MLLA Parton Spectra Compared to ARIADNE

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Abstract: The parton spectra as predicted by the ARIADNE Monte Carlo generator, for both e^+e^- annihilation and deep inelastic scattering, are compared to the QCD MLLA calculations.

1 Introduction

The perturbative QCD approach to describing the inclusive energy spectra, via the modified leading log approximation (MLLA) in conjunction with local parton hadron duality (LPHD), has been very successful in both e^+e^- annihilation and deep inelastic scattering experiments [1]. Using LPHD, the non-perturbative effects of such distributions are reduced to a simple factor of normalisation that relates the hadronic distributions to the partonic ones. Perturbative features of these distributions are calculated by MLLA which accounts for both the double and single logarithmic effects. The MLLA approach has two free parameters: a running strong coupling, governed by a QCD scale Λ, and an energy cut-off, Q_0, below which the parton evolution is truncated.

The MLLA evolution equation allow the parton spectra for the logarithmic scaled energy spectra, ξ, to be calculated [2]. The variable ξ is defined as $\ln(E_0/E) \equiv \ln(1/x_p)$, where E_0 is the original energy of the jet and E is the parton's energy. The cut-off, Q_0, bounds the parton energy, $E \geq k_T \geq Q_0$, where k_T is the transverse energy of the decay products in the jet evolution. In order to reconstruct the ξ distributions one has to perform the inverse Mellin transformation:

$$\bar{D}(\xi, Y, \lambda) = \int_{\epsilon-\imath\infty}^{\epsilon+\imath\infty} \frac{d\omega}{2\pi\imath} x_p^{-\omega} D(\omega, Y, \lambda) \tag{1}$$

where the integral runs parallel to the imaginary axis on the right of all singularities in the complex $\omega-$plane, $Y = \ln(E_0/Q_0)$ and $\lambda = \ln(Q_0/\Lambda)$.

The Mellin-transformed distributions, $D(\omega, Y, \lambda)$, can be expressed [2] in terms of confluent hypergeometric functions, Φ:

$$\begin{aligned} D(\omega, Y, \lambda) \;&= \tfrac{t_1 A}{B(B+1)} \Phi(-A+B+1, B+2; -t_1) \Phi(A-B, 1-B; t_2) \\ &\quad + \left(\tfrac{t_2}{t_1}\right)^B \Phi(-A, -B; -t_1) \Phi(A, B+1; t_2), \end{aligned} \tag{2}$$

where

$$t_1 = \omega(Y + \lambda) \ , \quad t_2 = \omega\lambda. \tag{3}$$

In addition A and B are defined as:

$$A = 4N_c/b\omega, \qquad B = a/b, \tag{4}$$

where N_c is the number of colours, $a = 11N_c/3 + 2n_f/3N_c^2$, n_f is the number of flavours and $b = 11N_c/3 + 2n_f/3$.

Equation 1 is then calculated using a numerical integration in the complex $\omega-$plane.

The current region in the ep Breit frame is analogous to a single hemisphere of e^+e^- annihilation. In $e^+e^- \to q\bar{q}$ annihilation the two quarks are produced with equal and opposite momenta, $\pm\sqrt{s}/2$, where $\sqrt{s}$ is the positron-electron centre of mass energy. The fragmentation of these quarks can be compared to that of the quark struck from the proton; this quark has an outgoing momentum $-Q/2$ in the Breit frame, where Q^2 is the negative square of the four-momentum of the virtual exchanged boson in DIS. In the direction of this struck quark the scaled momentum spectra of the particles are expected by MLLA [3, 4, 5] to have a dependence on Q similar to that observed in e^+e^- annihilation at energy $\sqrt{s} = Q$, with no Bjorken-x dependence.

The ARIADNE Monte Carlo generator [6] is based on the colour dipole model, CDM [7]. In the CDM, all gluon emissions constituting the QCD cascade start as radiation from the colour dipole formed between the quark and the anti-quark in the case of e^+e^- annihilation or the struck quark and the proton remnant in the case of DIS. All subsequent radiation arises from independent colour dipoles formed either from $q\bar{q}$ pairs or softer gluons radiated by the previously produced gluons. In the DIS scenario, the proton remnant is treated as an extended object which results in a suppression of radiation [8], generally in the proton direction. In addition the struck quark is treated as extended, as the photon only probes it to a distance inversely proportional to the transferred momentum. Treating the remnant and the struck quark as extended objects, rather than point like, results in a reduction in the available phase space for gluon radiation in DIS.

