FINAL-STATE PROPERTIES IN THE LDC MONTE CARLO

Leif Lönnblad
Dept. of Theoretical Physics, Sölvegatan 14A,
S-223 62 Lund, Sweden
E-mail: leif@thep.lu.se

The Linked Dipole Chain model and Monte Carlo is presented and the resulting final-state properties in DIS is compared to other programs and to data.

1 Introduction

To leading logarithmic accuracy, the CCFM equation [1] is currently the only known way to correctly describe exclusive final-state properties in deeply inelastic lepton–hadron scattering. It achieves this by carefully considering cancellations between real and virtual contributions to the total cross-section and by limiting the splittings which contributes to the cross-section – the initial-state bremsstrahlung (ISB) – to an angular ordered region, treating all other splittings as final-state bremsstrahlung (FSB). It can be shown that the CCFM evolution equations reduces to the DGLAP [2] and BFKL [3] ones in the relevant limits of high Q^2 and small x respectively.

It has turned out, however, that it is highly non-trivial to implement the CCFM evolution in an event generator, and it has therefore been very difficult to compare the CCFM description of final-state properties with experimental result. The problem is related to the so-called non-Sudakov form factor, which multiplies each splitting

$$\Delta_{\mathrm{NS}}^{\mathrm{CCFM}} = \exp(-\bar{\alpha}\ln\frac{1}{z_i}\ln\frac{k_{i\perp}^2}{z_i q_{i\perp}^2}) \qquad (1)$$

and is a function, not only of the energy sharing variable z_i and the transverse energy $q_{i\perp}$ of the emitted gluon in each splitting, but also the transverse momentum of propagator after the emission - i.e. the vectorial sum of the transverse momenta of all previous emissions. This makes a straight-forward implementation very complicated and inefficient.

The linked dipole chain (LDC) model[4] is a reformulation of the CCFM evolution which solves this problem by redefining the separation between initial- and final-state bremsstrahlung. In the following a brief description of the LDC model and its implementation in the event generator program LDCMC [5] is given together with some comparisons to other event generators and to experimental data from HERA.

2 The LDC Model

In the CCFM evolution the division between initial- and final-state emission is defined by requiring subsequent ISB splittings (from the hadron side) to have increasing angles. Looking in the γ-hadron rest frame, the incoming hadron has momentum $(P^+, 0, \vec{0})$ in light-cone coordinates and the incoming virtual photon has momentum $(-xP^+, Q^-, \vec{0})$, with $Q^2 = xP^+Q^-$, $W^2 = (1-x)P^+Q^-$. Denoting the propagators and emitted gluons k_i and q_i respectively counting from the hadron, the angular ordering requirement can be written

$$k_{i\perp}^2 > z_i q_{i\perp}^2, \quad \text{with} \quad z_i = k_i^+/k_{i-1}^+. \tag{2}$$

It is clear that one can choose other divisions between ISB and FSB. Some requirements must be fulfilled, however. Firstly, emissions allowed as ISB must not be allowed as FSB and vice versa to avoid double counting. Conversely, no kinematically allowed emission should be excluded from both ISB and FSB. Thirdly, the FSB emissions must not disturb the initial state ladder, so that weight of a given ladder is significantly changed. Finally the emissions moved from the initial state must be summed over and the weight for the initial state splittings must be modified in order not to change the total cross-section.

The division in the LDC model is to define as FSB all emissions which may be emitted as dipole radiation (according to the colour dipole model [6]) from the dipole spanned by two consecutive gluons emitted in the initial state. The two first requirements above are then met by construction. The third requirement is achieved by limiting the transverse momentum of a gluon emitted as FSB to be less than that of the propagator between the ISB gluons making up the radiating dipole.

The new limit on ISB can be written $q_{i\perp}^2 > \min(k_{i\perp}^2, k_{i-1\perp}^2)$ and it is clear that more radiation is considered FSB in LDC as compared to the CCFM model. Summing up these emissions and reweighting the initial-state splittings, it turns out that the non-Sudakov factor in Eq. (1) is completely cancelled giving the corresponding factor in LDC

$$\Delta_{\mathrm{NS}}^{\mathrm{LDC}} = 1. \tag{3}$$

In other words, the factor in Eq. (1) can be considered as the no-emission probability in the phase-space region allowed by CCFM but disallowed by LDC.

Each splitting in the LDC model now contributes with a weight $d\omega_i \approx \frac{\bar{\alpha}}{\pi}\frac{dz_i}{z_i}\frac{d^2 q_{i\perp}}{q_{i\perp}^2}$, with $\bar{\alpha} = 3\alpha_s/\pi$, and integrating over the azimuth angle gives

$$d\omega_i \approx \bar{\alpha}\frac{dz_i}{z_i}\frac{dk_{i\perp}^2}{k_{i\perp}^2}\min(1, \frac{k_{i\perp}^2}{k_{i-1\perp}^2}). \tag{4}$$

One can now identify four distinct cases of three consecutive propagators:

- $k_{1\perp} < k_{2\perp} < k_{3\perp}$: DGLAP-like with DGLAP weight: $d\omega_2 d\omega_3 \propto \frac{1}{k_{2\perp}^2}\frac{1}{k_{3\perp}^2}$.
- $k_{1\perp} < k_{2\perp} > k_{3\perp}$: Gives the weight $d\omega_2 d\omega_3 \propto \frac{1}{k_{2\perp}^4}$ – a hard sub-collision with momentum transfer $\hat{t} \approx -k_{2\perp}^2$.
- $k_{1\perp} > k_{2\perp} < k_{3\perp}$: $d\omega_2 d\omega_3 \propto \frac{1}{k_{1\perp}^2 k_{3\perp}^2}$ has no $\frac{1}{k_{2\perp}^2}$ pole and is infrared safe.
- $k_{1\perp} > k_{2\perp} > k_{3\perp}$: DGLAP-like from photon side: $d\omega_2 d\omega_3 \propto \frac{1}{k_{1\perp}^2}\frac{1}{k_{2\perp}^2}$.

