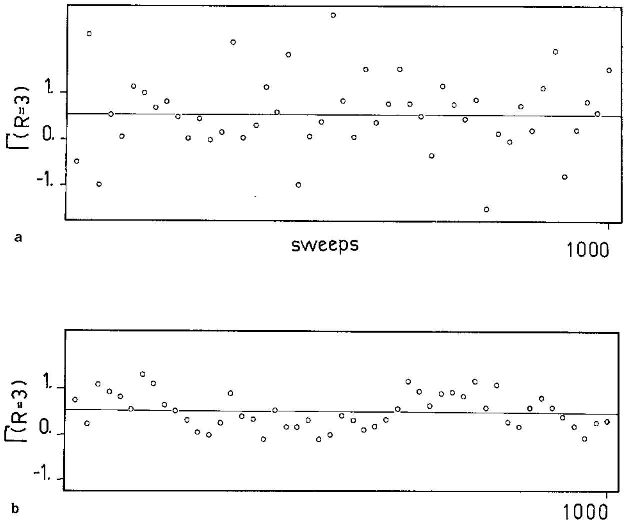

Fig. 1. **a** Values of the correlation $\Gamma(R) = \langle L(R)L(0)\rangle - \langle L(0)\rangle^2$ between Polyakov loops L located three lattice spacings apart on a 6×16^3 lattice. Each point is the average value over 20 sweeps. **b** Same as in Fig. 1a but with variance reduction implemented in the algorithm

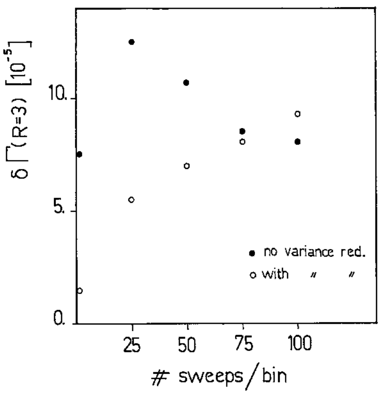

Fig. 2. Statistical errors in the correlation between Polyakov loops situated three lattice spacings apart on a 8×16^3 lattice as a function of the bin size used in the error analysis

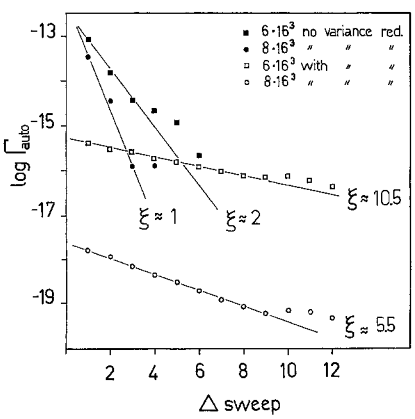

Fig. 3. Autocorrelations Γ_{auto} for the configuration average of the Polyakov loops as a function of the number of intermediate sweeps. Approximate correlations lengths ξ are given

updating of the lattice. Another method frequently used is to evaluate the local integrals by MC methods or to be more precise by applying a local multihit technique to measured links. We note that the four links situated at the corners of the Wilson loop cannot be modified since their neighbours include links involved in the loop itself. Compared to Wilson lines these links will induce larger fluctuations of the modified loops.

We allow for 5 sweeps between every measurement and use at least 5000 sweeps to thermalize the system. The number of sweeps used for measurements range from 800 to 3500 with emphasis on $\beta = 5.5$. Errors in the measured quantities are estimated by a standard procedure. Data are grouped together in bins of sizes up to ~ 100. The bins are regarded as independent measurements and standard errors calculated amongst them. Errors are only accepted if stable as the size of the bins are varied.

B. Autocorrelations

Using the above error analysis we find that long-range autocorrelations tend to survive the variance reduction and cause errors as large the original ones.

