Transverse Momentum Transfer and Low x Parton Dynamics at HERA

E. A. De Wolf and P. Van Mechelen

Universitaire Instelling Antwerpen, Department of Physics
Universiteitsplein 1. B-2610 Antwerpen, Belgium
e-mail : edewolf@uia.ua.ac.be, pierre@uia.ua.ac.be

Abstract: The transverse momentum transfer correlation is introduced as a sensitive probe that can be used to discrimate between models for parton dynamics in low-x deep-inelastic scattering. Expectations for uncorrelated models and models with short-range or long-range correlations are discussed and confronted to results obtained from the Lepto and Ariadne Monte Carlo simulation programmes.

1 Introduction

The successful description of the nucleon structure function by perturbative QCD, using the DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) parton evolution equations [1] constitutes a major success of QCD. However, at very small Bjorken-x, these equations are expected to become invalid. An alternative ansatz for the small-x regime is the BFKL (Balitsky-Fadin-Kuraev-Lipatov) equation [2]. At lowest order, the BFKL and DGLAP equations resum the leading logarithmic $(\alpha_s \ln 1/x)^n)$ or $(\alpha_s \ln Q^2/Q_0^2)^n)$ contributions, respectively, with Q^2 being the virtuality of the exchanged photon in deep-inelastic neutral current ep collisions. The leading-log DGLAP ansatz implies strong ordering $(Q_0^2 \ll {k_T}_1^2 \cdots \ll {k_T}_i^2 \ll \cdots Q^2)$ of the transverse momenta $(k_T)_i$ in the parton cascade. Here, $(k_T)_i$ is the transverse momentum (measured relative to the incident proton direction) of the i-th emitted "final state" parton (see Fig. 1). According to BFKL kinematics, the transverse momenta are no longer strongly ordered but may be thought of as forming, roughly speaking, a random walk with $(k_T^2)_i \sim (k_T^2)_{i+1}$.

Measurements of the hadronic final state resulting from the initial-state parton cascade should be sensitive to the type of evolution. For example, without strong k_T-ordering, more transverse energy E_T is to be expected from the BFKL than from the DGLAP type of evolution in a rapidity interval between the proton fragmentation region and the current fragmentation region. A similar expectation holds for the inclusive transverse momentum distribution of hadrons [3, 4]. So far, however, none of the studied observables (including recent measurements of forward jet and large-p_T charged and neutral pions [5]) are directly probing the correlation structure of the parton cascade. To allow additional discrimination between a strictly k_T-ordered and an unordered scenario, it is necessary to go beyond single-particle observables and consider directly measures of the transverse momentum correlations in the parton cascade.

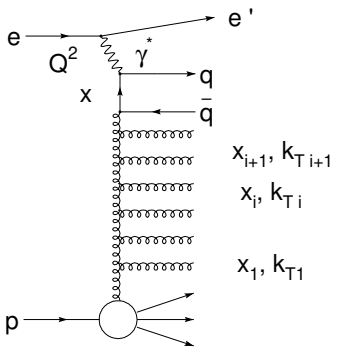

Figure 1: Parton evolution in the ladder approximation. The longitudinal fractional momenta x_i and transverse momenta k_{Ti} of subsequently emitted partons are indicated.

2 The transverse momentum transfer

In this paper we consider the transverse momentum transfer across rapidity y, $\vec{\Pi}(y)$. This quantity is defined as the vector-sum of the transverse momenta $\vec{k_i}$ of all particles with rapidities smaller than y in an event with n particles. For simplicity, we consider in the following only one of the components of the transverse momentum vectors (denoted by k_i) and define

$$\Pi(y) = \sum_{i=1}^{n} k_i \, \theta(y - y_i), \tag{1}$$

where $\theta(x) = 0$ for $x < 0$ and $\theta(x) = 1$ for $x \geq 0$. $\Pi(y)$ is a random function, varying from event to event. A set of k-th order moment-functions can be constructed defined as[1]

$$\langle \Pi(y_1) \, \Pi(y_2) \ldots \Pi(y_k) \rangle, \tag{2}$$

where $\langle\rangle$ means that the average is taken over an ensemble of events. From the moment-functions, one can further define a set of correlation functions by taking a cluster decomposition (see eg. [7]). The two-point correlation function $D^2(y_1, y_2)$ is then given by

$$D^2(y_1, y_2) = \langle \Pi(y_1) \, \Pi(y_2) \rangle - \langle \Pi(y_1) \rangle \, \langle \Pi(y_2) \rangle. \tag{3}$$

