

Exercises 5

(To be completed before 08.10.08)

1. Use Euler's method to solve $y'(x) = -y$, $y(0) = 1$.
 - (a) Determine y after n steps of size h , y_n . What is the exact solution?
 - (b) For which values of h is the sequence $y_0, y_1, \dots$ bounded?
 - (c) Let $y(x, h)$ denote the estimate of $y(x)$ obtained for step size h . Determine the global error $y(x, h) - y(x)$ to leading order in h , for fixed x .
2. The midpoint method applied to the equation $y'(x) = -y$ gives the recursion formula $y_{n+1} = y_{n-1} - 2hy_n$, which has a general solution of the form $y_n = A\lambda_1^n + B\lambda_2^n$. Determine λ_1 och λ_2 .
3. Show that the last step in the modified midpoint method,

$$y(x_0 + Nh, h) = \frac{1}{2} (y_N + y_{N-1} + hf(x_N, y_N)) ,$$

can be written as

$$y(x_0 + Nh, h) = \frac{1}{4} (y_{N-1} + 2y_N + y_{N+1}) ,$$

where $y_{N+1} = y_{N-1} + 2hf(x_N, y_N)$ (corresponding to one extra step with the non-modified midpoint method). This shows that an oscillating component (see exercise 2) is damped by this last step.