Let us illustrate this with measurements of Polyakov loops on a 6.16^3 lattice at $\beta = 2.25$, which is far from the finite temperature phase transition. In Fig. 1a, b values of the correlation between Polyakov loops during 1000 sweeps are shown, with and without variance reduction. It is clearly seen that whereas short-range fluctuations are damped out long-range ones are still present after the variance reduction. The corresponding autocorrelations are given in Fig. 2, where we also compare with results from a 8.16^3 lattice. Finally we give the result of an error analysis for Polykov loop values in Fig. 3. Increasing the bin size we find that the real error is not decreased by the variance reduction method. We conclude that a real gain in statistics can be obtained only for rather large objects. In fact, we do find an improvement for large enough Wilson loops.

Partly because of this observation, and partly because we are looking for long-range phenomena, we have made a choice of the measuring frequency so as to increase statistics for larger loops at the cost of the smaller ones. This feature will allow for stable fits in the analysis presented here.

4 Confrontation with data

A. General procedure

In order to see if the data reveal systematics that can be adhered to the appearance of a roughened string, we have used two different methods. As a first step we have performed *least-square fits*. Excluding data points probing distances smaller than the critical distance

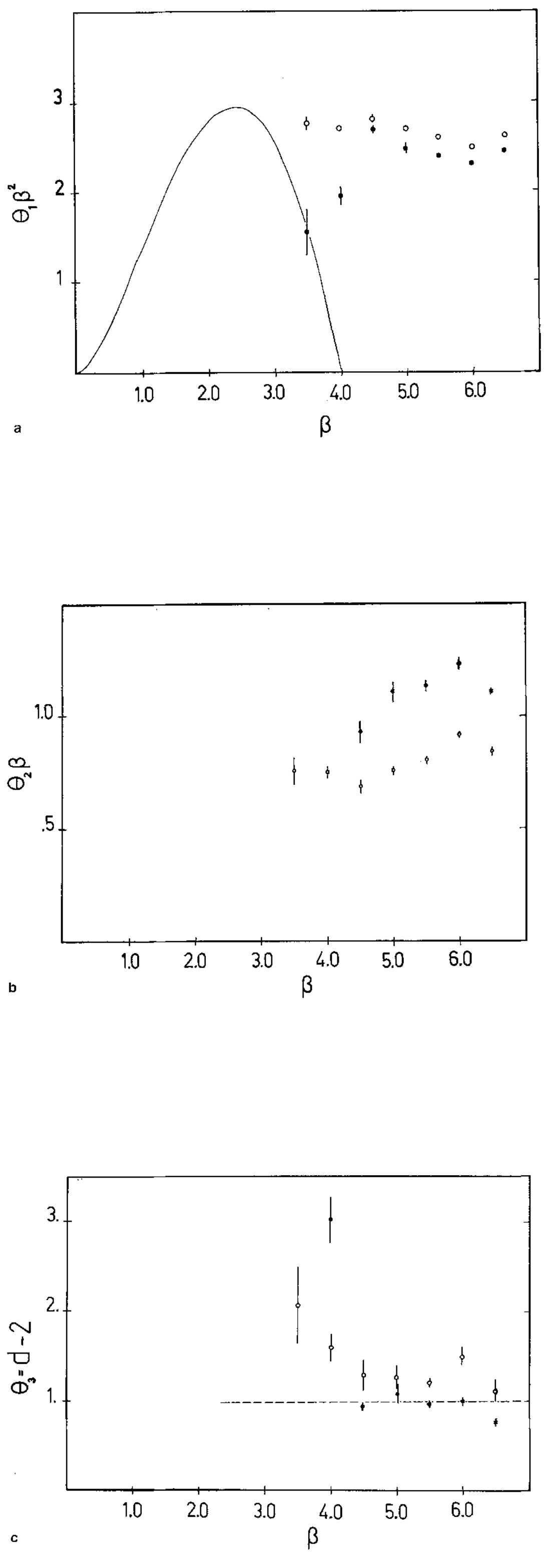

Fig. 4a–c. Values of the parameters θ_1, θ_2 and θ_3, defined in (14), for our $SU(2)_3$ data. Open points are from fits to a Dirac string (10), whereas filled points are from fits to a bosonic string (5). In order to check scaling we have plotted the quantity $\theta_1\beta^2$ instead of θ_1 in **a**. The curve indicates the lowest order strong coupling behaviour. In **b** the coefficient θ_2 in front of the perimeter term is scaled according to its perturbative origin. The dashed line in **c** corresponds to $\theta_3 = d - 2 = 1$

