Multiple vector boson production at NLO QCD

Stefan Prestel

Institut für theoretische Physik
Universität Karlsruhe (TH)

06.10.08
Outline

- Some words on multiple boson production
- Calculational framework of triboson production processes
- NLO corrections
- Results of triple W production processes
Multiple vector boson production

Multiple vector boson production processes span a huge class of processes at hadron colliders, ranging from Drell-Yan and Drell-Yan like processes \((V \in \{W^\pm, \gamma, Z\})\),

Vector Boson Fusion (VBF) or Gluon Fusion (GF) processes

to Drell-Yan like processes with an extra jet

and many other processes - each interesting in it's own right. In this talk, let us focus on elektroweak gauge bosons in the final state.
Motivation to study multiboson processes

All of these processes are interesting at the LHC:

- VBF and GF are excellent production mechanisms for a Higgs boson $h$.
  $\Rightarrow$ Precise knowledge of higgs production and backgrounds necessary.

\[ h \quad Z, \gamma \]
Motivation to study multiboson processes

All of these processes are interesting at the LHC:

• VBF and GF are excellent production mechanisms for a Higgs boson $h$

• Diboson processes show large gauge cancellations between different topologies
  $\Rightarrow$ Ideal to check for anomalous couplings of three gauge bosons.

Diboson processes are furthermore interesting backgrounds for BSM trilepton searches (e.g. gaugino pair production).
  $\Rightarrow$ Precise knowledge of diboson production as signal and background necessary.
Motivation to study multiboson processes

All of these processes are interesting at the LHC:

• VBF and GF are excellent production mechanisms for a Higgs boson $h$
• Diboson processes interesting for anomalous couplings and BSM searches.
• Triple weak boson production processes show large gauge cancellations between different topologies and incorporate Higgsstrahlung diagrams. Furthermore, they are background to BSM processes with $p_T$ and multilepton final states (SUSY, seesaw signals).

⇒ Precise knowledge of triboson production as signal and background neccessary.
Electroweak triboson production

QCD corrections to triboson processes are on the experimentalist’s wishlist
[QCD, EW and Higgs working group; hep-ph/0604120]

However, until recently, electroweak triboson processes were only known at LO QCD.

⇒ Since $\mu_R$ dependence is absent, scale dependence of LO results does not even give a measure for remaining QCD uncertainties.
• Some words on multiple boson production
• Calculational framework of triboson production processes
• NLO corrections
• Results of triple W production processes
O.k., let’s calculate a triboson production process . . .

. . . analytically:

- **Pro:** All divergencies cancel exactly
- **Contra:** Huge number of Feynman diagrams make calculation nearly impossible
- **Contra:** PDFs not known analytically at every scale

. . . numerically:

- **Contra:** Some effort has to be made to cancel divergencies
- **Pro:** Efficient integration and amplitude calculation
- **Pro:** Easy to histogram events
- **Pro:** PDFs not known analytically at every scale

⇒ Use numerical methods
Steps in a NLO calculation

A general NLO calculation using numerical method might be structured like:

• Randomly generate momenta of incoming and outcoming particles
• Calculate LO $2 \rightarrow 6$ processes, thereby optimize the probability density with which momenta are created (e.g. with a VEGAS-algorithm)
• Calculate NLO $2 \rightarrow 6$ processes using the optimalization and only one integration
• Calculate NLO $2 \rightarrow 7$ processes, thereby optimize the probability density with which momenta are created
• Sum everything at the end

As an example, let us have a look at the process with largest cross section for intermediate massive vector bosons – WWW production.
Phasespace generation

Momentum configurations are created in three steps:

1. Parton momenta are created,

2. $W$ momenta are created using Euler angles and Dalitz variables and paying attention to a double Higgs resonance,

3. Lepton momenta are created Breit-Wigner distributed around $M_W$. 

Only generated when calculating real emissions.

![Diagram showing the process of generating momentum configurations](image-url)
There are essentially three different topologies:

- 85 ways to get to the same final state
- 8 different initial states ($u\bar{d}, c\bar{s}, u\bar{s}, c\bar{d}$ and crossing related one’s for $W^+ W^- W^+$)
Triple W matrix elements

Even within helicity methods, calculation of the whole amplitude for every initial state is quite slow.
⇒ Calculate electroweak parts ("leptonic tensors") only once per psp!

\[
\begin{align*}
\mathcal{J}_{W^+}^\mu & \quad \mathcal{J}_{W^-}^\mu & \quad \mathcal{J}_{W^+W^-}^\mu \\
\nu_{\ell_1} & \quad \ell_2^+ & \quad \nu_{\ell_3} \\
\bar{\nu}_{\ell_2} & \quad \ell_3^+ & \quad \bar{\nu}_{\ell_1}
\end{align*}
\]

