Exam, FYTA14, 2020-08-29, 8.00-13.00

Allowed material: One a4 sheet with notes, writing material.

30 points total, 15 points to pass, 24 points for distinction.

The tasks are *not* sorted in order of difficulty.

Read the text *carefully* before you start to solve a problem.

Present partial results, even if your solution is incomplete.

Many sub-problems can be solved independently of previous sub-problems.

Cartesian coordinates in an inertial frame are used, unless a rotating system is explicitly referred to.

1. Velocity Field (8p) A velocity field, given in non-dimensional variables, is:

$$\boldsymbol{v} = (\alpha xz + \beta t,\ \gamma x^2,\ -z^2). \tag{1}$$

The field contains three constants α, β and γ.

a) [4p] Determine the constants so that the field is consistent with a steady, irrotational, ideal flow above a floor at $z = 0$. Establish if any constants remain undetermined. For the given flow, find the pressure field p, given zero gravity, constant density $\rho = 2$ and boundary condition $p = p_0$ at $x = y = z = 0$. Both ρ and p are given as non-dimensional quantities.

b) [4p] Ignoring the previous task, instead determine the constants so that the field is consistent with an incompressible, viscous flow above a floor lying at rest at $z = 0$. Establish if any constants remain undetermined. Given a constant viscosity $\eta = 1$, find the vertical shear force per area, σ_{xz} on the floor. Both η and σ are given as non-dimensional quantities.

2. Geostrophic Wind (5p)

Four weather-stations (A, B, C and D) are located in the corners of a square with side 20 km. Station A and B are 20 km south of C and D. Station A and C are 20 km west of B and D. Simultaneous pressure measurements are $p_A = 1002.0\,\mathrm{hPa}$ ($1\,\mathrm{hPa} = 100\,\mathrm{Pa}$), $p_B = 1002.4\,\mathrm{hPa}$, $p_C = 1002.2\,\mathrm{hPa}$ and $p_D = 1002.6\,\mathrm{hPa}$.

The region is at 45° north, and the air density is $1.2\,\mathrm{kg/m^3}$. We assume geostrophic balance and a constant pressure gradient.

a) [2p] Draw a sketch illustrating the direction of the geostrophic wind!

b) [3p] Approximately, what is the magnitude of the wind?