R_C, (9), we have fitted our $SU(2)_3$ data to the expression

$$-\log W(R,T) = \theta_1 RT + \theta_2(R+T) + \theta_4 + \theta_3 q(R,T) \tag{14}$$

with $q(R,T)$ given by (5), (8) or (10). The parameter θ_3 corresponds to the effective number of transverse dimensions $d-2$. We have also tested the inclusion of two-loop contribution with yet one parameter. With a fixed value of the $d-2$ parameter we have checked the χ^2-values for different choices of $q(R,T)$. These are shown in Table 1. It is seen that the values are considerably lowered when a scalar or a Dirac string is assumed, as compared to the case of vanishing $q(R,T)$. However, no major difference between these two strings appears (this conclusion differs from [10]).

Because of the universality of the T/R term one might want to use different parameters in front of the different terms in $q(R,T)$. Another and more direct way of obtaining more detailed information on the deviations from the area law behaviour is to consider the *second lattice derivative* of log W with respect to R

$$D_R^2 \log W(R,T) = \log\{W(R-1,T)W(R+1,T)/W(R,T)^2\}. \tag{15}$$

Assuming (4), this quantity will be sensitive only to the roughening contribution $q(R,T)$. As a function of T, for fixed R, it asymptotically tends to a straight line with slope and intercept determined by the T/R term and the $\log R$ term, respectively. These can be found, even by eye, from a plot and compared with theory. Furthermore, the onset of finite lattice size effect as well as the small deviations from the straight line behaviour induced by the last term in $q(R,T)$ at $T \approx R$ is clearly seen and does not affect the extraction, see Fig. 5e, f. The spread of the points allow us to estimate reasonable errors of the coefficients despite the strong correlation in R and T between W-loops.

B. $SU(2)_3$

This theory is well suited for an in-depth string model study. One can obey the Alvarez bound, (9), for a wide range of β values and still have enough large sized loops available for a meaningful string model comparison. Actually we are probing distances, with little effort, as large as those obtained in the best available data on $SU(3)$ on a 24^4 lattice when measured in terms of correlation lengths! We are able to test couplings from $\beta = 3.5$ to 6.5. In Table 2 we give our data and statistics. The resulting fitted parameters are shown in Fig. 4a–c for different couplings and assumed string functionals. Our results for $\beta \geqslant 4.5$ are in good agreement with effective string prediction, with a slight preference for the bosonic alternative. Moving towards stronger coupling we observe a dramatic breakdown of the string picture at $\beta \approx 4.0$. The $d-2$ parameter raises rapidly and the fit deteriorates. For the fermionic string the breakdown is displaced towards somewhat lower β-value.

