Improving parton showers with multi-jet NLO calculations

Stefan Prestel

(in collaboration with Leif Lönnblad)

(Lund University)

Theory seminar, SLAC, June 20, 2012
Outline

• Introduction: CKKW-L\textsuperscript{1} matrix element merging in parton showers.
• Example: One jet above a scale $\rho_c$.
• The NL\textsuperscript{3} prescription\textsuperscript{2} to move towards NLO accuracy: A (reasonably?) short how-to.
• Results for $W + 0$ and $W + 1$ at NLO.

\textsuperscript{1}For Catani Krauss Kuhn Webber – Lönnblad.
\textsuperscript{2}For Nils Lavesson and Leif Lönnblad.
Improving parton showers with “multi”-jet NLO calculations

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Problem: To describe soft/collinear and hard jets together, virtues of both Fixed Order and Resummation are needed, since

- Matrix Elements (ME) accurate to fixed order far away from phase space boundaries, but breaks down e.g. in infrared region.
- Parton Shower (PS) resummation constructed to work in collinear region, with some improvements for soft gluon resummation.

⇒ Approaches (somewhat) complementary.
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  → Use ME above a cut $t_{\text{MS}}$, and PS below $t_{\text{MS}}$.
- This introduces another problem: Cut dependence.
  → Apply the same weights above and below the cut.
- This means reweighting the multi-jet ME with
  (a) : $\alpha_s$ factors for $\alpha_s$-running in the PS,
  (b) : PDF ratios for backward evolution,
  (c) : No-emission probabilities, e.g. $\Pi_{S+0}(\rho_0, \rho_1)$ for no emissions before the first emission scale $\rho_1$. 
Example:
One jet above $\rho_c$

($\rho$: evolution $p_T$

$z$: Auxiliary variables

$t_{ms}$: Merging scale)

Take (a) from +1 jet matrix element $\mathcal{T}_1$. Reweight with the PS weight, i.e. pick this state with weight

$$
\left[ f_1(\mu_1) \alpha_s(\mu_R) \mathcal{T}_1 \right] d\Phi_1^{ME} \times \mathcal{W}_{Path} \times \frac{f_0(\rho_0)}{f_1(\mu_1)} \\
\times \frac{\alpha_s(\rho_1)}{\alpha_s(\mu_R)} \frac{f_1(\rho_1)}{f_0(\rho_1)} \Pi_{S+0}(\rho_0, \rho_1) \Pi_{S+1}(\rho_1, \rho_c)
$$

Take (b) from +0 jet matrix element $\mathcal{T}_0$, with one shower splitting, i.e. with weight

$$
\left[ f_0(\mu_0) \mathcal{T}_0 \right] d\Phi_0^{ME} \times \frac{f_0(\rho_0)}{f_0(\mu_0)} \\
\times \alpha_s(\rho_1) \frac{f_1(\rho_1)}{f_0(\rho_1)} P(z) d\rho_1 dz_1 \Pi_{S+0}(\rho_0, \rho_1) \Pi_{S+1}(\rho_1, \rho_c)
$$
One jet above $\rho_c$

Combining this, the merged approximation to the inclusive zero-jet cross section is

$$d\sigma^{CKKW} = f_0(\rho_0) \left\{ \right.$$

$$T_1 d\Phi_1^{ME} w_{Path} \alpha_s(\rho_1)$$

$$\Theta(t(S_{+1,me}) - t_{MS})$$

$$\frac{f_1(\rho_1)}{f_0(\rho_1)} \Pi_{S+0}(\rho_0, \rho_1) \Pi_{S+1}(\rho_1, \rho_c)$$

$$+ T_0 d\Phi_0^{ME} P(z) d\rho_1 dz_1 \alpha_s(\rho_1)$$

$$\Theta(t_{MS} - t(S_{+1,ps}))$$

$$\frac{f_1(\rho_1)}{f_0(\rho_1)} \Pi_{S+0}(\rho_0, \rho_1) \Pi_{S+1}(\rho_1, \rho_c) \left\} \right.$$

