An update on WWW-production at hadron colliders

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Outline

• Some words on $WWW$-production and LO results
• NLO corrections and results
There are essentially three different topologies: 

for triple W production; m2line contains three-boson, m1line four-boson couplings. In total, these are 87 diagrams.
Problems due to diagonal ckm matrix?

Processes are implemented with diagonal CKM matrix. Is this a suitable approximation?
Yes, while setting all quark masses to zero:
Massless quarks $\rightarrow$ mass eigenstates $\equiv$ flavour eigenstates
$\rightarrow$ CKM matrix diagonal

The error made approximating the top quark massless does only affect propagators in m3line:

$$\frac{1}{\rho_k - m_k} = \frac{1}{\rho_2 - \rho_{W_2^+} - m_k} = \frac{1}{\rho_2 - \rho_{W_2^+}} \cdot \frac{1}{1 - (\rho_2 - \rho_{W_2^+})^{-1} \cdot m_k}$$

The deviation of the last factor from one for typical LHC momenta is $\mathcal{O}(10^{-1})$. This gets multiplied by $V_{ij} \cdot V_{jk}^* \cdot V_{kl}$ which is at least of $\mathcal{O}(10^{-4})$ if there is a virtual top, so this deviation can be neglected in $|M|^2$. 
Structure of computation

Computation is structured as follows:

- LO calculation is performed in $n$ iterations
- Calculation of vertex correction ($\propto \sigma_{\text{Born}}$) is done using $(n - 1)th$ grid of LO computation
- Calculation of boxes using $(n - 1)th$ grid of LO computation
- Calculation of pentagons using $(n - 1)th$ grid of LO computation
- Calculation of real emission is performed in $m$ iterations
Phasespace and Higgs resonance

The phasespace generator used is optimized to generate events with a higgslike resonance, which will occur in

\[ W^+ d_l W^- W^+ \Phi \]

This, on the other hand, disfavours phasespace regions where there is no resonance - 'box'-topologies. Since the boxes are numerically small anyway this effect (higher statistics needed for boxes) is only a minor flaw.
LO results

At leading order accuracy in $\alpha_s$ ($\mathcal{O}(\alpha_s^0)$) cross sections are:

<table>
<thead>
<tr>
<th></th>
<th>$\sigma(pp \rightarrow W^+W^-W^+) \text{ (ab)}$</th>
<th>$\sigma(pp \rightarrow W^-W^+W^-) \text{ (ab)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>449.8 ± 0.2</td>
<td>263.0 ± 0.9</td>
</tr>
<tr>
<td></td>
<td>205.5 ± 0.9</td>
<td>121.6 ± 0.4</td>
</tr>
</tbody>
</table>

Table: LO computation: $\mu_R = 3M_W$, $M_H = 120\text{GeV}$; black: no cuts; green: $p_{T,\text{Lepton}} = 10\text{GeV}$, $y_{\text{Lepton, max}} = 2.5$, $R_{LL,\text{min}} = 0.4$

Leading order results were tested against MadEvent and HELAC at the cross section level and against MadGraph at the matrix element level, real emission cross sections (without divergent parts) were as well checked against MadEvent and MadGraph, finding good agreement.
Next-to-leading-order corrections: Real corrections

The NLO corrections to $WWWW$-Production are actually leading order QCD, namely a gluon loop and the real emission of a gluon. The (finite) real emission corrections (210 diagrams)

can be calculated in a straightforward way. The IR-divergent parts of this correction cancel analytically with the IR-divergent parts of the virtual contributions. This is exploited in a numerical integration within the substraction method of Catani and Seymour. Dipoles needed for this are identical in $VVVV$-production processes.
Next-to-leading-order corrections: Virtual corrections

The corrections to topologies with one boson attached to the fermion line are just:

\[ W^+ \rightarrow W^+_1 W^-_1 W^+_2 \]

These are proportional to \( \mathcal{M}_{\text{Born}} \):

\[
\mathcal{M}_V^1 = \mathcal{M}_B C_F \left( \frac{4\pi\mu^2_R}{Q^2} \right)^\varepsilon \frac{\alpha_s(\mu_R)}{4\pi} \cdot \Gamma(1 + \varepsilon) \cdot \left[ -\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + c_{\text{virt}} + O(\varepsilon) \right]
\]

where \( c_{\text{virt}} = \frac{4\pi^2}{3} - 8 \).

