Correcting the Dipole Cascade

with fixed order

QCD Matrix Elements

- The problem
- A new algorithm
- Some results
- Conclusions and Outlook

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The best of both worlds: ME+PS

We want to build a generator which uses *both* MEs, for the correlations between well separated jets, and PS for the inter- and intra-jet structure. Start with MEs, add on PS.

This is not trivial

Implementations exist for e.g. $\mathrm{e^+e^-} \rightarrow 3$ jets, $\mathrm{ep} \rightarrow 2+1$ jets and $\mathrm{pp} \rightarrow \mathrm{W}+1$ jet, but no satisfactory implementation for higher order MEs exists.

There are some recent suggestions for a general procedure. The best one is from Webber[†] et al.

[†]Catani, Krauss, Kuhn, Webber hep-ph/0109231

The problem

Let's look at $\mathrm{e}^+\mathrm{e}^- \to n$-jets to $\mathcal{O}(\alpha_{\mathrm{s}}^2)$. We need a resolution scale Q_0 to separate the ME and PS.

We can generate inclusive 2-, 3- and 4-parton states with a ME generator. We can the add on PS below Q_0, but PS are exclusive:

PS expects e.g. a 4-parton state modified with the probability that there was no other emission above Q_0.

Enter the Sudakov form factor.

The n-jet rates to $\mathcal{O}(\alpha_s^2)$ according to the ME:

$$
\begin{aligned}
R_2(Q,Q_0) &= 1+\alpha_s C_{0,1}(Q,Q_0)+\alpha_s^2 C_{0,2}(Q,Q_0),\\
R_3(Q,Q_0) &= \alpha_s C_{1,1}(Q,Q_0)(1+\alpha_s C_{1,2}(Q,Q_0)),\\
R_4(Q,Q_0) &= \alpha_s^2 C_{2,2}(Q,Q_0),
\end{aligned}
$$

The $C_{n,m}$ are divergent as $Q_0 \to 0$. R_3 becomes negative. Q_0 should be small to take advantage of ME, but it can't be too small.

In a PS, the large logarithms in $C_{n,m}$ are resummed to all orders in (modified) leading log accuracy (MLLA):

$$\begin{aligned} R_2(Q,Q_0) &= \mathbf{\Delta}_{\mathrm{S}_2}(Q,Q_0), \\ R_3(Q,Q_0) &= \alpha_{\mathrm{s}} C^{\mathrm{PS}}_{1,1}(Q,Q_0)\mathbf{\Delta}_{\mathrm{S}_3}(Q,Q_0), \\ R_4(Q,Q_0) &= \alpha_{\mathrm{s}}^2 C^{\mathrm{PS}}_{2,2}(Q,Q_0)\mathbf{\Delta}_{\mathrm{S}_4}(Q,Q_0). \end{aligned}$$

The Sudakov form factors, $\mathbf{\Delta}_{\mathrm{S}_\mathrm{n}}(Q,Q_0)$, corresponds to a no-emission probability and can be written:

$$\mathbf{\Delta}_{\mathrm{S}_\mathrm{n}}(Q,Q_0) = e^{-\alpha_{\mathrm{s}} \int_{Q_0}^{Q} dq \Gamma_n(q)}.$$

We can expand the Sudakovs:

$$
\begin{aligned}
R_2(Q,Q_0) &= 1+\alpha_{\mathrm{s}}C^{\mathrm{PS}}_{0,1}(Q,Q_0)+\alpha_{\mathrm{s}}^2C^{\mathrm{PS}}_{0,2}(Q,Q_0)+\ldots,\\
R_3(Q,Q_0) &= \alpha_{\mathrm{s}}C^{\mathrm{PS}}_{1,1}(Q,Q_0)(1+\alpha_{\mathrm{s}}C^{\mathrm{PS}}_{1,2}(Q,Q_0)+\ldots),\\
R_4(Q,Q_0) &= \alpha_{\mathrm{s}}^2C^{\mathrm{PS}}_{2,2}(Q,Q_0)(1+\alpha_{\mathrm{s}}C^{\mathrm{PS}}_{2,3}(Q,Q_0)+\ldots).
\end{aligned}
$$

We want to use the full matrix element, but correct with the Sudakovs replacing the first few terms with the full fixed order virtual terms.

