Low x news for Monte Carlo

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Invited talk at Monte Carlo Workshop, DESY, 1999.

Abstract: This talk is a review of news in low x physics which, I think, would be useful for writing Monte Carlo codes. The following topics are discussed here: (i) the next-to-leading order BFKL Pomeron; (ii) two indications for shadowing corrections (SC) in DIS from HERA data; (iii) matching of "soft" and "hard" photon - proton interactions; and (iv) survival probability for large rapidity gap (LRG) processes in hadron-hadron scattering and DIS. I hope, that our current understanding of these topics will allow us to narrow the gap between the MC codes and our microscopic theory - QCD.

1 Introduction

Goals: The main goal of this talk is to share with you my understanding of the current situation in low x physics looking at this subject from the angle of possible improvement of existing Monte Carlo codes. Everyone knows that the typical MC code for deep inelastic scattering (DIS) contains three parts: the perturbative QCD cascade, the hadronization stage and the matching between "soft" and "hard" processes. Since we have no solid theoretical understanding of the hadronization stage and the "soft" processes, there is a danger to write a MC code that will describe the experimental data, but in which the clear connection with our microscopic theory - QCD, will be broken. This is the reason that MC experts should follow all theoretical and experimental news, and implementing in the MC everything that we have learned both theoretically and experimentally. My goal is to provide them with a guide to the recent achievements in low x physics, which is a meeting point for all the complicated problems of perturbative and non-perturbative QCD.

Topics: The choice of the topics, that I would like to cover here, is dictated by my personal interests as well as by the considerable progress that has recently been made in their understanding. They are: (i) the next-to-leading order BFKL Pomeron; (ii) two indications for shadowing corrections (SC) in DIS from HERA data; (ii) matching of "soft" and "hard" photon - proton interactions; and (iv) survival probability for large rapidity gap (LRG) processes in hadron-hadron scattering and DIS. You can see that all above topics are closely related to our main problem - the matching between long distance (non-perturbative) physics and short distance (perturbative) approaches.

2 The next-to-leading order BFKL Pomeron

Why do we love the BFKL Pomeron? I think, we can list here three main reasons for our love to the BFKL Pomeron:

1. The BFKL Pomeron [1] is a high energy asymptotic approach in perturbative QCD which we call leading log (1/x) approximation (LL(x)A). In the LL(x)A we select the diagrams using $\alpha_S \ll 1$, $\alpha_S \ln Q^2 \ll 1$ but $\alpha_S \ln(1/x) \approx 1$. The LL(x)A is quite different from the leading log (Q^2) approximation (LL(Q^2)A , $\alpha_S \ll 1$, $\alpha_S \ln(1/x) \ll 1$, $\alpha_S \ln Q^2 \approx 1$) in which the DGLAP evolution equations [2] were derived. We believe, that the BFKL Pomeron gives:

1. A guide for the matching of perturbative QCD (pQCD) with non-perturbative one (npQCD), at least for high energy scattering amplitude in DIS;

2. A possibility to get the "soft" Pomeron contribution by taking into account the npQCD corrections to the BFKL Pomeron.

We have several examples which support our hopes [3, 4].

2. The BFKL Pomeron is the only theoretical way to prove that at sufficiently high energy (low x) the density of partons becomes large. In other words, the BFKL Pomeron shows that the matching of the perturbative QCD parton cascade with the non-perturbative high energy asymptotics, goes through a new non-perturbative QCD stage i.e. a high parton density QCD (see, for example, review [5] for details). My personal opinion is that we do not need any theoretical arguments for a high parton density QCD, since the HERA experimental data show that the gluon density is high [6, 7]. However, the same experimental data can be used as the experimental confirmation of the BFKL Pomeron;

3. The new HERA data shows that the BFKL Pomeron is needed to describe the forward jet production [8]. These data change the entire attitude to the BFKL Pomeron, which from a theoretical toy, becomes a tool for the description of the experimental data and, therefore, a part of DIS phenomenology which should be included in the MC codes. The question still remains could the experimental data of Ref. [8] be described without the BFKL Pomeron? You will answer this question better than me.

The map of disaster: It is well known that in the leading order the BFKL Pomeron leads to Regge-like asymptotics:

$$\sigma_{tot}(\,BFKL\ \ LO\,) \quad \propto \quad e^{\omega_L^{LO}\,(y-y_0)\,-\,\frac{(r-r_0)^2}{4D^{LO}(y-y_0)}}\,, \tag{1}$$

where $y - y_0 = \ln(s/s_0)$ and $r - r_o = \ln(Q^2/Q_0^2)$ (see Fig. 1-a for notations).

This simple asymptotic behaviour in the leading order is based on: (i) the time structure of the parton cascade (see Fig. 1-b); (ii) the separation between the longitudinal and transverse degrees of freedom; (ii) the fact that gluon emission leads to $\sigma \ \propto \ \frac{1}{x^{\omega_L}}$; and (iv) the diffusion in log of transverse momenta.

Recently, the next-to-leading order corrections to the kernel of the BFKL equation has been calculated [9, 10] and it turns out that these corrections are so large that they could change the entire understanding of the structure of the parton cascade in pQCD. Indeed, in the NLO:

