Comments on the lectures for chapter 18

18.0-18.2

Local angular velocity
The main achievement in the beginning of the chapter is perhaps not to reach eq. (18.7), but to then approximate it!

The centrifugal force can be absorbed into the acceleration field $g$, with negligible numerical consequences. As I discussed on the lecture, including the centrifugal term into $g$ will make $g$ point vertically to the local earth surface.

The angular velocity vector $\Omega$ has one vertical component $\Omega_z$ and one horizontal $\Omega_y$, pointing north. The effects of the horizontal $\Omega$ component can also be neglected in most applications (and always in this course). The cross product $\Omega_y \times v_y $ points in the vertical direction, and is usually negligible compared to $g$. The cross product $\Omega_y \times v_z \hat{e}_z$ points in the west-east direction and is negligible compared to other terms, because “$v_z$ tends to be small”. That is a fair statement in the atmosphere and ocean applications discussed in this chapter.

The only fictitious acceleration field we need to consider is therefore $-2\Omega_z \times v$.

Important: Until eq. (18.7), the book uses $\Omega$ for the total angular velocity. In the next section and onwards, it instead uses notation $\Omega_0$ for the total, and lets $\Omega$ denote the local angular velocity. In lecture notes and extra problems, I will use notation $\Omega_z$ for the local angular velocity vector.

Rossby number
Very important and beautiful is the introduction of the Rossby number, that allows us to neglect the non-linear advective term. Horray! This means that we will not apply eq (18.10) anywhere, but immediately approximate it into (18.12).

Equation (18.12) is the fundamental equation for geostrophic balance, written in a way independent of coordinate system. (When we use notation $\Omega_z$, we have specified a $z$-direction, though.) As often, this general equation is a bit far from finding answers to specific problems. The text, from (18.12) to the example on Great Danish Belt, illustrates how to go from the general equation to more specific (but also not generally valid!) equations.

I did not have time to show the Taylor-Proudman theorem. The derivation involves some gymnastics with $\nabla$ and cross products. I hope to return to this when we talk about the Ekman layer.

18.3 Ekman Layer

Extra-curricular parts
Anything marked with *. Also, the Taylor columns (bottom of p. 314). The section leading
to eq. 18.27 need not be reproduced, but the student is expected to be able to confirm that
the equation is a solution to its problem, and analyze its properties.

Since this is an introductory course, we tend to focus on very simple systems, where boundary
conditions are easily treated. Trying to do that with geostrophic flow, we reach strange
results (there cannot be any wind, for example). Thus, the Ekman layer is our representative
of an important procedure in fluid dynamics, where the simple solutions are applied in their
respective domain, and a boundary layer that interpolates between the solutions is studied
in more detail after. If the boundary layer is found to have reasonable thickness, we confirm
a posteriori that the procedure was valid.