Somewhat less ambitious: Take the real emissions from the full MEs, but the resummed virtual corrections from the PS.

In fact we need the differential version of:

$$\begin{aligned} R_2(Q,Q_0) &= \Delta^{\rm PS}_{\rm S_2}(Q,Q_0), \\ R_3(Q,Q_0) &= \alpha_{\rm s} C^{\rm ME}_{1,1}(Q,Q_0)\Delta^{\rm PS}_{\rm S_3}(Q,Q_0), \\ R_4(Q,Q_0) &= \alpha_{\rm s}^2 C^{\rm ME}_{2,2}(Q,Q_0)\Delta^{\rm PS}_{\rm S_3}(Q,Q_0). \end{aligned}$$

- Choose a parton multiplicity according to tree-level R_n.
- Generate 2-, 3- and 4-parton states according to the full tree-level MEs.
- Reconstruct a possible parton cascade history of the produced states.
- Calculate the Sudakovs and running $\alpha_{\rm s}$.
- Continue with parton cascade.

The new algorithm

Use any tree-level ME generator to generate 2-, . . . , N-parton states. Regularized with some cutoff scale Q_0, given e.g. by the invariant mass of any two partons, or some jet resolution measure.

We want to use the Colour-Dipole cascade Model, CDM as implemented in ARIADNE for the parton cascade.

We use a modified version of the DICLUS jet algorithm to reconstruct a possible sequence of dipole-emissions.

In the CDM all emissions are $2 \to 3$ and all partons are always on-shell. In DICLUS 3 jets are clustered into 2, and all jets are always on-shell.

The evolution/resolution scale in CDM and DICLUS is a Lorentz-invariant $p_\perp$ of one parton w.r.t. two neighbouring ones. This need not be the same as the resolution scale used to regularize the ME generator.

A given partonic state may have several possible dipole-cascade histories. We choose one according to a probability given by a product of the relevant dipole splitting functions.

We have now reconstructed a sequence of intermediate partonic states S_2, ..., S_{n-1} and a corresponding sequence of emission scales $p_{\perp 1}^2$,...,$p_{\perp n-2}^2$.

Webber et al. uses an approximate analytic expression for the Sudakovs, e.g. for a 3-parton state:

$$
\begin{aligned}
\Delta_{S_3}(Q,q,Q_0) &= \Delta_{S_2}(Q,q) \times \Delta_{S_3}(q,Q_0) \\
&= \Delta_{S_\mathrm{q}}(Q,q)\Delta_{S_{\bar{\mathrm{q}}}}(Q,q) \\
&\quad \times\Delta_{S_\mathrm{q}}(q,Q_0)\Delta_{S_{\bar{\mathrm{q}}}}(q,Q_0)\Delta_{S_\mathrm{g}}(q,Q_0) \\
&= \Delta_{S_\mathrm{q}}(Q,Q_0)\Delta_{S_{\bar{\mathrm{q}}}}(Q,Q_0)\Delta_{S_\mathrm{g}}(q,Q_0)
\end{aligned}
$$

The last equivalence assumes that the phase space for emission off the quark is not altered after having emitted one gluon.

We will not only use the reconstructed scales, but also the intermediate states.

The Sudakov veto algorithm

Assume again that we have a 3-parton state from the ME

- Make a trial emission with the dipole cascade from the reconstructed state S_2.
- If the emission was above the reconstructed scale $q = p_{\perp 1}$ then veto the whole state and generate a new one with the ME. The state is kept with a probability $\Delta_{S_2}(Q, q)$.
- Make a trial emission from S_3.
- Veto if it was above Q_0. This gives $\Delta_{S_3}(q, Q_0)$.