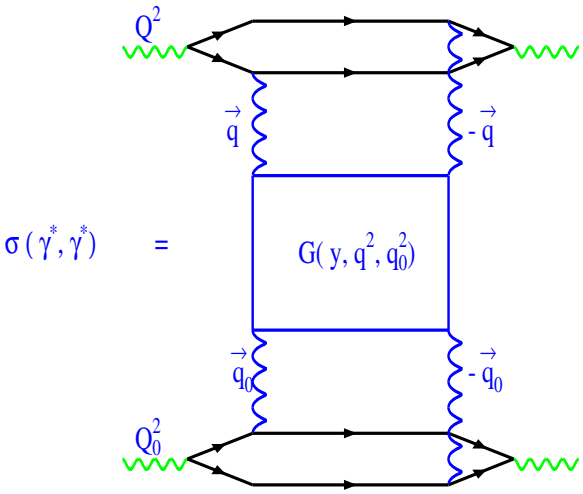

Figure 1-a

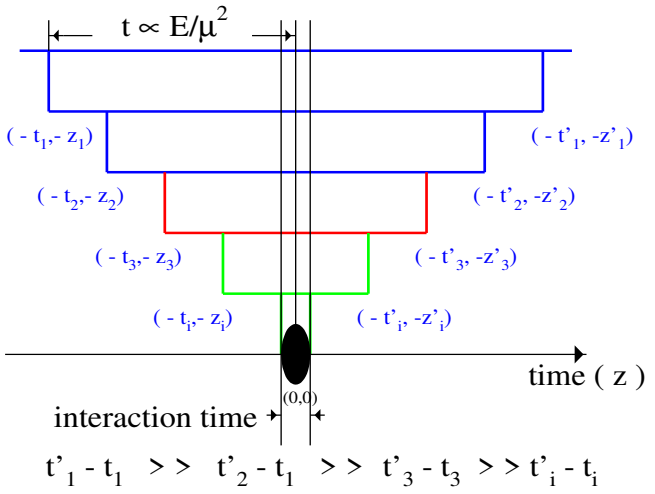

Figure 1-b

Figure 1: $\gamma^*\gamma^*$ - *interaction in the BFKL approach (a) and the time structure of the BFKL Pomeron (b).*

1. $\omega_L < 0$ at $q_0^2 \lesssim 30\,GeV^2$ [9–11] (see notations in Fig. 1-a). It means that in a large kinematic region, the BFKL emission is suppressed, and only the Born term contributes;

2. For $q_0^2 \lesssim 3\cdot 10^6\,GeV^2$ $D < 0$ [12]. It means that there is no diffusion in log of transverse momenta in the NLO BFKL. It also turns out that $D < 0$ leads to oscillation of the total cross section [3,12] which shows that the NLO BFKL has a serious pathology;

3. In NLO, the Regge-like asymptotics of Eq.(1) is valid only in the limited range of energy $y - y_0 = \ln(s/qq_0) \leq (\alpha_S)^{-\frac{5}{3}}$ [13,14]. For large values of $y - y_0$ an extra term appears in exponent of Eq.(1) which is proportional to $\alpha_s^5\,(y-y_0)^3$ [3,14]. Therefore, we cannot hope that in NLO BFKL we will get Regge-like asymptotics [3,14,15].

Even a slight glance at the difficulties, listed above, shows that the main properties of the LO BFKL are broken in the NLO.

A possible way out - the main idea of resummation: The BFKL equation in the NLO can be written in the form:

$$\frac{\partial G(y,\vec{q},\vec{q}_0)}{\partial y} \;=\; \delta^{(2)}(\vec{q}-\vec{q}_0) \;+\; \int d^2q'\,K(\vec{q},\vec{q}')\,G(y,\vec{q}',\vec{q}_0)\;, \tag{2}$$

where the NLO kernel $K(\vec{q},\vec{q}')$ can be written as

$$K(\vec{q},\vec{q}') \;=\; \alpha_S(q^2)\,\{\,K^{LO}(\vec{q},\vec{q}') \;+\; K^{NLO}(\vec{q},\vec{q}')\,\} \;= \tag{3}$$

$$\frac{r_0}{r}\,K_{conf}(\vec{q},\vec{q}') \;=\; \frac{r_0}{r}\,\{\,\alpha_S(q_0)\,K^{LO} \;+\; \alpha_S^2(q_0)\,K^{NLO}\,\}\;.$$

In Ref. [9] it was found that the largest contributions to K^{NLO} with the Fadin-Lipatov choice of the energy variable $y - y_0 = \ln(s/qq_0)$ is $K^{NLO} \;\propto\; \alpha_S^2\,\ln^2(q^2/q'^2)$. One can see that such a term appears as a change of the energy scale in the normal double log contribution. Indeed,

the term $\alpha_S^2\ \ln(q^2/q_0^2)\ \ln^2(s/q\,q_0)$ leads to $\alpha_S^2\ \ln^2(q^2/q'^2)\ \ln(s/q_0^2)$ -contribution if we change the energy scale from $s_0 = q\,q_0$ to $s_0 = q_0^2$.

The above example shows that in the NLO we lost the very important property of the LO BFKL Pomeron, namely, matching the BFKL approach with the DGLAP one, for both $q^2\ \gg\ q_0^2$ and $q_0^2\ \gg\ q^2$. The main idea , suggested by G.Salam [16] and by M.Ciafaloni [17], is to replace the NLO kernel of the BFKL equation by the new kernel in which subleading α_S^n - corrections to the BFKL equation have been resummed in a such way that

$$K^{BFKL}_{resummed}\ \ \longrightarrow\ \ K^{DGLAP}\ , \tag{4}$$

for $q^2\ \gg\ q_0^2$ and for $q_0^2\ \gg\ q^2$.

A possible way out - a toy model: Actually, the influence of the energy scale on the BFKL equation has been studied by Andersson, Gustafson and Samuelson [18] in their Linked Dipole Chain Model which reproduces the DGLAP double logs $\ln(1/x)\ \ln(q^2/q_0^2)$ for $q^2\ \gg\ q_0^2$ and for $q_0^2\ \gg\ q^2$.

The key equation of this model

$$\frac{\partial G(y,\vec{q},\vec{q}_0)}{\partial y}\ =\ \frac{\alpha_S N_c}{\pi}\ \int\ \frac{d^2l}{\pi\,l^2}\ \{\ G(x',q'^2)\ -\ G(x',q^2)\,\Theta(q-l)\ \} \tag{5}$$

with $\vec{l}\ =\ \vec{q}\ -\ \vec{q}'$ and

$$y\ =\ \ln(1/x)\ ;\quad x\ =\ \frac{q^2}{s}\ ;\quad x'\ =\ \max(x, x\,\frac{q'^2}{q^2})\ .$$

This equation is the BFKL one if $x' = x$. With a new definition of the energy variable x', the equation sums some of the next-to-leading order corrections to the BFKL kernel.