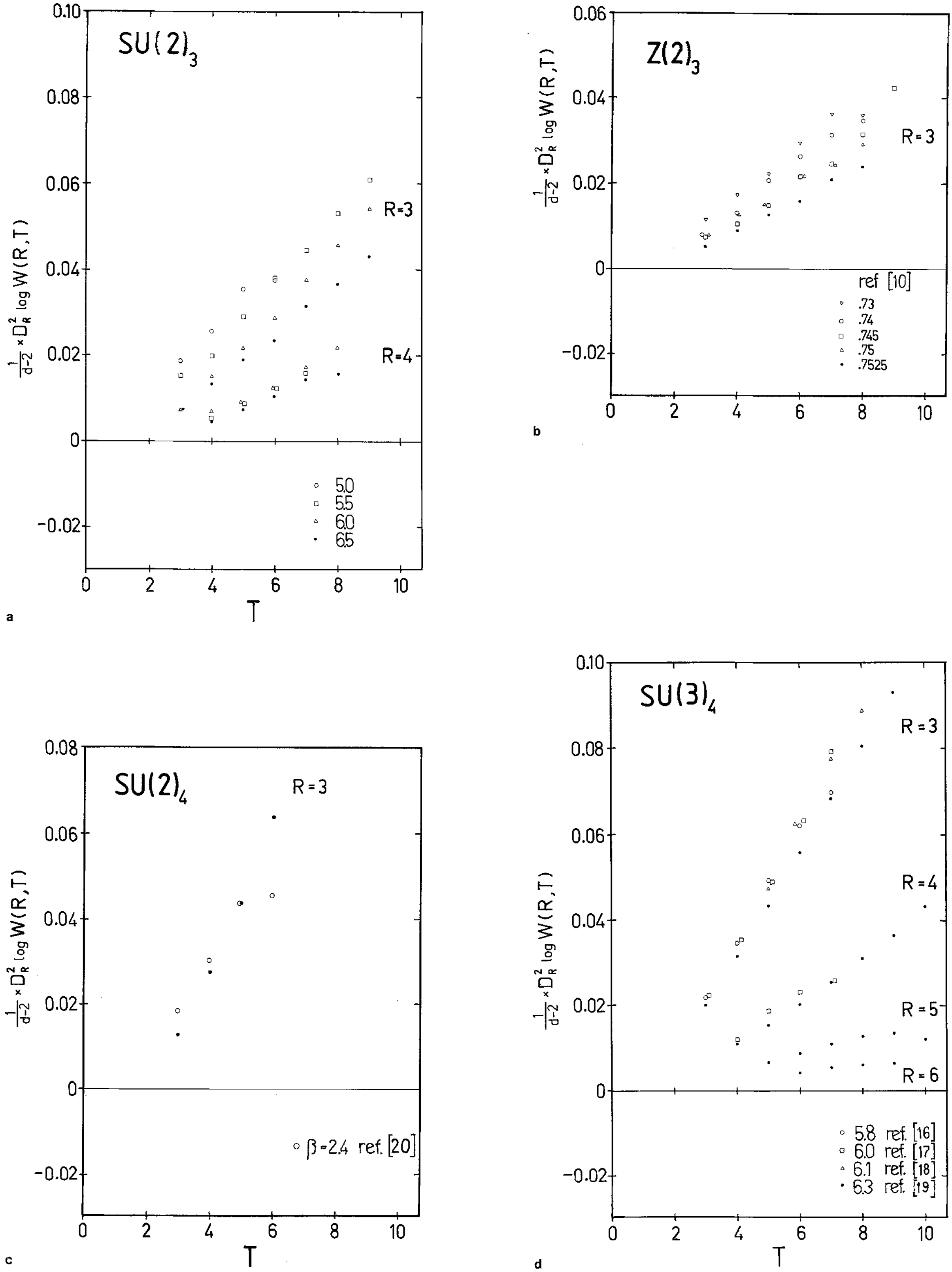

Fig. 5a–g. The second lattice derivative of log $W(R, T)$ with respect to R as a function T for different fixed values of R (see (14)). In **e** and **f** the contributions from the one-loop expressions (5), (8) and (10) are given. The two-loop contribution from the Nambu string is shown in **g** for $d = 4$. In both **e** and **g** full and dashed lines correspond to ζ-function and lattice regularized expressions, respectively. The actual behaviour of the data is shown in **a–d**. Due to strong correlations between Wilson loops of similar sizes a meaningful error analysis is difficult and error bars are therefore omitted

