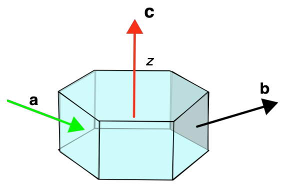
Divergent-Light Halos. Algorithm and Manual

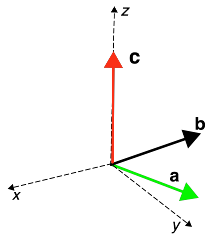
Description of the algorithm

We generate a number of raw events (5 000 000). For each event we use the following procedure:

1. Generate an incident ray vector **a** with random direction.

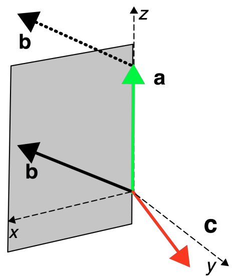


2. Let this ray hit a hexagonal crystal with the crystal axis vector **c** in the positive *z* direction (upwards) and one prism face in the *x* direction, the observer being located in the *xz* plane.



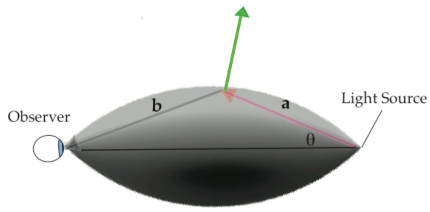
3. Raytrace the ray through the crystal with a standard raytracing algorithm taking in account Fresnel reflection and transmission coefficients.

4. The resulting scattered ray vector is **b**. The vector triad **a**, **b**, and **c** is used throughout the following discussion.



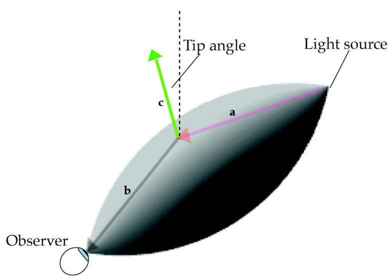
5. Rotate the vector triad such that the incident vector **a** points in the positive *z* direction and the scattered vector **b** lies in the *xz* plane.

6. We now use the fact that if the scattered ray is to reach the eye of the observer, the vectors **a** and **b** have to be located on a *Minnaert cigar*, determined by the scattering angle ω with $\cos\omega = \mathbf{a}\cdot\mathbf{b}$. A point on the cigar is determined by two angles θ and ϕ, where θ determines the location of the point along the axis of the cigar and ϕ the rotation around the axis.



7. Choose θ randomly in the interval $[0,\omega]$ and ϕ randomly in the interval $[0, 2\pi]$.
8. Rotate the triad such that **a** goes from the source apex to the chosen point on the cigar. **b** will then automatically go from this point to the opposite apex of the cigar.

9. Finally rotate the cigar with the triad such that the axis is aligned between the observer and the light source.



10. The angle between the crystal axis, **c**, and the vertical determines the tip angle τ which in turn determines the probability that of this event being plotted in a direction given by the vector **b**. The probability is given by a normal distribution with mean zero and a standard deviation equal to the target tip angle τ_0 set by the sliders in the parameter window (see below:

$$p(\tau) = e^{-\tau^2/2\tau_0^2}$$

11. A scattering event with scattering angle ω has a weight W given by

$$W = \frac{\omega}{\sin\omega}$$

We have $W \to 1$ when $\omega \to 0^\circ$ and $W \to \infty$ when $\omega \to 180^\circ$. The latter situation corresponds to that either the crystal is a) very far away or b) lies directly behind and screened by the light source or c) directly behind and shadowed by the observer. Assuming we have a finite extension of the cloud if ice crystals, case a) is not realized and b) and c) events not observable.

The program implements this by repeating steps 6 to 10 a number of times, N, essentially proportional to this weight:

$$N = \begin{cases} 0 & \text{if } \omega \le 0.3^\circ \\ K & \text{if } 0.3^\circ < \omega \le 1^\circ \\ K\dfrac{\omega}{\sin\omega} & \text{if } 1^\circ < \omega \le 175^\circ \\ 0 & \text{if } \omega > 175^\circ \end{cases}$$

where K is the numerical setting of the recycle slider $1 \le K \le 999$.