Fig. 2-a shows the Mellin image of the kernel for the BFKL case ($x = x'$, $\chi^{(0)}$), for the next-to-leading order correction to the BFKL equation ($\chi^{(0+1)}$) and for the kernel of Eq.(5) (all orders). One can see, that the NLO gives a minimum at $\nu = 0$ which corresponds to oscillation in the total cross section. However, the resummed kernel (all orders) has the same qualitatively behaviour as the LO BFKL kernel. In Fig. 2-b the BFKL intercept is plotted for the solution of Eq.(5). One can see that the negative intercept for $\alpha_S\ > 0.2$ in the NLO is irrelevant for the intercept of Eq.(5), which is about twice smaller than the intercept of the LO BFKL equation, but is definitively positive.

Therefore, we can conclude from this toy model that the principle, formulated in the previous subsection (see Eq.(4)), works and the resummation has a chance to heal our difficulties in the NLO BFKL equation.

A possible way out - an example of resummation:

G.Salam [16], using the principle of Eq.(4), suggested how to deal with unphysical logs of $\alpha_S^2\ \ln^2(q^2/q_0^2)$ - type. He demonstrated how to extend (resum) the NLO BFKL kernel so as to guarantee the concellation of unphysical logs to all orders and to provide the matching of the BFKL Pomeron with the DGLAP evolution equation. Unfortunately, the prescription for resummation is ambiguous and Salam suggested four realizations (schemes) of his ideas. It should be mentioned that other schemes were advocated [10] as well as the arguments for Salam's schemes 3 and 4 were suggested [10, 19–21]. In Refs. [19, 20] the criteria was suggested

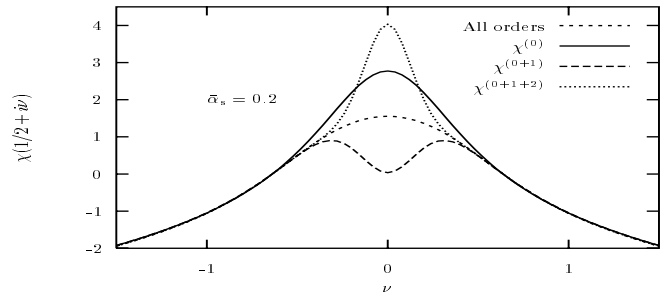

Figure 2-a

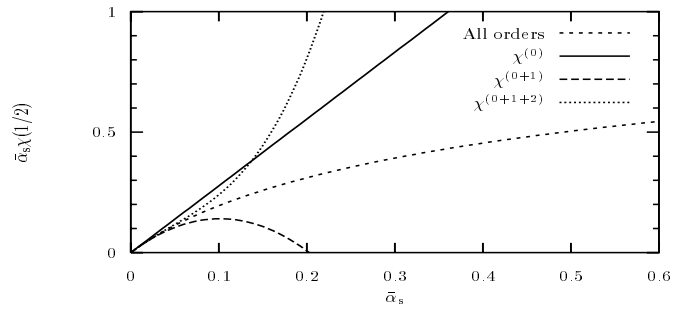

Figure 2-b

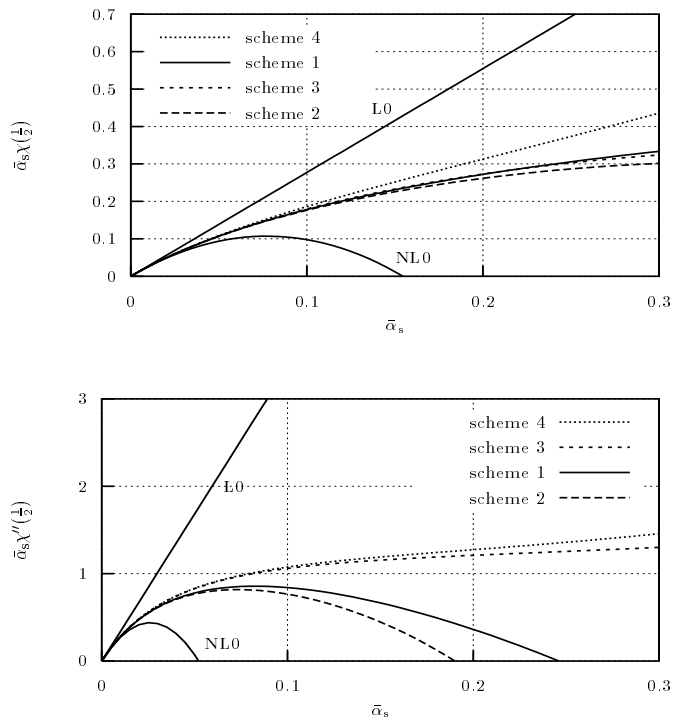

Figure 2-c

Figure 2: *The Mellin image of the toy kernel (a) and the intercept of the toy-Pomeron (b)and the BFKL Pomeron (c) intercept ($\omega_L = \bar{\alpha}_S\,\chi(\frac{1}{2})$ and the width of the BFKL Pomeron ($D = \bar{\alpha}_S\,\chi''(\frac{1}{2})$). See text for detail description.*

following the idea of Lipatov: the independence of the resummed result on the energy cutoff. This criteria is in the same spirit as the independence of the result on the factorization scale and it can be a good first step in understanding of the accuracy of the resummed BFKL Pomeron.

Fig. 2-c shows the behaviour of the BFKL intercept ($\omega_L = \bar{\alpha}_S\,\chi(\frac{1}{2})$) in Eq.(1) and the width of the BFKL Pomeron ($D = \bar{\alpha}_S\,\chi''(\frac{1}{2})$). One can see that the BFKL Pomeron intercept is positive in contrast with the NLO approximation. D is also positive, but only for schemes 3 and 4. The second important conclusions that we can derive from Fig. 4, is the similarity of the resummed NLO BFKL kernel with the toy model which has been discussed.