Then calculate wavefunctions of incoming particles, form vertices and contract with effective electroweak currents, multiply with PDFs.
⇒ Roughly 7 times faster than MADGRAPH generated code.
LO results – cross sections for the LHC

<table>
<thead>
<tr>
<th></th>
<th>$\sigma(W^+W^-W^+)$ (fb)</th>
<th>$\sigma(W^-W^+W^-)$ (fb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>0.226</td>
<td>0.134</td>
</tr>
</tbody>
</table>

Before continuing to NLO corrections, let us pause here to get an idea about LO ($\mathcal{O}(\alpha_s^0)$) results.

Parameters and Cuts used:

\[
\mu_F = \mu_R = 3M_W, \quad E_{CM} = 14 \text{ TeV}, \quad M_H = 120\text{GeV}
\]

\[
p_{T,Lepton} > 10 \text{ GeV}, \quad y_{Lepton,max} < 2.5
\]

If we are only interested in $e^\pm$ and $\mu^\pm$ in the final state and don’t distinguish these, we have to multiply the results by a factor $F = 4$. 
• Some words on multiple boson production
• Calculational framework of triboson production processes
• NLO corrections
• Results of triple W production processes
The NLO corrections to $WWW$-Production are actually leading order QCD, namely a gluon loop and the real emission contribution:

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int_m d\sigma^C$$

Here, $\int_{m+1} d\sigma^R$ and $\int_m d\sigma^V$ are separately IR-divergent in 4 dimensions, though $\sigma^{NLO}$ is finite.
NLO corrections in numerical integration

In numerical implementations, this cancellation can be exploited within the Catani-Seymour subtraction method:

\[ \sigma^{NLO} = \int_{m+1} \left[ d\sigma^R - d\sigma^A \right] + \int_m \left[ d\sigma^V + \int_1 d\sigma^A \right] \]

\[ \text{integrable in 4 D} \quad \text{poles cancel analytically} \]

\[ + \int_m d\sigma^C \]

\[ \text{additional finite term} \]

If the subtraction term \( d\sigma^A \) is introduced as local counter-term to \( d\sigma^R \), real emission diagrams can be regularised.
In WWW-production, there are 210 real emission diagrams (a):
Real corrections: Subtraction for quarks in initial state

Divergencies in these diagrams can be cancelled by subtraction terms.

For incoming quarks emitting a gluon, these terms are:

\[
d\sigma^A_{1,2} = \frac{8\pi\alpha_s C_F}{2xp_a \cdot p_g} \cdot \left(\frac{1 + x^2}{1 - x}\right) \cdot \left|\mathcal{M}_B (\tilde{p}_a, p_b, \tilde{k}_{\ell_1}, \ldots)\right|^2
\]

\[+ \quad a \longleftrightarrow b\]

with \( x = \frac{p_a p_b - p_g p_a - p_g p_b}{p_a p_b} \), \( \tilde{p}_a = xp_a \)
Real corrections: Subtraction for a gluon in initial state

For diagrams with an initial state gluon splitting into a $q\bar{q}$-pair, subtraction terms read

$$d\sigma_{3,4}^A = \frac{8\pi\alpha_s T_R}{2xp_g \cdot p_{q_1}} \cdot [1 - 2x (1 - x)] \cdot |\mathcal{M}_B (\tilde{p}_g, p_{q_2}, \tilde{k}_{\ell_1}, \ldots)|^2$$

so that the sum of subtraction terms, integrated over the one particle phase space is

$$\langle I(\epsilon) \rangle = |\mathcal{M}_B|^2 C_F \cdot \left(\frac{4\pi\mu}{Q^2}\right)^\epsilon \frac{\alpha_s}{2\pi} \cdot \Gamma(1 + \epsilon) \cdot \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{4\pi^2}{3} + 10 + \mathcal{O}(\epsilon)\right].$$

This is exactly the structure we will find in virtual corrections.
NLO corrections: Virtual corrections

The corrections to topologies with one boson attached to the fermion line are:

\begin{equation}
W^+ \rightarrow W_1^+ W_1^- W_2^+
\end{equation}

These are proportional to $\mathcal{M}_{\text{Born}}$:

\begin{equation}
\mathcal{M}^{(i)}_{V_1} = \mathcal{M}^{(i)}_B C_F \left(\frac{4\pi \mu_R^2}{Q^2}\right)^\epsilon \frac{\alpha_s(\mu_R)}{4\pi} \cdot \Gamma(1 + \epsilon) \cdot \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{4\pi^2}{3} + \mathcal{O}(\epsilon) \right]
\end{equation}
NLO: Virtual corrections

