

Exercises 6.

(To be completed before 08.10.14)

1. Consider a one-dimensional harmonic oscillator with energy

$$H(x, p) = \frac{1}{2} (p^2 + x^2)$$

($m = 1$, $V = x^2/2$).

- (a) Write the leapfrog equations

$$\begin{aligned} x(h/2) &= x(0) + \frac{h}{2}p(0) \\ p(h) &= p(0) - hV'(x(h/2)) \\ x(h) &= x(h/2) + \frac{h}{2}p(h) \end{aligned}$$

in the vector-matrix form

$$\begin{pmatrix} x(h) \\ p(h) \end{pmatrix} = \mathbf{T} \begin{pmatrix} x(0) \\ p(0) \end{pmatrix},$$

and compute the determinant of the 2×2 -matrix $\mathbf{T}$.

- (b) Same thing for the Euler method.
- (c) Same thing for the implicit Euler method.
- (d) Same thing for the exact solution.