FYTN13: Part II Assignment

To be submitted by May 9th, before 5 pm

Justify all your steps.

Question 1 [**3p**] We have seen the Baker-Campbell-Hausdorff formula

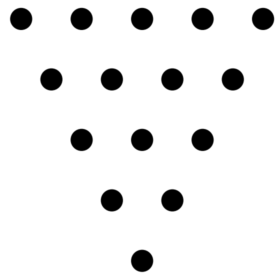
$$e^a e^b = e^{a+b+\frac{1}{2}[a,b]+\cdots}\ .$$

Assume that a and b are small, $a = \epsilon A$, $b = \epsilon B$ and prove that the Baker-Campbell-Hausdorff formula holds to order ϵ^2.

Question 2 [**7p**] Consider the tensor product of two spin-$\frac{1}{2}$ representations of $SU(2)$, $D^{(1/2)} \otimes D^{(1/2)}$.

- **(a)**[**1p**] Find the Clebsch-Gordan series for the tensor product, i.e., state the possible overall representations in $D^{(1/2)} \otimes D^{(1/2)}$.
- **(b)**[**1p**] Draw the weight diagram. Explicitly include different linearly independent states which can result in each eigenvalue m of the total J_z operator.
- **(c)**[**1p**] Find the quadratic Casimir operator, J^2, in the representation given by the direct product, i.e., before diagonalizing.
- **(d)**[**1p**] Find the highest weight state of the largest representation, and use the step operator J_- to find the other states within that representation.
- **(e)**[**1p**] Find the state(s) in the other representation(s).
- **(f)**[**1p**] Derive the matrix (a matrix) which diagonalizes J^2.
- **(g)**[**1p**] Verify that the diagonalized version of J^2 has the expected form.

Question 3 [4p] Consider the 15-plet state in QCD with weight diagram



$(4,0)$.

Find the Clebsch-Gordan series of $\mathbf{15} \otimes \mathbf{3}$, i.e., find the representations in the direct sum. (You are not asked to write down the corresponding states.)

Question 4 [6p] The Poincaré group is the group of Minkowski spacetime isometries. For non-relativistic scenarios the corresponding group is the Galilei group.

(a)[2p] Find all continuous group transformations of the Galilei group. How may generators are needed?

(b)[2p] Which quantities are preserved by the group transformations?

(c)[2p] Just as we did for the Lorentz group, define the $\mathsf{Gal}^{\uparrow}$ to be the proper orthochronous Galilei group. Write all the group transformations as a group element of $\mathsf{GL}(4;\mathbb{R})$. Show that $\mathsf{Gal}^{\uparrow}$ is not semi-simple.