The corrections to topologies with two bosons attached to the fermion line are:

These can be written as:

\[
\mathcal{M}^{(i)}_{V_2} = \mathcal{M}^{(i)}_B C_F \left( \frac{4\pi\mu_R^2}{Q^2} \right) \frac{\alpha_s(\mu_R)}{4\pi} \Gamma(1 + \epsilon) \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{4\pi^2}{3} + \mathcal{O}(\epsilon) \right] \\
+ \frac{\alpha_s(\mu_R)}{4\pi} C_F g_{Wf} g_{V_{f_2}} \cdot \tilde{\mathcal{M}}^{(i)}_2(q_W, q_V)
\]

The new part to compute here is \(\tilde{\mathcal{M}}^{(i)}_2\), which is finite. This is computed using Passarino-Veltman reduction to scalar integrals.
NLO: Virtual corrections

The corrections to topologies with three bosons attached to the fermion line are:

\[
\begin{align*}
M^{(i)}_{V_3} &= M^{(i)}_B C_F \left(\frac{4\pi \mu_R^2}{Q^2}\right) \frac{\alpha_s(\mu_R)}{4\pi} \Gamma(1 + \epsilon) \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{4\pi^2}{3} + O(\epsilon) \right] \\
&\quad + \frac{\alpha_s(\mu_R)}{4\pi} C_F g_{\bar{W}_f}^3 \cdot \widehat{M}_3^{(i)}(q_{W_1}, q_{W_2}, q_{W_3})
\end{align*}
\]

where the new part \(\widehat{M}_3^{(i)}\) is finite. Pentagons are computed using Denner-Dittmaier reduction to scalar integrals.
Gauge invariance

As a trick, gauge invariance can be used to save computation time. It is always possible to shift the polarisation vector:

\[
\varepsilon_{\mu}^{\text{new}} = \varepsilon_{\mu}^{\text{old}} - x_V \cdot q_{V,\mu} \quad \text{with}
\]

\[
x_V = \frac{(\varepsilon_V^{\text{old}} \cdot Q)}{(q_V \cdot Q)}, \quad \text{where} \quad Q^\mu = q_{W_1}^\mu + q_{W_2}^\mu + q_{W_3}^\mu
\]

\[
\rightarrow \quad \varepsilon_{V,\text{new}} \cdot Q = 0
\]

- $\varepsilon_{\mu}^{\text{new}}$ has vanishing time component in CMS of the three bosons $\Rightarrow$ less pentagon coefficients to be calculated
- $q_{V,\mu}$-term gives sum of boxes when contracted with pentagons $\Rightarrow$ fast box routines can be used
Putting everything together

All in all, we get ($\tilde{M}_V$ are finite terms):

$$
\tilde{M}_V = M_B C_F \left( \frac{4\pi \mu^2_R}{Q^2} \right)^\epsilon \alpha_s(\mu_R) \frac{\Gamma(1+\epsilon)}{4\pi} \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{4\pi^2}{3} + O(\epsilon) \right]
$$

Therefore, the matrix element squared of the $2 \rightarrow 6$ process is

$$
|\tilde{M}_B + \tilde{M}_V|^2
= |M_B|^2 + 2 \text{Re}\{\tilde{M}_V M_B^*\} + O(\alpha_S^2)
= |M_B|^2 + 2 \text{Re}\{\tilde{M}_V M_B^*\} + O(\alpha_S^2)
+ |M_B|^2 C_F \left( \frac{4\pi \mu^2_R}{Q^2} \right)^\epsilon \alpha_s(\mu_R) \frac{\Gamma(1+\epsilon)}{2\pi} \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{4\pi^2}{3} + O(\epsilon) \right]
$$
Remember: Integrated subtraction terms are

\[ \langle I(\epsilon) \rangle = |\mathcal{M}_B|^2 \, C_F \cdot \left( \frac{4\pi \mu}{Q^2} \right)^\epsilon \frac{\alpha_S}{2\pi} \cdot \Gamma(1 + \epsilon) \cdot \left[ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{4\pi^2}{3} + 10 + \mathcal{O}(\epsilon) \right]. \]

Adding the $\alpha_s^1$ parts of real and virtual corrections, we get:

\[ \langle I(\epsilon) \rangle + 2 \text{Re}\{\mathcal{M}_V \mathcal{M}_B^*\} = \frac{C_F \alpha_S}{2\pi} 2|\mathcal{M}_B|^2 + 2 \text{Re}\{\widetilde{\mathcal{M}}_V \mathcal{M}_B^*\} \]

⇒ Everything is finite now, Catani-Seymour subtraction works.
• Some words on multiple boson production
• Calculational framework of triboson production processes
• NLO corrections
• Results of triple W production processes
Following these steps, we are able to compute every triboson process (dipoles, tensor reductions can be reused). Within VBFNLO WWW, WWZ, WZZ, WWγ and Wγj will among other processes be publicly available soon.