The algorithm step by step

1. First the number of partons, $n \leq N$, to be generated is chosen according to the integrated jet rates R_n from the tree-level MEs. Note that $\sum R_n$ in general is larger than 1 since we do not include virtual corrections.

2. Then the momenta of the n partons are generated according to the $\mathcal{O}(\alpha_s^{n-2})$ tree-level ME.

3. Now, all the intermediate states S_2, . . . , S_{n-1} and scales $p_{\perp 1}^2$, . . . , $p_{\perp n-2}^2$ are reconstructed according to the modified DICLUS algorithm.

4. The generated event is kept with a probability given by

$$\frac{1}{\alpha_{s0}^{n-2}} \Pi_{i=1}^{n-2} \alpha_s(p_{\perp i}^2)$$

otherwise goto 1.

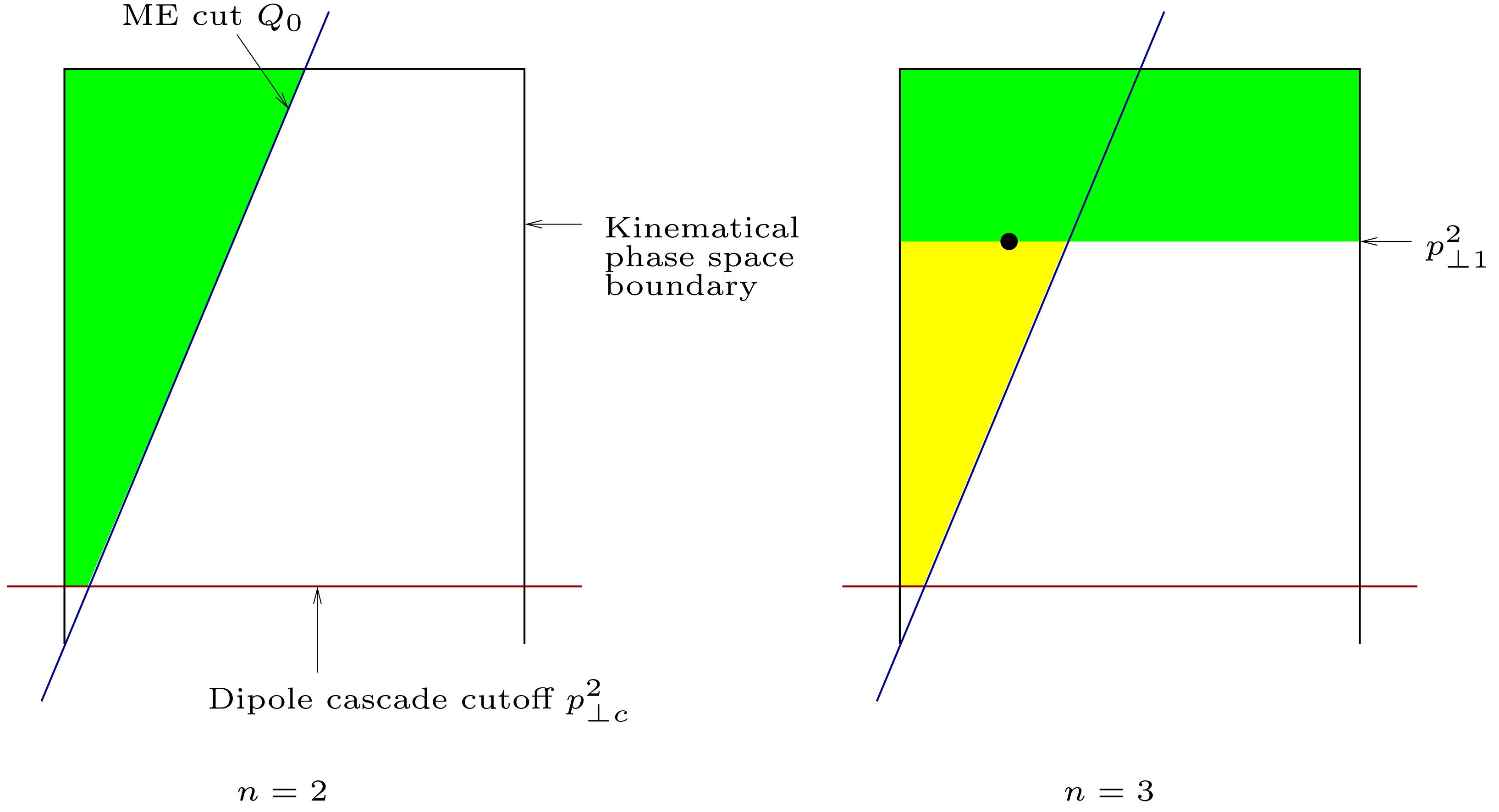
5. We now make a trial emission with the dipole cascade from the state S_2 starting from the maximum scale limited by E_{cm}. If this emission is at a scale above $p_{\perp 1}^2$ and all partons pass the invariant mass cut Q_0, the event is rejected and we restart from step 1. In any case, the trial emission will be discarded.