→ Multiply \( \mathcal{M}_B \) with a constant factor for this part.
Next-to-leading-order corrections: Virtual corrections

The corrections to topologies with two bosons attached to the fermion line are:

\[ W_1^+ Z/\gamma \rightarrow W_2^+ W_1^- \]

These are:

\[ M_{V}^2 = M_{V}^1 + \frac{\alpha_s(\mu_R)}{4\pi} C_F g_{Wf} g_{Vf} \cdot M_{WV}(q_W, q_V) \]

The new part to compute here is \( M_{WV} \), which is finite. This is computed using either Passarino-Veltman reduction to scalar integrals (Carlo Oleari’s code) or Denner-Dittmaier reduction (Francisco Campanario’s code). Both calls were implemented.
Next-to-leading-order corrections: Virtual corrections

The corrections to topologies with three bosons attached to the fermion line are:

\[ M_{WWW}^2 = M_{V}^1 + \frac{\alpha_s(\mu_R)}{4\pi} C_F g_{Wf}^3 \cdot M_{WWW}(q_{W_1}, q_{W_2}, q_{W_3}) \]

where the new part \( M_{WWW} \) is finite.
Gauge invariance

This can also be computed using either PV or DD reduction - both calls have been implemented.

The virtual corrections are always tested on gauge invariance, e.g. reducing pentagons to boxes and checking the result. Gauge invariance can also be used to save computation time by shifting parts of the pentagons to boxes. Since boxes tend to have higher errors, this might not prove too useful.
Results at NLO accuracy

Corrections to NLO accuracy stem from real emission amplitudes and interference terms of born and virtual amplitudes:

\[
\begin{align*}
\sigma(W^+ W^- W^+) \text{ (ab)} & \quad \sigma(W^- W^+ W^-) \text{ (ab)} \\
\text{LO} & \quad 449.8 \pm 0.2 \quad 263.0 \pm 0.9 \\
& \quad 205.5 \pm 0.9 \quad 121.6 \pm 0.4 \\
\text{NLO1} & \quad 528.9 \pm 0.4 \quad 315.8 \pm 1.3 \\
& \quad 248.6 \pm 1.0 \quad 150.5 \pm 0.6 \\
\text{BOX} & \quad -7.6 \pm 0.2 \quad -3.0 \pm 0.2 \\
& \quad -4.8 \pm 0.7 \quad -2.7 \pm 0.3 \\
\text{PENT} & \quad 49.6 \pm 0.2 \quad 25.8 \pm 1.1 \\
& \quad 22.4 \pm 0.9 \quad 11.8 \pm 0.3 \\
\text{RE} & \quad 96.9 \pm 0.1 \quad 55.9 \pm 0.6 \\
\text{Result (NLO)} & \quad \text{not available yet} \quad \text{not available yet}
\end{align*}
\]

Table: Black: no cuts; Green: $p_T,l = 10\text{GeV}$, $y_{l,\text{max}} = 2.5$, $R_{ll,\text{min}} = 0.4$
Comparison with Binoth et alii

In arXiv: 0805.2152v2 [hep-ph] Binoth et alii presented NLO corrections to $W^+W^-W^+$ production in narrow width approximation (NWA), against which my results were tested, finding good agreement in cross sections and $p_T$-distributions. There is still an issue to be resolved concerning $y$-distributions. 'Integrating’ the vbfnlo $y$-distribution yields the correct cross section.
$p_T$ distributions
y distributions

![Graphs showing y distributions with data points and curves for different scales and log scales.](image-url)
Scale dependance: $\mu_R$ and $\mu_F$ varied together

While the LO result should only depend on $\mu_F$, NLO results depend on $\mu_F$ and $\mu_R$:

The NLO result differs by 8% changing $\mu = 120$ GeV to 360 GeV. Due to a lack of time, a thorough investigation of scale dependence must be postponed.
Dependence on higgs mass

This plot shows the change in $W^+ W^- W^+$ total cross sections while varying $M_H$ between 60 and 180 GeV.

