

# Homework set 5, FYTN04, Autumn 2018

Due: Friday 15 December 2018, 10.15

1. a) The measured  $\tau$  lepton lifetime is  $2.90 \times 10^{-13}$  s, known to an accuracy of about 0.4%. How does that compare with the prediction of the standard model? Remember the calculation of  $\tau_\mu$  in the lectures, and the number of channels.  
b) QCD corrections modify the decay width to quarks roughly by a factor  $1 + \alpha_s/\pi$ . Use the data above to estimate  $\alpha_s(M_\tau^2)$ .

2. (a) Show that any coupling strength  $\alpha_i(Q^2)$  (at least in the Standard Model) satisfy an equation of the form

$$\frac{\partial \alpha_i(Q^2)}{\partial [\ln(Q^2/\mu^2)]} = B\alpha_i^2 .$$

What does  $B$  depend on?

- (b) If  $\Lambda_{QCD}$  is measured to be 225 MeV, what is  $\alpha_3$  at  $Q^2 = M_\Upsilon^2$  and at  $M_Z^2$ ? (Reminder:  $\Upsilon$  is the  $b\bar{b}$  state equivalent to  $J/\psi$  of  $c\bar{c}$ . See Appendix C for mass.)

3. The  $\alpha$  coupling of QED increases with  $Q^2$ . Disregarding the rest of the standard model, it becomes singular at some scale. This phenomenon is called a Landau pole. Estimate the energy scale of the pole. As extremes, either include only the electron in the theory, or three full generations of fermions. You may simplify by assuming that all fermions have the electron mass and that  $\alpha(m_e^2) = 1/137$ . Compare with the Planck energy,  $E_{Pl} = 1/\sqrt{G_N}$ , where  $G_N$  is Newton's gravitational constant.

4. Explain why the decay  $D^0 \rightarrow K^-\pi^+$  is allowed, while  $D^0 \rightarrow K^+\pi^-$  is suppressed. Derive an expected ratio of branching ratios, based on the coupling structure alone. Compare with data, where the branching ratio for the former is observed to be  $3.89 \cdot 10^{-2}$  and for the latter  $1.48 \cdot 10^{-4}$ .

5. The  $B^0 - \bar{B}^0$  and  $B_s^0 - \bar{B}_s^0$  meson systems oscillate, similarly to the  $K^0 - \bar{K}^0$  one. The oscillations are slower, however, in the sense that the oscillation times are not too dissimilar from the decay times. Therefore it is meaningful to ask whether a particle produced e.g. as a  $B^0$  remains as that at a later time, or has oscillated to an  $\bar{B}^0$  one. This has been studied e.g. at LEP1. The  $B$  mesons produced there take most of the energy of their jet, so  $E_B \approx 40$  GeV.

- a) Calculate the typical distance a  $B^0$  meson travels before it decays and compare this with the  $B^0 - \bar{B}^0$  oscillation length. Some numbers:  $m_B = 5.279$  GeV,  $\Delta m_B = 0.464 \times 10^{12} \hbar s^{-1}$ , and  $\tau = 1.56 \times 10^{-12}$  s. The decay rates of the two

states is the same.

b) Repeat the exercise above for the  $B_s^0$  system. Here  $m_{B_s^0} = 5.366$  GeV,  $\Delta m_{B_s^0} = 17.8 \times 10^{12} \hbar s^{-1}$  and  $\tau = 1.47 \times 10^{-12}$  s.

c) For each of the two particles, calculate how often an original  $B^0/B_s^0$  decays as a  $\bar{B}^0/\bar{B}_s^0$ , integrated over all times.

d) Propose a suitable experimental signature for the presence of decays of the “wrong” particle.