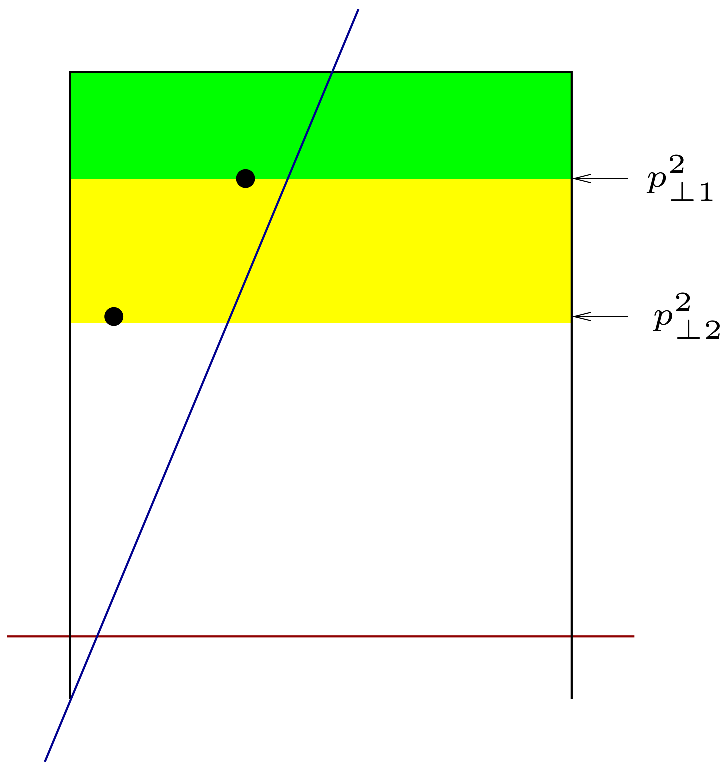
5. (cntd.) If not, a trial emission is performed from the state S_3 starting from the scale $p_{\perp 1}^2$. If this emission is at a scale above $p_{\perp 2}^2$ and all partons pass the invariant mass cut Q_0 the event is rejected and we restart from step 1. This procedure is repeated for all states down to S_{n-1}. If no rejection has been made, a trial emission is made from the ME-generated n-parton state starting from the scale $p_{\perp n-2}^2$. There are now two cases

6a. If $n = N$ the trial emission is always kept and the dipole cascade is allowed to continue down to the cutoff $p^2_{\perp c}$ and the event is accepted.