Resume and recommendations: Let me share with you to-day (20.04.1999) my understanding of the BFKL Pomeron standing.

1. Large NLO corrections to the BFKL kernel left us without any theoretical predictions for both the BFKL Pomeron intercept (ω_L in Eq.(1)) and the BFKL Pomeron width (D in Eq.(1)). We can introduce both of them as phenomenological parameters in a MC code, which should be extracted from the experimental data. The resummed BFKL kernel can be used to check how reasonable these extracted values of ω_L and D will be;

2. For the running QCD coupling constant (α_S) the Regge-like asymptotic of Eq.(1) can be justified only in a limited range of energies. Therefore, it is better to use the numerical solution of the BFKL equation than the analytical expression of Eq.(1);

3. The main qualitative properties of the parton BFKL cascade remain the same as in LO BFKL, for the NLO resummed BFKL equations. This is the very important conclusion for MC, which is the most striking theoretical result;

4. It is better to use the Linked Dipole Chain (LDC) Monte Carlo [22] than the BFKL equation itself. However, we first need to check that the LDC MC describes the LDC model [18][1].

3 Two indications for shadowing corrections from HERA data

HERA puzzle: The wide spread opinion is that HERA experimental data for $Q^2 \gtrsim 1\,GeV^2$ can be described using only the DGLAP evolution equations, without any other ingredients such as shadowing corrections, high twist contributions and so on (see, for example reviews [6, 7]). On the other hand, the most important HERA discovery is the fact that the density of gluons (gluon structure function) becomes large in HERA kinematic region [23].

The value of the gluon density turns out to be so large that the parameter

$$\kappa \;=\; \frac{3\,\pi^2\alpha_S}{2Q^2}\;\times\;\frac{xG(x,Q^2)}{\pi\,R^2}\;, \tag{6}$$

which determines the value of SC [24–27], reaches unity ($\kappa(x,Q_0^2(x)) = 1$) in HERA kinematic region (see Fig. 3a)

[1] I am very thankful to L. Lönnblad for discussions with me regarding the LDC MC. I realised that the MC experts have a problem in understanding why the LDC MC describes the experimental data worse than other approaches. I am very certain that as a result of such understanding will be a MC code which will include more from the BFKL cascade.

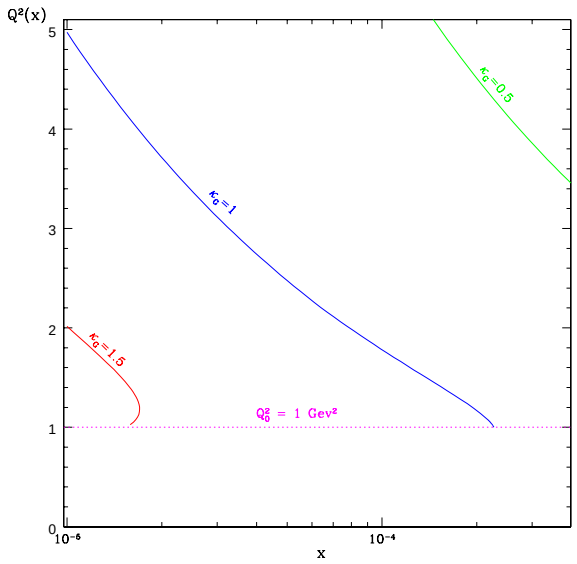

Figure 3-a

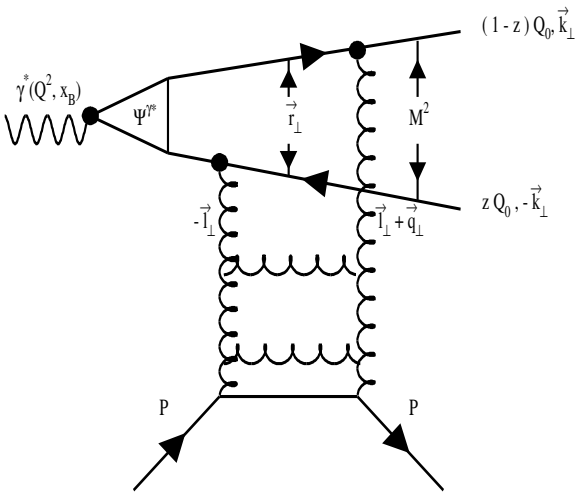

Figure 3-b

Figure 3: *Kinematic plot with lines $\kappa = 0.5$, 1 and 1.5 (Fig. 3a) and the diffractive production of quark - antiquark pair with mass M (Fig. 3b)*

It means that in large kinematic region where $\kappa \gtrsim 1$ (to the left from line $\kappa = 1$ in Fig. 3a), we expect that the SC should be large and important for the description of the experimental data. At first sight such expectations are in clear contradiction with the experimental data. Certainly, this fact gave rise to the suspicion (or even mistrust) that our theoretical approach to the SC is not consistent. However, the revision and re-analysis of the SC have been completed [27–32] with the result that κ is the parameter which is reponsible for the value of SC.

Therefore, we face a puzzling question: *where are SC in DIS?* I am happy to tell you that we now have two indications in the HERA experimental data that the SC are rather large and important. These indications are:

1. x_P - behaviour of the cross section of the diffractive dissociation (σ_{DD}) in DIS [2] ;
2. Q^2 - behaviour of F_2-slope ($\frac{\partial F_2(x_B,Q^2)}{\partial \ln Q^2}$).

Here, we would like to discuss both phenomena and their relation to the SC.

