Homework set 2, FYTN04, Autumn 2018

Due: Friday November 23 2018, 10.15

1 Fill in the table where appropriate:

Particle	Spin (0,L,R,1)	Mass	Colour (singlet,triplet, octet)	Weak Isospin: I and T_3^f $(0, 1/2, 1)(0, \pm 1/2, (0, \pm 1))$	Electric Charge
ν_{eL}					
e_L^-					
e_R^-					
u_L					
d_L					
u_R					
d_R					
$W^\pm$					
Z^0					
γ					
g					
W^3					
B					
H					

2 a) Show that

$$\sum_{f=L,R} \bar{e}_f \gamma_\mu e_f (T_3^f - Q^f \sin^2\theta_W) = -\frac{1}{4}\bar{e}\gamma(a + b\gamma_5)e\,. \tag{1}$$

What are a and b in terms of $\sin^2\theta_W$? This is the coupling to the Z-boson or the neutral current. Compare with the equivalent result for the W-boon or the charged current.
b) Could a Standard Model Higgs boson decay to $\bar{\nu}\nu$ if neutrinos are massless: (b1) Any term that was allowed in the Lagrangian could occur in nature. Could an effective Lagrangian (whatever its origin e.g. from loops) $\mathcal{L} = GH^0\bar{\nu}_L\nu_L$ (G some constant) mediate the decay $H^0 \to \bar{\nu}\nu$? H^0 is the Higgs field. (b2) Repeat (b1) for $\mathcal{L} = G(\partial^\mu H^0)\bar{\nu}_L\gamma_\mu\nu_L$.

3 a) In the QCD Lagrangian we have several vertices and as explained also interactions with the gluons alone. The lagrangian is $\mathcal{L}_g = -1/2\mathrm{tr}(G_{\mu\nu}G^{\mu\nu})$ with $G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - ig_S[G_\mu, G_\nu]$ and $G_\mu = \lambda^a G_\mu^a/2$. G_μ^a are the 8 gluon fields. Write down an expression for the interaction lagrangian for 3 gluon fields and for 4 gluon fields. Either with the λ^a-notation or using the structure constants of $SU(3)$, see App. B.4 for their definition. (Do not perform the sums over $a = 1, \ldots, 8$ etc., leave them in as summed over

indices)
b) Draw the Feynman diagrams for quark-gluon scattering, i.e. $qG \to qG$. (Hint: there are three.)

4 In working out the Higgs mechanism for the Standard Model, we used the covariant derivative (8.38), and the Higgs field near its vacuum expectation value (8.32) and (8.33), in the Higgs Lagrangian. We put $h(x) = 0$ to find the mass terms for W and Z. Repeat with $h(x) \neq 0$ and obtain the interactions of h with $W^{\pm}$ and Z and γ. I.e. we want the terms that contain h once or twice in addition to one or more of $W^{\pm}, Z^0, \gamma$.