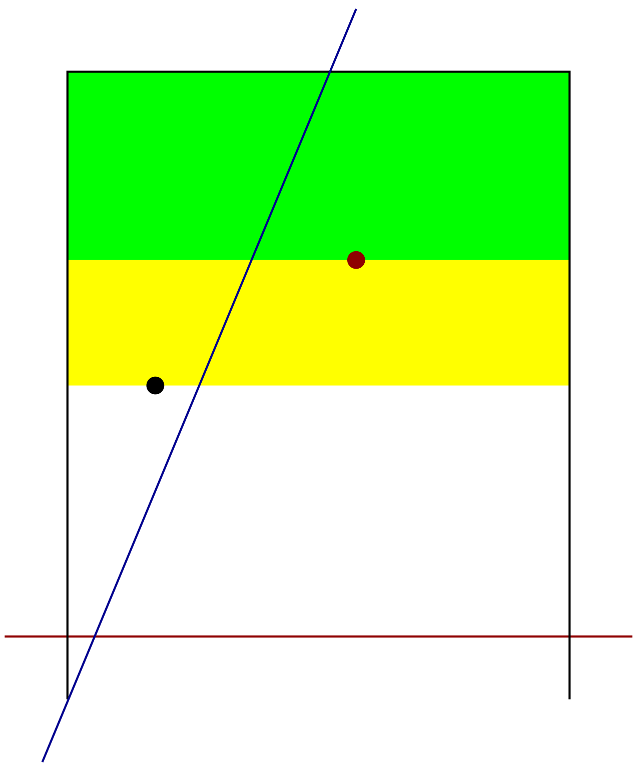
6b. If $n < N$, and all partons passes the ME cut, Q_0, the event is rejected and we restart from step 1. If any of the partons fail the cut, the trial emission is accepted and the dipole cascade is allowed to continue down to the cutoff $p^2_{\perp c}$ and the event is accepted.

Abstract 2-dimensional phase space:





