Exercises 1, FYTN04, Autumn 2018

Due: Friday November 16 2018, 10.15

We will have 5-6 sets of hand-in exercises with 4 or 5 exercises each. One of those exercises, chosen randomly, will be graded. This way you can get a maximum of 5 points. The written exam is 40 points and you need a total of 23 to pass, so if you get all 5 points from the home-work you need only 18 points on the written exam to pass. We reserve the right to still fail if the oral exam is too bad.

This weeks exercises are:

1 **a)** If a cross-section is calculated to be $\sigma = 10^{-3}/M_W^2$, (M_W is the mass of the W particle) what is it in cm^2 ? If a lifetime $\tau = 1\ GeV^{-1}$, what is it in sec ?
b) What is Newton's constant G_N in natural units (i.e. the corresponding power of GeV) ?
Note: We can in principle also remove the last dimension (e.g. the GeV units) by setting $G_N = 1$ in addition to $\hbar = 1$ and $c = 1$.

2 **a)** Consider the gauge transformation $\psi' = e^{-ig\chi}\psi$ and $A'_\mu = A_\mu - \partial_\mu\chi$. If ψ satisfies the equation $\mathcal{D}^\mu\mathcal{D}_\mu\psi + m^2\psi = 0$, where $\mathcal{D}^\mu = \partial^\mu - igA^\mu$, what equation does ψ' satisfy ?
b) Suppose ψ and A_μ transform according to $\psi' = U\psi$, $A'_\mu = (-i/g)(\partial_\mu U)U^{-1} + UA_\mu U^{-1}$. The transformations can be non-Abelian, and they hold for a set of states $\psi_1, \psi_2, \ldots$ and $A_{\mu 1}, A_{\mu 2}, \ldots$. Do the transformations of A_μ form a representation of the group ? (i.e. are these transformations compatible with the group product) Do the transformations on ψ form a representation of the group ? If yes, and if it is an $SU(n)$ group, which representation are the ψ in. Note that the U themselves are already elements of the group so you do not need to prove that.

3 **a)** if $\vec{v}^\mu$ is a "vector in isospin space" and a Lorentzfourvector with the three isospin components v_1^μ, v_2^μ and v_3^μ, write $\mathcal{D}^\mu = \partial^\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{v}^\mu$ explicitly in two-by-two matrix form.
b) if G_a^μ is a vector under $SU(3)$ with $a = 1, \ldots, 8$ write explicitly the covariant derivative $\mathcal{D}^\mu = \partial^\mu - ig\frac{\lambda_a}{2}G_a^\mu$ as a three-by-three matrix.
Note: this exercise helps in constructing the full standard model Lagrangian later and to make you understand the notation.

4 **a)** Calculate explicitly P_L^2, P_R^2, P_LP_R and $P_L + P_R$ (in terms of gamma matrices and γ_5 and remove γ_5^2. Do not solve in explicit four-by-four matrices).
b) Show for a general solution of the Dirac equation that $\bar{u}_L\gamma^\mu u_R = \bar{u}_R\gamma^\mu u_L = 0$ and $\bar{u}_L\gamma^\mu u_L \neq 0$ (hint: try some of the explicit solutions). What about $\bar{u}_Lu_L$, $\bar{u}_Ru_L$, $\bar{u}_Ru_R$ and $\bar{u}_Lu_R$. (hint, remember that P_L and P_R are projection operators). Note: this exercise is helpful in showing which type of fermion-antifermion bilinears can participate in mass terms and interactions.