Exam, FYTA14, 2015-06-05, 09.15-15.15

Allowed material: One a4 sheet with notes, writing material.

30 points total, 15 points to pass, 24 points for distinction.

The tasks are *not* sorted in order of difficulty.
Read the text *carefully* before you start to solve a problem.
Present partial results, even if your solution is incomplete.
Many sub-problems can be solved independently of previous sub-problems.
Cartesian coordinates in an inertial frame are used, unless a rotating system is explicitly referred to.

1. Navier–Stokes Basics (5p)

a) [1p] Write down the Navier–Stokes equation in vector notation. Name the included fields and constants.

b) [3p] Which terms vanish if the flow is:
. . . incompressible?
. . . steady?
. . . ideal?

c) [1p] How would you modify the Navier–Stokes equation if it was expressed in a coordinate system rotating with a constant angular vector $\mathbf{\Omega}$ relative to an inertial frame?

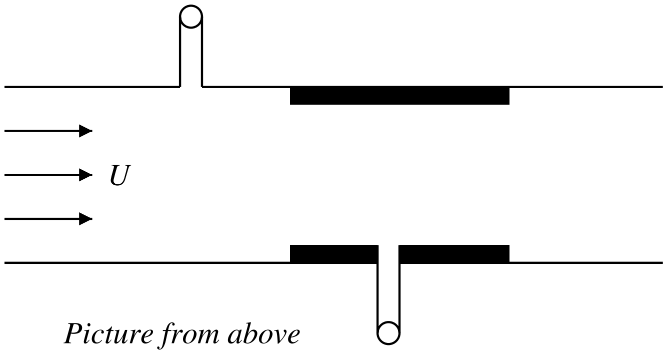
2. Surface Gravity Waves (6p)

a) [1p] Make a sketch of a surface gravity wave and the relevant physical quantities that are important for its propagation.

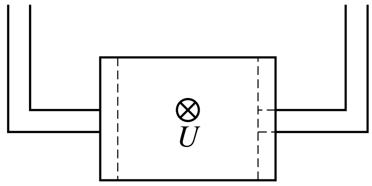
b) [2p] Give a qualitive description of what drives the propagation of a surface gravity wave.

c) [3p] Describe the difference between the properties of a shallow-water wave and a deep-water wave.

3. Velocity Measurement (6p)



Picture from above



Picture from inlet

Consider steady water flow filling a rectangular pipe with constant height. The width of the pipe is mostly constant, but in one region the inner of the pipe is narrowed to 80% of the width. The water velocity in the wide region is 1 m/s.

Two side tubes are attached on the walls, one in the narrow region and one in a wide region. The tubes bend upwards and water entering from the pipe reaches different heights in the two tubes. The tubes are open on top and in contact with air.

a) [1p] Making reasonable assumptions, which of the two side tubes has the higher water surface?

b) [5p] What is the height difference? As approximations, you may use the gravitational acceleration $\approx 10\,\mathrm{m/s}$, water density $\approx 1000\,\mathrm{kg/m^3}$ and air pressure $\approx 100\,\mathrm{kPa}$.

4. Flow on a Plane (7p)

Consider steady water flow in the x-direction on a plane perpendicular to the z-direction, such that $\mathbf{v} = (v_x(z), 0, 0)$. The water surface is at constant height above the plane. The plane is lying still. Assume constant water density ρ_0 and a constant pressure p_0 at the surface.

Let the plane (and the coordinate system) be tilted with respect to the earth vertical, so that the gravitational acceleration is $\mathbf{g} = (g_x, 0, g_z)$, where g_x and g_z are constants.

a) [1p] Verify that the proposed velocity field is consistent with a constant density.

b) [2p] Show that the pressure in the fluid is independent of x. *Hint:* Use the z-component of the Navier–Stokes equations and the pressure boundary condition at the surface.

c) [1p] Knowing that the pressure is independent of x (even if you have not proven it), find an expression for $v_x''(z)$.

d) [3p] Find an expression for $v_x(z)$ and determine all integration constants with the help of two boundary conditions:
1) The velocity at the plane
2) The shear forces at the surface. Assume that air imposes negligible shear forces on the water, so that the stress tensor component σ_{xz} is 0 at the water surface.

5. The Gulf Stream (6p)

The Gulf Stream runs counter-clockwise in the northern Atlantic Ocean (as a very simplified description). According to Wikipedia it is typically 100km wide and flows with a velocity of about 1 m/s. We assume that the current can be described by *geostrophic balance*, and that the water surface lies at constant pressure p_0.

a) [2p] Describe why there will be a height difference between the inner and outer edges of the current, and determine which edge that has the highest surface. Furthermore, assuming (boldly) that the width and speed of the flow is constant, where on earth would the height difference be maximized?

b) [2p] Deeper down, the Gulf stream meets other water layers with different motion. At the lower regions of the Gulf stream there is therefore an Ekman layer where the flow disagrees with geostrophic balance. Qualitatively, what is the direction of the flow in the Ekman layer, and how does that affect the height difference between the outer and inner water surfaces?

c) [1p] If the stream instead had run clock-wise, with preserved numerical values, would the height difference change in size or direction? Would the effects of the Ekman layer change? For the "yes" answers: what is the change?

d) [1p] *A side-remark stated as a sub-problem*: When the counter-clockwise Gulf stream turns left, you may expect a pressure gradient to drive the change of velocity direction. With a typical speed U and radius R of the turn, the acceleration "left-wards" is U^2/R. With reasonable assumptions on R, show that this acceleration is negligible compared to other effects in geostrophic balance. Thus, this acceleration need not be considered in the rest of the problem!