The merging scale dependence vanishes if the red probabilities are equal, and if the no-emission probabilities $\Pi_{S^+}$ are identical (because then the blue $\Theta$-functions add to one).
CKKW-L merging prescription

Easily extendible to arbitrary number of additional jets: \( n \)-parton MEs will be reweighted with \( w_{\text{CKKW-L}} \) to produce the exclusive (\( n \)-jet) cross section

\[
d\sigma^{\text{CKKW}}_n = f_n(\mu_f) T_n d\Phi_n^{\text{ME}} \\
\left[ \prod_{i=1}^{n} \frac{\alpha_s(\rho_i)}{\alpha_s(\mu_r)} \frac{f_{i-1}(\rho_{i-1})}{f_{i-1}(\rho_i)} \right] \prod_{i=1}^{n} \left[ \frac{f_n(\rho_n)}{f_n(\mu_f)} \right] \left[ \prod_{i=1}^{n} \frac{\alpha_s(\rho_i)}{\alpha_s(\mu_r)} \frac{f_{i-1}(\rho_{i-1})}{f_{i-1}(\rho_i)} \right] \Pi_{S+n}(\rho_n, \rho_{\text{MS}})
\]

\( w_{\text{CKKW-L}} \)

Compare to the parton shower exclusive cross section

\[
d\sigma^{\text{PS,ex}}_n = f_0(\rho_0) T_0 d\Phi_0^{\text{ME}} \\
\left[ \prod_{i=1}^{n} \frac{\alpha_s(\rho_i)}{\alpha_s(\mu_r)} \frac{f_{i-1}(\rho_{i-1})}{f_{i-1}(\rho_i)} \right] \Pi_{S+n}(\rho_n, \rho_{\text{MS}})
\]

\( \Rightarrow \) After some PDF shuffling, you can see that CKKW-L replaces the product of splitting kernels with the full tree-level predictions.
Results of CKKW-L merging at the LHC

Figure: Number of jets in $W+$jets events as measured by ATLAS. Three additional jets (above $t_{\text{MS}} = 30$ GeV) were merged with PYTHIA8.
Results of CKKW-L merging at the LHC

In the ATLAS data, the inclusive jet multiplicity (electron channel) was measured as follows:

\[ \sigma(W + \geq N_{\text{jet}} \text{ jets}) \text{ [pb]} \]

<table>
<thead>
<tr>
<th>(N_{\text{jet}})</th>
<th>MC/data</th>
<th>ATLAS data</th>
<th>PYTHIA8</th>
<th>ME3PS ( t_{\text{MS}} = 30 \text{ GeV} )</th>
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<td>0</td>
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<td></td>
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</tr>
<tr>
<td>3</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure: Number of jets in \(W +\)jets events as measured by ATLAS. Three additional jets (above \(t_{\text{MS}} = 30 \text{ GeV}\)) were merged with PYTHIA8.

⇒ For more exclusive observables, CKKW-L does better than the default shower.
But is that enough?

![Graph showing p_T(Z) distributions for Z (Drell-Yan) at 1960 GeV ppbar]

**Figure: 1960 GeV ppbar Z (Drell-Yan)**

- **p_T(Z) (muon channel)**
  - D0
  - Hw++ UE7-2 (PwHg)
  - Sherpa

**Legend:**
- D0
- Hw++ UE7-2 (PwHg)
- Sherpa

**Note:**
- mcplots.cern.ch
- 6.3M events
- Rivet 1.8.0, Herwig++ Powheg 2.5.2, Sherpa 1.3.1

**Graph Details:**
- Y-axis: dσ/dp_T(Z) [pb/GeV]
- X-axis: p_T(Z) [GeV]
- Ratio to D0

**Legend Note:**
- D0_2010_S8671338
  - Herwig++ Powheg 2.5.2, Sherpa 1.3.1
But is that enough?

- Sorry about the colours!
But is that enough?

