# ColorFull and other tools for color space

- Motivation
- Trace type "bases"
- ColorFull
- Orthogonal bases
- ColorMath
- Conclusions and outlook

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# Motivation

- With the LHC follows an increased demand of accurately calculated processes in QCD
- This is applicable to NLO calculations and resummation
- ...but my perspective is from a parton shower point of view
- First SU(3) parton shower in collaboration with Simon Plätzer JHEP 07(2012)042, arXiv:1201.0260 color structure treated using my C++ ColorFull code

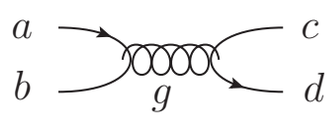
# Dealing with color space

- We never observe individual colors
  $\rightarrow$ we are only interested in color summed/averaged quantities
- For given external partons, the color space is a finite dimensional vector space equipped with a scalar product

$$\langle A, B\rangle = \sum_{a,b,c,\ldots} (A_{a,b,c,\ldots})^* B_{a,b,c,\ldots}$$

Example: If

$$A = \sum_g (t^g)^a{}_b (t^g)^c{}_d =$$



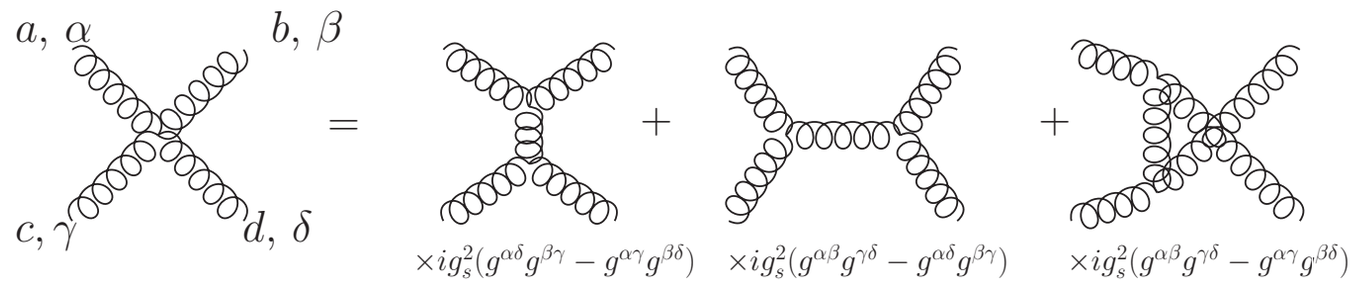

,

then $\langle A|A\rangle = \sum_{a,b,c,d,g,h} (t^h)^b{}_a (t^h)^d{}_c (t^g)^a{}_b (t^g)^c{}_d$

- One way of dealing with color space is to just square the amplitudes as one encounters them
- Alternatively, we may use any basis (spanning set)

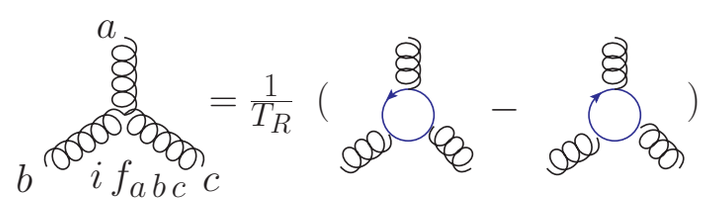
# The standard treatment: Trace bases

- Every 4g vertex can be replaced by 3g vertices:

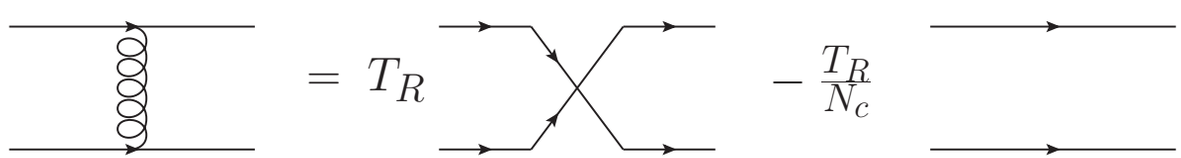



(read counter clockwise)

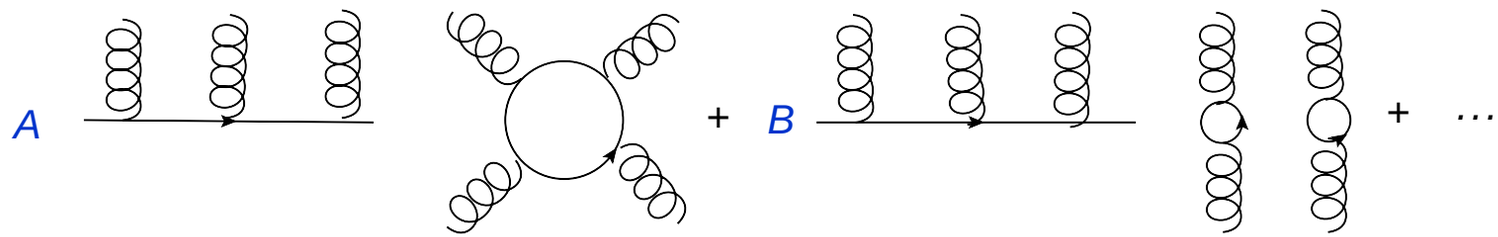
- Every 3g vertex can be replaced using:



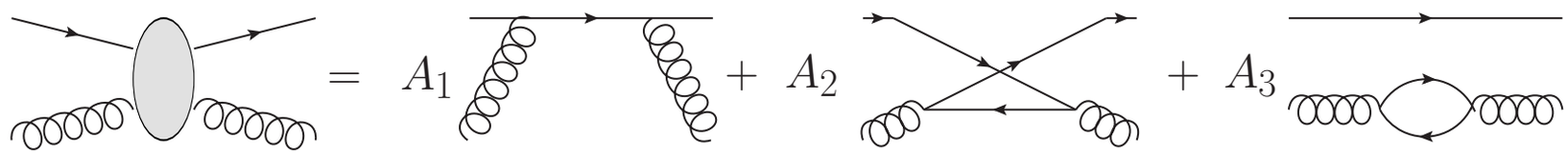

- After this every internal gluon can be removed using:



- This can be applied to any QCD amplitude, tree level or beyond
- In general an amplitude can be written as linear combination of different color structures, like




- For example for 2 (incoming + outgoing) gluons and one $q\bar{q}$ pair

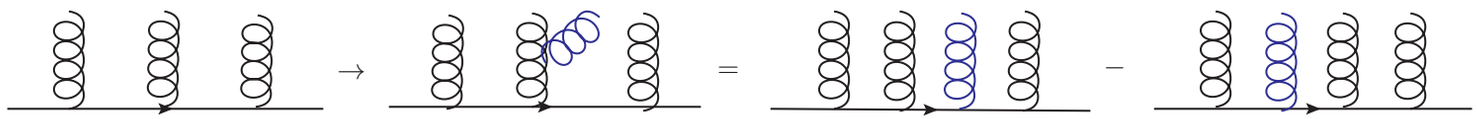



(an incoming quark is the same as an outgoing anti-quark)

The above type of color structures can be used as a spanning set, a trace basis. (Technically it's in general overcomplete, so it is rather a spanning set.)

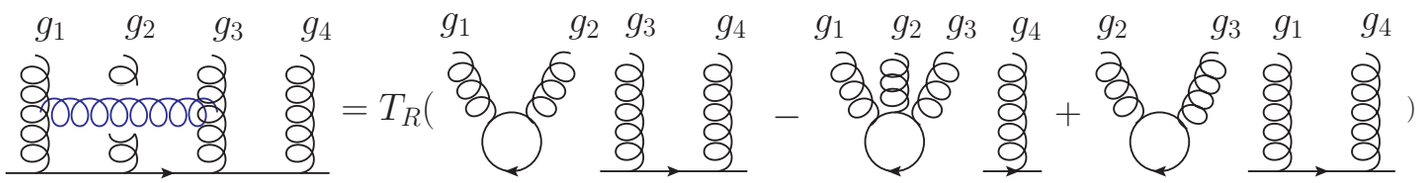
These bases have some nice properties

- The effect of gluon emission is easily described:



Convention: + when inserting after, minus when inserting before.

- So is the effect of gluon exchange:




Convention: + when inserting after, - when inserting before

# ColorFull

For the purpose of treating a general QCD color structure I have written a C++ color algebra code, ColorFull, based on this type or "trace basis".

ColorFull is used in the color shower with Simon Plätzer, JHEP 07(2012)042, arXiv:1201.0260.

ColorFull is interfaced to Herwig++ ($\geq$ 2.7) via Matchbox, and is now publicly available in a **pre-release** version (0.90) at `colorfull.hepforge.org`.