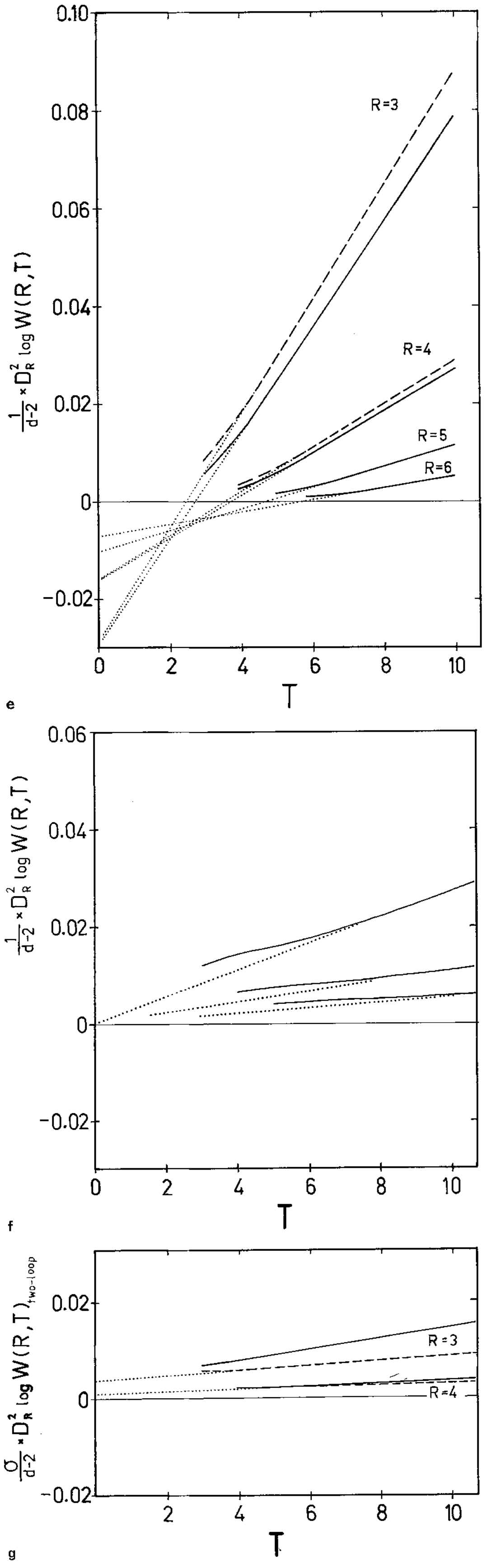

Fig. 5e–g

Table 1. χ^2-values for fits of the data to eq. (14) with $q(R, T)$ given by (5) (χ_C^2), (8) ($\chi_{L_2}^2$) or (10) (χ_D^2), normalized to the value χ_0 obtained for vanishing $q(R, T)$. The parameter θ_3 has been fixed to $d-2$ which means that the number of parameters is three for all fits

Theory	β	χ_C^2/χ_0^2	χ_L^2/χ_0^2	χ_D^2/χ_0^2
$SU(2)_3$	3.0	0.96	0.96	0.96
	3.5	0.49	0.48	0.80
	4.0	0.54	0.46	0.56
	4.5	0.67	0.59	0.32
	5.0	0.50	0.50	0.37
	5.5	0.16	0.14	0.19
	6.0	0.09	0.09	0.19
	6.5	0.33	0.37	0.10
$SU(2)_4$	2.25	0.48	0.34	0.46
$SU(3)_4$	6.3	0.65	0.62	0.32

The second derivative, see Fig. 5a–g, reveals that the agreement between data the lowest order scalar string prediction is not perfect. Due to their sign, the deviations cannot be explained by including the two-loop contribution of the Nambu string (compare with Fig. 6a–c).

C. $SU(2)_4$

In order to test the $d-2$ dependence, we perform a MC-calculation also for the four-dimensional case at $\beta = 2.25$ using the same algorithm and methods as in the 3-dimensional case, see Table 1. Guided by the behaviour of $SU(2)_3$ we have chosen a rather small β-value. Thus R_C is small. Using a value $\sigma = .20$ we obtain $R_C = 1.5a$. At this σ the second derivative is only reliable at $R = 3$, see Fig. 5c.

D. $Z(2)_3$

Data on $Z(2)_3$ were obtained and analyzed in [10] and were found to be well described by a Dirac string. We add to their investigation a study of $D_R^2 \log W(R, T)$, the result of which is shown in Figs. 5b and 6c. Neither a bosonic nor a Dirac string is quite compatible with this result. The discrepancies from a lowest order bosonic string picture is however not surprising if one takes into account the fact that the data are taken close to the critical point at which the correlation length becomes large. Using the string tension values given in [10] one finds that $R_C \approx 2.5$, 3, 4, 5 and 5a respectively for the analyzed coupling values. Since all the measured Wilson loops have $R \leqslant 4$ we cannot expect them to be described by the bosonic string.

E. $SU(3)_4$

Finally we will for comparison also reanalyze the very high statistics data from [16, 19] on $SU(3)$. As discussed in [4] the value of R_c is rather high, ranging

Table 2.

