LUND UNIVERSITY

Universal Quantum Mechanics — The Universe as a Quantum Mechanical System

- Quantum mechanics the textbook way
- Axioms of quantum mechanics, the Copenhagen interpretation
- For the universe we have no external observer
- Universal quantum mechanics, what if everything is quantum mechanics?
- Decoherence, deriving classical behavior

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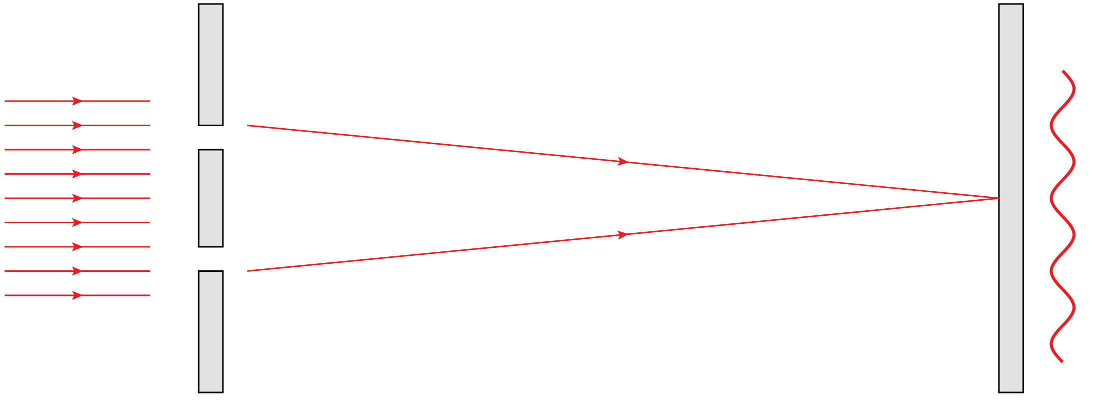
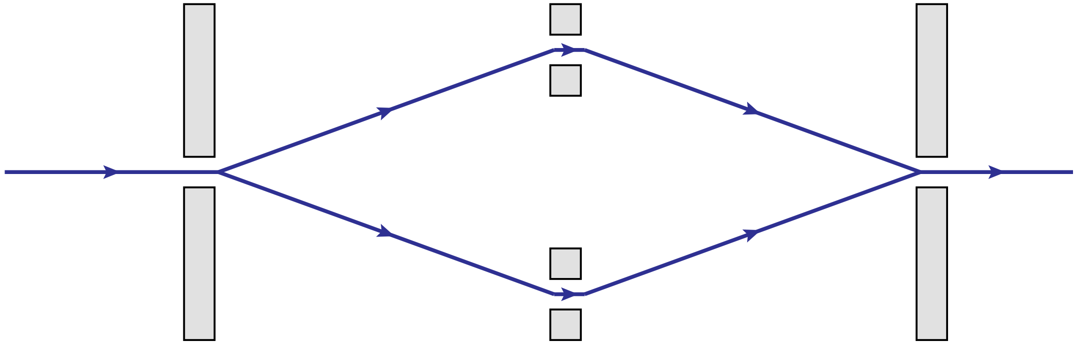
Quantum mechanics - the textbook way

In quantum mechanics courses we are taught that the world sometimes behaves in a quantum mechanical way with interference and sometimes in a classical way without interference.

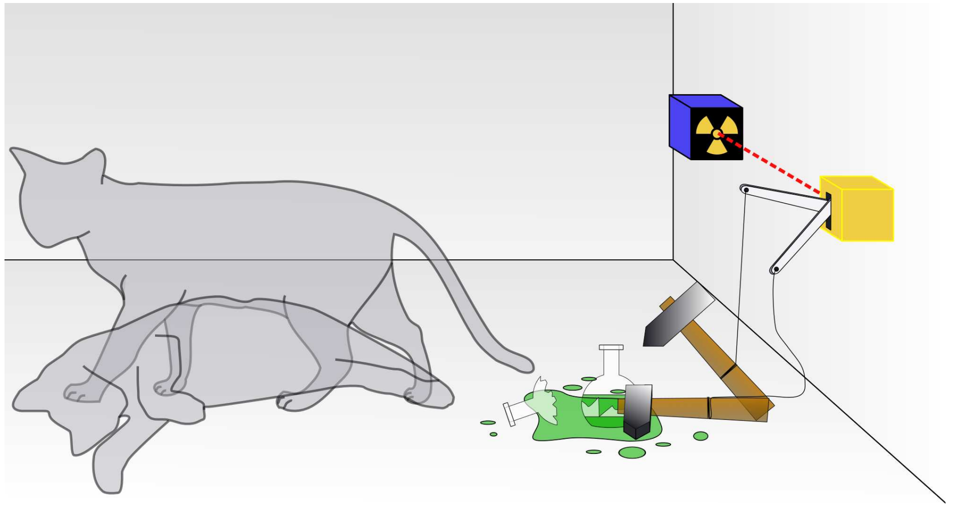
- A famous example is the double slit experiment where light may pass through two narrow slits and is detected on a screen on the other side. As long as it is not measured which way light takes the screen will display an interference pattern, but as soon as it is known which slit the light passes, the pattern disappears.



