Comments on the litterature for chapter 12

Chapter 12

Extra-curricular parts

Section 12.5 and sections marked with star $*$ are extra-curricular. I briefly mentioned the displacement field, since it may be used when we discuss waves.

Flow visualization

It really helps to be able to visualize flows. The best way to get a feel for it is through the exercises. The most important message is perhaps that particle trajectories, stream lines, and streak lines all coincide for *static flow* ($\partial \mathbf{v}/\partial t = 0$).

The Math

The back-bone of this chapter is made up by eqs. (12.12), (12.5), (12.19) and (12.26).

No material rate of change of mass

The material time derivate, or comoving derivative, (12.19) is important and often applied. To reach eq. (12.17) without relying on chapter 7, note the powerful argument after eq. (12.18): our definition of material particle i implies $\frac{D}{Dt}M_i = 0$, from which we can derive (12.17). The same relation is used to get from Newton II, $\frac{\mathrm{d}}{\mathrm{d}t}[M_i\mathbf{v}] = \mathbf{F}_i$ to Cauchy's equation (12.26).

The effective force density

The effective force density $\mathbf{f}^*$ in eq. (12.26) can have contributions that are external (for example from gravity, $\rho\mathbf{g}$), and internal (for example from a pressure gradient ∇p). These are the contributions we have discussed so far, but once we introduce friction, there will be more, and therefore eq. (12.25) involves notation we have not met yet. To our delight, we do not need to specify $\mathbf{f}^*$ to reach (12.26).

Also, ficticious forces in a rotating coordinate frame will contribute to $\mathbf{f}^*$. That is our next lecture.

Incompressibility

We revisited 12.3, the equation of continuity. It immediately follows that incompressible fluids obey $\nabla \cdot \mathbf{v} = 0$. Note that eq. (12.5) is not generally applicable! Remember to read the "fine print" in the text preceding an equation, to see when it applies.

Note that the word "incompressible" is a bit vague. It could refer to the fluid itself, so that $\rho = \rho_0$. It could mean $\rho(\mathbf{x}, t) = \rho(\mathbf{x})$. In this case, the fluid can vary its density, but the flow happens to be such that the density never changes in a point. It could also refer to vanishing co-moving derivative of density, $\frac{D\rho}{Dt} = 0$, which again implies $\nabla \cdot \mathbf{v} = 0$. In old exams the wording "incompressible fluid" (in contrast to "incompressible flow") always implies $\rho = \rho_0$.