- Sorry about the colours!
- Merged prediction gets shape right.
- POWHEG NLO matched PS gives good normalisation for mid/low $p_T$. 
What can we learn from that?

1. We need NLO accuracy for a good normalisation.
2. For a better description of the tail, we would like Z+1 jet at NLO.
3. For a better description of the low $p_T$ end, we need better logarithmic accuracy for Z+0 jet (also related to 1. and 2.). Skipped here, sorry.

⇒ We want NLO multi-jet merging.
The $\text{NL}^3$ prescription to perform NLO merging.
Multi-jet NLO + PS: How do we get there?

- Avoid double counting states.  
  ⇒ Define NLO exclusive cross section.

- Avoid double counting orders of $\alpha_s$.  
  ⇒ Expand CKKW cross section in $\alpha_s$, remove things we want to NLO accuracy, and add back true NLO.

- Define merging conditions to ensure NLO accuracy.

- Implement, and check.
Multi-jet NLO + PS: General idea

0 jet, tree

\[ Q_{\text{MS}} \]

0 jet, virtual

\[ Q_{\text{MS}} \]

1 jet, tree

\[ \Delta(\text{Shower}) - \alpha_s\text{-term} \]

1 jet, virtual

\[ \alpha_s\text{-term} \]

2 jet, tree

\[ \Delta(\text{Shower}) \]

\[ \text{Correct } \alpha_s\text{-term + higher orders} \]
Tree level configurations for $n$ partons can contain $n - 1$ resolved jets.
NLO configurations for $n$ partons can contain $n - 1$ and $n + 1$ resolved jets.
⇒ To avoid counting states with $n + 1$ resolved jets twice, we need to define an NLO $n$-jet cross section that contains exactly $n$ resolved jets.
⇒ Need an NLO weight for $n$-resolved jets phase space points.
**NL³ prerequisites: Exclusive NLO cross sections (\(\bar{B}\))**

\[
d\sigma_{n,\text{ex},f_{b1}}^{NLO} = d\phi_n J_n(\phi_n) T_{n,f_{b1}} + d\phi_n J_n(\phi_n) \left[ \mathcal{V}_{n,f_{b1}} + \sum_{\alpha_r \in \{\alpha_r \mid f_{b1}\}} \mathcal{I}_{n+1,\alpha_r} \right]
\]

\[
+ d\bar{\phi}_n \sum_{\alpha_r \in \{\alpha_r \mid f_{b1}\}} \int_{\rho_{\text{MS}}} d\Phi^{(0)} \mathcal{J}_n(\bar{\phi}_n) (R_{n+1,\alpha_r} - D_{n+1,\alpha_r})
\]

\[
+ d\bar{\phi}_n \sum_{\alpha_r \in \{\alpha_r \mid f_{b1}\}} \int_{\rho_{\text{MS}}} d\Phi^{(1)} \left\{ \mathcal{J}_{n+1}(\phi_{n+1}) (R_{n+1,\alpha_r}) - \mathcal{J}_n(\bar{\phi}_n) D_{n+1,\alpha_r} \right\}
\]

The red part contains resolved \(n + 1\) parton states, and should be zero (by vetoing ). \(n + 1\)-parton phase space points will be included in the next higher multiplicity.  
⇒ NLO cross section “exclusive” in the same way that tree-level is.