- Another famous example is the Stern-Gerlach experiment. Here a spin 1/2 particle, which can have either spin up or spin down, is forced by magnets to move along two different paths depending on its spin. As long as it is not measured which path the particle takes the two component can be brought together and give rise to interference, but once the path is known, the interference disappears.



- A third example, chosen to illustrate how absurd quantum mechanics is, is Schrödinger's cat. In an gedankenexperiment a cat is placed in a container with some radioactive substance that has 50 % chance of decaying within the time the box is kept closed. If there is a decay, a toxic container will open, and the cat will die. If not, the cat will still be alive when the box is opened. It is claimed that the cat is both dead and alive until the box has been opened.



The Axioms of Quantum Mechanics

A common set of axioms for quantum mechanics is

(1) Hilbert space: The properties of a quantum mechanical system are completely defined by its state vector $|\psi\rangle$. The state vector is a (normalized) element of a complex Hilbert space (think vector space with scalar product). If two systems are described by Hilbert spaces $\mathcal{H}_\alpha$, $\mathcal{H}_\beta$ the composite systems is described by the tensor product $\mathcal{H}_\alpha \otimes \mathcal{H}_\beta$.

(2) Unitary evolution: The evolution of a closed system is unitary. The state vector $|\psi(t)\rangle$ at time t is derived from the state vector $|\psi(t_0)\rangle$ at time t_0 by applying a unitary operator $U(t, t_0) = \exp(-iHt)$, for some Hermitian operator H, known as the Hamiltonian, $|\psi(t)\rangle = U(t, t_0)\,|\psi(t_0)\rangle$.

(3) Observable: The expectation value of an observable (spin, position, etc) is obtained from an Hermitian operator A. The set of possible outcomes is the set of eigenvalues of A.

(4) Wave function collapse: After the measurement the quantum mechanical state is collapsed to the component corresponding to the measured eigenvalue.

(5) Born rule: The probability for finding a system in state $|\psi_\lambda\rangle$, corresponding to eigenvalue λ, is given by $|\psi_\lambda|^2$. (*Together* with the unitary evolution this means that probability is conserved.)

The Copenhagen interpretation

According to the standard, so-called Copenhagen interpretation of quantum mechanics this is how the world works: There is a unitary evolution according to the Schrödinger equation (or corresponding), until a measurement is done, whereupon the wave function collapses into one state. This idea relies on the existence of an observer performing the measurement and the existence of a collapse of the wave function. The universe is split into a classical and a quantum mechanical regime.

A Quantum Mechanical Universe

- Let's assume that the universe is all that exists, i.e., we have no external observer (no god, no meta-physicist sitting outside the universe, no external observer)
- But this means that the universe as a whole is a *closed system*
- Let's go back to the axioms...
- We find that axiom (4) contradicts axiom (2): According to (2) we should have unitary evolution all the time, but according to (4) we have wave function collapses every now and then when a measurement is made!
- Aside note: Standard quantum mechanics never defines when a measurement takes place ($\rightarrow$ you may conclude it's not a well-defined theory)

- One way out is clearly if there never is any collapse, but instead universal quantum mechanics, i.e., the whole universe is quantum mechanical
- Then we would not have to be concerned about defining when the collapse should happen, because it never happens!
- Instead the whole universe is described by some (gigantic) wave function
- But then we have to explain why we only see one reality, why is Schrödingers cat dead *or* alive? Rather: Why would an observer perceive the cat as dead or alive?