So let’s look at some results.
All of the following results are produced with VBFNLO, using the very minimal cuts

\[ p_{T_{\ell}} > 10 \text{ GeV}, \quad y_{\ell} < 2.5. \]

Parameters are set to

\[ M_H = 120 \text{ GeV}, \quad \mu_F = \mu_R = 2M_W \]

All fermions are assumed to be massless.
Cross sections for WWW production

<table>
<thead>
<tr>
<th></th>
<th>$\sigma(W^+ W^- W^+) ,(\text{fb})$</th>
<th>$\sigma(W^- W^+ W^-) ,(\text{fb})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>$0.2257 \pm 1 \cdot 10^{-4}$</td>
<td>$0.1341 \pm 5 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>NLO</td>
<td>$0.3593 \pm 2 \cdot 10^{-4}$</td>
<td>$0.2162 \pm 9 \cdot 10^{-5}$</td>
</tr>
</tbody>
</table>

NLO corrections to triple W production processes

$$pp \rightarrow \nu_{\ell,1} \ell_1^+ \ell_2^- \bar{\nu}_{\ell,2} \nu_{\ell,3} \ell_3^+ + X$$

$$pp \rightarrow \ell_1^- \bar{\nu}_{\ell,1} \nu_{\ell,2} \ell_2^+ \ell_3^- \bar{\nu}_{\ell,3} + X$$

are quite sizeable:
The overall K-factor is

$$K = \left\{ \begin{array}{ll}
1.59 & \text{for } pp \rightarrow W^+ W^- W^+. \\
1.61 & \text{for } pp \rightarrow W^- W^+ W^-.
\end{array} \right.$$
At NLO, scale dependence of $\sigma$ is due to $\mu_R$-dependence. The newly opened channel with an initial state gluon that drives the scale variation. The overall scale variation is $\approx 7\%$ (LO variation is $\approx 5\%$).
The cross sections of triboson processes are heavily dependent on $M_H$ and can be enhanced by a factor of $\approx 5$.

At NLO, other contributions are enhanced compared to Higgs contribution.

$\Rightarrow$ K-factor drops when Higgs contribution is dominant.

$\Rightarrow$ NLO $M_H$ dependence cannot be mimicked by simply rescaling LO results.
In general, K-factors

\[ K(x) = \frac{d\sigma^{NLO}(x)/dx}{d\sigma^{LO}(x)/dx} \]

are heavily phase space dependent.
⇒ We cannot mimic NLO distributions by multiplying LO results with a constant K-factor.
$m_{ll}$ distributions

Similar phase space dependences have already been noted in VBF and Diboson processes.
Kinematical reason

At NLO, it is possible for an emitted weak boson to recoil against a jet, increasing the mean $p_T$ of the leptons. So, with a suitable choice of a jet veto, the variation of the K-factors for specific observables can be imitated: The k-factor

$$K \left( p_T^\ell, p_T^{\text{veto}} \right) = \left[ \frac{d\sigma_{\text{NLO}}}{dp_T^\ell} - \frac{d\sigma_{\text{LO,jet}}}{dp_T^\ell} \left( p_{T,\text{min}}^{\text{jet}} = 50\text{GeV} \right) \right] \cdot \left[ \frac{d\sigma_{\text{LO}}}{dp_T^\ell} \right]^{-1}$$

is flat and close to one.
⇒ NLO results can be reproduced using

$$\frac{d\sigma_{\text{NLO}}}{dm} \approx \frac{d\sigma_{\text{LO}}}{dm} + \frac{d\sigma_{\text{LO,jet}}}{dm} \left( p_{T,\text{min}}^{\text{jet}} = p_T^{\text{veto}} \right)$$

So, good news: For special cases, we can omit the timeconsuming calculation of virtual corrections!
Conclusions

- Multiboson processes may be interesting signal and background processes
- Most of the triboson processes have by now been calculated
- To make these calculations efficient, one has to pull some tricks
- At the LHC, $\mathcal{O}(\text{few fb})$ of cross section are expected, very much depending on the actual Higgs mass
- For certain observables ($p_T$, $m_{\ell\ell}$ ...), there is no need to compute timeconsuming virtual corrections when using appropriate jet cuts on LO results.
The VBFNLO code is available at
http://www-itp.particle.uni-karlsruhe.de/~vbfnloweb/.