In particular, for $y_1 = y_2 = y$ we define $D^2(y) = D^2(y, y)$. This quantity measures the variance of the transverse momentum transfer distribution across the rapidity "boundary" y. $D^2(y)$ provides a measure of the rapidity structure of the correlations between the transverse momenta as can be seen from the relation [8]

$$D^2(y) = -\int_{-\infty}^{y} dy_1 \int_{y}^{\infty} dy_2 \int d^2k_1 \, d^2k_2 \, \vec{k}_1 \cdot \vec{k}_2 \; \rho_2(\vec{k}_1, y_1; \vec{k}_2, y_2), \tag{4}$$

where $\rho_2(\vec{k}_1, y_1; \vec{k}_2, y_2)$ is the inclusive two-particle density.

With reference to the parton-level diagram of Fig. 1, it should be noted that $\Pi(y)$ is in that case equal to the (propagator) transverse momentum exchanged between the vertices i and $i + 1$ if the position y is located between particles i and $i + 1$ on the rapidity axis. Transverse

[1] See [6] for a discussion of the theory of random functions

momentum transfer correlations are, therefore, a direct measure of the correlations between the exchanges.

In models of uncorrelated production, the transverse momentum of a given particle can be compensated by that of any other particle or group of particles. Consequently, the correlation length describing the transverse momentum correlations, which follow from momentum conservation alone, is of the order of half the available rapidity range and should thus increase with energy or W in DIS. For uncorrelated transverse momenta $D^2(y)$ is expected to increase when y moves from the edge of the rapidity space to $y = 0$ since, for independent random variables, the variance of a sum increases proportionally to the number of terms included. For the same reason also $D^2(0)$ should increase with the number of particles produced.

Going beyond the uncorrelated production assumption, one could assume that the relevant part of the diagram in Fig. 1 (neglecting any dependence on longitudinal momenta) can be schematically written as a product of nearest-neighbour correlated "links"

$$|T_n|^2 \approx \prod_i V(\vec{\Pi}_i, \vec{\Pi}_{i+1}), \tag{5}$$

where the functions V represent the (iterated) kernel of the diagram. In that case, an important role is played by the eigenvalues and eigenfunctions of the kernel, considered as a transfer integral operator [9]. For a sufficiently long ladder, it can be shown that the function $D^2(y_1, y_2)$ is controlled by the largest eigenvalue λ_{01} leading to a behaviour

$$D^2(y_1, y_2) \approx \exp\left\{-\rho(1 - \lambda_{01})|y_1 - y_2|\right\}, \tag{6}$$

with ρ the mean parton multiplicity per unit of rapidity. In this short-range model, the correlation length is therefore given by $1/\rho(1 - \lambda_{01})$.

In models with short-range correlations, by definition, particles far away in rapidity cannot be correlated. As a result, the compensation of transverse momenta is local, i.e. it is compensated by its neighbours. The corresponding correlation length λ is energy independent. These general properties are reflected in the behaviour of $D^2(y)$. Only particles in the rapidity interval $\Delta = (-\lambda + y, y + \lambda)$ contribute to the transverse momentum transfer at y. Therefore, $D^2(y)$ is expected not to increase faster with energy than the particle density in the interval Δ (for constant λ) which is known to increase only logarithmically.

In a k_T-ordered cascade, correlations will be long-range. The correlation length can be expected to increase with the 'length of the cascade' in rapidity space, i.e. proportional to $\ln 1/x$ or $\ln W$ (at fixed Q^2) in DIS. For the case of the BFKL scenario, it can further be expected that the correlation length is related to the dominant eigenvalues of the BFKL kernel (cfr. Eq. 6)

3 A Monte Carlo study

Predictions for the cases of the ordered and unordered cascades are obtained from DIS Monte Carlo models, which incorporate the QCD evolution in different approximations and utilize phenomenological models for the non-perturbative hadronization phase. The MEPS model (Matrix Element plus Parton Shower) [10], incorporates the QCD matrix elements up to first order, with additional soft emissions generated by adding leading log parton showers. In the

colour dipole model (CDM) [11, 12] radiation stems from colour dipoles formed by the colour charges. Both programs use the Lund string model [13] for hadronization. The CDM description of gluon emission is similar to that of the BFKL evolution, because the gluons emitted by the dipoles do not obey strong k_T ordering [14]. In MEPS the partons are strongly ordered in k_T, because they are based upon leading log DGLAP parton showers.