The QCD cascade in ARIADNE is governed by a number of parameters in the Monte Carlo models. Two of the most important are the QCD scale, Λ, (PARA(1)) and the parameter that determines the k_T cut off for the shower (PARA(3)). An additional parameter, PARA(28), also allows the user to bound the lower energy of the emitted parton as well. For this study ARIADNE version 4.10 has been used.

2 Comparisons with MLLA

Before investigating the evolution of the shower in DIS, the evolution in the simpler case of $q\bar{q}$ pair production in e^+e^- annihilation was studied. The spectra for both MLLA and ARIADNE were generated with a $\Lambda = 150$ MeV and a cut-off $Q_0 = 2\Lambda = \text{PARA(3)} = \text{PARA(28)}$. Below $\xi \approx 1$ there are instabilities in the numerical integration of equation 1 so all subsequent comparisons are for $\xi > 1$. Except for an overall normalisation discrepancy (a factor of 1.4 greater parton multiplicity in ARIADNE) the ξ spectra are in very good agreement as illustrated in Figure 1. This normalisation discrepancy is constant, independent of the $\sqrt{s}$ at which

Q	x
14.8	5×10^{-3}
20.9	1×10^{-2}
29.3	5×10^{-2}
41.7	5×10^{-2}
59.1	0.1
91.2	0.2

Table 1: The (x,Q) analysis bins.

the events were generated at. There is a slight tendency for the MLLA calculation to fall off quicker at large values of ξ than the ARIADNE predictions.

Using LEPTO [9] to generate the electroweak cross section and colour flow configuration for DIS, ARIADNE was then used to generate the subsequent QCD cascade. The event was boosted to the Breit frame and those partons in the current fragmentation region selected. The DIS events were generated with fixed kinematics that are accessible in the HERA regime. The corresponding value of Bjorken-x with Q are shown in Table 1. Using the same values of Λ and Q_0 that was used for the e^+e^- annihilation study, the MLLA prediction is again compared to the ARIADNE generated spectra.

Figure 2 shows the default version of ARIADNE for DIS compared to the MLLA predictions. As Q^2 increases the discrepancy between ARIADNE and the MLLA calculations becomes more pronounced. The ξ distribution of ARIADNE peaks at higher values than the MLLA calculation. In addition, the MLLA calculations are narrower than the ARIADNE predictions. Again the parton multiplicity of the two distributions are different. Unlike the e^+e^- situatuion this normalisation factor seems to exhibit a Q dependence. At low Q the height of the peak for ARIADNE compared to MLLA is a factor of 1.1 higher whilst in the highest Q bin it is 1.1 lower.

In the default ARIADNE, the mechanism for soft suppression of radiation due to the extended source of the proton remnant results in a suppression of radiation in the current region of the Breit frame at high Q^2. Figure 3 shows the high Q^2 modified version of ARIADNE [10] for DIS, where this suppression in the current region is removed, compared to the MLLA predictions. As expected, this modification to ARIADNE leads to a much better agreement between the MLLA calculations and ARIADNE. The situation with the parton multiplcity is similar. The $Q-$dependence of the ratio of the peak heights is less than the default ARIADNE, with ARIADNE being a factor $1.2 - 1.3$ higher.

In both options of the ARIADNE program there are discrepancies evident in the lower (x, Q) bins compared to the MLLA predictions. One possible explaination of this discrepancy is given in Ref. [11], where it shown that high p_T emissions in DIS can lead to the situation where the current region of the Breit frame is depopulated.

3 Conclusions

The QCD cascade as implemented in the ARIADNE Monte Carlo program is in good agreement with the shape of the MLLA prediction in the simple scenario of $q\bar{q}$ production in e^+e^-

annihilation. In the more complex situation of DIS the agreement is not as good, unless account is taken of the additional suppression introduced into the model in the current fragmentation region caused by the suppression of phase space due to the extended nature of the proton remnant.

