

Exercises 2

(To be completed before 08.09.17)

1. Calculate the first three cumulants $\langle X^n \rangle_c$ ($n = 1, 2, 3$) for a normally distributed random number X with mean μ and variance σ^2 . Use the fact that the characteristic function is the Fourier-transform of the probability distribution and that the cumulants are defined in terms of the Taylor expansion of the logarithm of the characteristic function.

2. Show how a random number with frequency function

$$p_a(x) = \begin{cases} (1+a)x^a & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (a > -1)$$

can be obtained from a rectangularly distributed random number.

3. Construct a normally distributed random number with mean μ and standard deviation σ from a normally distributed random number with mean 0 and unit variance.
4. Show how a random number with frequency function

$$p(x) = \begin{cases} 4/[\pi(1+x^2)] & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

can be obtained from rectangularly distributed random numbers by (a) the transformation method, and (b) the accept/reject method.

5. If R_1 och R_2 are independent and uniformly distributed random numbers between 0 and 1, then $X_1 = \sqrt{-2 \ln R_1} \cos 2\pi R_2$ and $X_2 = \sqrt{-2 \ln R_1} \sin 2\pi R_2$ are independent and normally distributed random numbers with mean 0 and unit variance (the Box-Muller method). Suppose R_1 and R_2 are consecutive numbers obtained by the linear congruential method

$$\begin{aligned} R_j &= I_j/m \\ I_j &= aI_{j-1} + c \pmod{m} \end{aligned}$$

where a , c , m and all I_j are integers. Show that the corresponding points (X_1, X_2) fall onto the curve

$$\begin{cases} x_1 = \rho \cos \phi(\rho) \\ x_2 = \rho \sin \phi(\rho) \end{cases}$$

where $\phi(\rho) = 2\pi[a \exp(-\rho^2/2) + c/m]$, $\rho \geq 0$.