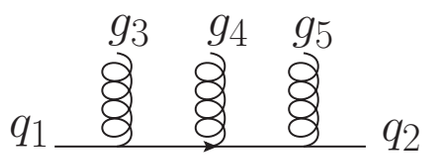
ColorFull can automatically create a "trace basis" for any number and kind of partons, and to arbitrary order in $\alpha_s$. For example we may create a basis for 1 $q\bar{q}$-pair, 3 gluons and 0 loops (in pure QCD):

```
Trace_basis MyBasis(1,3,0);
MyBasis.write_out_Col_basis_to_cout();
MyBasis.write_out_Col_basis("ColorResults/MyBasis");
```

This results in a basis with basis vectors:

```
0     [{1,3,4,5,2}]
1     [{1,3,5,4,2}]
2     [{1,4,3,5,2}]
3     [{1,4,5,3,2}]
4     [{1,5,3,4,2}]
5     [{1,5,4,3,2}]
```

Here [{1,3,4,5,2}]$=(t^{g3}t^{g4}t^{g5})^{q1}{}_{q2}=$



Having a basis one probably wants to calculate the scalar product matrix

```
MyBasis.scalar_product_matrix();
```

The Polynomial and numerical scalar product matrices are then calculated and saved in `MyBasis.P_smp` and `MyBasis.d_spm`

The result is Mathematica readable:

```
{ { 1 Nc CF^(3), 1*-1 TR CF^(2), 1*-1 TR CF^(2),1 TR^(2) Nc^(-1) CF,1 TR^(2) Nc^(-1) CF,(1 TR^(2) Nc CF + 1 TR^(2)
Nc^(-1) CF)}, { 1*-1 TR CF^(2), 1 Nc CF^(3),1 TR^(2) Nc^(-1) CF,(1 TR^(2) Nc CF + 1 TR^(2) Nc^(-1) CF), 1*-1 TR
CF^(2), 1 TR^(2) Nc^(-1) CF}, { 1*-1 TR CF^(2),1 TR^(2) Nc^(-1) CF, 1 Nc CF^(3), 1*-1 TR CF^(2),(1 TR^(2) Nc CF +
1 TR^(2) Nc^(-1) CF), 1 TR^(2) Nc^(-1) CF}, {1 TR^(2) Nc^(-1) CF,(1 TR^(2) Nc CF + 1 TR^(2) Nc^(-1) CF), 1*-1 TR
CF^(2), 1 Nc CF^(3),1 TR^(2) Nc^(-1) CF, 1*-1 TR CF^(2)}, {1 TR^(2) Nc^(-1) CF, 1*-1 TR CF^(2),(1 TR^(2) Nc CF +
1 TR^(2) Nc^(-1) CF),1 TR^(2) Nc^(-1) CF, 1 Nc CF^(3), 1*-1 TR CF^(2)}, {(1 TR^(2) Nc CF + 1 TR^(2) Nc^(-1) CF),1
TR^(2) Nc^(-1) CF,1 TR^(2) Nc^(-1) CF, 1*-1 TR CF^(2), 1*-1 TR CF^(2), 1 Nc CF^(3)} }
```

ColorFull can also square any color amplitudes and calculate any interference,

```
Col_amp Ca1("[{1,3,2}(4,5)]"); // (t^{g3})^{q1}_{q2} Tr(t^{g4} t^{g5})
Col_amp Ca2("[{1,3,4,5,2}]"); // (t^{g3} t^{g4} t^{g5})^{q1}_{q2}
Col_functions Col_fun;
Col_fun.scalar_product(Ca1,Ca1);
Col_fun.scalar_product(Ca1,Ca2);
```

giving the results `TR Nc^2 CF^2` and `TR Nc CF^2`, respectively.

In the context of parton showers we like to be able to describe the effect of gluon emission:

```
Col_fun.emit_gluon(Ca1,3,77);
```

(recall `Ca1=[{1,3,2}(4,5)]` ) resulting in:

```
[{1,3,77,2}(4,5)] + 1*-1[{1,77,3,2}(4,5)]
```

Convention +, when inserting before, - when inserting after

and gluon exchange:

```
Col_fun.exchange_gluon(Ca1,1,4);
```

giving:

```
1 TR[1,4,5,3,2] + 1*-1 TR[1,5,4,3,2]
```

For resummation type calculations one may also like to calculate the soft anomalous dimension matrices. To do so, we need the full basis

```
Trace_basis MyBasisFull(1,3); // Create an all order basis
MyBasisFull.Col_gamma(1,4); // Effect of gluon exchange
```

results in

```
{{ 1 TR Nc, 0, 0, 0, 0, 0, 0, 0, 1 TR, 1 TR, 0},
{ 0, 0, 0, 0, 0, 1*-1 TR, 0, 0, 0, 0, 0}}
{{ 0, 0, 0, 0, 0, 0, 0, 0, 1*-1 TR, 0, 0},
{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1*-1 TR},
{ 0, 0, 1 TR Nc, 0, 0, 0, 0, 0, 1 TR, 0, 1 TR},
{ 0, 0, 0, 1 TR Nc, 0, 0, 1 TR, 0, 0, 1 TR, 0},
{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 1*-1 TR, 0},
{ 0, 0, 0, 0, 0, 0, 1*-1 TR, 0, 0, 0, 0},
{ 0, 0, 0, 0, 0, 1*-1 TR, 0, 0, 0, 0, 0},
{ 0, 1 TR, 0, 0, 1 TR, 0, 0, 1 TR Nc, 0, 0, 0},
{ 1*-1 TR, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{ 0, 0, 0, 0, 1*-1 TR, 0, 0, 0, 0, 0, 0},
{ 0, 1*-1 TR, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

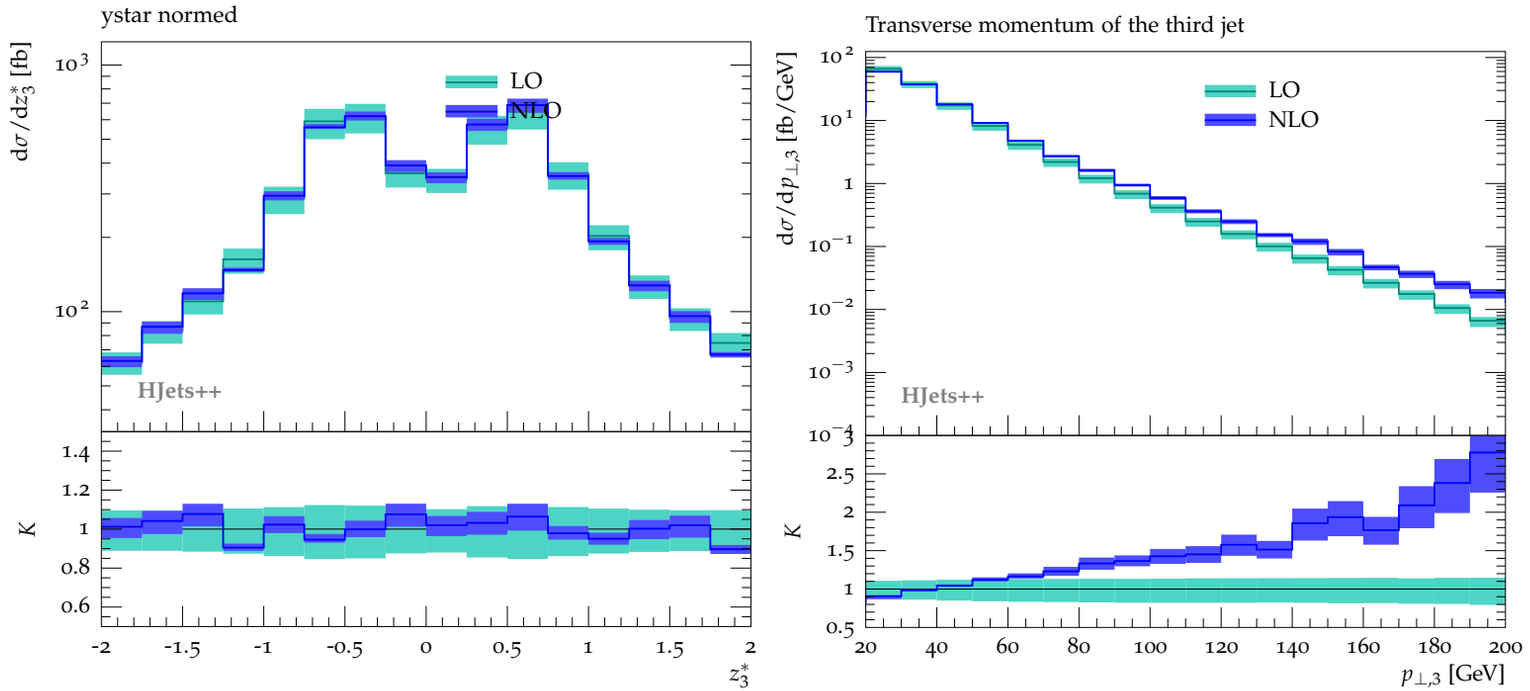
Apart from this ColorFull can also

- Read in and use orthogonal bases
- Create tree level gluon bases (using charge conjugation invariance)
- Be used with arbitrary $N_c$
- Be used with any $T_R = \mathrm{Tr}(t^a t^a)$ (no sum)
- Take the strict leading $N_c$ limit
- Take the leading $N_c$ limit, but keep $C_F$ to it's $N_c = 3$ value (shower like)

To learn more download from `colorfull.hepforge.org`, and see the toy program ColorPlay, or browse the doxygen documentation

# Use case: NLO electroweak Higgs + 3 jet production

In collaboration with Francisco Campanario, Terrance Figy and Simon Plätzer, arXiv:1308.2932, accepted for publication in PRL




Here, normalized centralized rapidity distribution of third jet w.r.t. two hardest jets (left) and transverse momentum of third jet (right).

However...

- This type of "basis" is non-orthogonal and overcomplete (for more than $N_c$ gluons plus $q\bar{q}$-pairs)
- ... and the number of spanning vectors grows as a factorial in $N_g + N_{q\bar{q}}$
  $\rightarrow$ when squaring amplitudes we run into a factorial square scaling
- Hard to go beyond $\sim$ 8 gluons plus $q\bar{q}$-pairs
- Would be nice with minimal orthogonal basis

# Minimal orthogonal bases for color spaces

In collaboration with Stefan Keppeler

- Want orthogonal minimal basis for color space
- Basis vectors can be enumerated using Young tableaux multiplication, for example for $gg \to gg$


and constructed if projection operators are known

- The problem is the construction of the corresponding projection operators; the Young-tableaux operate with "quark-units" but the physical particles include anti-quarks and gluons

- One may think that the problem of constructing group theory based multiplet bases should have been solved a long time ago
- The $2g \to 2g$ case was solved in the 60's ($N_c = 3$)
- However, until recently only a few cases had been dealt with, those for which (loosely speaking) nothing more complicated than two gluon projection operators is needed
- About one year ago me and Stefan Keppeler presented a general recipe for constructing gluon projection operators. From these we also know how to construct orthogonal bases for any number and kind of partons, JHEP09(2012)124, arXiv:1207.0609

- For many partons the size of the vector space is much smaller for $N_c = 3$ (exponential), compared to for $N_c \to \infty$ (factorial)

| Case | Vectors $N_c = 3$ | Vectors, general case |
|---|---|---|
| 4 gluons | 8 | 9 |
| 6 gluons | 145 | 265 |
| 8 gluons | 3 598 | 14 833 |
| 10 gluons | 107 160 | 1 334 961 |

Number of basis vectors for $N_g \to N_g$ gluons

*without* imposing vectors to appear in charge conjugation invariant combinations

- Multiplet bases can potentially speed up exact calculations in color space very significantly, as squaring amplitudes is very quick
- ... but before squaring, the amplitudes must be decomposed in the bases (master thesis of Johan Thorén)
- For resummation and higher order calculations one also wants to quickly compute the effect of gluon exchange (master thesis of Fritiof Persson)
- It seems that both these issues can be dealt with relatively quickly $\rightarrow$ I'm optimistic

# ColorMath

- Calculations are done using my Mathematica package, ColorMath, Eur. Phys. J. C 73:2310 (2013), arXiv:1211.2099
- ColorMath is an easy to use Mathematica package for color summed calculations in QCD, SU($N_c$)
- Repeated indices are implicitly summed