$n = 4$

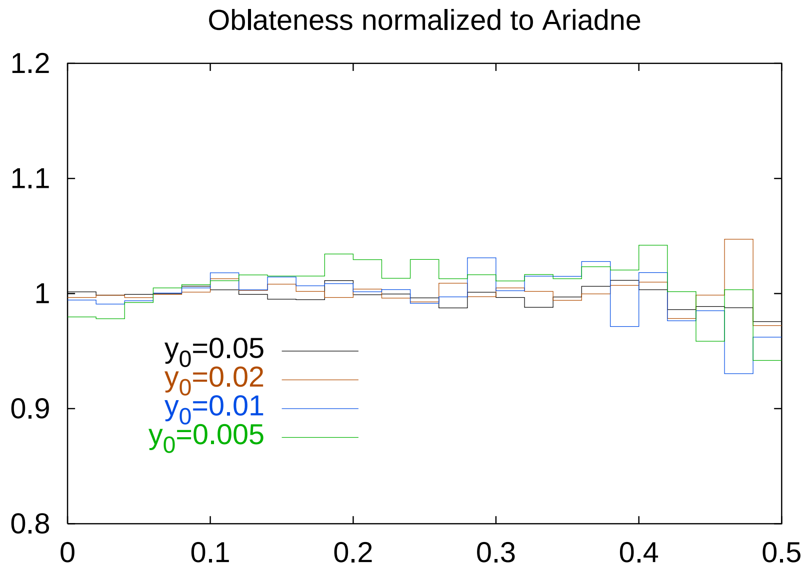


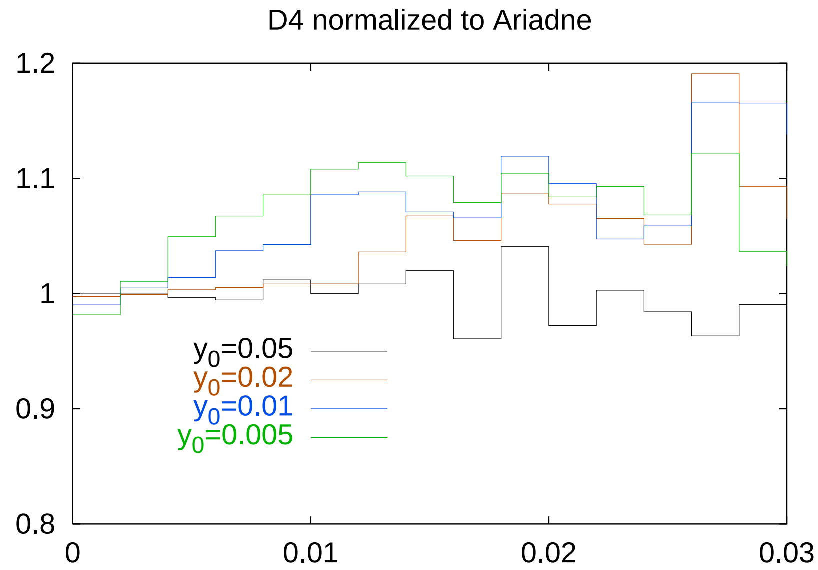
$n = 2$

Some results

It is important that the unphysical dependence on $y_0 = Q_0^2/E_{\rm cm}^2$ cancels. Let's look at standard event shapes. Not compared to data because we need some re-tuning of hadronization parameters.

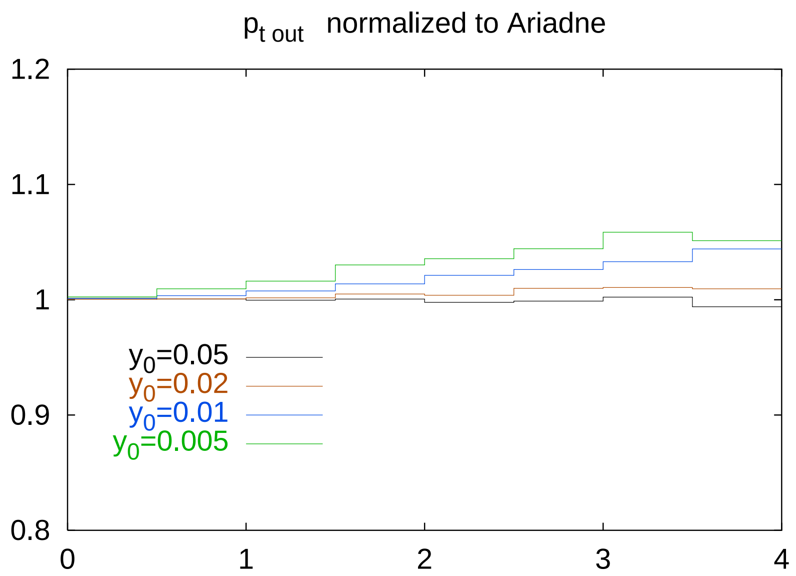
Compare to standard dipole cascade in ARIADNE:

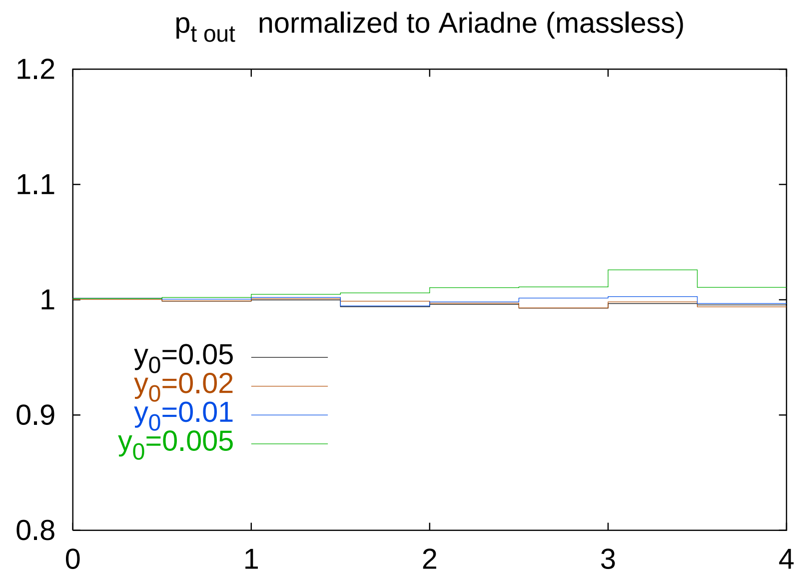




ARIADNE is already corrected for $\mathcal{O}(\alpha_{\rm s})$ ME, so we look at 4-jet related distributions.

