Comments on the lectures for chapter 6 and 15

Extra-curricular parts
In chapter 6, anything referring to solids (look for words like "deformation" or "rupture"). Also, the discussion on the most general situations where σ is symmetric is extra-curricular. An alternative motivation for symmetric stress tensor in a Newtonian fluid was given in lectures and pdf problem 11. Section 6.5 is not part of the course.

In chapter 15, the sections marked with * **actually are included** in the course. However, the two small sub-sections on "Viscous dissipation" are extra-curricular.

Local interactions
Assuming *local* internal interactions (thus keeping away from internal electromagnetism) the internal contribution to the force on a material particle must be possible describe as a surface integral over the particle boundary. Using integral theorems we then find that the internal force density $\boldsymbol{f}$ must be possible to express as a derivative of a matrix (2-rank tensor), $\boldsymbol{f} = \nabla \cdot \boldsymbol{\sigma}^\top$, so that the force on a (very small) surface $\mathrm{d}S$ perpendicular to a unit vector $\mathbf{n}$ is $\mathrm{d}\boldsymbol{F} = \boldsymbol{\sigma} \cdot \mathbf{n}\mathrm{d}S$.

The 9 components of $\boldsymbol{\sigma}$ are thus interpreted such that σ_{xy} is the force per area in the x-direction acting on a surface perpendicular to the y-direction.

We discussed transformation properties. A set of 3 numbers qualify as components of a vector only if they **transform** correctly when we change our coordinate system. The 9 numbers of a 2-rank tensor are also subject to transformation properties. Etc.

It may feel like $\boldsymbol{\sigma}$ is a detour to reach the force density $\boldsymbol{f}$, but it is the stress tensor that is constrained by boundary conditions.

Newtonian fluid
For a fluid, friction can only arise due to velocity differences. The simplest way to introduce friction is to assume that the stress tensor is linear in the first-order derivatives of the velocity, like $\frac{\partial v_i}{\partial x_j}$, and independent of higher-order derivates. This is called a Newtonian fluid. Problem 11 on the extra pdf shows that an isotropic, Newtonian fluid must have a symmetric stress tensor.

The most general $\boldsymbol{\sigma}$ for an *isotropic* Newtonian fluid (one which does not define any particular direction in space) is then given by eq. (15.29). No other terms can be added to $\boldsymbol{\sigma}$ if it is to transform as a proper 2-rank tensor.

Navier–Stokes
The stress tensor introduces 2 new scalar fields, the viscosity η and the "bulk viscosity" ζ. If we assume these to be constant (or rather, that their variation is negligeble in the small reagions where friction matters), differentiation of the stress tensor leads to Navier–Stokes equations (15.30). The Navier–Stokes equations are not quite as fundamental as Newton II, since the viscosity terms depend on all the simplifying assumptions of a Newtonian fluid.

Navier–Stokes together with the Equation of Conitinuity, and a state equation for density and/or pressure, we have the starting point to tackle any dynamical problem in fluid dynamics involving a “Newtonian fluid”.

Applications
The book has very nice examples, with everyday phenomena discussed, on steady planar flow and sliding of bodies on thin layers of fluids. We also looked at the strong dampening of transverse waves (shear waves) in contrast to more far-reaching longitudinal waves (sound waves).

We also noted that viscosity will lead to diffusion of velocities, so that a concentrated current of high velocities gets smeared out. In the process, momentum is preserved, but kinetic energy is lost to heat.

The part in the book on ideal gases is interesting. It gives a microscopic motivation for newtonian fluids and also emphasizes that viscosity may be temperature dependent, and thus formally a field varying over space. It also illustrates how kinetic energy is transformed into heat.

Boundary conditions
To solve problems we need boundary conditions, and what we can use is very clearly listed in chapter 15. Note that in the application of viscous friction (p.246-247), we use the known surrounding velocities to calculate the velocity field of the fluid. Then we calculate the stress tensor for the fluid, and use the boundary condition “backwards” to get the frictional forces on the surrounding. Nice!

Reynold’s number
Looking up the kinematic viscosity in a table and setting “typical” velocity and length scales of the problem, we can determine if the flow is dominated by viscosity (“creeping”) or nearly ideal. For a really high Reynold’s number, we need to consider turbulence (later in the course).

Final thoughs
Intimidating as Navier–Stokes equation may be, it is perhaps better to start with it in its full glory, together with the equation of continuity, and cut away terms that dissappear in different applications, rather than memorizing one set of equations for each condition.

For ideal fluids, we have $\eta = \zeta = 0$. (That removes a lot!) Furthermore, we abandon the “no-slip” boundary condition.

For incompressible fluids we have $\rho = \rho_0$ and $\nabla \cdot \mathbf{v} = 0$. (That removes the messiest bit!)

For steady flow, we have $\frac{\partial \mathbf{v}}{\partial t} = 0$.

If the Reynold’s number is far from 1, that information can be used to neglect terms.

In many applications, we can simplify further using information about absence of external forces, or simple external forces. We should also put thought behind our choice of coordinate system, so the equations (and boundary conditions!) in component form become as simple as possible.