Two samples of one million events have been generated using the latest versions of the models (Lepto 6.5 for MEPS, Ariadne 4.08 for CDM) with the parton density parametrisation GRV-94 [15]. The fraction of events undergoing a Soft Colour Interaction [16] was set to zero. In all figures the transverse momentum transfer was calculated using final state hadrons, including neutrals so that transverse momentum is conserved for the full event.

Figure 3 shows the variance of the transverse momentum transfer, $D^2(y)$, scaled to the average hadron transverse momentum squared in different bins of Q^2 and x. Large differences between models are observed at low x. Because the variance is scaled, this effect is not due to the larger p_T-flow produced by Ariadne. As expected, the transverse momentum transfer fluctuations are larger in the unordered scenario.

The correlation function, computed as the ratio $D^2(y_1, y_2)/D(y_1)D(y_2)$, is shown in Fig. 2. The correlation length (shown in Fig. 4) is calculated along slices perpendicular to the diagonal $y_1 = y_2$. Within these slices the correlation decreases exponentially with $|y_1 - y_2|$ and the correlation length is defined as the slope of an expontial fit. The models used in this simulation show no strong dependence of the correlation length on x or Q^2. Again, differences are observed in the behaviour of the correlation length between the Lepto and Ariadne Monte Carlo models. In particular, Ariadne predicts a symmetric behaviour in the proton and photon fragmentation hemisphere with a double-peaked structure for the correlation length, while Lepto predicts longer correlation lengths in the photon fragmentation hemisphere.

4 Conclusion

It has been demonstrated that the transverse momentum transfer correlation is a theoretically attractive variable to discriminate between models for QCD evolution in parton cascades. The two considered models, Lepton and Ariadne, show large differences in the behaviour of the transverse momentum transfer fluctuation and correlation length. However, further work is needed to investigate the feasibility of an experimental measurement.

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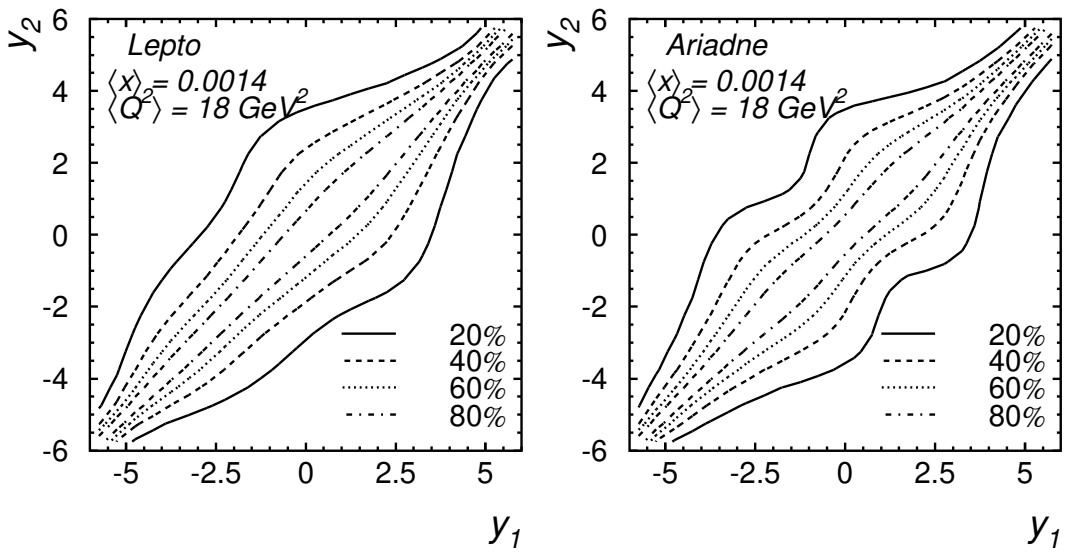

Figure 2: The two-dimensional correlation function, defined as the ratio $D^2(y_1, y_2)/D(y_1)D(y_2)$, is shown as a contour graph for Lepto (top) and Ariadne (bottom).