3.1 x_P - dependence of σ_{DD}

Data: Both collaborations (H1 and ZEUS) found that

$$\sigma_{DD} \;\propto\; \frac{1}{x^{2\,\Delta_P}} \tag{7}$$

where $\Delta_P = \alpha_P(0) - 1$. The values of $\alpha_P(0)$ are:

[2] x_P is the fraction of energy carried by Pomeron $x_P = 1 - x_F$, where x_F is the energy fraction of the recoiled proton (see Fig. 3-b)

- **H1 [33]:** $\alpha_P(0) = 1.2003 \pm 0.020\ (\text{stat.}) \pm 0.013\ (\text{sys.})$;
- **ZEUS [34]:** $\alpha_P(0) = 1.127 \pm 0.009\ (\text{stat.}) \pm 0.012\ (\text{sys.})$.

It is clear that the Pomeron intercept for the diffractive processes in DIS is higher than the intercept of the "soft" Pomeron.

Why is it surprising and interesting? To answer this question let us consider the simplest diffractive process - the diffractive production of the quark - anti quark pair with mass M (see Fig. 3b)

As it was shown (see Refs. [36–38]) the cross sections of this process are proportional to:

$$x_P \frac{d\sigma^T_{DD}(\gamma^* \to q + \bar{q})}{dx_P dt} \propto \int_{Q_0^2}^{\frac{M^2}{4}} \frac{dk_\perp^2}{k_\perp^2} \times \frac{\left(\alpha_S\ x_P\, G(x_P, \frac{k_\perp^2}{1-\beta})\right)^2}{k_\perp^2} \tag{8}$$

for transverse polarized virtual photon and

$$x_P \frac{d\sigma^L_{DD}(\gamma^* \to q + \bar{q})}{dx_P dt} \propto \int_{Q_0^2}^{\frac{M^2}{4}} \frac{dk_\perp^2}{Q^2} \times \frac{\left(\alpha_S\ x_P\, G(x_P, \frac{k_\perp^2}{1-\beta})\right)^2}{k_\perp^2} \tag{9}$$

for the longitudinal polarized photon. In these formulae we used the notation (see Fig. 3b): Q^2 is the photon virtuality; M is the produced mass; $x_P = \frac{Q^2+M^2}{s}$, where s is the energy of virtual photon proton collisions; and $\beta = \frac{Q^2}{Q^2+M^2}$.

The same factor also enters a more complicated process as, for example, a diffractive production of the quark - antiquark pair and one extra gluon [38].

One can see that for the transverse polarized photon the diffractive cross section stems from the low values of the transverse momenta $k_\perp$ (see Eq.(8)) if we use for xG the solution of the DGLAP equations. However, the situation changes significantly if we take into account the SC in expression for xG. For estimates of the value of the SC, we use the Glauber - Mueller approach [26] for xG, namely [38, 39]:

$$x_P G^{SC}(x_P, k_\perp^2) = \frac{2}{\pi^2} \int_{x_P}^1 \frac{dx'}{x'} \int_{\frac{1}{k_\perp^2}}^\infty \frac{dr^2}{r^4} \int db_t^2 \{\ 1 - e^{-\kappa^{DGLAP}(x', r^2)\, S(b_t)}\ \} \tag{10}$$

where $S(b_t) = e^{-\frac{b_t^2}{R^2}}$.

From Eq.(10) we can see that in the limit of low x

$$x_P G(x_P, k_\perp^2) \implies k_\perp^2\, R^2 \int_x^1 \frac{dx'}{x'} \ln \kappa(x', \frac{1}{k_\perp^2}) \tag{11}$$

if $k_\perp^2 \ll Q_0^2(x)$, where $Q_0^2(x)$ is plotted in Fig. 6. Therefore, the substitution of xG^{SC} in Eq.(8) leads to the situation that the integral over $k_\perp^2$ is not infrared divergent, and the main contribution comes from the region of $k_\perp^2$ of the order of $Q_0^2(x_P)$. It means that

$$x_P \frac{d\sigma_{DD}}{dx_P dt} \implies \frac{1}{Q_0^2(x_P)} \times \left(x_P\, G(x_P, Q_0^2(x_P))\right)^2 \propto \frac{1}{x_P^{2\lambda_{eff}(Q^2)}} \tag{12}$$

Eq.(12) allows us to find the typical distances which are reponsible for the diffraction dissociation in DIS. Comparing the experimental values of $\lambda_{eff}(Q^2)$ with the ones calculated

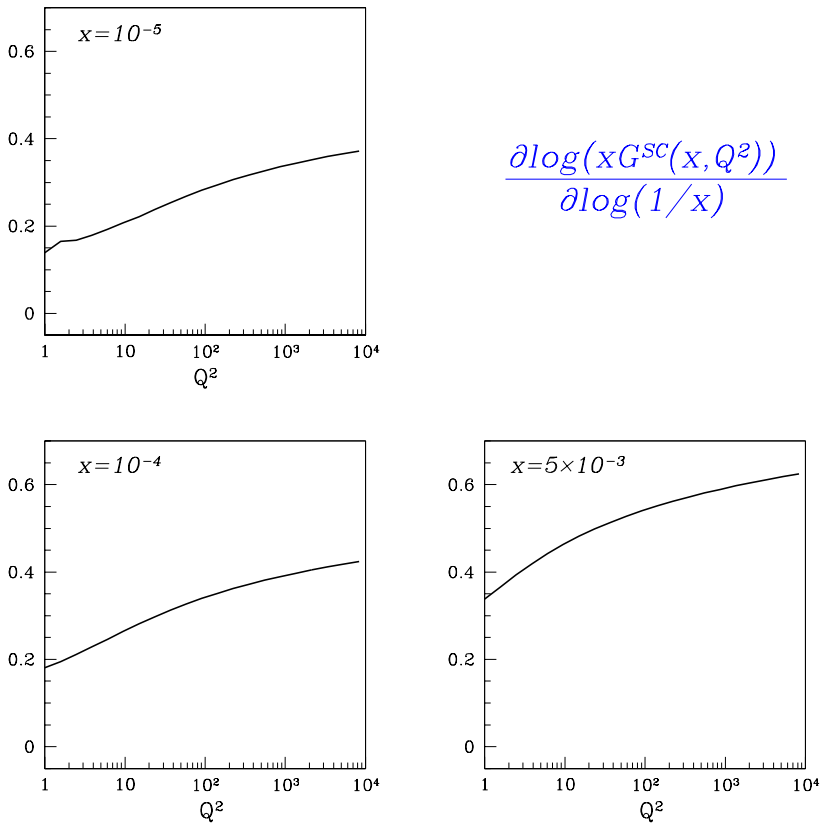

Figure 4: $\lambda(Q^2) = \frac{1}{x_P\, G^{SC}(x_P,k_\perp^2)} \frac{dx_P\, G^{SC}(x_P,k_\perp^2)}{d\ln(1/x_P)}$ *calculated using Eq.(10).*