Figure: Impact of higgs mass on total cross section

with $p_{T,l} = 10\text{GeV}$, $y_{l,\text{max}} = 2.5$, $R_{ll,\text{min}} = 0.4$ as cut values. The changes are substantial.
Outlook and To Do:

- Solve problems in comparison with Binoth et alii
- Check if there are interesting observables
- If there is time, a comparison to SUSY-cascades might be interesting
- Start writing everything down
Thank you for your attention
Processes for triple W production have ID’s (ProcID) 73XX. At the moment processes

\[ pp \rightarrow \nu_e \ e^+ \ \mu^- \ \bar{\nu}_\mu \ \nu_\tau \ \tau^+ \ (\text{ProcID: 7313}) \] and

\[ pp \rightarrow \nu_e \ e^+ \ \mu^- \ \bar{\nu}_\mu \ \tau^- \ \bar{\nu}_\tau \ (\text{ProcID: 7331}) \]

implemented and tested at NLO. To check if there are large interference effects,

\[ pp \rightarrow \nu_e \ e^+ \ \mu^- \ \bar{\nu}_\mu \ \nu_e \ e^+ \ (\text{ProcID: 7321}) \]

was tested, giving the same result as 7313, making it seem reasonable to neglect interference terms between final state leptons.
Interference between different VVV processes

The structure of vbfnlo - classifying processes in terms of intermediate bosons rather than final state leptons - does not easily allow to include and check interferences between

\[ pp \rightarrow W^+ W^- W^+ \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu \nu_\mu \mu^+ \] and
\[ pp \rightarrow W^+ ZZ \rightarrow \nu_e e^+ \mu^- \mu^+ \nu_\mu \bar{\nu}_\mu . \]

Since the leptonic tensors for both processes are the same, one only has to worry about m3line - but when m3line of WWW is large (all W’s are resonant), m3line of WZZ is small (Z’s are nonresonant) and vice versa. Hence interference effects of this type can be neglected.
The need for a NLO computation

Study of this channel interesting, since

- measurement might give insight into three and four boson couplings (large gauge theory cancellations between m2line and m1line, changing couplings slightly might result in testable deviations)

- measurement might give conclusions concerning higgs sector: longitudinal W production might be enhanced

→ Therefore a precise calculation is desirable. Furthermore it is wanted to complete VVV processes in vbfnlo. It may be background to SUSY cascade decays as well.
Problems due to diagonal ckm matrix?

Processes are implemented with diagonal CKM matrix. Is this a suitable approximation?
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Processes are implemented with diagonal CKM matrix. Is this a suitable approximation?

Yes, while setting all quark masses to zero it is reasonable: m1line, m2line will only be multiplied by a factor $V_{il}$ which will give $\delta_{il}$ when squared. For m3line:

$$m3line \propto V_{ij} \cdot \frac{1}{p_j - m_j} \cdot V^*_{jk} \cdot \frac{1}{p_k - m_k} \cdot V_{kl}$$

$$\rightarrow \quad V_{ij} \cdot V^*_{jk} \cdot V_{kl} \cdot \frac{1}{p_1 - p_{W_1^+}} \cdot \frac{1}{p_2 - p_{W_2^+}}$$

$$\rightarrow \quad V_{il} \cdot \frac{1}{p_1 - p_{W_1^+}} \cdot \frac{1}{p_2 - p_{W_2^+}}$$