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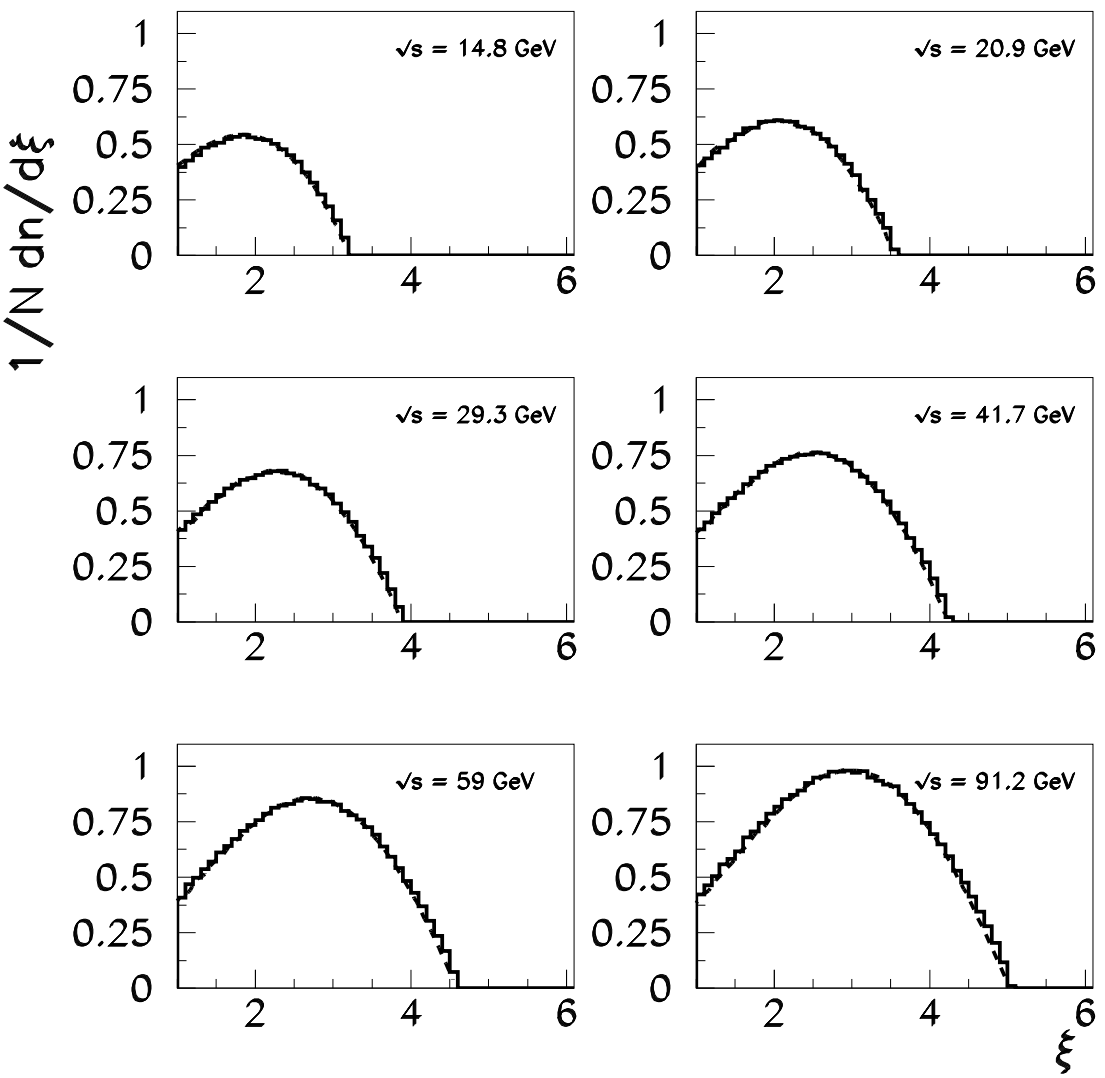

Figure 1: The logarithmic scaled momentum distributions, ξ, for the ARIADNE Monte Carlo generator (full line) and MLLA calculation (dashed line) for e^+e^- annihilation. The ARIADNE prediction has been scaled down by a factor 1.4.