using Eq.(10) (see Fig. 4)[3] , one can conclude that the typical $k_\perp^2$ is not small, but rather $k_\perp^2 \approx 1 - 2\,GeV^2$ [38,39], if we take $x_P \approx 10^{-3}$, or even larger.

It should be stressed that the diffraction dissociation for longitudinally polarized photon (see Eq.(9)) for quark - antiquark production comes from long distances and its cross section is proportional to

$$x_P \frac{d\sigma^L_{DD}(\gamma^* \to q + \bar{q})}{dx_P dt} \propto \left(\alpha_S\, x_P\, G(x_P, \frac{M^2}{4(1-\beta)}) \right)^2 \tag{13}$$

3.2 The effect of screening on Q^2 - dependence of the F_2 - slope.

Data: The experimental data [45] for the F_2 - slope $\frac{dF_2(x,Q^2)}{d\ln Q^2}$ are shown in Fig. 5 (Caldwell plot). These data give rise to a hope that the matching between "hard" (short distance) and "soft" (long distance) processes occurs at sufficiently large Q^2. Indeed, the F_2 - slope starts to deviate from the DGLAP predictions around $Q^2 \approx 5 - 8\,GeV^2$.

[3]Doing this comparison, we have to take into account that $\lambda_{eff}(Q^2) = \lambda(Q^2)(Fig.\ 4) - \frac{1}{2}\frac{d\ln Q_0^2(x)}{d\ln(1/x)}$. $Q_0^2(x)$ in Fig. 3a can be described by simple formula $Q_0^2(x) = 1\,GeV^2\,(\frac{x_0}{x})^{0.52}$.

Such a large value of this separation parameter between short and long distance processes leads to a natural question:"Could the experimental behaviour of the F_2 - slope as a function of Q^2 be a manifestation of the shadowing corrections or, better to say, of the saturation of the gluon density in DIS ?". The answer [39,46,47] is: "Yes".

F_2 - slope and SC: In Refs. [39,47] the influence of the SC on the behaviour was calculated using Eq.(10) and a similar formula in quark-antiquark sector. The result is given in Fig. 11 where the F_2 - slope in ALLM parameterization [48] is also plotted. Comparison with the experimental data as well as with the ALLM predictions shows that the SC can give a plausible explanation of the effect.

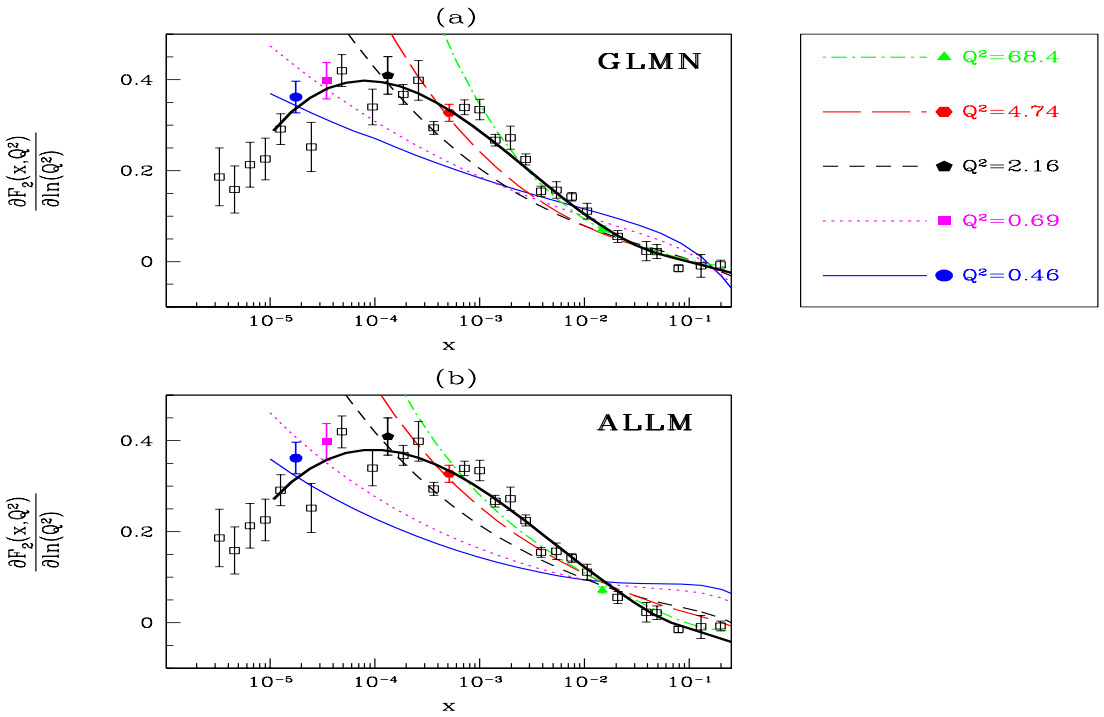

Figure 5: *The F_2 - slope in QCD calculations taking into account SC (GLMN) [39] and in ALLM'97 parameterization [48], which describes the matching between "soft " and "hard" interactions in DIS.Data are taken from [45]*

We would like to draw your attention to the fact that at fixed Q^2 the SC do not lead to qualitative change of the x - dependence of the F_2-slope (see lines with fixed Q^2 in Fig. 11). However, the saturation of the gluon density gives $F_2 \propto Q^2 R^2$ at $Q^2 < Q_0^2(x)$. Therefore, the F_2 - slope shows a decrease at small Q^2. The calculations support the idea that the experimental selection of the data on the F_2 - slope shows the predicted Q^2-dependence. However, it is important to stress that only data on F_2 - slope at fixed energy (or x) would clarify how much of the experimentally observed effect is related to the Q^2 - dependence. It is worthwhile mentioning that the ALLM'97 parameterization which describes phenomenologically the matching between "hard" and "soft" interactions in DIS, predicts the same behaviour of the F_2 - slope as the SC calculations (see Fig. 5).