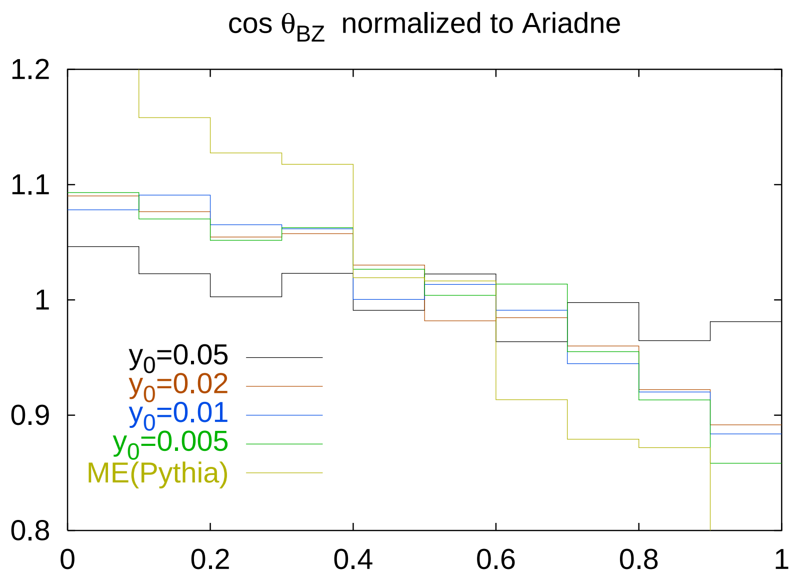
The dependence on y_0 is very small. Compared to hadronization uncertainties it is nothing. But it does not seem to go away.





The problem has to do with massive quarks – we know that both the ME generator in PYTHIA and the cascade in ARIADNE have shortcomings here. Comparison with fully massive CompHEP is under way.

When looking at 4-jet angular correlations we want things to change:



Not exactly like the pure ME result. But we expected some smearing from the parton cascade.

No problem with y_0 dependence – $y_c = 0.03$ for the jet reconstruction.

Conclusions and outlook

- A general method for interfacing QCD ME generators with parton cascades.
- A non-intrusive interface as far as ME generator is concerned
- It works.(although a bit slow for small Q_0)
- Key ingredients are reconstruction of the intermediate states and correcting MEs with Sudakov form factors.
- So far only for $\mathrm{e}^+\mathrm{e}^-\to n$-jets, but also incoming hadrons should be possible.
- Inclusion of exact $\mathcal{O}(\alpha_\mathrm{s}^n)$ virtual corrections are difficult but possible.

$$
\begin{aligned}
R_2(Q,Q_0) &= \mathbf{\Delta}_{\mathrm{S}_2}(Q,Q_0) + \alpha_{\mathrm{s}}\delta C_{0,1}^{\mathrm{MEPS}}(Q,Q_0) + \alpha_{\mathrm{s}}^2\delta C_{0,2}^{\mathrm{MEPS}}(Q,Q_0), \\
R_3(Q,Q_0) &= \alpha_{\mathrm{s}} C_{1,1}(Q,Q_0)(\mathbf{\Delta}_{\mathrm{S}_3}(Q,Q_0) + \alpha_{\mathrm{s}}\delta C_{1,2}^{\mathrm{MEPS}}(Q,Q_0)), \\
R_4(Q,Q_0) &= \alpha_{\mathrm{s}}^2 C_{2,2}(Q,Q_0).
\end{aligned}
$$

where $\delta C_{n,m}^{\mathrm{MEPS}} = C_{n,m} - C_{n,m}^{\mathrm{PS}}$. The δC coefficients are free from singularities and the jet rates would come out finite and positive unless Q_0 becomes too small.

This would break the nice separation between ME and PS.