	$SU(2)_3$								$SU(2)_4$
$R \times T$	beta = 3.0	beta = 3.5	beta = 4.0	beta = 4.5	beta = 5.0	beta = 5.5	beta = 6.0	beta = 6.5	beta = 2.25
2 × 2	0.044279(279)	0.247098(703)	0.320755(479)	0.383557(663)	0.437562(621)	0.483611(322)	0.519464(1145)	0.55889(59)	0.158059(261)
2 × 3	0.009564(93)	0.130336(495)	0.193193(329)	0.252367(511)	0.306607(591)	0.355840(310)	0.395080(954)	0.43923(63)	0.072269(139)
2 × 4	0.002019(44)	0.069014(339)	0.117097(292)	0.166996(383)	0.216583(494)	0.263645(260)	0.301569(803)	0.34636(61)	0.033425(55)
2 × 5	0.000463(21)	0.036694(279)	0.071067(252)	0.110524(361)	0.152946(496)	0.195414(268)	0.230723(781)	0.27369(66)	0.015571(36)
2 × 6	0.000100(7)	0.019342(157)	0.043038(173)	0.073629(242)	0.108172(389)	0.144808(178)	0.176654(566)	0.21650(57)	0.007243(17)
2 × 7	0.000023(2)	0.010279(85)	0.026078(109)	0.048952(144)	0.076703(249)	0.107406(119)	0.135379(377)	0.17111(41)	0.003392(9)
2 × 8	0.000005(1)	0.005478(64)	0.015798(86)	0.032499(122)	0.054258(214)	0.079660(108)	0.103674(336)	0.13534(39)	
2 × 9				0.021586(101)	0.038341(187)	0.059078(95)	0.079416(295)	0.10713(36)	
2 × 10				0.013758(243)	0.026293(404)	0.044025(187)	0.061189(854)	0.08481(33)	
3 × 3	0.000966(46)	0.052792(641)	0.095124(453)	0.141531(777)	0.188568(1006)	0.235226(584)	0.274406(1435)	0.31872(13)	0.025471(131)
3 × 4	0.000089(17)	0.021492(291)	0.047697(260)	0.080336(381)	0.118178(598)	0.157913(300)	0.191971(769)	0.23302(81)	0.009210(33)
3 × 5	0.000007(7)	0.008889(205)	0.024015(217)	0.045630(315)	0.073995(539)	0.106083(286)	0.135014(703)	0.17099(86)	0.003399(19)
3 × 6	0.000005(3)	0.003649(103)	0.012129(141)	0.026339(198)	0.046574(387)	0.071240(188)	0.095099(470)	0.12607(68)	0.001251(9)
3 × 7	0.000000(1)	0.001515(53)	0.006067(77)	0.015118(116)	0.029382(231)	0.047943(118)	0.067118(293)	0.09278(46)	0.000467(5)
3 × 8		0.000652(37)	0.003039(57)	0.008682(94)	0.018471(185)	0.032268(100)	0.047287(249)	0.06840(41)	
3 × 9				0.004999(73)	0.011585(144)	0.021712(84)	0.033332(209)	0.05048(36)	
3 × 10				0.002585(181)	0.006876(345)	0.014666(154)	0.24033(580)	0.03729(32)	
4 × 4		0.006850(240)	0.020107(296)	0.039303(379)	0.066163(690)	0.096497(329)	0.124068(824)	0.15890(100)	0.002682(21)
4 × 5		0.002221(104)	0.008590(165)	0.019269(204)	0.037098(422)	0.059293(204)	0.080745(508)	0.10887(71)	0.000810(9)
4 × 6		0.000765(59)	0.003674(95)	0.009667(131)	0.020825(294)	0.036417(143)	0.052694(345)	0.07516(58)	0.000245(5)
4 × 7		0.000296(29)	0.001540(49)	0.004861(80)	0.011650(172)	0.022382(91)	0.034561(218)	0.05183(40)	0.000072(3)
4 × 8		0.000136(20)	0.000647(37)	0.002428(60)	0.006575(131)	0.013788(75)	0.022583(178)	0.03586(34)	
4 × 9				0.001226(46)	0.003694(103)	0.008485(61)	0.014772(146)	0.02484(29)	
4 × 10				0.000556(83)	0.002002(197)	0.005238(93)	0.009743(287)	0.01722(24)	
5 × 5		0.000611(95)	0.003112(184)	0.008130(212)	0.018550(483)	0.033439(238)	0.048737(594)	0.06983(98)	0.000201(8)
5 × 6		0.000187(36)	0.001169(65)	0.003538(97)	0.009228(217)	0.018852(113)	0.029560(269)	0.04529(54)	0.000050(3)
5 × 7		0.000055(19)	0.000428(33)	0.001536(59)	0.004620(121)	0.010618(71)	0.018111(166)	0.02949(35)	0.000011(2)
5 × 8		0.000018(13)	0.000176(24)	0.000678(45)	0.002348(93)	0.005989(57)	0.011025(132)	0.01911(29)	
5 × 9				0.000287(34)	0.001220(75)	0.003350(46)	0.006731(106)	0.01238(24)	
5 × 10				0.000114(64)	0.000478(150)	0.001832(71)	0.004062(223)	0.00805(20)	
6 × 6		0.000035(29)	0.000419(52)	0.001244(89)	0.004011(195)	0.009768(110)	0.016713(255)	0.02758(63)	0.000010(2)
6 × 7			0.000146(21)	0.000438(37)	0.001781(80)	0.005043(50)	0.009560(114)	0.01687(30)	0.000002(1)
6 × 8			0.000058(15)	0.000145(28)	0.000823(62)	0.002592(40)	0.005437(90)	0.01025(24)	
6 × 9				0.000050(20)	0.000386(49)	0.001303(32)	0.003114(71)	0.00620(20)	
6 × 10				−0.000006(26)	0.000133(60)	0.000631(40)	0.001783(109)	0.00374(16)	
7 × 7		0.000011(12)	0.000030(18)	0.000118(35)	0.000729(74)	0.002385(48)	0.005059(108)	0.00971(31)	0.000001(1)
7 × 8			0.000018(10)	0.000022(19)	0.000330(43)	0.001108(27)	0.002670(601)	0.00553(18)	
7 × 9				0.000002(14)	0.000131(35)	0.000495(22)	0.001421(478)	0.00309(14)	
7 × 10				−0.000007(16)	0.000081(41)	0.000208(23)	0.000775(535)	0.00171(11)	
8 × 8		0.000006(6)	0.000020(11)		0.000151(50)	0.000449(31)	0.001318(66)	0.00290(19)	
8 × 9					0.000066(29)	0.000171(17)	0.000659(36)	0.00146(11)	
8 × 10					0.000077(33)	0.000053(19)	0.000386(41)	0.00070(8)	
9 × 9					0.000035(32)	0.000056(20)	0.000293(40)	0.00059(12)	
9 × 10					0.000044(25)	0.000012(15)	0.000174(33)	0.00019(7)	