Thank you for your attention.
Backup slides
Difficulties in triboson processes
Difficulties in triboson production processes

How come that triboson processes were not calculated before?

- Detectors only see leptons, photons, jets
  ⇒ Leptonic decays of $V$’s necessary
  ⇒ Many Feynman diagrams for $2 \rightarrow 6$ processes
Difficulties in triboson production processes

How come that triboson processes were not calculated before?

- Many Feynman diagrams for $2 \rightarrow 6$ processes
- Complicated phasespace structure, e.g. in WWW-production, three W’s may become resonant and there is a double Higgs resonance.

$\Rightarrow$ Optimized phasespace generation necessary.
Difficulties in triboson production processes

How come that triboson processes were not calculated before?

- Many Feynman diagrams for $2 \rightarrow 6$ processes
- Complicated phasespace structure
- Virtual corrections to 3-, 4- and 5-point functions have to be calculated. Especially reduction of 5-point functions (pentagons) is very timeconsuming.

⇒ Fast & stable implementations of tensor reductions needed
Difficulties in triboson production processes

How come that triboson processes were not calculated before?

- Many Feynman diagrams for $2 \rightarrow 6$ processes
- Complicated phasespace structure
- Virtual corrections to 3-, 4- and 5-point functions
- Lots of new diagrams for real emission contributions in $\mathcal{O}(\alpha_S)$. Virtual and real corrections are separately divergent.
  $\Rightarrow$ For numerical calculations, regularise virtual and real part separately.
Difficulties in triboson production processes

How come that triboson processes were not calculated before?

- Many Feynman diagrams for $2 \rightarrow 6$ processes
- Complicated phasespace structure
- Virtual corrections to 3-, 4- and 5-point functions
- Lots of new diagrams for real emission
- Cross sections without final state photons are really small for Tevatron energies (e.g. WWW production is roughly 24 ab at NLO QCD).

As an example, let us have a look at the process with largest cross section for intermediate massive vector bosons – WWW production.
Leading order dependences
LO results – scale dependence

Let’s have a look at the scale dependence, so that we can estimate the remaining QCD uncertainties!

⇒ Scale dependence stems only from $\mu_F$-dependence of PDFs. Variation of the total cross section is $\approx 5\%$
LO results – Higgs mass dependence

... and what about the Higgs mass dependence?

⇒ Higgs contribution might increase the cross section by a factor of $\approx 5$ where the Higgs primarily decays into $W$ pairs.
Pentagon shift
Shifting the pentagons

For the pentagons, this shift was implemented:

⇒ Pentagons are split into less complicated pentagons and boxes, saving computation time in the pentagons and enhancing box contributions.
Cross sections for different NLO parts
<table>
<thead>
<tr>
<th>Beitrag</th>
<th>$\sigma(pp \rightarrow W^+W^-W^+) \ [fb]$</th>
<th>$\sigma(pp \rightarrow W^-W^+W^-) \ [fb]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{LO}$</td>
<td>$0.2257 \pm 1 \cdot 10^{-4}$</td>
<td>$0.1341 \pm 5 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\sigma_{VERT}$</td>
<td>$0.2750 \pm 2 \cdot 10^{-4}$</td>
<td>$0.1659 \pm 7 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\sigma_{BOX}$</td>
<td>$0.0029 \pm 2 \cdot 10^{-5}$</td>
<td>$0.0010 \pm 8 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>$\sigma_{PENT}$</td>
<td>$0.0160 \pm 1 \cdot 10^{-4}$</td>
<td>$0.0091 \pm 5 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\sigma_{REAL}$</td>
<td>$0.0654 \pm 7 \cdot 10^{-5}$</td>
<td>$0.0401 \pm 3 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\sigma_{NLO}$</td>
<td>$0.3593 \pm 2 \cdot 10^{-4}$</td>
<td>$0.2162 \pm 9 \cdot 10^{-5}$</td>
</tr>
</tbody>
</table>

- $\sigma_{LO}$: LO cross section
- $\sigma_{VERT}$: LO cross section + vertex correction
- $\sigma_{BOX}$: cross section of Box parts
- $\sigma_{PENT}$: cross section of ”true” pentagon parts
- $\sigma_{REAL}$: cross section for real corrections
- $\sigma_{NLO}$: full NLO cross section