```
In[2]:= Amplitude = I f[g1, g2, g] t[{g}, q1, q2]
```

Out[2]= $i\, t^{\{g\}\,q1}{}_{q2}\, f^{\{g1,g2,g\}}$

```
In[3]:= CSimplify[Amplitude Conjugate[Amplitude /. g → h]]
```

Out[3]= $2\,Nc\,\left(-1+Nc^2\right)\,TR^2$

- The package and tutorial can be downloaded from `http://library.wolfram.com/infocenter/MathSource/8442/` or `www.thep.lu.se/~malin/ColorMath.html`

# Conclusions and outlook

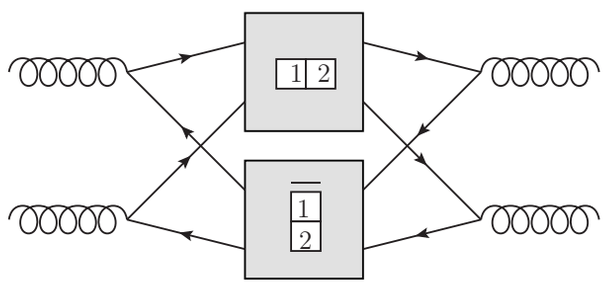
- One way of dealing with color space is to use "trace bases"
- This method is pursued in my and Simon Plätzer's $N_c = 3$ parton shower (JHEP 07(2012)042, arXiv:1201.0260)
- This is also the type of basis ColorFull, `colorfull.hepforge.org`, is built on
- Alternatively one may want to use orthogonal bases (JHEP09(2012)124, arXiv:1207.0609)
- This may have the potential to very significantly speed up exact calculations in the color space of SU($N_c$), but some work still remains...
- I have also written a Mathematica package ColorMath for color summed calculations of moderate complexity (Eur. Phys. J. C 73:2310 (2013), arXiv:1211.2099)

# Backup: Full program

```
// -*- C++ -*-
#include <iostream>
#include "Trace_basis.h"
using namespace ColorFull;
using namespace std;

int main(){

  Trace_basis MyBasis(1,3,0);
  cout << "Constructed the basis: \n";
  MyBasis.write_out_Col_basis_to_cout();
  MyBasis.write_out_Col_basis("ColorResults/MyBasis");

  MyBasis.scalar_product_matrix();
  cout << MyBasis.P_spm;

  Col_amp Ca1("[{1,3,2} (4,5)]");
  Col_amp Ca2("[{1,3,4,5,2}]");
  Col_functions Col_fun;
  cout << "Ca1.Ca1 " << Col_fun.scalar_product(Ca1,Ca1) << endl;
  cout << "Ca1.Ca2 " << Col_fun.scalar_product(Ca1,Ca2) << endl;
  cout << "Col_fun.emit_gluon(Ca1,3,77)" << Col_fun.emit_gluon(Ca1,3,77) << endl;
  cout << "Col_fun.exchange_gluon(Ca1,1,4)" << Col_fun.exchange_gluon(Ca1,1,4) << endl;