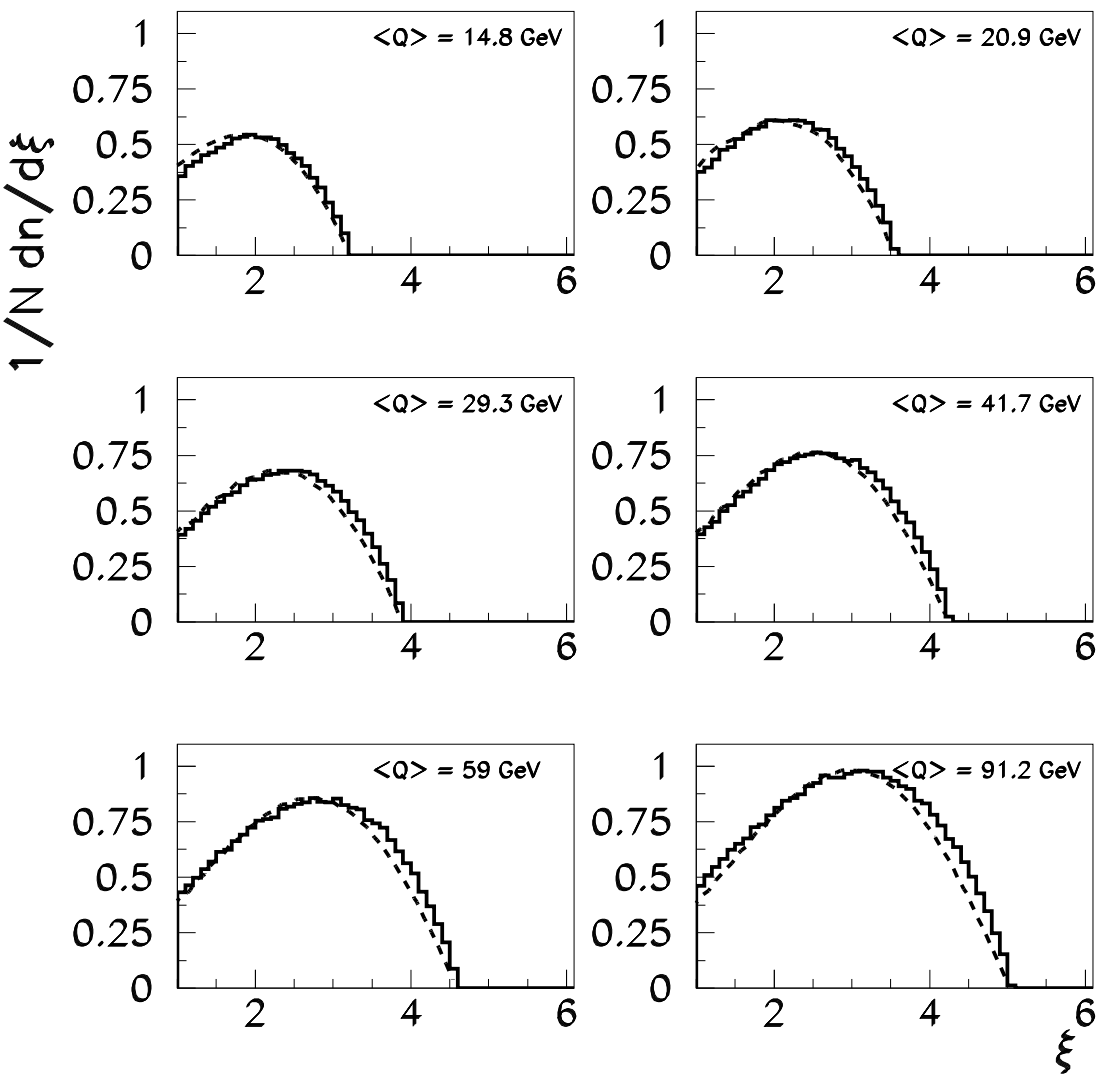

Figure 2: The logarithmic scaled momentum distributions, ξ, for the default ARIADNE Monte Carlo generator (full line) and MLLA calculation (dashed line) for DIS. The distributions have been normalised to the peak height of the MLLA calculation.