Decoherence

- We would like to *derive* apparent classical behavior from quantum mechanics, i.e., we would like to explain why a spin in the Stern-Gerlach experiment is up *or* down, why Schrödinger's cat is dead *or* alive, why I appear to stand in one place *or* another and not both at the same time, etc.
- For a long time, recall QM is from the early 1900s, it was believed to be impossible, and generations of physicists were taught not to ponder the absurdities of quantum mechanics, to quote Richard Feynman:

  "Do not keep saying to yourself, if you can possibly avoid it, "But how can it be like that?" because you will get "down the drain", into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that."

- Some physicists kept asking these questions
- In the 50's Hugh Everett presented the idea that the whole universe is described by one universal wave function, and that you perceive spins as being say up, and cats as being say alive simply because the particular version of you that you call you is in a component of the wave function where the spin is up. In another component of the wave function another version of you will see spin down

$$
\begin{aligned}
|\text{Universe}\rangle &= \sum_i |\uparrow\rangle \otimes |\text{you}\rangle_i \otimes |\text{all the rest}\rangle_i \\
&+ \sum_j |\downarrow\rangle \otimes |\text{another version of you}\rangle_j \otimes |\text{another version of all the rest}\rangle_j
\end{aligned}
$$

- This theory is known as the relative state formalism, or the universal wave function theory, or the many worlds interpretation

- But saying that this is the case is not enough, we want to *derive* classical probabilities
- Work in this direction was done from the 70's and onward using decoherence (Zeh 1970, Zureck, Tegmark)

- Define the density matrix (operator) for a pure state as $D = |\psi\rangle\langle\psi| = \psi\psi^\dagger$
- Then the expectation value of an observable is given by $\langle A\rangle = \langle\psi| A |\psi\rangle = \sum_{ij} \psi_i^\dagger A_{ij}\psi_j = \sum_{ij} A_{ij}\psi_j\psi_i^\dagger = \sum_{ij} A_{ij}D_{ji} = \mathrm{Tr}[AD]$
- Similarly if we have a composite system, $|\psi\rangle \otimes |\Phi\rangle = |\psi\rangle |\Phi\rangle$, of a small system $|\psi\rangle$ (spin 1/2 particle) and a large system $|\Phi\rangle$ (detector), and want the expectation value of an observable $\mathcal{A} = A \otimes 1$ which only depends on the small system (the spin of the particle)

$$\langle\mathcal{A}\rangle = (\langle\psi| \langle\Phi|)(A \otimes 1)(|\psi\rangle |\Phi\rangle) = \langle\psi| A |\psi\rangle \langle\Phi| 1 |\Phi\rangle = \mathrm{Tr}[AD],$$

i.e., as expected, since the small system is not correlated with the large system, the large system doesn't matter

- But now if we let the different states of the small system be correlated with different states of the large system, s.t.

$$|\psi\rangle |\Phi\rangle \to \sum_k a_k |\psi_k\rangle |\Phi_k\rangle$$

(the sum may for example go over spin up and spin down) then we get for the expectation value

$$\begin{aligned}
\langle \mathcal{A} \rangle &= \left(\sum_k a_k^* \langle \psi_k| \langle \Phi_k| \right) (A \otimes 1) \left(\sum_l a_l |\psi_l\rangle |\Phi_l\rangle \right) \\
&= \sum_{k,l} a_k^* a_l \langle \psi_k| A |\psi_l\rangle \langle \Phi_k| 1 |\Phi_l\rangle = \sum_{k,l,n} a_k^* a_l \langle \psi_k| A |\psi_l\rangle \langle \Phi_k|n\rangle \langle n|\Phi_l\rangle \\
&= \sum_{k,l,n} a_k^* a_l \langle n|\Phi_l\rangle \langle \Phi_k|n\rangle \, \mathrm{Tr}\Big[A |\psi_l\rangle \langle \psi_k| \Big] \\
&= \sum_{k,l,n} a_k^* a_l \langle n|\Phi_l\rangle \langle \Phi_k|n\rangle \sum_i (A |\psi_l\rangle \langle \psi_k|)_{ii}
\end{aligned}$$

$$= \sum_{k,l,n} a_k^* a_l \langle n|\Phi_l\rangle \langle \Phi_k|n\rangle \sum_i \left(A \,|\psi_l\rangle \langle \psi_k|\right)_{ii}$$

- Rearranging terms gives

$$\begin{aligned} \langle \mathcal{A} \rangle &= \sum_i \left(A \left[\sum_n \langle n| \left(\sum_l a_l \,|\psi_l\rangle \,|\Phi_l\rangle \sum_k a_k^* \,\langle \psi_k| \,\langle \Phi_k| \right) |n\rangle \right] \right)_{ii} \\ &\equiv \mathrm{Tr}\left[A \sum_n \langle n| \,\mathcal{D}\, |n\rangle \right] \quad \text{def. of } \mathcal{D} \end{aligned}$$

- Defining the reduced density matrix $D_\psi = \sum_n \langle n| \,\mathcal{D}\, |n\rangle$ we finally get

$$\langle \mathcal{A} \rangle = \mathrm{Tr}(A D_\psi)$$

This is thus the expectation value of the observable A which only depends on the small system. The degrees of freedom of the large system have been summed over.