3.3 Why these two facts are still only the indications of SC contributions?

I hope that I have convinced you that the SC provide a natural explanation of both experimental effects. Even more, the new experimental data give a possible way to understand and resolve the HERA puzzle, that has been formulated at the beginning of this section. We have expected such effects for a long time, and the SC received strong support from the experiment.

However, we cannot make a final conclusion because both the experimental sets of data on x_P-dependence of the diffractive cross section, and on Q^2 - dependence of the F_2 - slope can have an alternative explanation. For example, the MRST parameterization [49] gives the explanation for the F_2 - slope behaviour assuming a strange behaviour of the gluon structure function at the initial virtuality $Q^2 = Q_0^2$ - decreasing at low x.

Nevertheless, we would like to stress that the SC give a good agreement with the ALLM'97 parameterization in the wide range of Q^2 and x. This provides strong support for SC which, we hope, will be confirmed by the future experimental data.

We need to introduce SC in the Monte Carlo codes to be prepared for the future experiments.

4 Matching of "soft" and "hard" photon - proton interactions

Gribov's approach: The rich and high precision data on deep inelastic ep scattering at HERA [50, 51], covering both low and high Q^2 regions, lead to a theoretical problem of matching the non-perturbative ("soft") and perturbative ("hard ") QCD domains. This challenging problem has been under close investigation over the past two decades, starting from the pioneering paper of Gribov [52] (see also [53]).

Gribov suggested two stages of the $\gamma^* p$ interaction in any QCD description (see Fig. 12-a):

1. The γ^* converts into a hadron system (quark-antiquark pair to lowest order) well before the interaction with the target;

2. The quark-antiquark pair (or hadron system) then interacts with the target.

These two stages are expressed explicitly in the double dispersion relation suggested in Ref. [52]:

$$\sigma(\gamma^* N) = \frac{\alpha_{em}}{3\pi} \int \frac{\Gamma(M^2) dM^2}{(Q^2 + M^2)} \sigma(M^2, M'^2, s) \frac{\Gamma(M'^2) dM'^2}{(Q^2 + M'^2)}, \tag{14}$$

where M and M' are the invariant masses of the incoming and outgoing quark-antiquark pairs, $\sigma(M^2, M'^2, s)$ is the cross section of a $q\bar{q}$ interaction with the target, and the vertices $\Gamma^2(M^2)$ and $\Gamma^2(M'^2)$ are given by $\Gamma^2(M^2) = R(M^2)$, where $R(M^2)$ is the ratio:

$$R(M^2) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}, \tag{15}$$

which has been measured experimentally.

The key problem in all approaches utilizing Eq.(14) is the description of the cross section $\sigma(M^2, M'^2, s)$.

1. We introduce $M_0 \approx 1\,GeV$ in the integrals over M and M', which play the role of a separation parameter. For M, $M' > M_0$ the quark - antiquark pair are produced at short distances ($r_\perp \propto \frac{1}{M} < \frac{1}{M_0}$), while for M, $M' < M_0$ the distance between quark and antiquark is too long ($r_\perp \propto \frac{1}{M} > \frac{1}{M_0}$), and we cannot treat this $q\bar{q}$ - pair in pQCD. Actually, we cannot even describe the produced hadron state as a $q\bar{q}$ - pair;

2. For M, $M' < M_0$ we use the Additive Quark Model [54] in which

$$\begin{aligned} \sigma(M^2, M'^2, s) &= \sigma_N^{soft}(M^2, s)\,\delta(M^2 - M'^2) \\ &= (\,\sigma_{tot}(qN) + \sigma_{tot}(\bar{q}N)\,)\,\delta(M^2 - M'^2) \end{aligned} \tag{16}$$

3. For M, $M' > M_0$ we consider the system with mass M and/or M' as a short distance quark - antiquark pair, and describe its interaction with the target in pQCD. The exact formulae for $\sigma^{hard}(M^2, M'^2, s)$ is given in Ref. [55]), but the key property of these formulae that this interaction can be expressed through the gluon structure function, and it is *not* diagonal with respect to the masses, contrary to the "soft" interaction of a hadron system with small mass.

Descriptions of the experimental data: Starting from the paper of Badelek and Kwiecinski [57] Gribov's ideas have been implemented to describe the rich and precise experimental data on $\gamma^* p$ interaction in the wide region of Q^2 and x [55, 56, 58]. It has been shown that such an approach is able to provide a successful description of the experimental data on photon-proton interaction, over a wide range of photon virtualities $0 \leq Q^2 \leq 100\,GeV^2$, and energies $\sqrt{s}\,(= W) \leq 300\,GeV$. The key assumption on which this approach is based, is that the non-perturbative and the perturbative QCD contributions in the Gribov formula can be separated by the parameter M_0. The successful reproduction of the experimental data [58] (see Figs. 6) shows that this assumption appears to be valid.