from three to five lattice spacings. From the second derivative in fig. 5d we conclude that the data contain a T/R term with a coefficient differing no more than a few percent from $(d-2)\pi/24$, see Fig. 6b.As to $\beta = 6.3$ we only find deviations from this value at $R = 2$ which should certainly be in the perturbative region. Unlike the situation in $SU(2)_3$ the two-loop expression here works in right direction in order to explain the deviations from the one-loop prediction. However, such an explanation requires, due to the $1/\sigma$ dependence, that the deviation increases as β increases. Such a behaviour cannot be seen.

Note that the Neveu–Schwarz string, (11), also fits data reasonably well. As advocated in [9], this string gives a better prediction for $T_c/\sqrt{\sigma}$ than the simple scalar one.

5 Summary

In summary we have obtained $SU(2)_3$ data for W-loops in a MC calculation especially designed for the purpose of extracting an effective string potential. We point out the severe long-range auto-correlations appearing for small objects and W-lines when variance reduction techniques are applied. The data allow for a good fit to the bosonic roughened potential for all β values $\geqq 4.5$, but a closer analysis reveals that scaling is not perfect and data contain more systematic deviation than can be explained by bosonic effective strings of lowest order, or by Nambu string up to two-loop corrections. Performing a lattice regulated roughening calculation does not change this conclusion.