- Let's now see how this can lead to classical behavior
- Let the small system $|\psi\rangle$ be

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + |\downarrow\rangle \Big)$$

- When correlated with the large system (the measurement apparatus) we get

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + |\downarrow\rangle \Big) \to \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle |\Phi_\uparrow\rangle + |\downarrow\rangle |\Phi_\downarrow\rangle \Big)$$

- From this we get the reduced density matrix (operator)

$$
\begin{aligned}
D_\psi &= \sum_n \langle n| \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle |\Phi_\uparrow\rangle + |\downarrow\rangle |\Phi_\downarrow\rangle \Big) \frac{1}{\sqrt{2}}\Big(\langle\uparrow| \langle\Phi_\uparrow| + \langle\downarrow| \langle\Phi_\downarrow| \Big) |n\rangle \\
&= \frac{|\uparrow\rangle\langle\uparrow|}{2} \sum_n \langle n|\Phi_\uparrow\rangle \langle\Phi_\uparrow|n\rangle + \frac{|\uparrow\rangle\langle\downarrow|}{2} \sum_n \langle n|\Phi_\uparrow\rangle \langle\Phi_\downarrow|n\rangle \\
&+ \frac{|\downarrow\rangle\langle\uparrow|}{2} \sum_n \langle n|\Phi_\downarrow\rangle \langle\Phi_\uparrow|n\rangle + \frac{|\downarrow\rangle\langle\downarrow|}{2} \sum_n \langle n|\Phi_\downarrow\rangle \langle\Phi_\downarrow|n\rangle \\
&= \frac{|\uparrow\rangle\langle\uparrow|}{2} \langle\Phi_\uparrow|\Phi_\uparrow\rangle + \frac{|\uparrow\rangle\langle\downarrow|}{2} \langle\Phi_\downarrow|\Phi_\uparrow\rangle + \frac{|\downarrow\rangle\langle\uparrow|}{2} \langle\Phi_\uparrow|\Phi_\downarrow\rangle + \frac{|\downarrow\rangle\langle\downarrow|}{2} \langle\Phi_\downarrow|\Phi_\downarrow\rangle
\end{aligned}
$$

- In matrix form (in the basis $|\uparrow\rangle, |\downarrow\rangle$)

$$
D_\psi = \frac{1}{2}\begin{pmatrix} \langle\Phi_\uparrow|\Phi_\uparrow\rangle & \langle\Phi_\downarrow|\Phi_\uparrow\rangle \\ \langle\Phi_\uparrow|\Phi_\downarrow\rangle & \langle\Phi_\downarrow|\Phi_\downarrow\rangle \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & \langle\Phi_\downarrow|\Phi_\uparrow\rangle \\ \langle\Phi_\uparrow|\Phi_\downarrow\rangle & 1 \end{pmatrix}
$$

- If $\langle \Phi_\downarrow | \Phi_\uparrow \rangle$ and $\langle \Phi_\uparrow | \Phi_\downarrow \rangle$ were 0, the reduced density matrix would be diagonal and we would get

$$\langle \mathcal{A} \rangle = \mathrm{Tr}(A D_\psi) = \sum_i A_{ii} (D_\psi)_{ii}$$

- Let for example $A = \lambda_\rightarrow |\rightarrow\rangle \langle \rightarrow| + \lambda_\leftarrow |\leftarrow\rangle \langle\leftarrow|$ and recall $D_\psi = (1/2) \times \mathrm{Diagonal}[1,1]$, giving

$$\begin{aligned}
\langle \mathcal{A} \rangle &= \frac{1}{2}\Big(\lambda_\rightarrow \langle \uparrow | \rightarrow \rangle \langle \rightarrow | \uparrow \rangle + \lambda_\rightarrow \langle \downarrow | \rightarrow \rangle \langle \rightarrow | \downarrow \rangle \\
&\quad + \lambda_\leftarrow \langle \uparrow | \leftarrow \rangle \langle \leftarrow | \uparrow \rangle + \lambda_\leftarrow \langle \downarrow | \leftarrow \rangle \langle \leftarrow | \downarrow \rangle \Big) \\
&= \frac{1}{2}\Big(\lambda_\rightarrow \mathrm{Prob}[\rightarrow | \uparrow] + \lambda_\rightarrow \mathrm{Prob}[\rightarrow | \downarrow] \\
&\quad + \lambda_\leftarrow \mathrm{Prob}[\leftarrow | \uparrow] + \lambda_\leftarrow \mathrm{Prob}[\leftarrow | \downarrow]\Big)
\end{aligned}$$