  Trace_basis MyBasisFull(1,3);
  Poly_matr Cg14=MyBasisFull.Col_gamma(1,4);
  cout << "Cg14\n" << Cg14;
  return 0;
}
```

# Backup: 2 gluon solutions

- For two gluons, there are two octet projectors, one singlet projector, and 4 "new" projectors, $10$, $\overline{10}, 27$, and for general $N_c$, "0"
- It turns out that the new projectors can be seen as corresponding to different symmetries w.r.t. quark and anti-quark units, for example the decuplet can be seen as corresponding to

$\mathbf{P}^{10} \propto$  $-$ octet(s) $-$ (singlet)

Similarly the anti-decuplet corresponds to $\boxed{\begin{smallmatrix}1\\2\end{smallmatrix}} \otimes \overline{\boxed{1\,2}}$, the 27-plet corresponds to $\boxed{1\,2} \otimes \overline{\boxed{1\,2}}$ and the 0-plet to $\boxed{\begin{smallmatrix}1\\2\end{smallmatrix}} \otimes \overline{\boxed{\begin{smallmatrix}1\\2\end{smallmatrix}}}$

# Backup: 2 gluon projectors

- Problem first solved for two gluons by MacFarlane, Sudbery, and Weisz 1968, however only for $N_c = 3$
- General $N_c$ solution for two gluons by Butera, Cicuta and Enriotti 1979
- General $N_c$ solution for two gluons by Cvitanović, in group theory books, 1984 and 2008, using polynomial equations
- General $N_c$ solution for two gluons by Dokshitzer and Marchesini 2006, using symmetries and intelligent guesswork

# Backup: Could this work in general?

On the one hand side

$$g_1 \otimes g_2 \otimes .... \otimes g_n \subseteq (q_1 \otimes \bar{q}_1) \otimes (q_2 \otimes \bar{q}_2) \otimes ... \otimes (q_n \otimes \bar{q}_n)$$

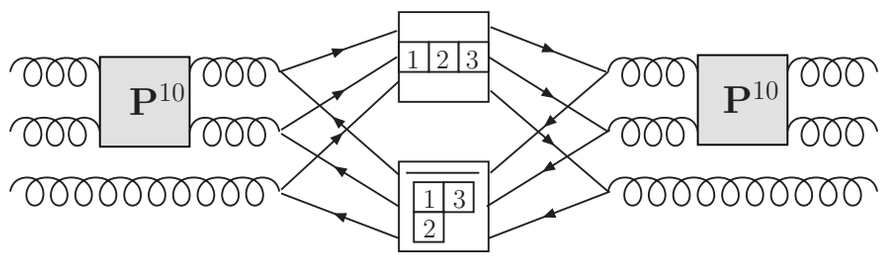
so there is hope...

On the other hand...

- Why should it?
- In general there are many instances of a multiplet, how do we know we construct all?

## Backup: Key observation:

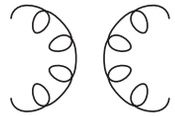
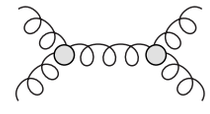
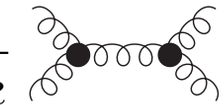
- Starting in a given multiplet, corresponding to some $q\overline{q}$ symmetries, such as 10, from $\begin{array}{|c|c|}\hline 1 & 2 \\ \hline \end{array} \otimes \overline{\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array}}$, it turns out that for each way of attaching a quark box to $\begin{array}{|c|c|}\hline 1 & 2 \\ \hline \end{array}$ and an anti-quark box to $\overline{\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array}}$, to there is at most one new multiplet! For example, the projector $\mathbf{P}^{10,35}$ can be seen as coming from

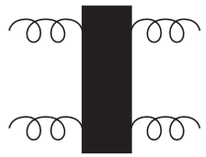