For β below 4.5 we see a rapid breakdown of the picture of a roughening modified linear potential. This

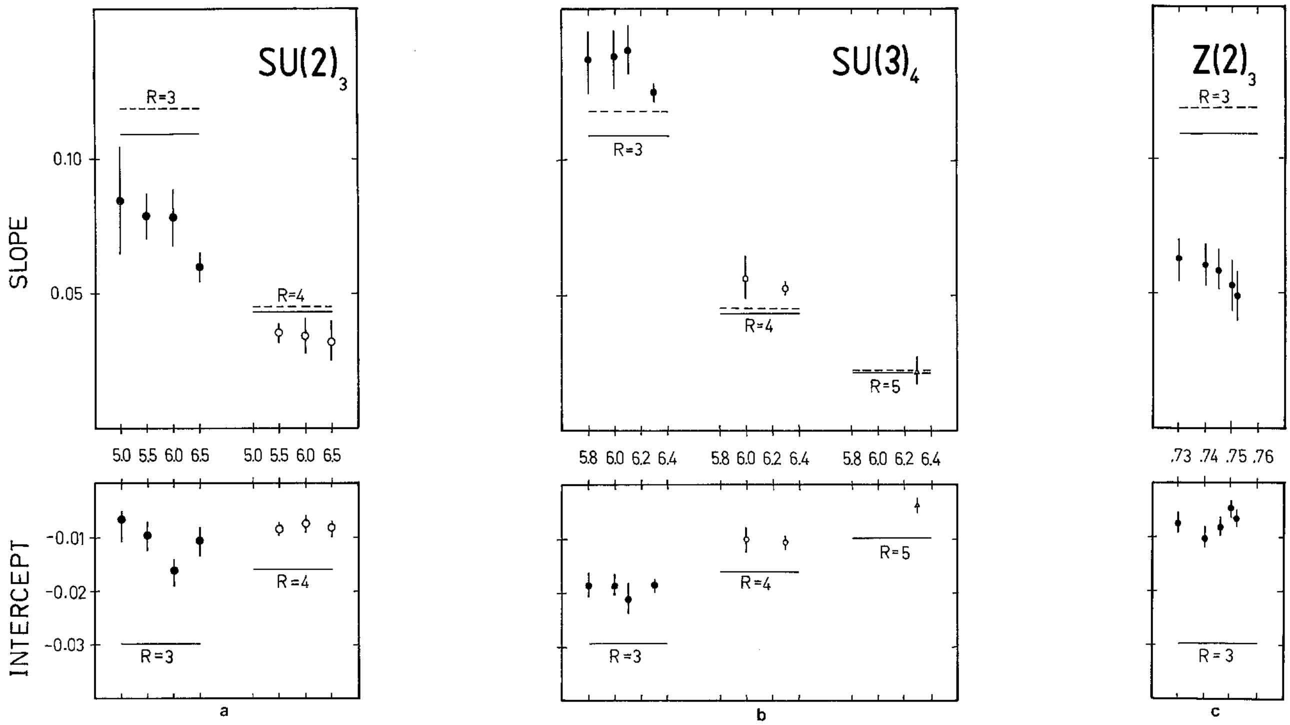

Fig. 6a–c. Slopes and intercepts as extracted from Fig. 5a, b and d

is clearly far beyond the transition seen in strong-coupling expansions and might signal the restoration of rotational symmetry. No shift into Dirac string can be seen.

The clear presence of a term constant in T and nonlinear in R signals either perturbative contributions from finite size boundary perimeter term, or, taking the string for granted at these distances, a typical bosonic $\log R$ contribution.

Comparing with existing data on $SU(3)$, a very similar picture emerges. Here distances in correlation lengths are smaller, of order 1. If a transition to a perturbative region is already present it is certainly very smooth. A T/R term is clearly present with a coefficient only few percent off its predicted value from roughening.

Finally we have reanalyzed available $Z(2)_3$ data where we find little evidence of the reported fermionic behaviour.

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