The blue part is what is collected in \(\bar{B}\) in POWHEG.
NL^3 prerequisites: Exclusive NLO cross sections (\(\bar{B}\))

\[
d\sigma_{n,ex,f_{b1}}^{NLO} = d\phi_n J_n(\phi_n) T_{n,f_{b1}} + d\phi_n J_n(\phi_n) \left[ \nu_{n,f_{b1}} + \sum_{\alpha_r \in \{\alpha_r|f_{b1}\}} I_{n+1,\alpha_r} \right] \\
+ d\bar{\phi}_n \sum_{\alpha_r \in \{\alpha_r|f_{b1}\}} \int_{\rho_{MS}} d\Phi_{rad} J_n(\phi_n) (R_{n+1,\alpha_r} - D_{n+1,\alpha_r}) \\
+ d\bar{\phi}_n \sum_{\alpha_r \in \{\alpha_r|f_{b1}\}} \int_{\rho_{MS}} d\Phi_{rad}^{(1)} \left\{ J_{n+1}(\phi_{n+1}) (R_{n+1,\alpha_r} - C_{n+1,\alpha_r}) \right\} \\
- J_n(\bar{\phi}_n) D_{n+1,\alpha_r}
\]

The red part contains resolved \(n + 1\) parton states, and should be zero (by vetoing or phase space subtraction). \(n + 1\)-parton phase space points will be included in the next higher multiplicity. 

\(\Rightarrow\) NLO cross section “exclusive” in the same way that tree-level is.

The blue part is what is collected in \(\bar{B}\) in POWHEG.
NL³ prerequisites: Rescaled CKKW-L cross sections

We now have a calculation exact to $\mathcal{O}(\alpha_s^{n+1})$.

⇒ Replace CKKW approximation of these orders by correct terms.
For this, remember the first few CKKW exclusive cross sections:

\[
\begin{align*}
\sigma_0^{\text{CKKW}} &= f_0(\rho_0) |M_{S_0}|^2 d\Phi_{\text{ME}}^0 K\Pi_{S+n}(\rho_0, \rho_{\text{MS}}) \\
\sigma_1^{\text{CKKW}} &= f_1(\rho_0)\alpha_s |M_{S_1}|^2 d\Phi_{\text{ME}}^1 K\frac{\alpha_s(\rho_1)}{\alpha_s} \frac{f_1(\rho_1)}{f_1(\rho_0)} \frac{f_0(\rho_0)}{f_0(\rho_1)} \Pi_{S+0}(\rho_0, \rho_1) \Pi_{S+1}(\rho_1, \rho_{\text{MS}})
\end{align*}
\]

Red to include $\alpha_s$-running, blue to amend backward branching probability, green to include probability for no emissions.

The CKKW weights (red $\times$ blue $\times$ green) incorporate the PS resummation. Cross sections have been rescaled by a $K$-factor (inspired by POWHEG, which rescales the “seed” cross section $B$ by $\bar{B}/B$).

We will of course remove the $\mathcal{O}(\alpha_s^1)$ term of $K$, $\Pi_{S+i}$, PDFs and $\alpha_s(\rho)$!
So let’s expand these to $O(\alpha_s^1)$

$$d\sigma_{0}^{\text{CKKW}} = f_0(\rho_0) |\mathcal{M}_{s+0}|^2 d\Phi_0^{\text{ME}} \left[ 1 + K_{|\alpha_s} \right]$$

$$- \frac{\alpha_s}{2\pi} \int_{\rho_{MS}} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho dz + \frac{1}{2} \left( \frac{\alpha_s}{2\pi} \right)^2 \left( \int_{\rho_{MS}} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho dz \right)^2$$

Replace blue terms by correct 0-jet NLO expression.
So let’s expand these to $\mathcal{O}(\alpha_s^1)$