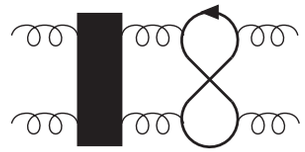


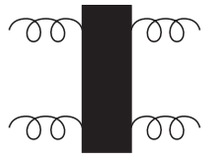
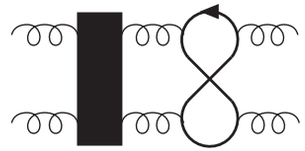

after having projected out "old" multiplets

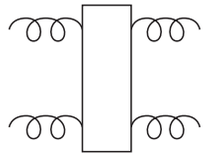
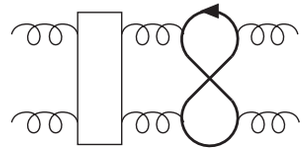
- In fact, for large enough $N_c$, there is precisely one new multiplet for each set of $q\overline{q}$ symmetries

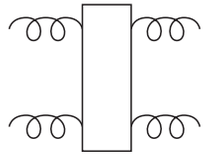
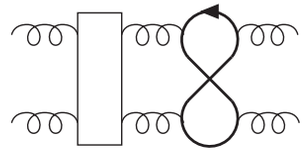
# Backup: 2 gluon projectors

$\mathbf{P}^{1} = \frac{1}{N_c^2-1}$  , $\mathbf{P}^{8s} = \frac{N_c}{2T_R(N_c^2-4)}$  , $\mathbf{P}^{8a} = \frac{1}{2N_cT_R}$  ,

$\mathbf{P}^{10} = \frac{1}{2}$  $+ \frac{1}{2T_R^2}$  $- \frac{1}{2}\mathbf{P}^{8a}$

$\mathbf{P}^{\overline{10}} = \frac{1}{2}$  $- \frac{1}{2T_R^2}$  $- \frac{1}{2}\mathbf{P}^{8a}$

$\mathbf{P}^{27} = \frac{1}{2}$  $+ \frac{1}{2T_R^2}$  $- \frac{N_c-2}{2N_c}\mathbf{P}^{8s} - \frac{N_c-1}{2N_c}\mathbf{P}^{1}$

$\mathbf{P}^{0} = \frac{1}{2}$  $- \frac{1}{2T_R^2}$  $- \frac{N_c+2}{2N_c}\mathbf{P}^{8s} - \frac{N_c+1}{2N_c}\mathbf{P}^{1}$

# Backup: Some 3g example projectors

$$\mathbf{P}^{8a,8a}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} = \frac{1}{T_R^2}\frac{1}{4N_c^2} if_{g_1\,g_2\,i_1} if_{i_1\,g_3\,i_2} if_{g_4\,g_5\,i_3} if_{i_3\,g_6\,i_2}$$

$$\mathbf{P}^{8s,27}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} = \frac{1}{T_R}\frac{N_c}{2(N_c^2-4)} d_{g_1\,g_2\,i_1}\mathbf{P}^{27}_{i_1\,g_3\,i_2\,g_6} d_{i_2\,g_4\,g_5}$$

$$\mathbf{P}^{27,8}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} = \frac{4(N_c+1)}{N_c^2(N_c+3)}\mathbf{P}^{27}_{g_1\,g_2\,i_1\,g_3}\mathbf{P}^{27}_{i_1\,g_6\,g_4\,g_5}$$

$$\mathbf{P}^{27,64=c111c111}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} = \frac{1}{T_R^3}\mathbf{T}^{27,64}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} - \frac{N_c^2}{162(N_c+1)(N_c+2)}\mathbf{P}^{27,8}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6}$$

$$- \frac{N_c^2-N_c-2}{81N_c\,(N_c+2)}\mathbf{P}^{27,27s}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6}$$

# Backup: Three gluon multiplets

| SU(3) dim | 1 | 8 | 10 | $\overline{10}$ | 27 | 0 |
|---|---|---|---|---|---|---|
| Multiplet | c0c0 | c1c1 | c11c2 | c2c11 | c11c11 | c2c2 |
| | $((45)^{8s}6)^{1}$ | $2 \times ((45)^{8s}6)^{8s}$ or $a$ | $((45)^{8s}6)^{10}$ | $((45)^{8s}6)^{\overline{10}}$ | $((45)^{8s}6)^{27}$ | $((45)^{8s}6)^{0}$ |
| | $((45)^{8a}6)^{1}$ | $2 \times ((45)^{8a}6)^{8s}$ or $a$ | $((45)^{8a}6)^{10}$ | $((45)^{8a}6)^{\overline{10}}$ | $((45)^{8a}6)^{27}$ | $((45)^{8a}6)^{0}$ |