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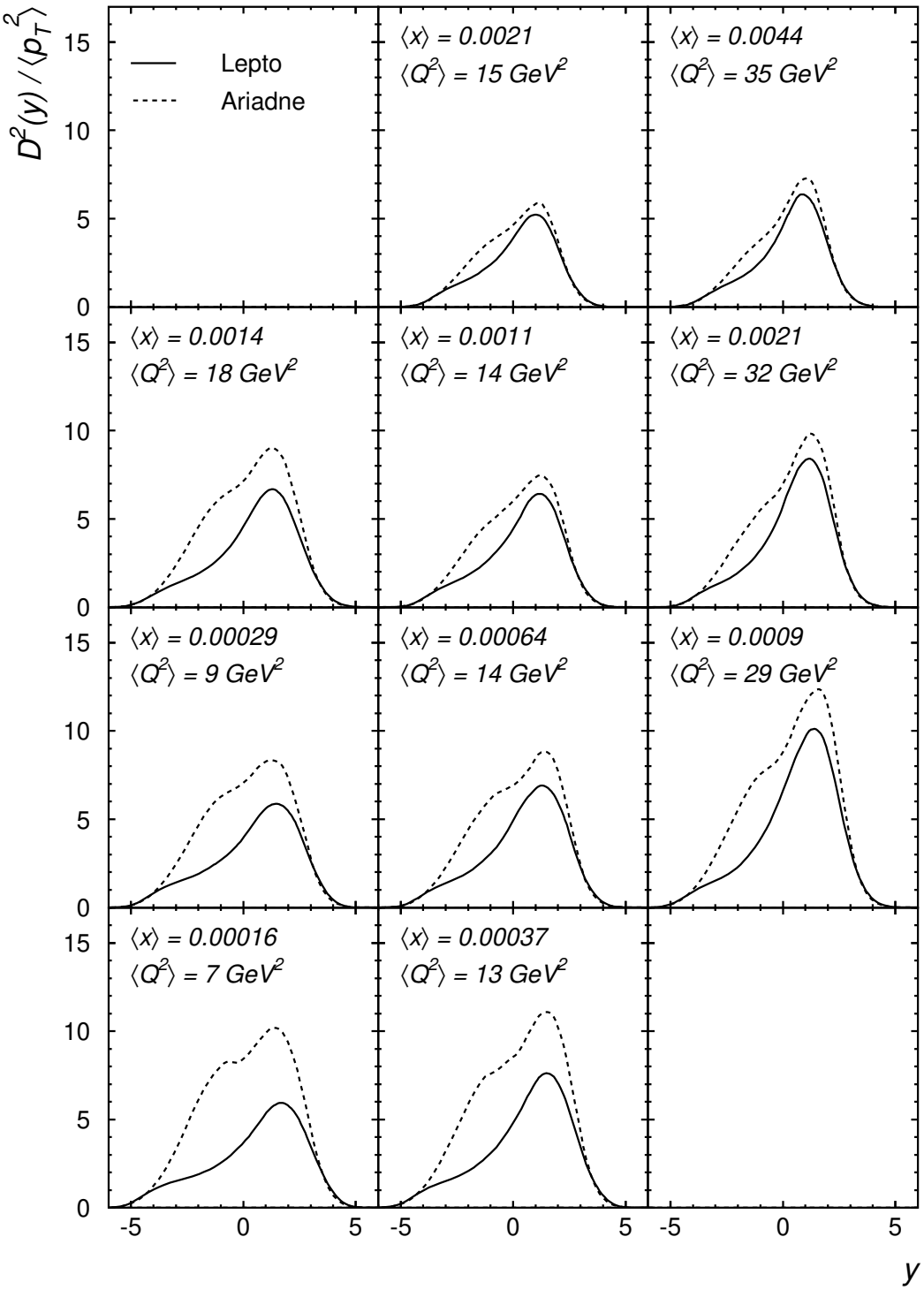

Figure 3: The fluctuation of the transverse momentum transfer, D^2, scaled to the average transverse momentum squared, $\langle p_T^2 \rangle$, is shown as a function of rapidity y in different bins of Q^2 and x for Lepto (full line) and Ariadne (dashed line).