\[
\begin{align*}
    d\sigma_0^{\text{CKKW}} &= f_0(\rho_0) |\mathcal{M}_{S_{+0}}|^2 d\Phi_0^{\text{ME}} \left[ 1 + K|_{\alpha_s} \right. \\
    &\left. - \frac{\alpha_s}{2\pi} \int_{\rho_{\text{MS}}} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho dz + \frac{1}{2} \left( \frac{\alpha_s}{2\pi} \right)^2 \left( \int_{\rho_{\text{MS}}} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho dz \right)^2 \right] \\
    d\sigma_1^{\text{CKKW}} &= f_1(\rho_0) \alpha_s |\mathcal{M}_{S_{+1}}|^2 d\Phi_1^{\text{ME}} \left[ 1 + K|_{\alpha_s} + \frac{\alpha_s}{4\pi} \beta_0 \ln\left( \frac{\rho_1}{\mu_r} \right) \right. \\
    &\left. + \frac{f_1(\rho_1)}{f_1(\rho_0)} |_{\alpha_s} + \frac{f_0(\rho_0)}{f_0(\rho_1)} |_{\alpha_s} \right. \\
    &\left. - \frac{\alpha_s}{2\pi} \int_{\rho_1}^{\rho_0} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho dz - \frac{\alpha_s}{2\pi} \int_{\rho_{\text{MS}}}^{\rho_1} \frac{f_2(\mu_f)}{f_1(\mu_f)} P_2(z) d\rho dz \right] 
\end{align*}
\]

Replace red terms by correct 1-jet NLO expression.
So let’s expand these to $\mathcal{O}(\alpha_s^1)$

\[
d\sigma_0^{CKKW} = f_0(\rho_0) |\mathcal{M}_{S+0}|^2 d\Phi_0^{\text{ME}} \left[ 1 + K|_{\alpha_s} \right]
\]

\[
- \frac{\alpha_s}{2\pi} \int_{\rho_{\text{MS}}}^{\rho_0} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho d\tau + \frac{1}{2} \left( \frac{\alpha_s}{2\pi} \right)^2 \left( \int_{\rho_{\text{MS}}}^{\rho_1} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho d\tau \right)^2
\]

\[
d\sigma_1^{CKKW} = f_1(\rho_0) \alpha_s |\mathcal{M}_{S+1}|^2 d\Phi_1^{\text{ME}} \left[ 1 + K|_{\alpha_s} + \frac{\alpha_s}{4\pi} \beta_0 \ln(\frac{\rho_1}{\mu_r}) \right]
\]

\[
+ \frac{f_1(\rho_1)}{f_1(\rho_0)} \bigg|_{\alpha_s} + \frac{f_0(\rho_0)}{f_0(\rho_1)} \bigg|_{\alpha_s}
\]

\[
- \frac{\alpha_s}{2\pi} \int_{\rho_0}^{\rho_1} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho d\tau - \frac{\alpha_s}{2\pi} \int_{\rho_{\text{MS}}}^{\rho_1} \frac{f_2(\mu_f)}{f_1(\mu_f)} P_2(z) d\rho d\tau
\]

\[
d\sigma_2^{CKKW} = f_2(\rho_0) \alpha_s^2 |\mathcal{M}_{S+2}|^2 d\Phi_2^{\text{ME}}
\]

Keep higher multiplicity ME if no NLO calculation available.
Merging conditions

Now that we have a feeling what we want to replace, we can set up conditions to ensure NLO accuracy, while taking all higher orders from the shower:

\[
\begin{align*}
\alpha_s^n t_n w_T + \alpha_s^{n+1} v_n w_V + \alpha_s^{n+1} r_n w_R &= \alpha_s^n t_n + \alpha_s^{n+1} (v_n + r_n) \\
&+ \alpha_s^n t_n \sum_{i=2}^{\infty} \alpha_s^i w_{PS,i} \\
\end{align*}
\]

and

\[
\alpha_s^{n+1} t_{n+1} w_H = \alpha_s^{n+1} t_{n+1} \sum_{i=0}^{\infty} \alpha_s^i w_{PS,i} ,
\]