Manual

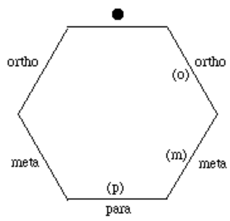
Start the application by double-clicking the application icon. Two windows will appear, one for the halo display and one for setting the parameters. The parameter window has four vertical sliders. The leftmost slider sets the elevation (Σ) of the source. Then next slider sets the tip angle and the following slider the c/a ratio of the crystal. The last slider sets a factor of proportionality for the weight; in practice this will be the “exposure time”.

With the three radio buttons you can choose between plate crystals, columns crystals and Parry columns (singly oriented columns).

In the Projection menu you can choose between Fisheye projection and Sun Centred projection.

You start the simulation by clicking the “Go” button. You will have to wait for some time before the halo display is computed. With a proportionality factor of one it takes about one minute, with the largest proportionality factor (999) it can take of the order of one hour depending on the settings of the other parameters. In the sun centred projection the horizon is marked by a red line.

Once the display is computed you can select any rectangular region of the display by drag-clicking that area with the mouse. The region will be marked and the program will compute the ray histories of the rays within the selected region giving the frequency of each kind of history. The result will be presented in a separate window. The contents of this window can be copied and pasted in other applications like Microsoft Word.



The convention for a ray history is the following: The ray history is represented by a string of letters. The first encountered prism face is denoted by 's' the first encountered basal face by 'E'. In the case of an external reflection one of these is the only character given. If the ray enters the crystal, the following letters tell the relation of the next encountered face to the previous face. For the prism faces, the relation scheme is given by the figure to the right where the 'previous face' is marked with a black dot. Second and following encounters with basal faces are denoted by 'P'.

For instance, ordinary parhelia rays are given by 'sm'. A refraction in the 90° edge of the crystal is 'Es' or 'sE'. For instance 'sEPPmPPm' is a ray that entered a prism face, was reflected in a basal face, then back and forth between the basal faces, then reflected in a prism face with relation *meta* to the first prism face, then gain back and forth between the basal faces, and finally exiting through a prism face having relation *meta* to the previous prism face.

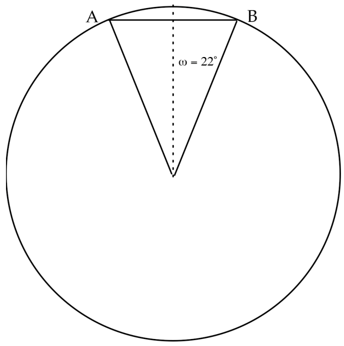
Once you have simulated a display, the Tool menu gets enabled. This menu has an item for drawing a stereo picture of the display. There is also an item for setting the preferences of the stereo picture. The cross-eyed alternative requires you to look at the picture with eyes crossed, i. e. the right eye looks at the left picture and the left eye looks at the right picture. The normal version can be used in a standard stereo viewer. You can also set your preferences for the sky background: black or white.

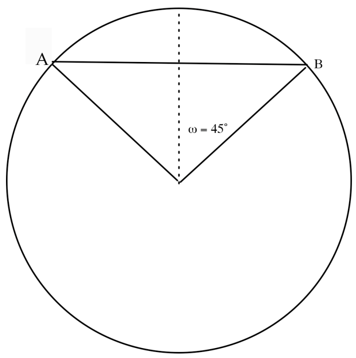
The display in the main window can be saved as a .jpg file using the menu File/Save Display. The stereo picture can likewise be saved by File/Save Stereo Picture.

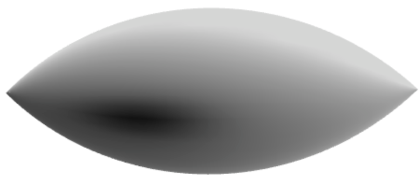
Finally the File menu has an option to save spatial location of the crystals that are involved in the display. The spatial information is save as a text file where each line gives the (X, Y, Z) coordinate of a crystal in units of the distance between the observer and the light source. The data file can be viewed in 3D by an application HaloViewer that is included with the main application. The 3D view can be manipulated in azimuth and elevation with two sliders and can be zoomed with the mouse wheel.