| | | $((45)^{10}6)^{8}$ | $((45)^{10}6)^{10}$ | $((45)^{\overline{10}}6)^{\overline{10}}$ | $((45)^{10}6)^{27}$ | $((45)^{10}6)^{0}$ |
| | | $((45)^{\overline{10}}6)^{8}$ | $((45)^{10}6)^{10}$ | $((45)^{\overline{10}}6)^{\overline{10}}$ | $((45)^{\overline{10}}6)^{27}$ | $((45)^{\overline{10}}6)^{0}$ |
| | | $((45)^{27}6)^{8}$ | $((45)^{27}6)^{10}$ | $((45)^{27}6)^{\overline{10}}$ | $((45)^{27}6)^{27}$ | $((45)^{0}6)^{0}$ |
| | | $((45)^{0}6)^{8}$ | $((45)^{0}6)^{10}$ | $((45)^{0}6)^{\overline{10}}$ | $((45)^{27}6)^{27}$ | $((45)^{0}6)^{0}$ |

| SU(3) dim | 64 | 35 | $\overline{35}$ | 0 |
|---|---|---|---|---|
| Multiplet | c111c111 | c111c21 | c21c111 | c21c21 |
| | $((45)^{27}6)^{64}$ | $((45)^{10}6)^{35}$ | $((45)^{\overline{10}}6)^{\overline{35}}$ | $((45)^{10}6)^{c21c21}$ |
| | | $((45)^{27}6)^{35}$ | $((45)^{27}6)^{\overline{35}}$ | $((45)^{\overline{10}}6)^{c21c21}$ |
| | | | | $((45)^{27}6)^{c21c21}$ |
| | | | | $((45)^{0}6)^{c21c21}$ |

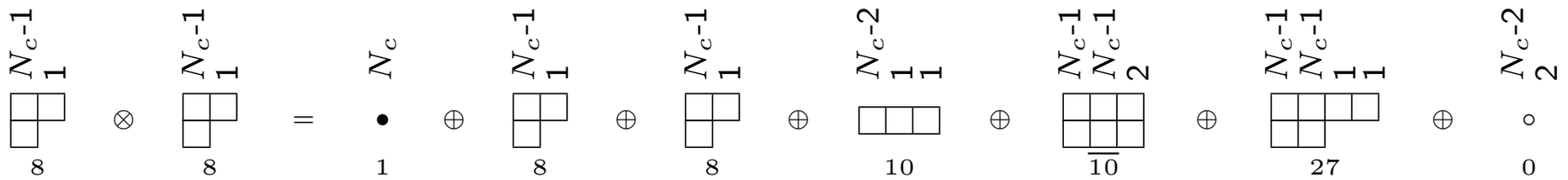
| SU(3) dim | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|
| Multiplet | c111c3 | c3c111 | c21c3 | c3c21 | c3c3 |
| | $((45)^{10}6)^{c111c3}$ | $((45)^{\overline{10}}6)^{c3c111}$ | $((45)^{10}6)^{c21c3}$ | $((45)^{\overline{10}}6)^{c3c21}$ | $((45)^{0}6)^{c3c3}$ |
| | | | $((45)^{0}6)^{c21c3}$ | $((45)^{0}6)^{c3c21}$ | |

Multiplets for $g_4 \otimes g_5 \otimes g_6$

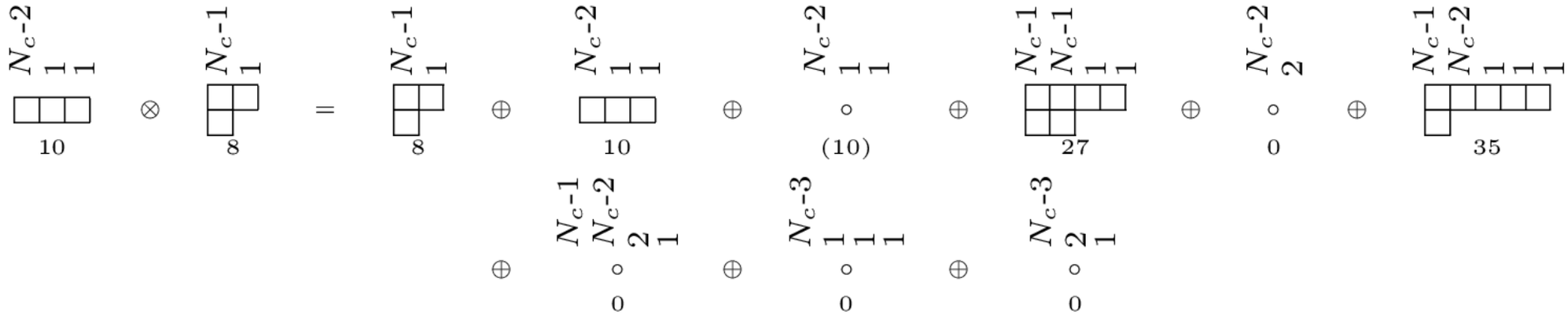
# Backup: Construction of 3 gluon projectors

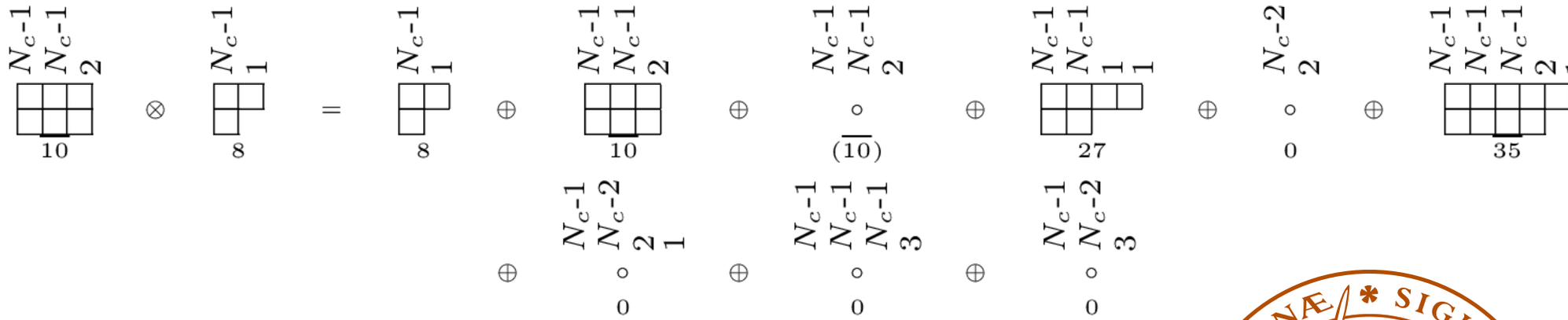
We start out by enumerating all projectors in $(8_1 \otimes 8_2) \otimes 8_3$

- Starting in a singlet, the result is trivial $1_{12} \otimes 8_3 = 8_{123}$
- If we start in an octet $8_{12}$, $8_{12} \otimes 8_3$ is known from before:

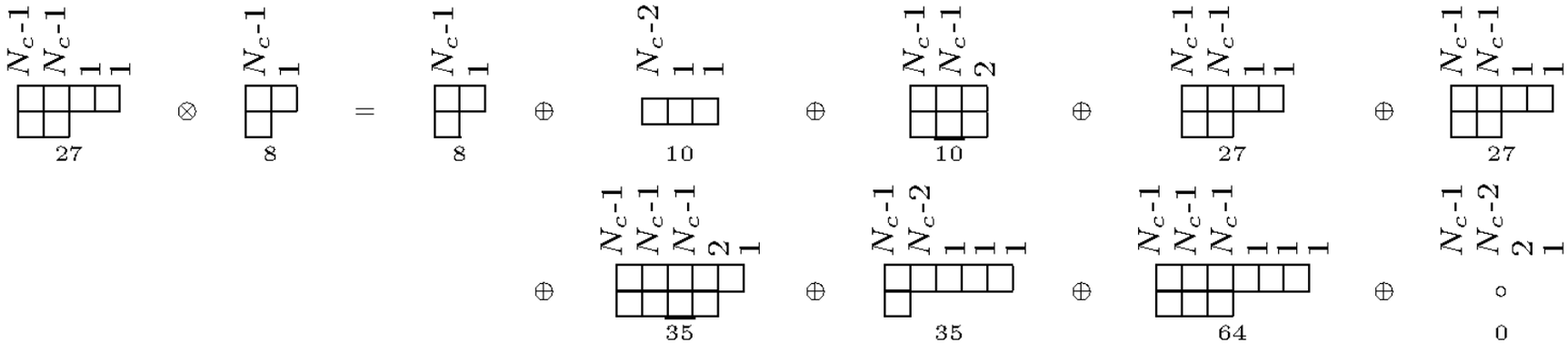


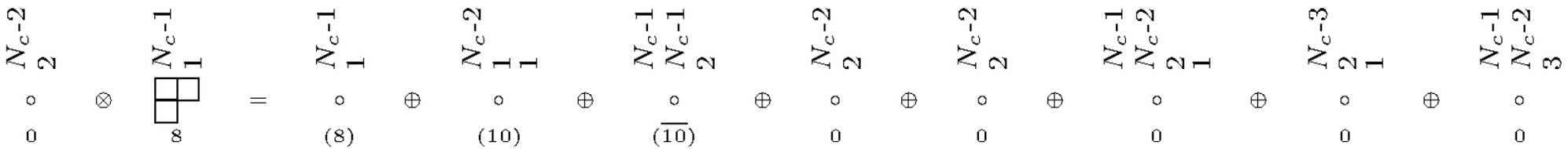
- The 3g multiplets from (anti-) decuplets



- The 3g multiplets from 27- and 0-plets

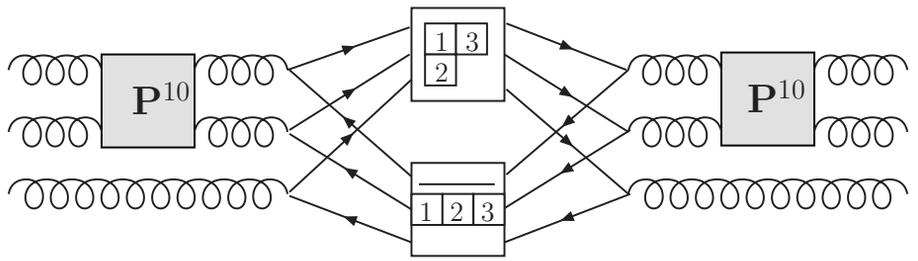


# Backup: Projector construction

- Construct projectors corresponding to "old" multiplets
- Construct the tensors which will give rise to "new" projectors

$\mathbf{T}^{10,35} \quad \propto$




- From these, project out "old" multiplets

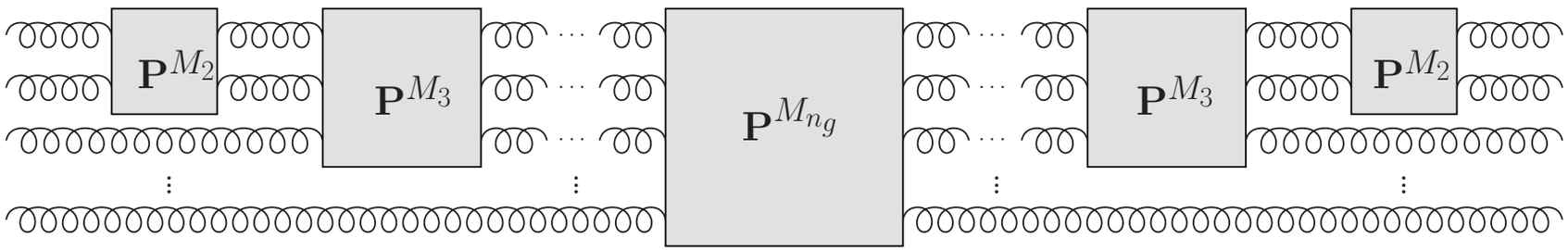
$$\mathbf{P}^{10,35} \propto \mathbf{T}^{10,35} - \sum_{m \subseteq 10 \otimes 8} \mathbf{P}^{m}\, \mathbf{T}^{10,35}$$

$\rightarrow$ "new" projectors