For (2), we can immediately put

\[
w_H = w_{CKKW-L}
\]
NLO merging weights

These conditions can be checked order by order. Then, from the ansatz

\[
w_{T,V,R} = a_{T,V,R,0} + \sum_{i=1}^{\infty} b_{T,V,R,i} \alpha_s^i + \sum_{i=1}^{\infty} c_{T,V,R,i} \left( \frac{1}{\alpha_s} \right)^i
\]

we find the solutions

\[
w_T = w_{\text{CKKW-L}} - w_{\text{CKKW-L}}|_{\alpha_s} + \sum_{i=1}^{\infty} c_{T,i} \left( \frac{1}{\alpha_s} \right)^i
\]

\[
w_V = 1 + \sum_{i=2}^{\infty} c_{V,i} \left( \frac{1}{\alpha_s} \right)^i, \quad w_R = 1 + \sum_{i=2}^{\infty} c_{R,i} \left( \frac{1}{\alpha_s} \right)^i
\]

of which

\[
w_T = w_{\text{CKKW-L}} - w_{\text{CKKW-L}}|_{\alpha_s}
\]

\[
w_V = 1 = w_R
\]

is a special case, as expected. The PS resummation is still encoded in the merging weight \(w_{\text{CKKW-L}}\). The weights \(w_{\text{CKKW-L}}|_{\alpha_s}\) are the coloured terms we have found in the expansion of the merging weight.
Consequences

- Algorithm works on exclusive NLO $n$-jet cross section.
  ⇒ Define a cut in the NLO calculation.
  If not possible (e.g. in the POWHEG-BOX), the calculation can be made exclusive by subtracting phase space points not passing the cut.$^3$

---

$^3$The phase space subtraction can be constructed by reclustering the next higher multiplicity tree-level events.
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- The condition $w_V = 1$ also means that the merging scale has to be defined by the jet algorithm ($\rho$) used for shower evolution.

- We want to keep the NLO 0-jet inclusive cross section fixed.  
  Merging multiple NLO calculations introduces terms $\propto \alpha_s^2 \ln^2 \frac{1}{\rho_{\text{MS}}}$, which are beyond the control of the PS.  
  $\Rightarrow$ Need to assess if these terms will prove a problem for reasonable $\rho_{\text{MS}}$ values.

---

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Putting it all together.

We can now write down an algorithm to merge multiple NLO calculations with a parton shower.
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For tree-level sample:

1. Generate events according to tree-level matrix element.
2. For each event, generate CKKW-L weight and subtract the $\mathcal{O}(\alpha_s^1)$ term $w_{\text{MS}}|_{\alpha_s}$.
3. Start shower from the last reconstructed scale $\rho_n$. Veto emissions above $\rho_{\text{MS}}$. 
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3. Start shower from the last reconstructed scale $\rho_n$. Veto emissions above $\rho_{\text{MS}}$.

For $\mathcal{O}(\alpha_s^1)$ (virtual + insertion + regularised unresolved real) sample:

1. Generate events according to NLO exclusive $n$-jet cross section. If the $\mathcal{O}(\alpha_s^1)$ sample was not generated with a cut on resolved real emissions, remove the $+1$-resolved jet phase space points.
2. For each event, start shower at $\rho_{\text{MS}}$. 
Implementation

To implement this scheme, we need to know how to generate the weights needed for the tree-level samples:

\[ K \]

\[ \frac{\alpha_s}{4\pi} \beta_0 \ln\left( \frac{\rho_1}{\mu_r} \right) \]

\[ \frac{f_i(\rho_j)}{f_i(\rho_k)} \bigg|_{\alpha_s} \]

\[ \frac{\alpha_s}{2\pi} \int_{\rho_i}^{\rho_{i-1}} \frac{f_1(\mu_f)}{f_0(\mu_f)} P(z) d\rho dz \]

\[ : \text{Calculate the (fixed) K-factor beforehand by dividing the cross sections.} \]
\[ : \text{Easily calculated by evaluating the logarithm.} \]
\[ : \text{Evolve } f_i(\rho_j) \text{ to } \rho_k \text{ according to (integrated) DGLAP equation, then use numerical integration to calculate integral.} \]
\[ : \text{Generated by counting the PS emissions between } \rho_{i-1} \text{ and } \rho_i, \text{ generated with fixed } \alpha_s \text{ and fixed PDF scales } \mu_f. \]
... and that’s what it looks like:
Let’s check some results!
Results: Rapidity of W-Boson, NL$^3_{0}$.