Hence for massless quarks there is no need to implement a nondiagonal CKM matrix (massless quarks $\rightarrow$ mass eigenstates $\equiv$ flavour eigenstates $\rightarrow$ CKM matrix diagonal)
The error made approximating the top quark massless does only affect the second (or first) propagator:

\[
\frac{1}{p_k - m_k} = \frac{1}{p_2 - p_{W^+} - m_k} = \frac{1}{p_2 - p_{W^+} \cdot \frac{1}{1 - (p_2 - p_{W^+})^{-1}} \cdot m_k}
\]

In the propagator, the deviation of the last factor from one for typical LHC momenta is \(O(10^{-1})\). This gets multiplied by \(V_{ij} \cdot V_{jk}^* \cdot V_{kl}\) which is at least of \(O(10^{-4})\) if there is a virtual top, so this deviation can be neglected in \(|M|^2\).
Gauge invariance

This can also be computed using either PV reduction or DD reduction - both calls have been implemented. The virtual corrections are always tested on gauge invariance, e.g. reducing pentagons to boxes and checking the result. Gauge invariance can also be used to save computation time: It is always possible to shift the polarisation vector

\[ \varepsilon_{\mu}^{\text{new}} = \varepsilon_{\mu}^{\text{old}} - x_V \cdot q_{V,\mu} \]  
\[ \varepsilon_{\mu}^{\text{new}} (q_{W_1} + q_{W_2} + q_{W_3})^\mu = 0 \]

with suitable \( x_V \). The second condition implies that \( \varepsilon_{\mu}^{\text{new}} \) has vanishing time component in CMS of the three bosons, resulting in less coefficients to be calculated in the tensor reduction.
For the pentagons, this shift was implemented:

\[
\varepsilon_{W_1^+}^{\mu} (q_{W_1^+}) + x_{W_1^+} : q_{W_1^+}^{\mu} + \varepsilon_{W_1^-}^{\mu} (q_{W_1^-}) + \varepsilon_{W_2^+}^{\mu} (q_{W_2^+})
\]

\[
\varepsilon_{W_1^-}^{\mu} (q_{W_1^-}) + \varepsilon_{W_2^+}^{\mu} (q_{W_2^+})
\]

which means splitting the pentagons into less complicated pentagons and boxes, saving computation time in the pentagons and enhancing box contributions. Since the boxes contributions are small and have comparatively large error, this may not be useful.
At different values of $\mu$ and $M_H$ cross sections are:

<table>
<thead>
<tr>
<th>$M_H =$60 GeV</th>
<th>90 GeV</th>
<th>120 GeV</th>
<th>150 GeV</th>
<th>180 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO : 0.1075</td>
<td>0.1115</td>
<td>0.2020</td>
<td>0.4539</td>
<td>0.4066</td>
</tr>
<tr>
<td>NLO: 0.1705</td>
<td>0.1890</td>
<td>0.2867</td>
<td>0.5898</td>
<td>0.4807</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\mu =$120.6 GeV</th>
<th>180.9 GeV</th>
<th>241.2 GeV</th>
<th>301.5 GeV</th>
<th>361.8 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO : 0.2026</td>
<td>0.2017</td>
<td>0.2020</td>
<td>0.2013</td>
<td>0.2038</td>
</tr>
<tr>
<td>NLO: 0.2987</td>
<td>0.2888</td>
<td>0.2867</td>
<td>0.2756</td>
<td>0.2748</td>
</tr>
</tbody>
</table>
$p_T$ distributions

Below, $p_T$ distributions for $W^+$ and $W^-$ in $W^+W^-W^+$ production are shown. These differ slightly though yielding identical integrated cross sections.

**Figure:** $p_T$ distributions (no cuts applied)
$p_T$ distributions

Below, $p_T$ distributions for $W^+$ and $W^-$ in $W^+W^-W^+$ production are shown. These differ slightly though yielding identical integrated cross sections.