# Backup: Projecting out "old" multiplets

This would give us a way of constructing all projectors corresponding to "new" multiplets, if we knew how to project out all old multiplets. In $g_1 \otimes g_2 \otimes g_3$, there are many 27-plets. How do we separate the various instance of the same multiplet?

- By the construction history!

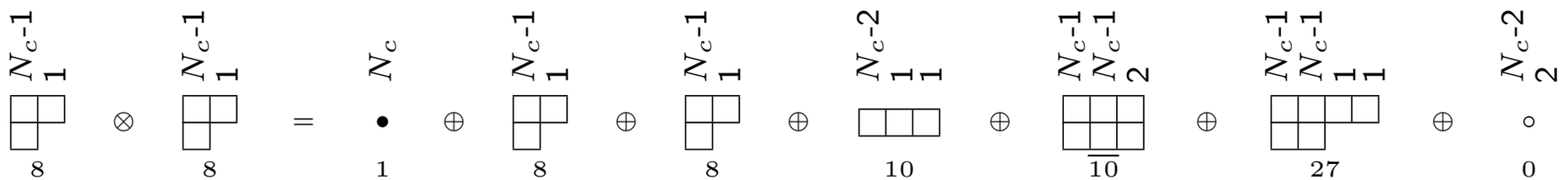



We make sure that the $n_g - \nu$ first gluons are in a given multiplet! Then the various instances are orthogonal as, at some point in the construction history, there was a different projector!

(More complicated for multiple occurrences...)

It turns out that the proof of this is really interesting:

- We find that the irreducible representations in $g^{\otimes n_g}$ for varying $N_c$ stand in a one to one, or one to zero correspondence to each other! (For each SU(3) multiplet there is an SU(5) version, but not vice versa.)
- Every multiplet in $g^{\otimes n_g}$ can be labeled in an $N_c$-independent way using the lengths of the *columns*. For example




I have not seen this column notation elsewhere... have you?

# Backup: Number of projection operators and basis vectors

In general, for many partons the size of the vector space is much smaller for $N_c = 3$, compared to for $N_c \to \infty$

| Case | Projectors $N_c = 3$ | Projectors $N_c = \infty$ | Vectors $N_c = 3$ | Vectors $N_c = \infty$ |
|---|---|---|---|---|
| 2g $\to$ 2g | 6 | 7 | 8 | 9 |
| 3g $\to$ 3g | 29 | 51 | 145 | 265 |
| 4g $\to$ 4g | 166 | 513 | 3 598 | 14 833 |
| 5g $\to$ 5g | 1 002 | 6 345 | 107 160 | 1 334 961 |

Number of projection operators and basis vectors for $N_g \to N_g$ gluons *without* imposing projection operators and vectors to appear in charge conjugation invariant combinations

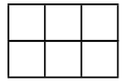
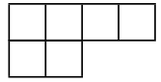
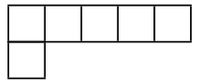
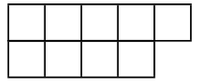
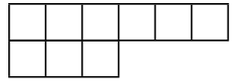
- The size of the vector spaces asymptotically grows as an exponential in the number of gluons/$q\bar{q}$-pairs for finite $N_c$
- For general $N_c$ the basis size grows as a factorial

$$N_{\text{vec}}[n_q, N_g] = N_{\text{vec}}[n_q, N_g - 1](N_g - 1 + n_q) + N_{\text{vec}}[n_q, N_g - 2](N_g - 1)$$

where

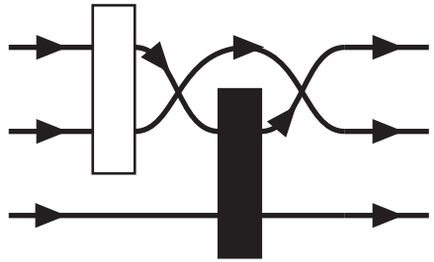
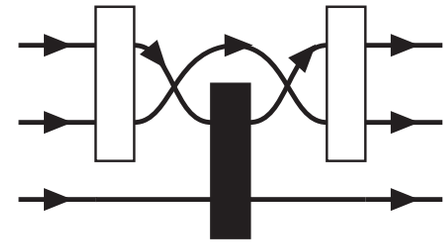
$$\begin{aligned} N_{\text{vec}}[n_q, 0] &= n_q! \\ N_{\text{vec}}[n_q, 1] &= n_q n_q! \end{aligned}$$

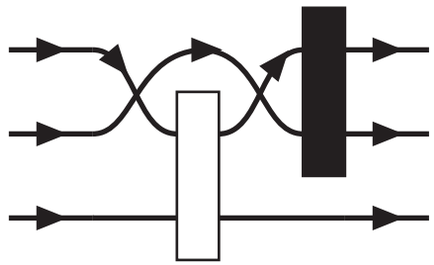
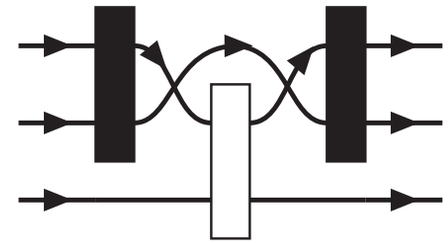
# Backup: First occurrence

| $n_f$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| SU(3) Young diagrams | $\bullet =$  |  |    |    |