Delightfully boring, since the POWHEG-BOX phase space mapping keeps the W-rapidity fixed.

Figure: W+0@NLO from POWHEG-BOX.
Results: $p_T$ of hardest jet, $\text{NL}_0^3$.

Delightfully boring, since the distribution is given completely by the $+1$-jet tree-level ME, for which the merging scale dependence cancels.

Figure: $W+0@\text{NLO}$ from POWHEG-BOX.
Results: $p_T$ of hardest jet, NL$_0^3$.

Delightfully boring, since the distribution is given completely by the $+1$-jet tree-level ME, for which the merging scale dependence cancels.

⇒ Yet another POWHEG W-production interface, but now with MENLOPS for free.

Figure: $W+0@NLO$ from POWHEG-BOX.
Results: $p_T$ of the hardest jet, NL$_{01}^3$.

New: Combined W and W + 1 jet at NLO.

The increase in the tail partly from the W+2 jet tree-level ME (25% at 100 GeV), and partly the effect of the $p_T$-dependence of the W+1 NLO cross section.

Figure: W+0@NLO from POWHEG-BOX and W+1@NLO from POWHEG-BOX.
Conclusions and Outlook

• For a consistent description of the shape of soft/collinear and multiple hard jets, we need to combine Matrix Elements and Parton Showers.

• CKKW-L merging is implemented in Pythia8 (public since version 8.157).

• Tree-level merging has disadvantages over NLO matching schemes.

• CKKW-L can be extended to allow for the inclusion of virtual corrections. This can be achieved by removing the $\mathcal{O}(\alpha_s^1)$-term of the CKKW-L weight and adding back NLO corrections.

• We have implemented a modified NL$^3$ scheme in Pythia8.

• So far, we have checked $W +$ jets as test case.

• We hope we will be able to check the method further and publish the code in autumn.
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Thank you for your time.
Improving parton showers with multi-jet NLO calculations

Stefan Prestel

(in collaboration with Leif Lönnblad)

(Lund University)

Theory seminar, SLAC, June 20, 2012
Back up
Exclusive NLO cross sections (\(\bar{B}\)), notation

\[ I_n(\phi_n) \] : Jet observable, measured at phase space point \(\phi_n\), giving an \(n\)-jet prediction.

\[ T_{n,f_{b1}} \] : Tree level ME with \(n\) partons, flavour \(f_{b1}\).

\[ V_{n,f_{b1}} \] : Virtual correction with \(n\) partons, flavour \(f_{b1}\).

\[ I_{n+1,\alpha_r} \] : Integrated subtraction for \(n\) partons and flavour \(f_{b1}\) (derived from approximate \(n + 1\) with flavour \(\alpha_r\)).

\[ R_{n+1,\alpha_r} \] : Real emission ME with \(n + 1\) partons and flavour \(\alpha_r\).

\[ D_{n+1,\alpha_r} \] : Subtraction terms for \(n + 1\) partons with flavour \(\alpha_r\).
Rescaled CKKW-L cross sections, K-factor in POWHEG