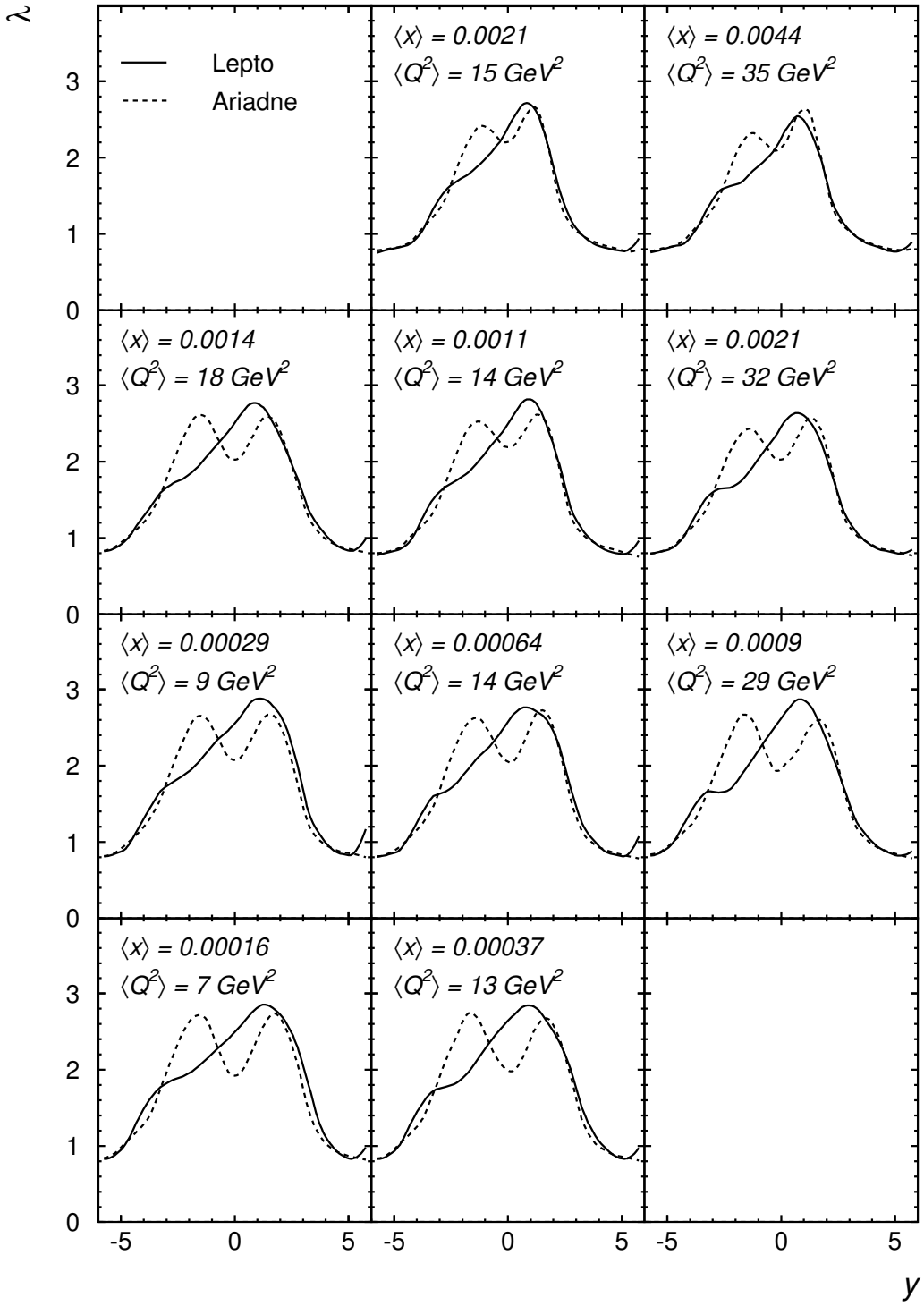

Figure 4: The correlation length, λ, defined as in the text, is shown as a function of rapidity y in different bins of Q^2 and x for Lepto (full line) and Ariadne (dashed line).