- This has the form of a classical expectation value!

- So if the off-diagonal elements vanish we can *derive* classical behavior!
- $\rightarrow$ we want to understand what happens to $\langle \Phi_\uparrow | \Phi_\downarrow \rangle$ once $|\Phi_\uparrow\rangle$ and $|\Phi_\downarrow\rangle$ describe a large system with many degrees of freedom (such as a detector)
- For each degree of freedom f (place of a particle in the detector, electron shell etc.) there is in general some overlap $|\langle (\Phi_\uparrow)_f | (\Phi_\downarrow)_f \rangle| \leq 1$, but in total the overlap is

$$\langle \Phi_\uparrow | \Phi_\downarrow \rangle \sim \prod_f \langle (\Phi_\uparrow)_f | (\Phi_\downarrow)_f \rangle$$

  where each term has an absolute value ≤ 1 (This is for product states, in general we would have a sum of terms)
- We will find $\langle \Phi_\uparrow | \Phi_\downarrow \rangle \approx 0$ as soon as we have many degrees of freedom (such as in a detector) which are affected by the spin
- $\rightarrow$ We will get classical probabilities!

- In essence what happens is that the degrees of freedom of the small system (spin 1/2) become correlated with so many different degrees of freedom of the large system that the interference (between spin up and spin down) disappears for all practical purposes
- In principle the same argument can be applied to larger objects like Schrödinger's cat, but in this case, a real cat in a real box would already have been sufficiently correlated with the degrees of freedom of the outside world to be either dead or alive before the box has been opened

Cleaning up among the axioms

- If there is no wave function collapse, and classical behavior is something that can be *derived*, then clearly we can throw away the wave function collapse axiom, axiom 4
- But it turns out that we can also get rid of axiom 3, we can derive that observables are given by Hermitian operators (von Neumann)
- On top of that, there are various attempts to derive the Born rule axiom, but they all assume something extra, for example that there are probabilities in the theory (Gleason 1957)

What does the relative state formalism imply?

- The relative state formalism implies that there are separate components of the wave function existing side by side, and if they are "too different" the overlap is so small that they for all practical purposes will never interfere again
- This means that there are components of the wave function living side by side (almost) without interfering, meaning that we do have parallel realities, "many worlds"
- We have assumed a quantum mechanical universe, so this applies to everything inside the universe, for example it applies to you

But this is absurd ...

- Yes, the first times you hear about it, but ...
- So is the fact that not all observers agree on one time in special relativity
- So is the fact that a mirror world would not behave in precisely the same way as our world (parity breaking)
- $\rightarrow$ Using the fact that something seems absurd as a way of arguing that it is false has *not* been a way forward in physics

How should we judge a scientific theory

- In principle a matter of taste
- A common taste is to like Occam's razor: "Among competing hypotheses that predict equally well, the one with the fewest assumptions should be selected."
- But what are the assumptions? Some people say they dislike the many worlds interpretation because it assumes the existence of lots of parallel worlds, which is not minimal.

- Others (me) would disagree with that because the parallel realities are not part of the assumptions, they are merely a consequence, as parts of the wave function of the universe cease to interfere. We do not dismiss the standard model of particle physics for its complex manifestations (plants, cats, humans), we like it because from a small set of assumptions we can derive a lot. Similarly we should not, in the spirit of Occam, dismiss the relative state formalism because of its complex manifestation
- The assumptions are the axioms, and they are simpler and as predictive for the relative state formalism

Conclusion

- One way of reducing the number of axioms and getting rid of the undefined moment in which a measurement takes place is to assume that it's "quantum mechanics all the way up"
- If we want to view the whole universe as one isolated quantum mechanical system we need something like this (no external observer...)
- This will lead to the relative state formalism of quantum mechanics, also known as universal quantum mechanics or the many worlds interpretation