It is clear that the formalism is symmetric w.r.t. evolution from the photon and from the hadron side, and there is a very smooth transition from DIS to photo production for decreasing Q^2. In the program it is not completely symmetric, however. As an example, the case where a propagator drops below the infrared cutoff, only the perturbative system close to the photon is generated, and everything else is considered to be a part of the input parton density of the hadron.

3 The LDC Monte Carlo

Just as the CCFM formalism, the LDC model only considers $g \to gg$ splittings to leading logarithmic accuracy. To implement the model in an event generator it is necessary to also include quark splittings and some other non-leading terms. The LDC Monte Carlo is described in detail elsewhere [5]. Some of the main issues are the inclusion of (massive) quarks and sub-leading terms by using AP splitting functions and full $2 \to 2$ matrix elements (where appropriate), running $\alpha_s(q_\perp^2)$, conservation of energy and momentum at each vertex, the correction of the vertex closes to photon with $\mathcal{O}(\alpha\alpha_s)$ matrix elements and the interface to Ariadne [7] to get final-state emissions, and to Jetset [8] for hadronization.

It should be noted that the cancellation of the non-Sudakov form factor in Eq. (3), although formally only proven for purely gluonic leading-log chains, is assumed to hold also after inclusion of non-leading effects.

It is clear that the activity of the final state is dependent on where the evolution starts. Starting at large x_0 for small x, there is a large rapidity interval where perturbative radiation is allowed. It is therefore necessary to fix the input parton densities used. Parameterizations of these can be fitted to the total cross section, and with most other parameters fixed from e^+e^- data, predictions for the hadronic final-states in DIS can be made. In the current implementation it is only possible to fit to F_2 data, and so far only a crude four-parameter fit has been made [5] with an acceptable $\chi^2 \sim 1.4$/D.o.F.

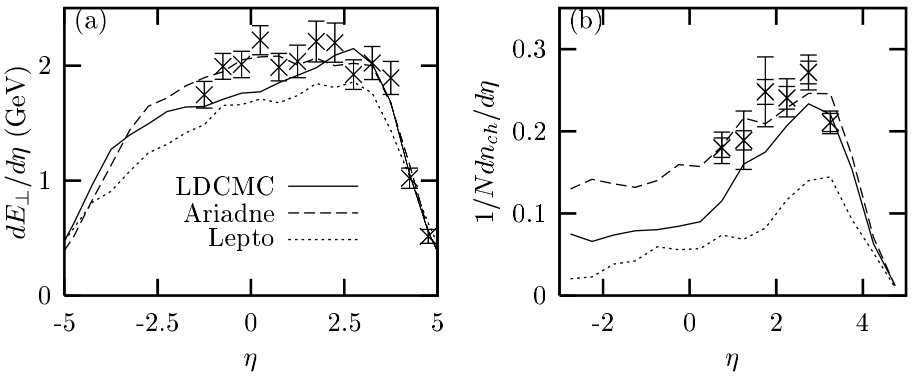

Figure 1: Comparison of the event generators LDCMC, Ariadne and Lepto [9] (all with default settings) to data from H1. In (a) is shown the transverse energy flow as a function of pseudo-rapidity [10] for events with $0.0001 < x < 0.0002$, $5 < Q^2/\mathrm{GeV}^2 < 10$. In (b) is shown the flow of charged particles with transverse momentum larger than 1 GeV as a function of pseudo-rapidity [11] for events with $0.0002 < x < 0.0005$, $6 < Q^2/\mathrm{GeV}^2 < 10$.

4 Comparison with data from HERA

Armed with parameterizations of the input parton densities, it is now possible to compare our model with data from HERA. In Fig. 1 two typical distributions is shown. One is the $E_\perp$-flow as a function of pseudo-rapidity η in the hadronic rest frame for a bin with small x and Q^2, and the other is the flow of charged particles with transverse momentum larger than 1 GeV in the hadronic rest frame, also as a function of η. The $E_\perp$ flow is rather well reproduced by LDCMC, while there are not enough high-$p_\perp$ particles in the forward region. Comparing with other event generators, Ariadne and Lepto, LDCMC is typically a bit closer to data than is Lepto, but not as close as Ariadne.

5 Conclusions and outlook

It is clear that the LDCMC gives more perturbative activity in the forward region than DGLAP based models such as Lepto, but not as much as is indicated by data. It is not possible, however, to say that this is a failure of the LDC model as such, or indeed of the CCFM formalism, since there are so many additional assumption entering in the event generator besides the underlying model. These assumptions have been investigated to some extent [5], among other things the results were found to be insensitive to the details in the fitting of input parton densities, variations in the infrared cutoff, and to variations of the cutoff in the splitting variable z, which together with a Sudakov form factor is used to regularize the $1/(1-z)$ pole in some splitting functions.

There are still some things that need to be investigated. Among other things, the Sudakov form factor above is currently only treated approximately, and better approximations are being tried out. Also there may be some problems with the implementation of the $2 \to 2$ matrix elements used to correct splittings corresponding to local sub-collisions in photo production-like events. One problem here is to correctly treat terms corresponding to different colour configurations of the outgoing partons which influences the the way the the final-state dipoles are connected. Here it could also be interesting to introduce colour *reconnections* to investigate the possibilities of obtaining events with large rapidity gaps.

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