

Exercises 1.

(To be completed before 08.09.10)

1. Calculate

$$S_a(x) = \sum_{i=1}^N L_i(x) \quad \text{and} \quad S_b(x) = \sum_{i=1}^N x_i L_i(x)$$

where

$$L_i(x) = \prod_{\substack{j=1 \\ j \neq i}}^N \frac{x - x_j}{x_i - x_j} \quad (x_i \neq x_j \text{ if } i \neq j).$$

(Hint: Lagrange's interpolation formula)

2. Use the Euler-Maclaurin summation formula to show that

$$\sum_{m=1}^n m = \frac{1}{2}n(n+1) \quad \text{and} \quad \sum_{m=1}^n m^2 = \frac{1}{6}n(n+1)(2n+1)$$

3. Calculate

$$\int_3^4 \frac{dx}{x-2}$$

with the trapezoidal rule for the step sizes 0.5, 0.25 och 0.125. Improve the values by Richardson extrapolation.

4. Calculate the integral in exercise 3 by using two-point Gaussian integration, after a change of variable to $t = 2x - 7$.
5. Determine an integration formula of the type

$$\int_{-1}^1 f(x)(1+x^2)dx \approx A_1 f(x_1) + A_2 f(x_2)$$

that is exact for all polynomials with degree less than 4.