Examples of SU(3) Young diagrams sorted according to their first occurrence $n_f$.

# Backup: The importance of Hermitian projectors

$\mathbf{P}_Y^{6,8} = \frac{4}{3}$  , $\mathbf{P}^{6,8} = \frac{4}{3}$ 

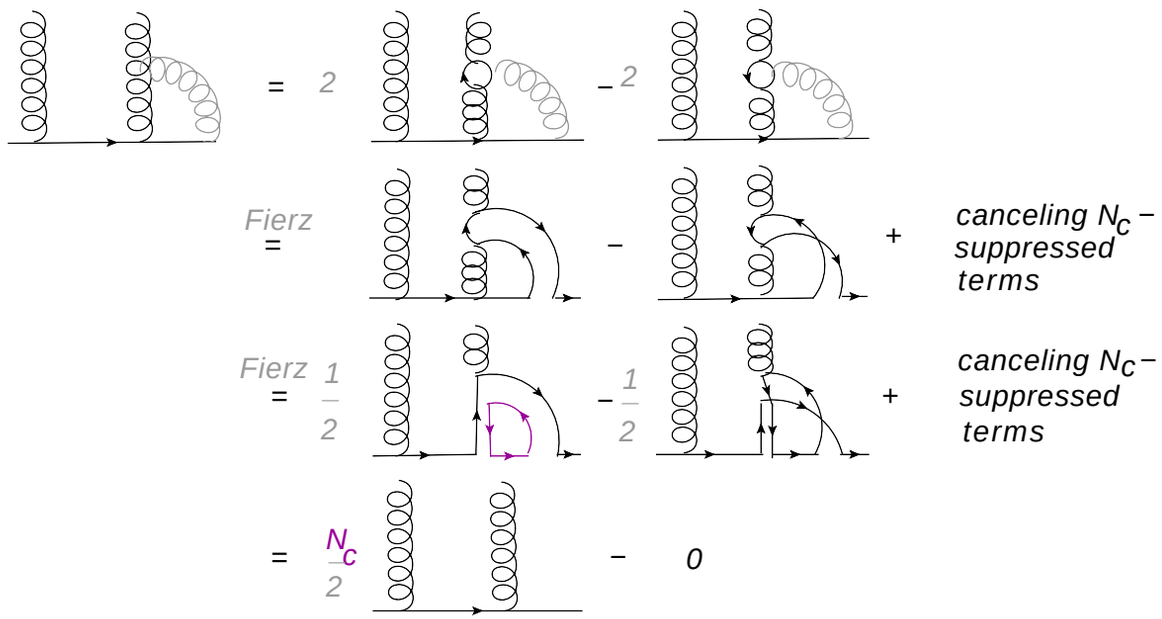
$\mathbf{P}_Y^{\bar{3},8} = \frac{4}{3}$  , $\mathbf{P}^{\bar{3},8} = \frac{4}{3}$ 

The standard Young projection operators $\mathbf{P}_Y^{6,8}$ and $\mathbf{P}_Y^{\bar{3},8}$ compared to their Hermitian versions $\mathbf{P}^{6,8}$ and $\mathbf{P}^{\bar{3},8}$.

Clearly $\mathbf{P}^{6,8\dagger}\mathbf{P}^{\bar{3},8} = \mathbf{P}^{6,8}\mathbf{P}^{\bar{3},8} = 0$. However, as can be seen from the symmetries, $\mathbf{P}_Y^{6,8\dagger}\mathbf{P}_Y^{\bar{3},8} \neq 0$.

# Backup: Gluon exchange

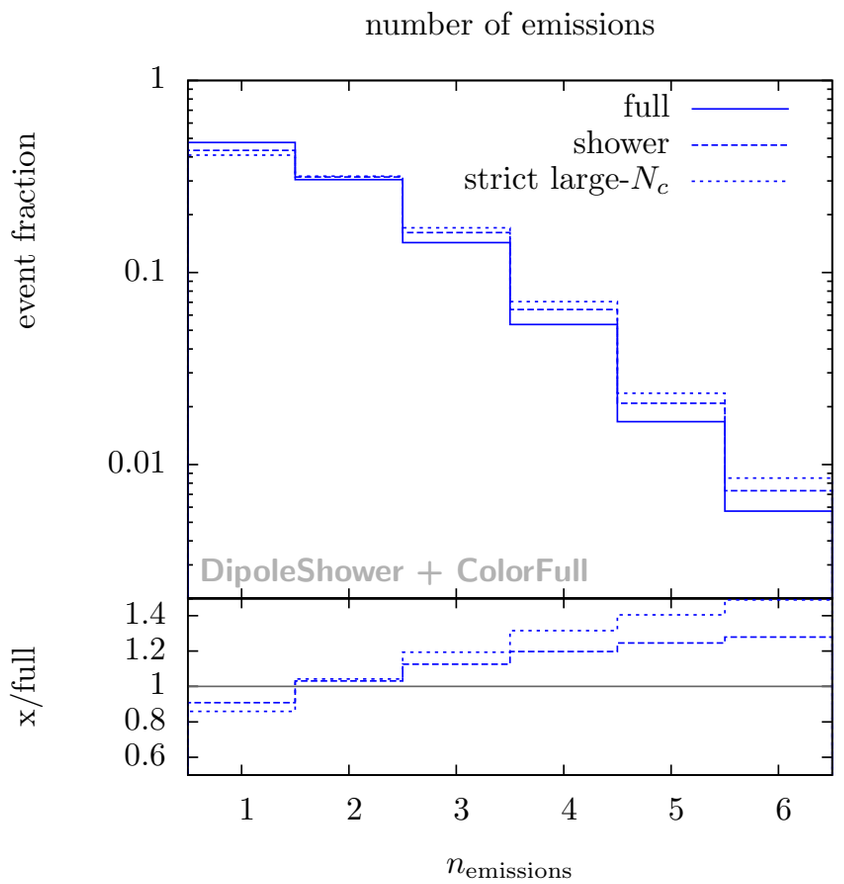
A gluon exchange in this basis "directly" i.e. without using scalar products gives back a linear combination of (at most 4) basis tensors




- $N_c$-enhancement possible only for near by partons
  $\rightarrow$ only "color neighbors" radiate in the $N_c \rightarrow \infty$ limit

# Backup: Number of emissions

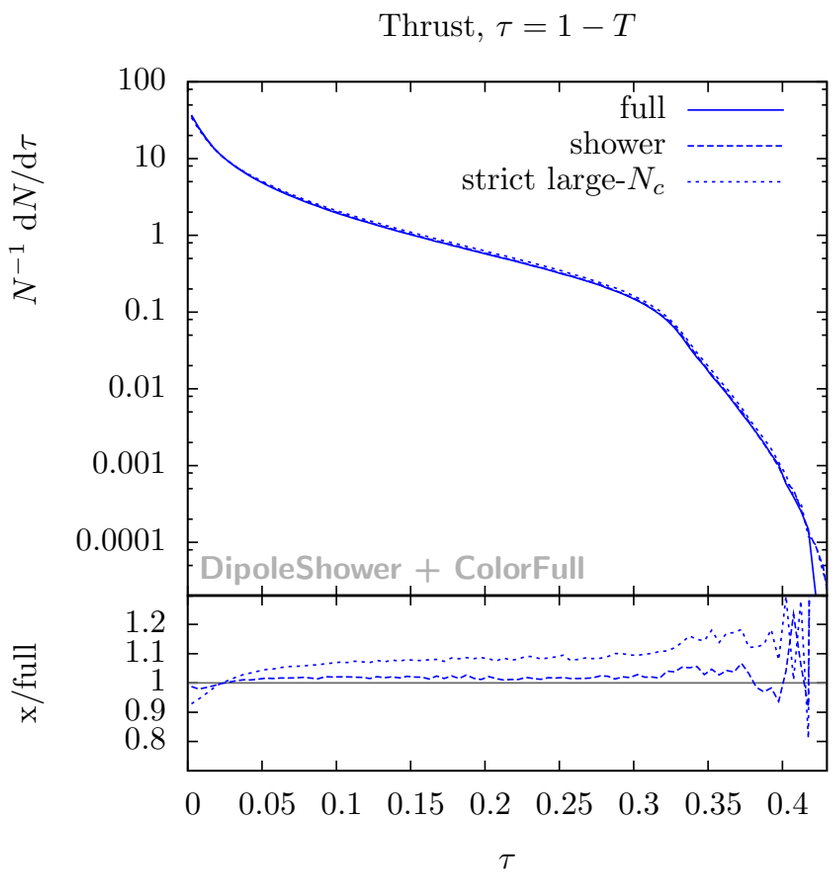
First, simply consider the number of emissions for a LEP-like setting




... this is not an observable, but it is a genuine uncertainty on the number of emissions in the perturbative part of a parton shower

# Backup: Thrust

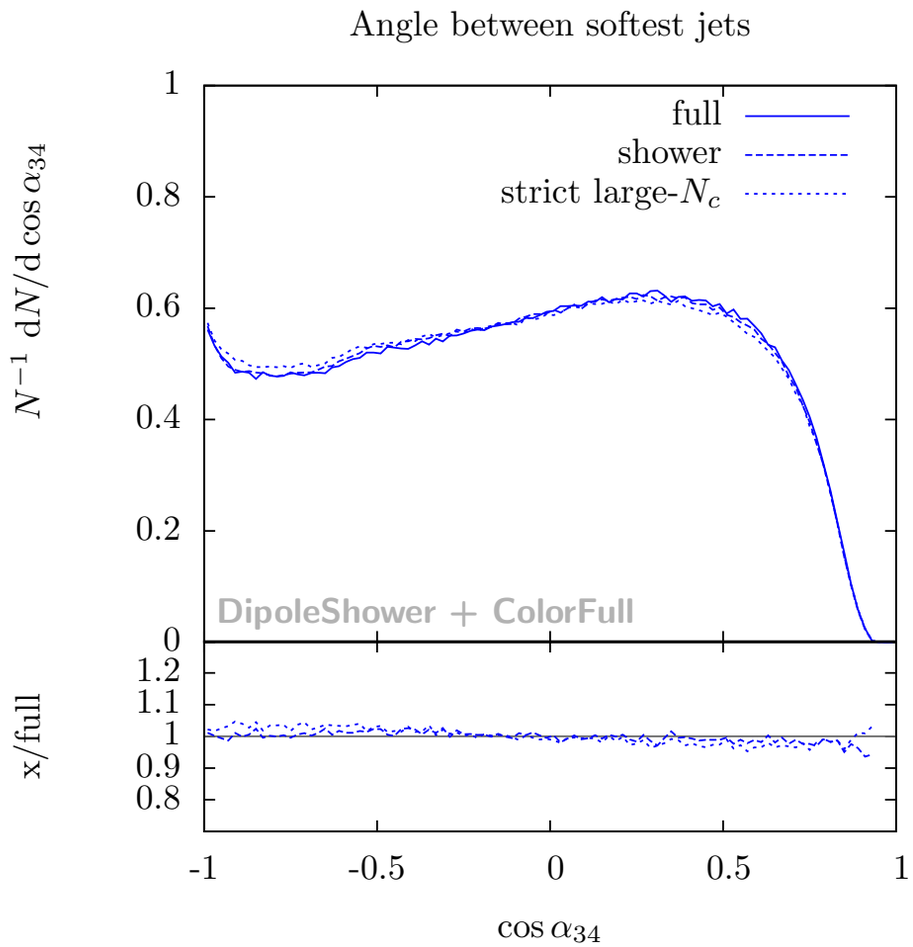
For standard observables small effects, here thrust $T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$




NOTE: Larger effects expected at LHC

# Backup: Angular distribution

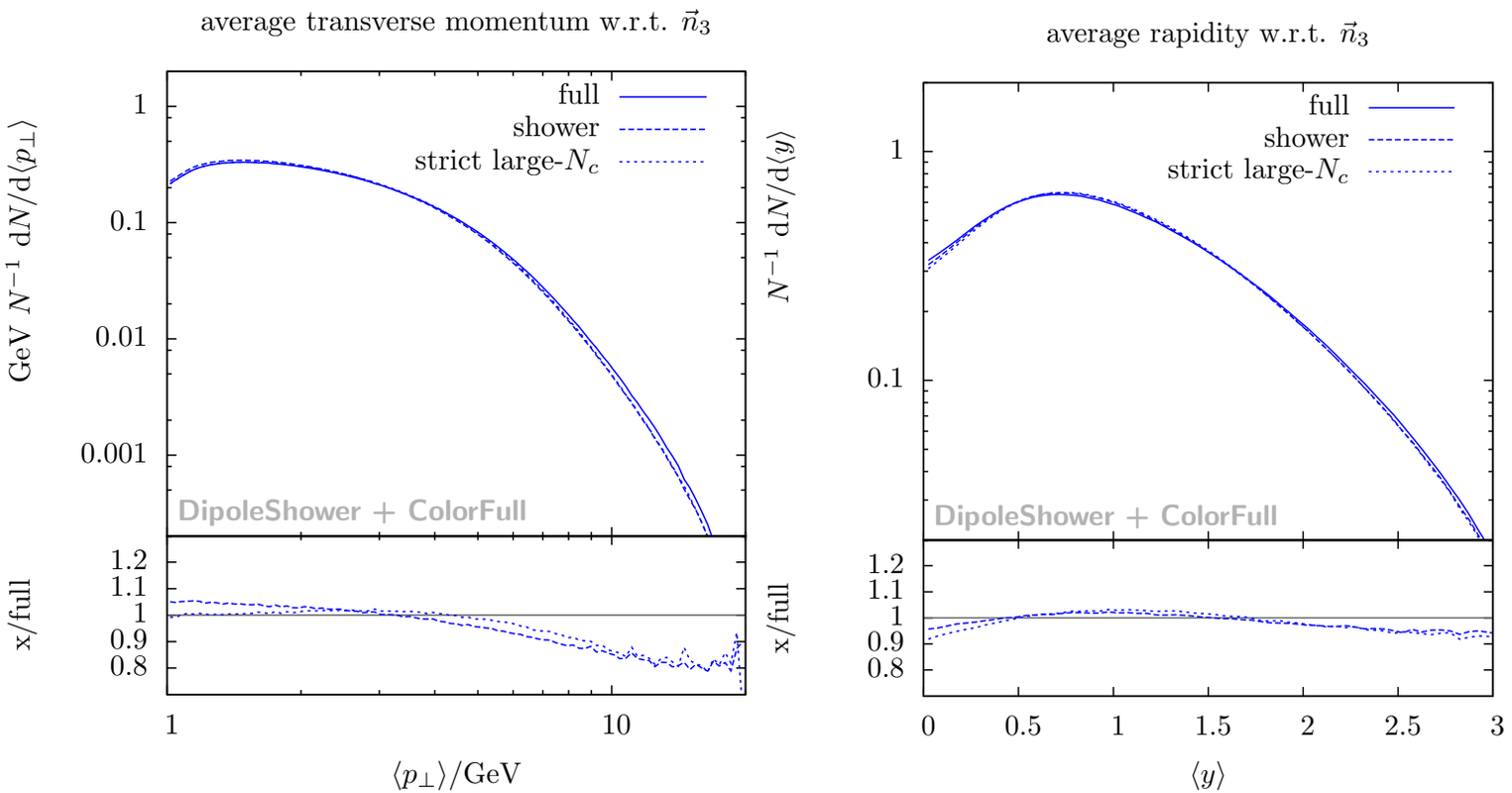
Cosine of angle between third and fourth jet




NOTE: Larger effects expected at LHC

# Backup: Some tailored observables

For tailored observables we find larger differences

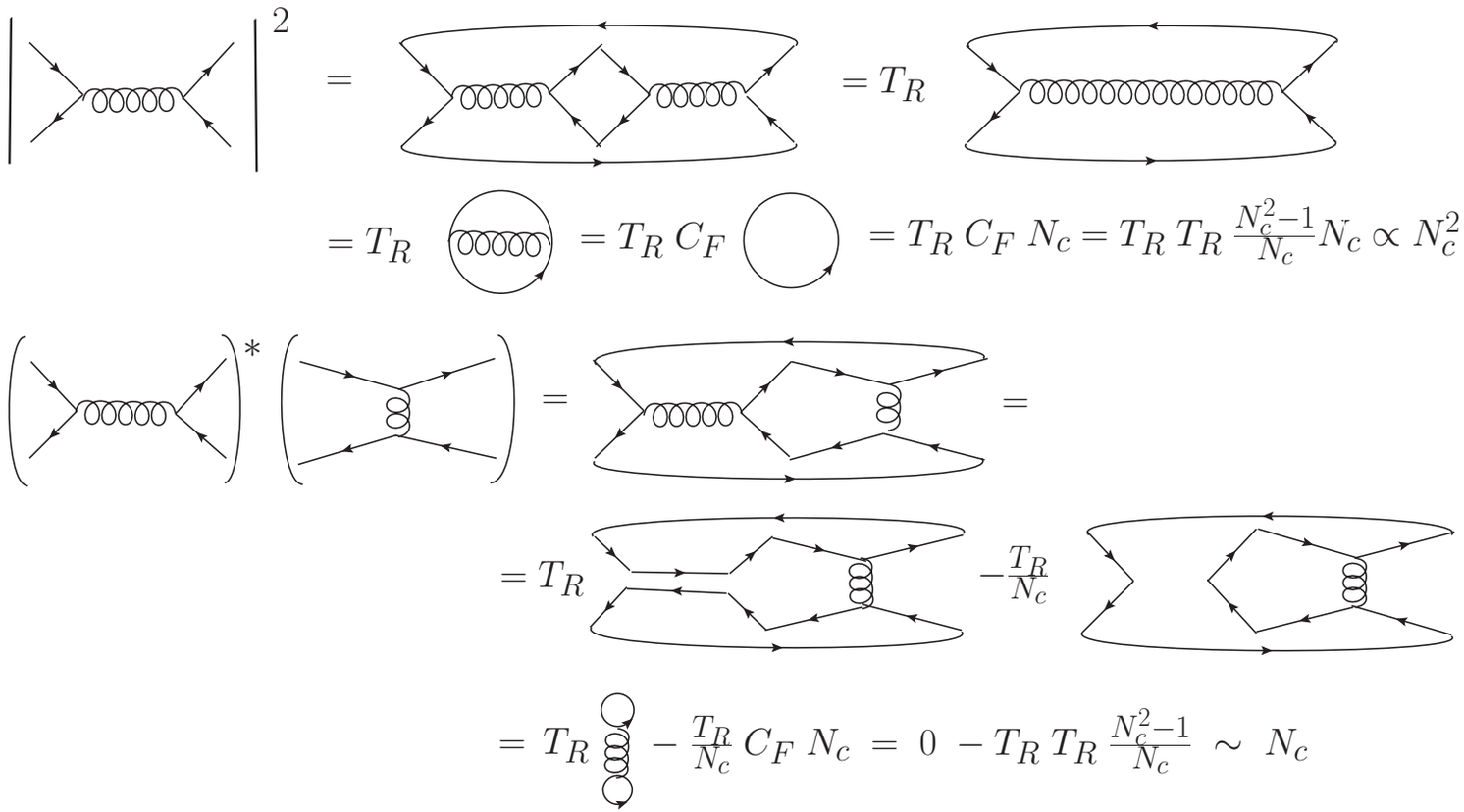



Average transverse momentum and rapidity of softer particles with respect to the thrust axis defined by the three hardest partons

NOTE: Larger effects expected at LHC

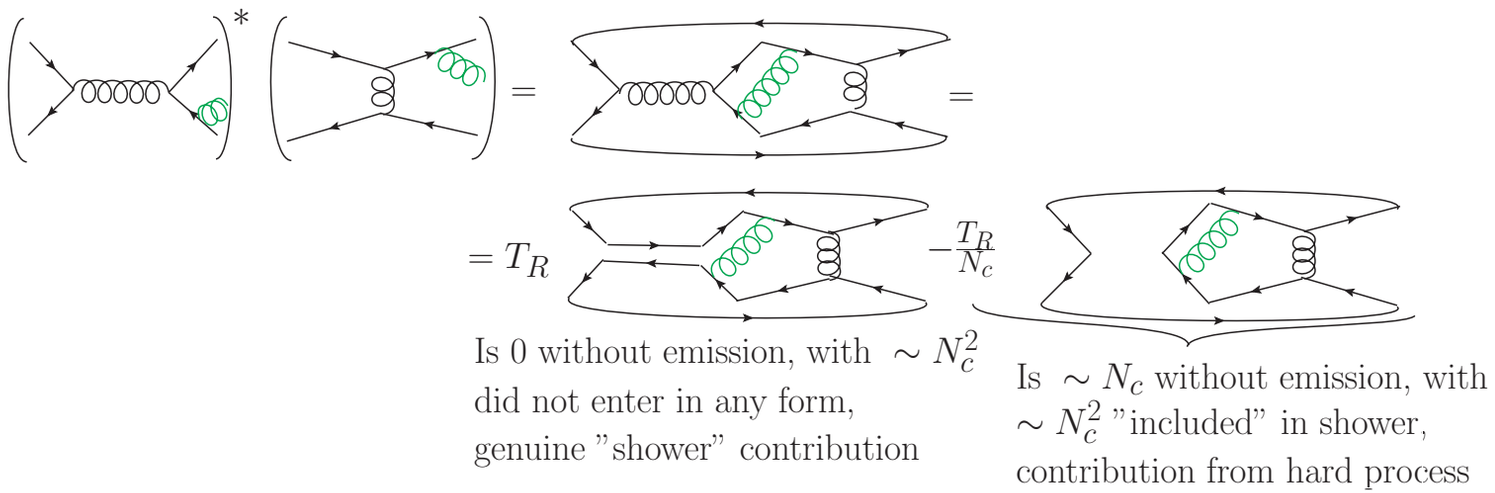
# Backup: $1/N_c$-suppressed terms

That non-leading color terms are suppressed by $1/N_c^2$, is guaranteed only for same order $\alpha_s$ diagrams with only gluons ('t Hooft 1973)



# Backup: $1/N_c$-suppressed terms

For a parton shower there may also be terms which only are suppressed by one power of $N_c$




The leading $N_c$ contribution scales as $N_c^2$ before emission and $N_c^3$ after