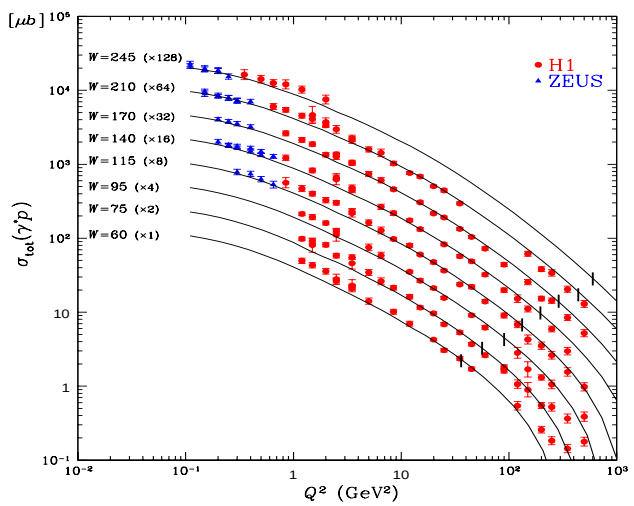

Figure 6-a

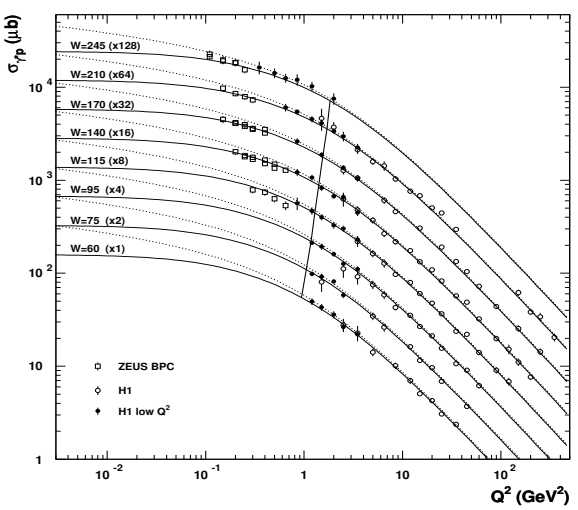

Figure 6-b

Figure 6: *The description of experimental data in Gribov approach, given in Ref. [58]and in the Golec - Biernat and Wüsthoff approach [40].*

A success in phenomenological description gives us hope, that the approach based on Gribov's ideas can be used for matching of "soft" and "hard" processes for more general observables than the total cross section and, finally, will lead to self-consistent Monte Carlo code for all values of the photon virtualities.

Golec - Biernat and Wüsthoff approach: The natural generalization of discussed attempts to describe the matching between long and short distances physics is to use the SC approach. The full description based on formulae of Eq.(10) - type has not been developed yet. However, Golec - Biernat and Wüsthoff [40, 41] suggested a simple phenomenological approach which incorporates the main qualitative and even quantitative features of the general SC calculations.

They use for the total cross section of photon - proton interaction the following formula [40]

$$\sigma(\gamma^* p) \;=\; \int\int d^2 r_\perp \, dz \, |\Psi(Q^2; z, r_\perp)|^2 \; \sigma(x, r_\perp) \; ; \tag{17}$$

where

$$\sigma(x, r_\perp) \;=\; \sigma_0 \; \{ \; 1 \; - \; e^{- \frac{r_\perp^2}{R^2(x)}} \; \} \; . \tag{18}$$

$R^2(x)$ is a new scale for the DIS,namely, $R^2(x) \;=\; 1/Q_0^2(x)$. Golec-Biernat and Wüsthoff use the phenomenological parameterization for $Q_0^2(x) \;=\; Q_0^2 \; (x/x_0)^\lambda$ instead of the calculation that lead to Fig. 6. With $\sigma_0 \;=\; 23.03 \; mb$,$\lambda \;=\; 0.288$ and $x_0 \;=\; 3.04 \, 10^{-4}$ Golec-Biernat and Wüsthoff successfully described both the total cross sections of the photon - proton interaction [40] as well as the cross sections of the diffractive production in DIS [41]. Fig. 6b shows what kind of description can be reached in such an approach.

This approach demonstrates that SC are a possible way of describing the matching of the "soft" and "hard" kinematic regions. It also gives a practical way to write the matching of pQCD and npQCD contribution into Monte Carlo codes.

5 Survival probability of Large Rapidity Gaps (LRG)

Unfortunately, I have no space to discuss the survival probability of LRG processes, but I gave a very detail discussions in the preprint version of this talk [59].

6 Conclusions

In this talk I tried to give you a picture of recent progress in low x physics. I chose the subject which, I hope, would be useful for Monte Carlo experts. Let me summarize in short the present situation in four subjects that I have discussed here:

1. We are on the right track in understanding of the next-to-leading BFKL Pomeron, but, at the moment, we cannot guarantee any calculations of the parameters of the BFKL Pomeron;

2. First experimental indications of the strong shadowing corrections have appeared in the energy behaviour of the diffraction dissociation cross sections in DIS and in the Q^2 - behaviour of the F_2 - slope (Caldwell plot). We hope, that these data will stimulate a new experimental

systematic search of SC, as well as the creation of new Monte Carlo codes, that will include the SC and the physics, induced by them;

3. Considerable progress, based on Gribov's ideas, has been achieved in describing the matching between the non-perturbative "soft" processes, and the perturbative "hard" ones in photon - proton interactions. We firmly believe that this progress will be useful both for future non-perturbative approaches in QCD and for creating a new Monte Carlo program, which takes this matching into account;

4. We can estimate the survival probability in hadron - hadron collisions only in rather primitive models, which can reproduce the basic experimental data on it. The prediction for the survival probability in DIS are more solid from theoretical point of view, but the experimental information is so poor that we cannot compare the main features of our calculations with the data. We think that time has come to include the decomposition of multiparticle production process in the series of multi parton showers production, which is closely related the the value of the survival probability [59] in Monte Carlo codes.

Acknowledgments: I am very grateful to E. Gotsman and U. Maor for permanent useful discussions on the subject which only partly were reproduced in our common papers. I would like to thank E. Naftali for preparing Fig. 5 specially for this talk. I would like also to thank all participants of the MC Workshop whose discussions were useful and supporting for me.

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