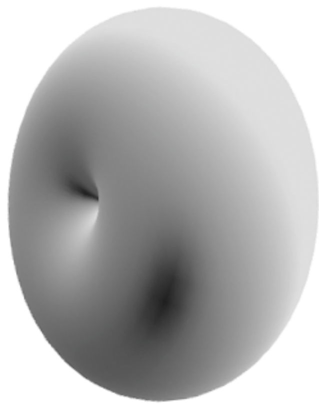
On Minnaert cigars

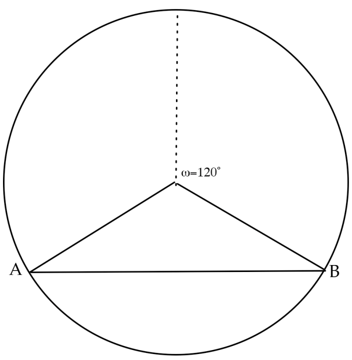
You construct a Minnaert cigar selecting a segment of a circle and then rotating the (upper) segment around the axis AB. The angle ω is the scattering angle. The pictures show the construction for scattering angles 22˚, 45˚ and 120˚ together with the resulting three-dimensional cigar. A scattering angle of 90˚ will obviously give a sphere.







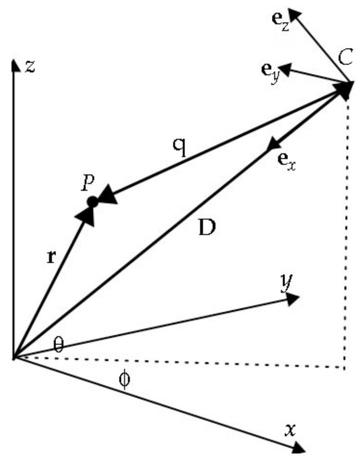




Camera projection mathematics

Given a set of spatial points $\mathbf{r}_i = (x_i, y_i, z_i), \quad i = 1,2 \ldots N$

We want to observe the set of points with a camera, *C*, aimed at the origin. The direction of the camera is specified by an azimuth angle $\phi \in [-180^\circ, 180^\circ]$ and an elevation angle $\theta \in [-90^\circ, 90^\circ]$, the dolly angles. The distance from the origin to the camera is *D*. See figure.



In the camera system we have three orthogonal base vectors:

$$\begin{aligned}
\mathbf{e}_x &= (-\cos\phi\cos\theta, -\sin\phi\cos\theta, -\sin\theta) \\
\mathbf{e}_y &= (-\sin\phi, \cos\phi, 0) \\
\mathbf{e}_z &= (-\cos\phi\sin\theta, -\sin\phi\sin\theta, \cos\theta)
\end{aligned}$$

$\mathbf{e}_x$ points to the origin, $\mathbf{e}_y$ is parallel to the xy-plane, and $\mathbf{e}_z$ lies in the plane spanned by the z axis and the vector **D.**

The vector **q**, the position of the specific point *P* in the camera system, is

$$\mathbf{q} = \mathbf{D} - \mathbf{r}, \quad \mathbf{r} = (x, y, z)$$

In the camera system the coordinates of the point *P* is

$$\begin{aligned}
X &= \mathbf{q}\cdot\mathbf{e}_x = (\mathbf{D} - \mathbf{r})\cdot\mathbf{e}_x = -D + x\cos\phi\cos\theta + y\sin\phi\cos\theta + z\sin\theta \\
Y &= \mathbf{q}\cdot\mathbf{e}_y = (\mathbf{D} - \mathbf{r})\cdot\mathbf{e}_y = x\sin\phi - y\cos\phi \\
Z &= \mathbf{q}\cdot\mathbf{e}_z = (\mathbf{D} - \mathbf{r})\cdot\mathbf{e}_z = x\cos\phi\sin\theta + y\sin\phi\sin\theta - z\cos\theta
\end{aligned}$$

The projection coordinates in a plane containing the origin and perpendicular to the camera direction *D* will then be

$$Y_p = \frac{D}{X} Y$$

$$Z_p = \frac{D}{X} Z$$

The dolly angles are given by the two slider settings. Projection coordinates are computed for the set of points and plotted, multiplied by a zoom factor that is read from the mouse wheel.