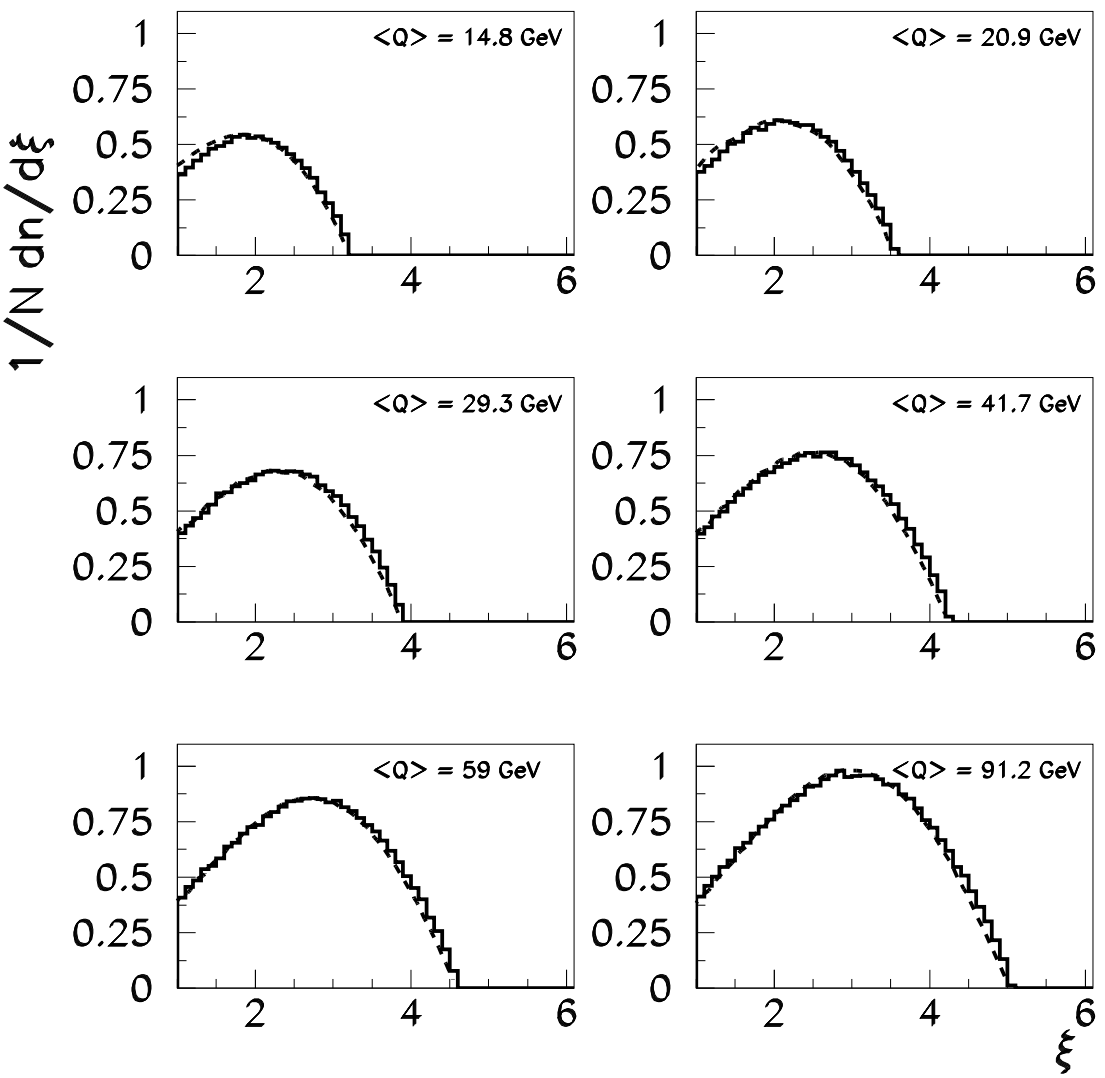

Figure 3: The logarithmic scaled momentum distributions, ξ, for the modified ARIADNE Monte Carlo generator (full line) and MLLA calculation (dashed line) for DIS. The distributions have been normalised to the peak height of the MLLA calculation.