POWHEG rescales the “seed” cross section $B$ by a phase space dependent K-factor $\bar{B}/B$). Schematically:

$$d\sigma^{PH} = d\phi_0 \bar{B} \times \left[ \Delta(\rho_{max}, \rho_c)O(\phi_0) + \int_{\rho_c} d\Phi_{rad} \frac{R}{\bar{B}} \Delta(\rho_{max}, \rho(\Phi_{rad}))O(\phi_1) \right]$$

$$= d\phi_0 B \frac{\bar{B}}{B} \times \left[ \Delta(\rho_{max}, \rho_c)O(\phi_0) + \int_{\rho_c} d\Phi_{rad} \frac{R}{B} \Delta(\rho_{max}, \rho(\Phi_{rad}))O(\phi_1) \right]$$

$$= d\phi_0 B \frac{\bar{B}}{B} \times \left[ \left( 1 - \int_{\rho_c} d\Phi_{rad} \frac{R}{B} + \frac{1}{2} \left( - \int_{\rho_c} d\Phi_{rad} \frac{R}{B} \right)^2 \right) O(\phi_0) \right.$$

$$+ \left. \int_{\rho_c} d\Phi_{rad} \frac{R}{B} \left( 1 - \int_{\rho_c} d\Phi_{rad} \frac{R}{B} \right) O(\phi_1) + O(\alpha_s^3) \right]$$

In the last lines, POWHEG rescales the $O(\alpha_s^2)$ terms. Since the subsequent showering is unitary, this “rescaled seed cross section” multiplies all approximate higher orders introduced by the shower.
Merging conditions, notation

\[
\begin{align*}
\alpha_s^n t_n &= \sum_{f_{b1}} d\phi_n \mathcal{J}_n(\phi_n) T_{n, f_{b1}} \\
\alpha_s^{n+1} v_n &= \sum_{f_{b1}} d\phi_n \mathcal{J}_n(\phi_n) \left[ V_{n, f_{b1}} + \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} \mathcal{I}_{n+1, \alpha_r} \right] \\
\alpha_s^{n+1} r_n &= \sum_{f_{b1}} \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} d\bar{\phi}_n \left\{ \int_{\rho_{MS}} d\Phi_{\text{rad}} \mathcal{J}_n(\bar{\phi}_n) [\mathcal{R}_{n+1, \alpha_r} - \mathcal{D}_{n+1, \alpha_r}] - \int_{\rho_{MS}} d\Phi_{\text{rad}} \mathcal{J}_n(\bar{\phi}_n) \mathcal{D}_{n+1, \alpha_r} \right\}
\end{align*}
\]
NLO merging weights

\[ w_T = w_{\text{CKKW-L}} - w_{\text{CKKW-L}} \big|_{\alpha_s} + \sum_{i=1}^{\infty} c_{T,i} \left( \frac{1}{\alpha_s} \right)^i \]

\[ w_V = 1 + \sum_{i=2}^{\infty} c_{V,i} \left( \frac{1}{\alpha_s} \right)^i , \quad w_R = 1 + \sum_{i=2}^{\infty} c_{R,i} \left( \frac{1}{\alpha_s} \right)^i \]

We do not want to include spurious \( \mathcal{O} \left( \alpha_s^{n-i} \right) \) terms, and do not want to disturb the intricate cancellations between \( \nu_n \) and \( \rho_n \):

\[ c_{T,1} = 0 \quad c_{T,i} + c_{V,i} + c_{R,i} = 0 \quad c_{V,i} = c_{R,i} \]

For example, we can also allow

\[ w_T = w_{\text{CKKW-L}} - w_{\text{CKKW-L}} \big|_{\alpha_s} - \sum_{i=2}^{\infty} 2 c_{V,i} \left( \frac{1}{\alpha_s} \right)^i \]

\[ w_V = w_R = 1 + \sum_{i=2}^{\infty} c_{V,i} \left( \frac{1}{\alpha_s} \right)^i \]

We choose the simplest form since we could not think of a useful, shower-producible factor that has an expansion in negative powers of \( \alpha_s \).
Merging scale variation for 2-jet CKKW-L merging.

![Graph showing the deviation in dσ/dpT,1 between CKKW-L and Pythia8 for different scale variations.](image)
$k_\perp$ dependence $\bar{B}$ for $W^+$ jet.